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A NEW TECHNIQUE FOR SOLVING A Z- TRAVELLING SALESMAN PROBLEM BY EMPLOYING BRANCH AND BOUND ALGORITHM

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Abstract

In the Travelling Salesman Problem (TSP), the primary goal is to reduce both the travel costs and the distance travelled by the salesman. Various algorithms such as Brute Force algorithm, Nearest Neighbour algorithm, Genetic Algorithm, and Ant Colony Optimization (ACO) have been developed to determine optimal solutions for the TSP. This article explores the TSP with pentagonal Z-number costs or distances using the Branch and Bound (BnB) algorithm.

Keywords Fuzzy number, Z-number, Branch and Bound algorithm, Travelling salesman problem, Pentagonal Z- number.

1. Introduction

The Travelling Salesman Problem (TSP) aims to determine the minimum route that allows the salesperson to travel each specified city exactly once and return to the starting city. In the Fuzzy Travelling Salesman Problem (FTSP), the cost or distance is represented using fuzzy numbers. Various researchers have presented different approach to solve the FTSP [1,2,5,6 9]. The foundational concepts of fuzzy set theory, which effectively address uncertainty, were initially proposed by Zadeh [12]. Kumar et al. [4] explored novel approaches for computing FTSP using diverse membership functions. Zimmermann [13] advanced the field by developing methods to solve fuzzy programming and linear programming problems with multiple objective functions. While fuzzy concepts are effective for handling uncertainty by assigning membership values to objects, Z-numbers offer increased reliability.

Many researchers have proposed various algorithms for ordering Z-numbers [7, 8]. This paper applies the median method to order pentagonal Z-numbers. Additionally, the BnB algorithm is used in this study to derive the optimal ZTSP solution.

2. Preliminaries

Definition 2.1: Formal definition of Z-number [12]

A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in [0,1].

Definition 2.2: Z-weight of an edge

A Z-graph is a graph with Z-number weighted edges.

The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 2.3: Z-length of the path

Consider a path P from a vertex u to vertex v. Suppose it consists of edges $e_1, e_2 \ldots e_n$. Suppose the weights of these edges are the Z- numbers $(A_1, B_1), (A_2, B_2) \ldots (A_n, B_n)$. The Z-length of the path P is denoted by ZL (P).

$$ZL(P) = (A_1, B_1)\ (+, \min)\ (A_2, B_2) \ldots\ (+, \min)\ (A_n, B_n)$$

The shortest path problem in Z- graphs is the problem of finding a path between two specified nodes such that the Z-length of the path is minimum.

Here our aim is to find the path between u and v whose Z-length is minimum. (Z- SP between u & v).

Definition 2.4: Operation on fuzzy numbers

Let $A = (a_1, a_2, a_3, a_4, a_5)$ and $B = (b_1, b_2, b_3, b_{4,}, b_5)$ be any two pentagonal fuzzy numbers. Then

$$(a_1, a_2, a_3, a_4, a_5) + (b_1, b_2, b_3, b_4, b_5) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$$

Definition 2.5: Binary operation on Z-numbers [11]

Let (A_1, B_1) and (A_2, B_2) be two pentagonal Z-numbers. Then

$$(A_1, B_1)(+, \min)(A_2, B_2) = (A_1 + A_2, \min(B_1, B_2))$$

3. Main Results

3.1 Mathematical Formulation of Z-Travelling salesman problem

To calculate a Z-shortest path in the ZTSP for a given list of cities, where the distances between each pair of cities are represented by Z- numbers and each must be visited exactly once before returning to the starting city, we can follow these steps:

1. We start with a list of cities $\{1, 2, 3 \ldots\ldots n\}$, where n is the total cities.

2. x_{ij} is a Z- number defined as follows

$$x_{ij} = \begin{cases} ((1,1),(1,1)) & \text{if salesman travels from city i to city j} \\ ((0,0),(1,1)) & \text{if salesman not travel from city i ti city j} \end{cases}$$

3. For each pair of cities *i* and *j,* there is a Z-number d_{ij} representing the distance or cost between

them.

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The ZTSP can be mathematically formulated as follows:

$$Y = Min\ (\ d_{11}x_{11}\ (+,\min)d_{12}x_{12}\ (+,\min)d_{13}x_{13}\ (+,\min)\ ...\ (+,\min)d_{nn}x_{nn}\ )$$

where $d_{ij} = (a_{ij}, b_{ij})$ and where d_{ij} is a Z − number representing cost or distance.

Subject to

$\sum_{j=1}^{n} x_{ij}$ =1, i=1, 2, …n,

$\sum_{i=1}^{n} x_{ij}$ =1, j=1, 2 …n,

$and\ \ x_{ij} = 0\ or\ 1$

Conversion to Fuzzy TSP:

The above ZTSP is converted to the following FTSP

$$Y = Min \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij}$$

Subject to

$\sum_{j=1}^{n} x_{ij}$ =1, i=1, 2 …n

$\sum_{i=1}^{n} x_{ij}$ =1, j=1, 2 …n,

$and\ \ x_{ij} = 0\ or\ 1$

For each Z-number $d_{ij} = (a_{ij}, b_{ij})$ representing the distance between city x_i and x_j, rank the Z-number based on its first component a_{ij} and set all second components (reliability factors) to be equal by taking minimum for consistency in the TSP formulation.(i.e.) the reliability of FTSP is $min\{b_{ij}\}$

Apply a BnB algorithm on the resulting FTSP instance to find the optimal tour solution. The optimal tour with the minimum total cost will be the solution to the ZTSP.

By using suitable ranking function, calculate $D_{ij} = R(a_{ij})$ and considering the minimum for the second componentb_{ij}. Then, we convert ZTSP into FTSP. The steps of the algorithm for the FTSP using BnB method as follows.

Step 0: Initialisation

From the table D of distances

$$D_{ij} = \begin{cases} d_{ij} & \text{if a path from i to j} \\ \infty & \text{else} \end{cases}$$

Step 1: Reduction of table

The table D is given**.**

- In every row, identify the smallest element $r_i = \min_j\{D_{ij}\}$
- Similarly, for every column locate$c_j = \min_i(D_{ij} - r_i)$

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- Compute the reduced table D' by applying the formula

$$D'_{ij} = D_{ij} - r_i - c_j$$

Step 2: Calculation of bounds

The lower bound for travelling time is calculated as$b = \sum_{i=1}^{n} r_i + \sum_{j=1}^{n} c_j$

Step 3: Calculation of penalties:

The penalty for not using the edge $(u, v)$is denoted by π_{uv}

$\pi_{uv} = \min_{i} \{D'_{iv}\} + \min_{j}\{D'_{uj}\}$

Then find (p, q) such that$\pi_{pq} = \max_{(i,j)}\{\pi_{ij}\}$

The maximum penalty π_{pq} is denoted as π.

Step 4: Branching

The branching is done in the tree as follows: -

$'(p, q)'$ branch and 'non(p, q)'branch.

Step 5: Calculation of labels in the tree

In the reduced tableD', remove p^{th} row and q^{th} column. Also mark the q^{th} row and p^{th} column as ∞ .Denote the new table by D''. Reduce D'' according to step 1.

Then, let $\sigma = \sum_{i=1}^{n} r_i + \sum_{j=1}^{n} c_j$

In the tree, assign the label $b + \pi$ to 'non(p, q)' branch and the label $b + \sigma$ to '$(p, q)'$ branch.

Step 6: Identification

Detect the node with the smallest cost in the adjacent branch in the tree. Now repeat the procedure starting at step 1.

Numerical Example:

Table 1. Example

	A	B	C	D	E
A	((∞,∞,∞,∞,∞), (1,1,1,1,1))	((5,6,7,8,9), (.6,.65,.7,.75,.8))	((4,5,6,7,8), (.7,.75,.8,.85,.9))	((6,7,8,9,10), (.5,.55,.6,.65,.7))	((2,3,4,5,6), (.7,.75,.8,.85,.9))
B	((5,6,7,8,9), (.7,.75,.8,.85,.9))	((∞,∞,∞,∞,∞), (1,1,1,1,1))	((6,7,8,9,10), (.8,.85,.9,.95,1))	((3,4,5,6,7), (.8,.85,.9,.95,1))	((4,5,6,7,8), (.6,.65,.7,.75,.8))
C	((4,5,6,7,8), (.6,.65,.7,.75,.8))	((6,7,8,9,10), (.7,.75,.8,.85,.9))	((∞,∞,∞,∞,∞), (1,1,1,1,1))	((7,8,9,10,11), (.5,.55,.6,.65,.7))	((5,6,7,8,9), (.8,.85,.9,.95,1))

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D	((6,7,8,9,10), (.7,.75,.8,.85,.9))	((3,4,5,6,7), (.8,.85,.9,.95,1))	((7,8,9,10,11), (.7,.75,.8,.85,.9))	((∞,∞,∞,∞,∞), (1,1,1,1,1))	((6,7,8,9,10), (.6,.65,.7,.75,.8))
E	((2,3,4,5,6), (.7,.75,.8,.85,.9))	((4,5,6,7,8), (.75,.8,.85,.9,.95))	((5,6,7,8,9), (.6,.65,.7,.75,.8))	((6,7,8,9,10), (.5,.55,.6,.65,.7))	((∞,∞,∞,∞,∞), (1,1,1,1,1))

Now, we convert ZTSP into FTSP. So, the reliability is

$Min\{(.5, .55, .6, .65, .7), (.6, .65, .7, .75, .8), (.7..75..8..85..9), (.75, .8, .85, .9, .95), (.8, .85, .9, .95, 1), (1,1,1,1$

$= (.5, .55, .6, .65, .7)$

Consider the ranking function $R(a, b, c, d, e) = \frac{a+b+c+d+e}{5}$

Iteration 1:

Step 0:

Calculate, $D_{ij} = R(a_{ij})$

Table 2. Rank of a_{ij}

	A	B	C	D	E
A	∞	7	6	8	4
B	7	∞	8	5	6
C	6	8	∞	9	7
D	8	5	9	∞	8
E	4	6	7	8	∞

Step 1:

Table 3. Locating the minimal elements in the i^{th} row

D_{ij}	A	B	C	D	E	r_i
A	∞	7	6	8	4	4
B	7	∞	8	5	6	5
C	6	8	∞	9	7	6
D	8	5	9	∞	8	5
E	4	6	7	8	∞	4

Table 4. Locating the minimal elements in the j^{th} column

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D_{ij}	A	B	C	D	E	r_i
A	∞	3	2	4	0	4
B	2	∞	3	0	1	5
C	0	2	∞	3	1	6
D	3	0	4	∞	3	5
E	0	2	3	4	∞	4
c_j	0	0	2	0	0	

Table 5. The reduction of the table D'

D_{ij}	A	B	C	D	E	r_i
A	∞	3	0	4	0	4
B	2	∞	1	0	1	5
C	0	2	∞	3	1	6
D	3	0	2	∞	3	5
E	0	2	1	4	∞	4
c_j	0	0	2	0	0	

Step 2:

The lower boundary $b = \sum ri + \sum cj$ = (4+5+6+5+4) + (0+0+2+0+0) = 26

Step 3:

Max penalty, $\pi_{pq} = \max_{(i,j)}\{\pi_{ij}\}$ where $\pi_{uv} = \min_{i}\{D'_{iv}\} + \min_{j}\{D'_{uj}\}$

Table 6: $.D'_{ij}\pi_{ij}$

	A	B	C	D	E
A			0+1=1		0+1=1
B					
C	1+0=1				
D		2+2=4			
E	0+1=1				

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Max penalty $\pi = \pi_{pq}$ =4 =π_{42}

Step 4&5:

Now remove the 4th row (D) and 2nd column(B) from the reduced table

Table 7. Reduced Table

	A	C	D	E	r_i
A	∞	0	4	0	0
B	2	1	∞	1	1
C	0	∞	3	1	0
E	0	1	4	∞	0
c_j	0	0	3	0	

$\sigma = \sum r_i + \sum c_j$ =4

Step 6:

For (D, B) branch assign, b+ $\sigma = 26 + 4 = 30$and for non (D, B) branch assign,

b+$\pi = 26 + 4 = 30$

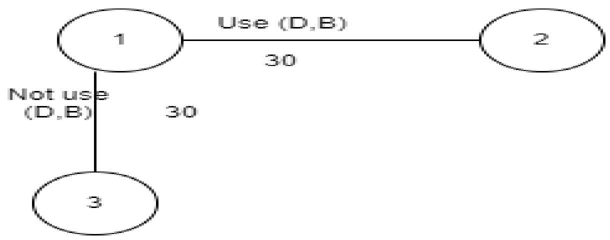

Figure 1. Output of iteration 1

In a similar way, we get,

Iteration 2:

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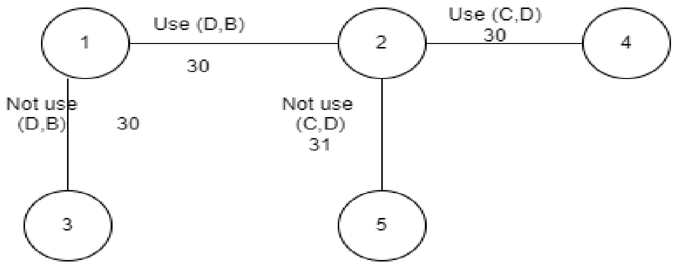

Figure 2. Output of iteration 2

Iteration 3:

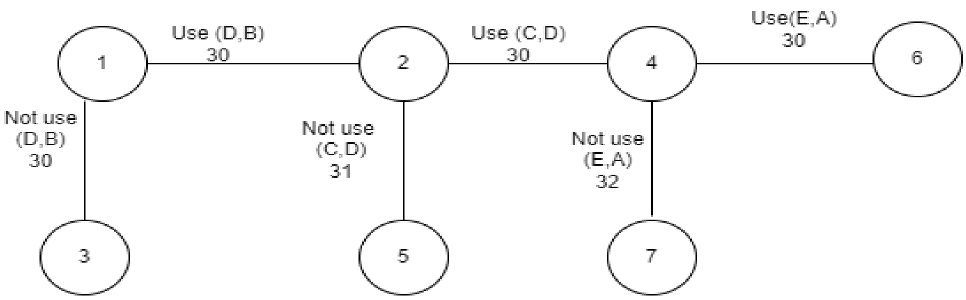

Figure 3. Output of iteration 3

Iteration 4:

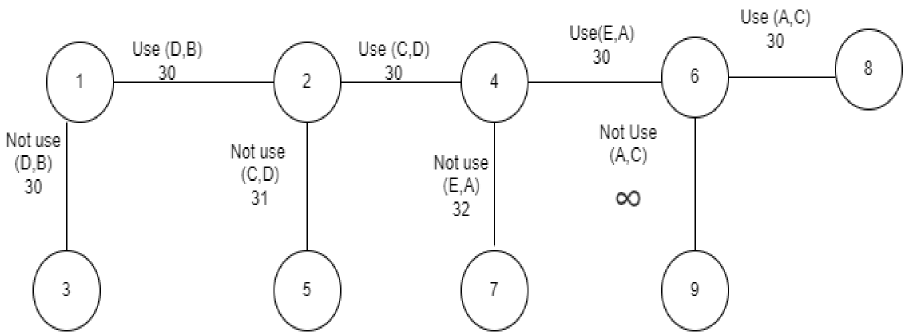

Figure 4. Output of iteration 4

Iteration 5:

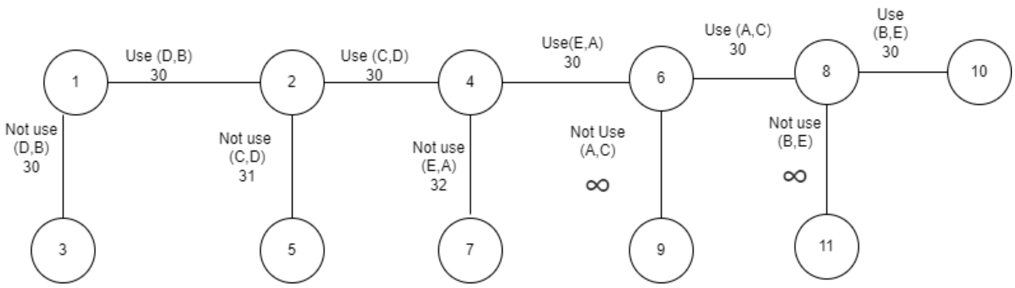

Figure 5. Output of iteration 5

Hence it is concluded that $\boldsymbol{B} \rightarrow \boldsymbol{E} \rightarrow \boldsymbol{A} \rightarrow \boldsymbol{C} \rightarrow \boldsymbol{D} \rightarrow \boldsymbol{B}$ is the optimal route for the **FTSP and the optimum value is 30.**

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Hence the corresponding **optimal value for ZTSP** is **((20,25,30,35,40), (.5,.55,.6,.65,.7))** and **the optimal path** is $\boldsymbol{B \rightarrow E \rightarrow A \rightarrow C \rightarrow D \rightarrow B}$

4. Conclusion

In order to calculate the best outcome, we modify the traditional Branch and Bound technique for the ZTSP in this study.

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