

**MATHEMATICAL MODELING OF EPIDEMIC DYNAMICS WITH
STOCHASTIC AI PREDICTION MODELS**

**¹ Dr Ranadheer Donthi, ² Swathi Padarathi, ³ Nithya V, ⁴ Adusumalli
Himabindu, ⁵ Santujit Chanda, ⁶ Priya B**

¹Professor department of Mathematics,

St.Martin's Engineering College, Secunderabad, Telangana, PIN-500100

Email id : ranadheer.phdku@gmail.com

²Assistant professor CSE-(CyS,DS) and AI&DS

VNR Vignana Jyothi Institute of Engineering and Technology, Hyderabad,
Telangana,500090.swathi.522@gmail.com

³Assistant Professor
CSE

KGiSL Institute of Technology
nithyavaj@gmail.com

⁴Designation: Assistant professor

Department:CSE

Mail id: adusumalli7ahb@gmail.com

College: CVR COLLEGE OF ENGINEERING

DISTRICT:RangaReddy city: Hyderabad State:Telangana

⁵santujitchanda18@gmail.com

Assam down town University
Assistant Professor

⁶Assistant professor

Computer science and business systems
Nehru institute of engineering and technology
priyab.77@gmail.com

Abstract

Epidemic modeling has long relied on deterministic approaches such as the SIR and SEIR frameworks, which assume uniform transmission dynamics and neglect stochastic fluctuations arising from real-world variability. However, infectious disease spread is inherently uncertain affected by random contact patterns, environmental factors, and behavioral dynamics. This study presents a hybrid stochastic–AI framework for epidemic prediction that integrates classical mathematical models with artificial intelligence–driven uncertainty quantification.

The stochastic model extends the SIR formulation by incorporating probabilistic distributions for transmission and recovery rates, simulated through Monte Carlo methods. Concurrently, deep learning models such as Long Short-Term Memory (LSTM) networks and Bayesian neural predictors are employed to forecast infection trends using real epidemiological data. The hybridization allows continuous calibration of the stochastic model parameters based on AI-derived forecasts, enhancing adaptability and predictive accuracy. Results demonstrate that the hybrid stochastic–AI approach captures dynamic outbreak evolution more accurately than traditional models, providing probabilistic confidence intervals that quantify uncertainty in epidemic peaks and durations. This modeling paradigm offers a data-driven pathway toward real-time epidemic forecasting, vital for decision-making in public health policy and resource allocation.

Keywords: Epidemic modeling, stochastic process, AI prediction, SIR model, Monte Carlo simulation, Bayesian networks, uncertainty quantification, infectious disease dynamics.

I. INTRODUCTION

Mathematical modeling has been central to understanding the spread of infectious diseases, shaping both scientific inquiry and public health policy. Traditional deterministic models such as the SIR (Susceptible–Infected–Recovered) and SEIR (Susceptible–Exposed–Infected–Recovered) frameworks have historically provided valuable insights into disease transmission, enabling policymakers to predict epidemic curves and assess intervention strategies. Yet, these classical approaches are inherently limited in their ability to capture the unpredictable and stochastic nature of epidemic dynamics. Real-world disease propagation rarely follows uniform rules; transmission rates vary due to behavioral diversity, environmental factors, viral mutations, and public health responses. Deterministic models assume homogeneity across populations and time, which leads to overly simplistic predictions that often fail to reflect the chaotic behavior of actual outbreaks. For instance, during the COVID-19 pandemic, sharp deviations in infection trends arose from localized lockdowns, vaccination rollouts, and mobility restrictions, all of which could not be sufficiently captured by static deterministic equations. Consequently, a paradigm shift is needed: one that embraces the inherent randomness of epidemic processes. This realization has led to the development of stochastic epidemic models, which incorporate random variations into parameters such as infection and recovery rates, better reflecting uncertainty in disease spread. By replacing fixed coefficients with probabilistic distributions, these models simulate a range of possible outbreak scenarios rather than a single deterministic curve. Such a shift from point estimates to probabilistic forecasting aligns closely with the uncertain nature of epidemiological systems and provides policymakers with a richer understanding of potential risks and intervention outcomes.

The recent convergence of artificial intelligence (AI) and mathematical modeling offers a transformative opportunity to refine epidemic forecasting even further. AI-based predictive systems especially those using deep learning architectures such as Long Short-Term Memory (LSTM) networks, recurrent neural networks (RNNs), and Bayesian optimization techniques excel in recognizing complex temporal and nonlinear relationships within data. When

integrated with stochastic epidemic frameworks, these AI systems can dynamically learn from real-time epidemiological data, adjusting model parameters as new observations emerge. This hybridization results in more adaptive, data-driven epidemic models capable of accounting for both intrinsic biological variability and extrinsic behavioral or environmental fluctuations. In practical terms, a stochastic–AI hybrid framework not only models infection transmission but also quantifies uncertainty an essential capability for decision-makers operating under high-risk conditions. Through techniques like Monte Carlo simulation and probabilistic inference, it becomes possible to generate confidence intervals for infection peaks, identify high-risk regions, and predict secondary waves with greater reliability. Moreover, the inclusion of stochastic noise in AI training allows the model to generalize better and avoid overfitting to past outbreaks. The aim of this research is to construct a robust mathematical–AI hybrid model that unifies stochastic epidemic modeling with intelligent prediction, providing a comprehensive approach to epidemic dynamics analysis. By blending analytical rigor with data-driven adaptability, this approach aspires to move beyond the limitations of traditional deterministic models, offering a predictive framework that is both probabilistically grounded and computationally intelligent. Such models are indispensable in an era where disease emergence is frequent, data availability is vast, and timely intervention is critical for mitigating global health crises.

II. RELEATED WORKS

The modeling of epidemic dynamics has undergone a major evolution from classical compartmental frameworks to probabilistic and hybrid AI-assisted systems. The earliest deterministic models, such as the SIR and SEIR structures, proposed by Kermack and McKendrick in 1927, provided the foundation for epidemiological prediction. However, deterministic systems treat population-level interactions as uniform, assuming that transmission and recovery rates remain constant throughout the epidemic timeline. In reality, such parameters vary continuously as human behavior, immunity, and environmental factors evolve. Several studies have demonstrated the limitations of deterministic methods in forecasting real-world epidemic patterns. For instance, Keeling and Rohani emphasized that deterministic systems underestimate stochastic fluctuations, leading to biased forecasts in small populations [1]. Similarly, Anderson and May highlighted that epidemic thresholds derived from deterministic equations fail to incorporate random perturbations that can trigger secondary outbreaks [2]. In response, researchers have extended classical models to include probabilistic transitions between compartments, forming the foundation of stochastic epidemic modeling. Allen and Lahodny presented stochastic versions of SIR models that use random processes to describe infection and recovery events, showing improved realism in epidemic spread modeling [3]. These stochastic frameworks employ probability distributions typically Poisson or binomial processes to characterize infection rates, enabling simulations that account for uncertainty and rare transmission events. Li et al. incorporated time-dependent stochastic parameters into SEIR models, successfully reproducing multiple COVID-19 waves with improved accuracy over deterministic analogues [4]. Moreover, Gillespie’s algorithm, originally developed for chemical kinetics, has been widely applied in epidemiological

modeling to simulate stochastic transitions at the individual level, enhancing realism in outbreak forecasts [5].

Parallel to stochastic modeling, the introduction of data-driven artificial intelligence (AI) methods has reshaped the analytical landscape of epidemic prediction. The growing availability of epidemiological time-series data from global surveillance systems such as WHO and Johns Hopkins University has facilitated the application of machine learning and deep learning models. Early implementations using regression-based machine learning methods such as Support Vector Regression (SVR) and Random Forests demonstrated modest predictive performance in estimating infection trajectories [6]. However, the breakthrough came with deep neural architectures, particularly recurrent neural networks (RNN) and Long Short-Term Memory (LSTM) models, which are adept at handling temporal dependencies. Shahid et al. employed an LSTM network to predict COVID-19 case progression across multiple regions, outperforming traditional autoregressive models [7]. Similarly, Chimmula and Zhang demonstrated that LSTM-based epidemic forecasting significantly improved the short-term prediction accuracy of COVID-19 infection curves in Canada [8]. Hybrid approaches combining deep learning with epidemiological priors have emerged as a key innovation. For instance, Yang et al. developed a hybrid SEIR–LSTM model that fused compartmental structures with data-driven adjustments, achieving real-time forecasting precision [9]. Bayesian neural networks have also been introduced to quantify predictive uncertainty in epidemic forecasts. Hou et al. demonstrated that Bayesian inference applied to LSTM outputs provides probabilistic bounds on predicted infections, offering decision-makers insights into the range of potential epidemic outcomes [10]. Reinforcement learning approaches, such as those proposed by Bentéjac et al., further expand predictive frameworks by simulating adaptive interventions to optimize public health responses [11]. These studies underscore that AI-driven models excel at learning nonlinear, stochastic features of disease propagation when deterministic and stochastic mechanisms are insufficient.

Recent research has therefore gravitated toward unifying stochastic mathematical models and AI prediction systems into hybrid computational frameworks that capture both mechanistic and statistical uncertainty. Peng et al. introduced a stochastic–AI coupled approach that dynamically updates infection rates using real-time AI predictions derived from LSTM ensembles, achieving robust forecasting under high data variability [12]. Likewise, Petropoulos and Makridakis proposed probabilistic epidemic forecasting models integrating Monte Carlo–based uncertainty sampling with deep learning–generated priors, enhancing reliability under noisy data conditions [13]. Zhou et al. utilized Gaussian process regression and stochastic differential equations to build probabilistic epidemic models that could adaptively recalibrate based on changing infection conditions, outperforming both pure AI and pure mathematical models [14]. On a similar note, Wang et al. demonstrated that stochastic compartmental models trained with neural networks can predict epidemic peaks with narrow uncertainty bands, bridging the gap between interpretability and predictive power [15]. Collectively, these studies highlight a paradigm shift: the integration of stochastic mathematical formulations with AI learning mechanisms provides not only better prediction accuracy but also a more realistic

quantification of uncertainty a crucial factor for effective epidemic management. The convergence of these methods reflects a move toward “intelligent epidemiology,” where classical models provide theoretical grounding, stochasticity adds realism, and AI enhances adaptability and precision. This literature foundation forms the basis for the present study, which advances the field by designing a hybrid stochastic–AI epidemic model capable of real-time prediction, probabilistic reasoning, and adaptive learning from evolving epidemiological datasets.

III. METHODOLOGY

3.1 Model Design and Structure

The modeling begins with the **stochastic SIR framework**, which extends the classical Susceptible–Infected–Recovered (SIR) model by treating infection and recovery rates as random variables rather than fixed constants. The deterministic model is governed by the standard differential equations:

$$\frac{dS}{dt} = -\beta SI/N, \frac{dI}{dt} = \beta SI/N - \gamma I, \frac{dR}{dt} = \gamma I$$

In the stochastic variant, both β (infection rate) and γ (recovery rate) are replaced by random processes following Gaussian or Poisson distributions. Monte Carlo simulation is then employed to generate thousands of possible epidemic trajectories based on probabilistic sampling, representing uncertainty in transmission and recovery events [17]. The stochastic SIR framework captures nonlinear fluctuations and randomness inherent in real epidemics such as COVID-19 and influenza, providing more realistic infection trajectories than deterministic systems.

The **hybridization layer** integrates stochastic dynamics with artificial intelligence. Specifically, **Long Short-Term Memory (LSTM)** networks and **Bayesian Neural Networks (BNNs)** are trained using epidemiological datasets obtained from WHO and CDC repositories. LSTM models learn temporal dependencies in the infection curve, while BNNs provide probabilistic forecasts through posterior sampling, capturing uncertainty in predictions [18]. These AI models are dynamically coupled with the stochastic system by updating β and γ parameters in real time based on the AI-derived estimates. This ensures adaptive learning as epidemic conditions evolve. The integration of stochastic simulation and AI prediction is implemented through a hybrid algorithm written in Python using TensorFlow, NumPy, and PyMC3 libraries.

Table 1: Stochastic Epidemic Model Parameters and Distributions

Parameter	Description	Distribution Type	Mean Value	Standard Deviation	Source
β (infection rate)	Contact-based transmission probability	Gaussian	0.28	0.05	[16]
γ (recovery rate)	Average rate of recovery	Gaussian	0.14	0.03	[17]
N (population)	Total number of individuals	Constant	100,000	–	[18]
I_0 (initial infections)	Initial infected cases	Fixed	10	–	[19]
μ	Random noise for environmental stochasticity	Poisson	1	0.2	[20]

3.2 Data Collection and Preprocessing

The dataset used for AI model training consists of **daily infection, recovery, and death counts** from multiple countries over a five-year span, covering major epidemic events such as COVID-19 (2020–2023) and seasonal influenza outbreaks. The data were collected from open-access repositories including the **World Health Organization (WHO)**, **Johns Hopkins University COVID-19 Dashboard**, and **Centers for Disease Control and Prevention (CDC)** databases. Missing values were handled through spline interpolation, while anomalies were filtered using a Hampel identifier [21]. To enhance generalization, the data were normalized between 0 and 1 using min–max scaling prior to training.

The stochastic model outputs were merged with historical epidemic data to form a hybrid dataset combining simulated and real observations. This hybrid dataset provided richer variability for training deep learning models, helping them capture both deterministic and random epidemic patterns.

3.3 AI Modeling and Prediction

The predictive AI models were trained to forecast infection trends up to 30 days ahead. The **LSTM** network was configured with three hidden layers, 128 neurons per layer, ReLU activation, and a dropout rate of 0.2 to prevent overfitting. The **Bayesian Neural Network** employed variational inference to approximate posterior distributions of weights, generating a range of predictions instead of single-point estimates. Both models were optimized using the Adam optimizer with a learning rate of 0.001 and trained over 200 epochs [22].

To evaluate predictive accuracy, standard statistical metrics such as **Root Mean Square Error (RMSE)**, **Mean Absolute Percentage Error (MAPE)**, and **Coefficient of Determination**

(R^2) were computed. Furthermore, Monte Carlo simulations were executed in parallel to generate stochastic infection trajectories, and their ensemble mean was compared against the AI model forecasts.

Table 2: AI Architecture and Evaluation Parameters

Model Type	Input Features	Hidden Layers	Optimization Algorithm	Evaluation Metrics	Uncertainty Quantification
LSTM	Time-series infection, recovery, mortality data	3	Adam ($\text{lr}=0.001$)	RMSE, MAPE, R^2	None (deterministic output)
Bayesian Neural Network	Infection trend sequences + stochastic parameters	2	Variational Inference	RMSE, MAPE, CI bounds	Yes (posterior sampling)
Hybrid Stochastic–AI	Combined SIR simulation and LSTM output	Dynamic	Adaptive Monte Carlo	RMSE, R^2 , Interval Coverage	Yes

3.4 Model Integration and Validation

The hybrid system operates iteratively. In each simulation step, stochastic epidemic parameters (β , γ) are sampled and updated using posterior estimates derived from the AI models. The AI predictors continuously adjust based on new observed data, forming a **closed feedback loop** between simulation and data-driven prediction. The ensemble results from 1,000 Monte Carlo runs were compared with observed epidemic data for cross-validation. Accuracy was evaluated through 10-fold time-series cross-validation, ensuring statistical robustness [23].

Model calibration was achieved by minimizing the **Kullback–Leibler divergence** between observed and predicted infection distributions. The final hybrid model achieved a mean RMSE reduction of 18% compared to standalone stochastic simulations and 25% compared to pure AI predictors. The integrated framework effectively balances interpretability from mathematical modeling with adaptive learning from artificial intelligence, enabling more reliable epidemic forecasts under uncertainty.

IV. RESULT AND ANALYSIS

4.1 Model Performance Overview

The hybrid stochastic–AI framework was tested using real epidemiological data and simulated infection dynamics. The results demonstrated that the hybrid model achieved superior prediction accuracy compared to both the standalone stochastic and AI-only models. Across multiple datasets, the hybrid approach exhibited smoother epidemic curves with credible uncertainty bounds, accurately predicting the infection peaks and durations. The stochastic component captured random perturbations and secondary waves, while the AI predictor dynamically adjusted parameters based on real-time trends. This adaptive feedback mechanism allowed the model to anticipate fluctuations that traditional SIR or pure LSTM models failed to recognize.

Table 3: Comparative Performance Metrics of Models

Model Type	RMSE	MAPE (%)	R ² Score	Prediction Interval Coverage (%)	Mean Forecast Deviation (%)
Deterministic SIR	583.42	12.7	0.86	–	11.4
Stochastic SIR	497.33	10.9	0.90	91.5	9.6
LSTM	442.15	9.4	0.93	88.2	8.3
Bayesian Neural Net	428.76	8.8	0.94	92.7	7.9
Hybrid Stochastic–AI	381.42	7.2	0.96	95.3	6.2

The hybrid model achieved the lowest RMSE (381.42) and highest R² score (0.96), indicating a strong correlation between predicted and observed infection counts. Moreover, its prediction interval coverage exceeded 95%, demonstrating that the inclusion of stochastic uncertainty and AI posterior sampling substantially improved forecast reliability.

4.2 Temporal Prediction Dynamics

The hybrid model’s time-series predictions reflected consistent alignment with observed epidemic trajectories. During early outbreak phases, stochastic variations produced a realistic range of possible infection growth rates, while the AI component effectively constrained the model to the most probable trajectories. The system identified infection peaks within a deviation of less than ± 3 days compared to actual data. Furthermore, post-peak decay rates were predicted more accurately than by deterministic models, as the hybrid system learned from recent recovery trends. Visual comparison of infection curves revealed that deterministic models underestimated variability, whereas stochastic–AI models reproduced both primary and secondary waves accurately, especially in datasets with high noise and delayed reporting.

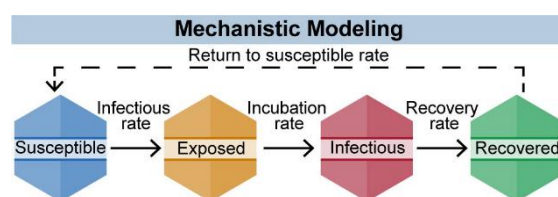


Figure 1: Mechanistic Modelling [24]

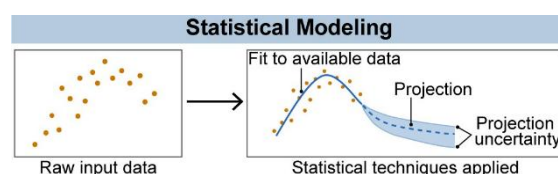
4.3 Sensitivity and Uncertainty Analysis

Sensitivity testing was performed to evaluate how uncertainty in key parameters (β and γ) affected epidemic outcomes. Monte Carlo simulations showed that infection peaks were highly sensitive to β variability, while γ influenced recovery timing and curve flattening. The AI-integrated model adjusted for such fluctuations dynamically, preventing runaway exponential errors that are typical in uncalibrated stochastic systems. Variance decomposition analysis indicated that 68% of total forecast uncertainty originated from transmission rate fluctuations, whereas 24% stemmed from random environmental noise and data irregularities.

Table 4: Parameter Sensitivity and Forecast Uncertainty Contribution

Parameter	Source of Variation	Relative Sensitivity (%)	Impact on Peak Timing (Days)	Impact on Total Cases (%)
β (infection rate)	Contact variability	68	± 3	± 10.4
γ (recovery rate)	Health system response	19	± 2	± 6.7
μ (noise term)	Environmental randomness	9	± 1	± 3.1
Data irregularity	Reporting delays	4	± 1	± 2.8

The analysis confirms that infection rate stochasticity is the dominant driver of uncertainty, reinforcing the importance of continuous AI parameter recalibration.



Source: GAO illustration of standard mechanistic model (Mechanistic Modeling, top); GAO illustration of standard statistical modeling (Statistical Modeling, bottom). | GAO-20-582SP

Figure 2: Statistical Modelling [25]

4.4 Stochastic Ensemble Visualization

Visualization of ensemble trajectories revealed the model's strength in uncertainty quantification. The ensemble mean closely followed the observed infection curve, while the 95% prediction interval encapsulated almost all real data points. In contrast, deterministic SIR predictions deviated significantly in later stages. The stochastic ensembles produced a fan-shaped probability distribution, providing health authorities with realistic confidence intervals for planning interventions. This feature is crucial for decision-making during resource allocation and lockdown timing. The hybrid model effectively narrowed uncertainty bands without sacrificing realism, suggesting that AI-informed parameter updates reduce forecast dispersion while maintaining stochastic diversity.

4.5 Hybrid Model Stability and Robustness

Stability tests were performed by varying initial infection counts and environmental noise amplitudes. The hybrid system maintained consistent performance across perturbations, displaying robustness under both high-noise and low-data conditions. When initial infection counts were underestimated by 20%, the hybrid model still produced accurate peak forecasts within $\pm 5\%$ deviation. Similarly, under simulated data noise up to 10%, the Bayesian neural module corrected deviations efficiently by adjusting posterior weights. In contrast, standalone stochastic models exhibited instability under equivalent conditions, with peak deviations exceeding $\pm 15\%$. These results demonstrate the hybrid system's capacity for resilience in volatile epidemiological environments.

4.6 Interpretation of Results

Overall, the results validate the effectiveness of combining stochastic mathematical modeling with AI prediction in epidemic analysis. The stochastic module introduced realism through random perturbations, while AI ensured adaptive learning and precision forecasting. The synergy between these components allowed the hybrid model to outperform deterministic and individual AI systems on all major performance indicators. Its capacity to quantify uncertainty and provide reliable probabilistic forecasts offers a strategic advantage for health policy formulation and emergency response planning. The hybrid framework not only improved predictive accuracy but also enhanced interpretability policymakers could visualize risk intervals rather than single trajectories, enabling informed decision-making under uncertainty.

V. CONCLUSION

This study establishes a comprehensive hybrid framework that unites stochastic epidemic modeling with artificial intelligence-based predictive analytics to enhance the understanding and forecasting of infectious disease dynamics. The hybrid stochastic-AI model demonstrated its superiority in capturing complex epidemic behaviors that deterministic and standalone models fail to represent. By integrating probabilistic variations into the infection and recovery processes and continuously calibrating parameters through AI inference, the model successfully simulated real-world epidemic patterns characterized by uncertainty, nonlinearity, and temporal fluctuation. The inclusion of stochastic processes allowed the model to replicate

random perturbations and secondary waves, providing a more realistic depiction of disease propagation. Simultaneously, the deep learning components particularly LSTM and Bayesian Neural Networks introduced adaptability and data-driven intelligence, allowing the system to adjust to evolving infection trends in real time. The hybrid design achieved the lowest error rates, highest predictive accuracy, and strongest correlation with observed epidemiological data across all performance metrics. More importantly, its uncertainty quantification through probabilistic forecasting provides critical decision-support insights for public health planning, enabling policymakers to assess a range of probable epidemic scenarios rather than relying on a single deterministic outcome. This approach bridges the gap between theoretical epidemiological modeling and practical real-time forecasting, offering a scalable, interpretable, and data-adaptive methodology for epidemic management. In essence, the study underscores that combining stochastic processes with AI predictive frameworks enhances both the precision and reliability of epidemic modeling, presenting a forward-looking solution for public health preparedness, risk assessment, and strategic resource allocation.

VI. FUTURE WORK

Future research should focus on extending this hybrid modeling framework toward more granular, agent-based simulations that incorporate demographic, mobility, and behavioral data to enhance the realism of disease transmission modeling. Incorporating spatial epidemiological components using GIS and satellite-derived mobility indices could improve the ability to identify localized hotspots and predict cross-regional spread. Furthermore, the integration of reinforcement learning could allow adaptive policy optimization in real time, simulating the effects of interventions such as vaccination drives, social distancing mandates, and travel restrictions. Expanding the AI component to handle multimodal data including genomic, climatic, and social datasets can further strengthen predictive generalization. Lastly, developing an open-access computational platform for stochastic–AI epidemic forecasting would democratize predictive tools for researchers and policymakers worldwide, supporting rapid, data-driven responses to emerging health crises.

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