International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

ON THE STAR DOMINATION NUMBER IN HYPERGRAPHS

Divya P.M[1], T.V. Ramakrishnan[2] and Shama K.S [3]

Department of Mathematical Sciences, Kannur University, Kannur, Kerala, India

[1] divyapmadhavan@gmail.com, [2] ramakrishnantvknr@gmail.com, [3] shamathariq@gmail.com

Abstract

The concept of the star domination number $\gamma^*(H)$ has been previously introduced as a natural extension of domination in graphs and hypergraphs. In this paper, we further investigate its structural properties by establishing general lower and upper bounds for $\gamma^*(H)$. Additionally, we examine the effect of vertex deletion—both weak and strong—on $\gamma^*(H)$ for various classes of graphs and hypergraphs, providing characterizations and examples. These results enhance the understanding of the relationship between vertex removal and star domination, offering insights relevant to both theoretical research and practical applications in network resilience.

2020 Mathematics Subject Classification: 05C65, 05C69

Keywords: Hypergraph, domination, dependence.

1 Introduction

Domination is a fundamental concept in graph theory, with extensive applications across computer science, social networks, biology, and more. A detailed treatment of domination in graphs can be found in the works of Chartrand and Lesniak [2], and comprehensive surveys appear in the books by Haynes et al. [6, 7].

A natural extension of graphs, *hypergraphs* offer a more general framework where edges—called *hyperedges*—can connect any number of vertices, not just pairs. Formally, a hypergraph H is an ordered pair $(V, \mathcal{E})$, where V is a nonempty finite set of vertices, and $\mathcal{E} \subseteq \mathrm{P}(V) \setminus \{\emptyset\}$ is a collection of nonempty subsets of V called hyperedges, such that $\cup_{e\in\mathcal{E}}\, e = V$. A hypergraph is said to be *simple* if no hyperedge is a proper subset of another, and it is called *r-uniform* if every edge contains exactly r vertices. Hypergraphs thus provide a natural generalization of graphs, which correspond to the special case where each hyperedge connects exactly two vertices (i.e., 2-uniform hypergraphs).

[1] Also, at Department of Mathematics, Sree Narayana College, Kannur, Kerala, India

[3] Also, at Government Women's Polytechnic College, Kozhikode, Kerala, India

Received: July 16, 2025

Volume 38 No. 2s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

A *Host graph* for a hypergraph is a connected graph on the same vertex set such that every hyperedge induces a connected subgraph of the host graph. A hypergraph $H = (V, \mathcal{E})$ is called a *hypertree* if there exist a host tree $T = (V, \mathcal{E})$, such that each edge $e \in$ E induces a subtree in T. $\mathcal{E}(\mathrm{v})$ denotes the set of all edges in E which contains v. A vertex $v \in V$ is called a transversal vertex if there exist an edge $\{v\}$ in $\mathcal{E}$. A vertex $x \in V$ is called hyperpendent if there exist a vertex $y \in V$ such that $\mathcal{E}(x) \subseteq \mathcal{E}(y)$.

Theorem 1.1. *[3] If $H = (V, \mathcal{E})$ is a hypertree with order $n \geq 2$, then it contains at least two vertices such that each of them is either transversal of hyper-pendant.*

In the context of hypergraphs, the concept of domination has evolved in several directions. Two prominent definitions are known in the literature. The classical one, introduced by Acharya [8], mirrors the notion from graph theory: a vertex is said to be dominated if it is adjacent to at least one vertex in the dominating set. A more recent formulation, introduced in [1], defines domination in terms of edge structure. This second definition—adopted in the present work—is referred to as the *star domination* model.

Definition 1.2. [1] *Let $H = (V, \mathcal{E})$ be a hypergraph. A subset $D \subseteq V$ is called a star dominating set of H if for every $v \in V \setminus D$, there exists an edge $e \in \mathcal{E}$ such that $v \in e$ and $e \setminus \{v\} \subseteq D$. The minimum cardinality of a dominating set is called the star domination number of H, denoted $\gamma^*(H)$.*

In this paper, we investigate structural properties of $\gamma^*(H)$ and explore constructions of hypergraphs that realize a prescribed star domination number. The goal is to deepen our understanding of how the structure of a hypergraph influences its domination characteristics under the star domination framework. The star domination number has potential applications in network design, where resources (represented by dominating vertices) must control or monitor all non-resource nodes through shared communication groups (hyperedges). It is also relevant in modelling scenarios such as sensor placement in systems with multi-node coverage or collaborative task allocation in distributed computing.

For graph-theoretic terminology, we follow Chartrand and Lesniak [2]; for hypergraph terminology, we refer to [3,4,5]. Further results on star domination in hypergraphs may be found in [1].

2 Main Results

To explore the flexibility and expressive power of the star domination number in hypergraphs, a natural question arises: can this parameter attain any prescribed positive integer? In other words, given an arbitrary $n \in N$, is it possible to construct a hypergraph whose star domination number is exactly n? The following result affirms this possibility, establishing that the star domination number is unbounded and can grow linearly with the order of the hypergraph.

Theorem 2.1. *For any $n \in N$, there exists a 3-uniform hypergraph H with $\gamma^*(H) = n$*

Received: July 16, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Proof. Let $G = (V, E)$be a graph with n vertices and m edges. We construct a hypergraph from G as follows. If $\{v_1, v_2\} \in$ E, we add a vertex u_1 to the edge $\{v_1, v_2\}$ and form a hyperedge $\{v_1, u_1, v_2\}$. In general, to each edge in G we add a vertex u_i and form a hyperedge with 3 vertices.

Let $H = (V', \mathcal{E})$ be the newly constructed hypergraph where $V' = V \cup \{u_1, u_2, \dots, u_m\}$ and $\mathcal{E} = \{\{v_i, u_t, v_j\}: \{v_i, v_j\} \in \text{E}, 1 \le t \le m\}$.

We claim that $\gamma^*(H) = n$. Clearly the vertex set of G will dominate H. So $\gamma^*(H) \le n$. Now suppose that S is a dominating set of H and S doesn't contain some $v_i \in V$. Then in order to dominate H, S must contain all $u_i s$ adjacent to $v_i \notin S$. Note that if $\{v_i, v_j\} \in E$, then either v_i or v_j belongs to every star dominating set since the newly added vertex u_i is a pendant vertex. That is corresponding to each vertex v_i that doesn't belong to S at least one u_i belong to S. So $|S| \ge n$. So $\gamma^*(H) \ge n$. Hence, we get $\gamma^*(H) = n$. □

We now present two complementary results that bound the star domination number $\gamma^*(H)$. The first theorem provides a general lower bound in terms of the order and size of H, while the second gives an upper bound under additional structural assumptions.

Theorem 2.2. *For a hypergraph* $H = (V, \mathcal{E})$ *with* $|V| = n$ *and* $|\mathcal{E}| = m$, *then* $n - m \le \gamma^*(H)$.

Proof. Let S be a minimum star dominating set of H. For each vertex $v \in V \setminus S$, choose one hyperedge $e_v \in \mathcal{E}$ such that $v \in e_v$ and $e_v \setminus \{v\} \subseteq S$. We claim that the edges e_v chosen for distinct vertices $v \in V \setminus S$ are pairwise distinct, for if $e_v = e_w$ for distinct v, w in $V \setminus S$, $w \in e_v \setminus \{v\}$ and $e_v \setminus \{v\}$ is not a subset of S. Consider a map $f : V \setminus S \to \mathcal{E}$ that brings $v \in V \setminus S$ into $e_v \in \mathcal{E}$. This is an injective map. Therefore

$$|V \setminus S| \le |\mathcal{E}| = m$$

$$|S| = n - |V \setminus S| \ge n - m$$

$$\text{Hence} \quad n - m \le \gamma^*(H) \qquad \square$$

Theorem 2.3. *Let* $H = (V, \mathcal{E})$ *be an r-uniform hypergraph of order n. Then* $\gamma^*(H) \le \frac{(r-1)n}{r}$.

Proof. Let $S \subseteq V$ be a minimum star dominating set of H, so that $|S| = \gamma^*(H) = x$. By the definition of star domination, for every vertex $v \in V \setminus S$, there exists an edge $e_v \in \mathcal{E}$ such that $v \in e_v$ and $e_v \setminus \{v\} \subseteq S$. Since H is r uniform, each edge contain r vertices and hence associated to each vertex $v \in V \setminus S$, presence of $r - 1$ vertices in S is guaranteed. This implies that the number of elements in S satisfies

$$|S| = x \le (r-1)(n-x) = (r-1)(n-x)$$

$$|S| \le \left(\frac{r-1}{r}\right) n$$

Received: July 16, 2025

Volume 38 No. 2s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Hence $\gamma^*(H) \leq \frac{(r-1)n}{r}$. □

2.1 Impact of Vertex Deletion on $\gamma^*(H)$

The notions of *weak deletion* and *strong deletion* of a vertex in hypergraphs are already defined in [3]. We adopt the same definitions and extend the associated concepts to our setting.

Let $H = (V, \mathcal{E})$ be a hypergraph. For a vertex $v \in V$, we denote the *weak deletion* of v by $H \setminus_w \{v\}$, which is obtained by removing v from the vertex set V and from every hyperedge in $\mathcal{E}$ that contains v. Similarly, the *strong deletion* of v is denoted by $H \setminus_s \{v\}$, which is obtained by removing v from $V(H)$ and deleting every hyperedge that contains v.

In graphs, the vertex set is often partitioned into V^0, $V^+$and V^- according to how the deletion of a vertex affects the domination number. Here, we extend this idea to hypergraphs under both weak deletion and strong deletion, using distinct notation for each case.

Weak deletion partition:

$$V_w^0 = \{v \in V | \gamma^*(H \setminus_w \{v\}) = \gamma^*(H)\},$$
$$V_w^+ = \{v \in V | \gamma^*(H \setminus_w \{v\}) > \gamma^*(H)\},$$
$$V_w^- = \{v \in V | \gamma^*(H \setminus_w \{v\}) < \gamma^*(H)\},$$

Strong deletion partition:

$$V_s^0 = \{v \in V | \gamma^*(H \setminus_s \{v\}) = \gamma^*(H)\},$$
$$V_s^+ = \{v \in V | \gamma^*(H \setminus_s \{v\}) > \gamma^*(H)\},$$
$$V_s^- = \{v \in V | \gamma^*(H \setminus_s \{v\}) < \gamma^*(H)\},$$

Next, we investigate the effect of weak deletion on the star domination number in hypergraphs. In particular, we examine how the removal of a vertex under weak deletion influences the partition of the vertex set into V_w^0, V_w^+ and V_w^-, and the resulting changes (or lack thereof) in $\gamma^*(H)$.

Observation 2.4.

1. *If $v \in V_w^-$, then $H' = H \setminus_w \{v\}$ has star domination number $\gamma^* - 1$. For if $\gamma^*(H') \leq \gamma^* - 2$ and S' is a minimal star dominating set of H', then $S' \cup \{v\}$ will be a star dominating set of H. So $\gamma^*(H) \leq \gamma^* - 1$, a contradiction.*
2. *If $v \in V_w^+$, removal of v can increase star domination number by more than one.*
3. *Removing a vertex outside a star dominating set will not increase star domination number, so if $v \in V_w^+$, then v is in every γ^* set of H*
4. *Obviously, every isolated vertex belongs to V^-.*

Theorem 2.5. *In the complete r uniform hypergraph K_n^r with $n \geq r$ and $r \geq 2$, every vertex belongs to V_w^-. In other words, for any vertex $v \in V(K_n^r)$, $\gamma^*(K_n^r \setminus_w \{v\}) = r - 2$.*

Received: July 16, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Proof. First note that $\gamma^*(K_n^r) = r - 1$ for every $n \geq r$. Indeed, any $(r-1)$ subset $S \subseteq V(K_n^r)$ star-dominates all vertices outside S, because for each $u \notin S$ the set $S \cup \{u\}$ is an edge of K_n^r. Hence $\gamma^*(K_n^r) \leq r - 1$.

Conversely, if $|S| \leq r - 2$, then there exists a vertex outside S that cannot be star-dominated via an edge formed by adding a single outside vertex to S; thus $\gamma^*(K_n^r) \geq r - 1$. Therefore $\gamma^*(K_n^r) = r - 1$.

Let $v \in V(K_n^r)$. After weak deletion of v, every edge of K_n^r that contained v becomes an $r-1$ subset of $V\backslash\{v\}$. Since K_n^r was complete, every $r-1$ subset of $V\backslash\{v\}$ arises in this way. Therefore $H' = K_n^r\backslash_w\{v\}$ contains all $r-1$ subset of $V\backslash\{v\}$ or in other words K_{n-1}^{r-1} is a spanning subhypergraph of H'. Hence $\gamma^*(H') = (r-1) - 1 = r - 2$. So $v \in V_w^-$.

□

Theorem 2.6. *Let H be a complete r-partite r-uniform hypergraph with partite classes* $P_1, P_2, \dots P_r$. *Then*

(i) *if* $v \in P_i$ *with* $|P_i| = 1$, *then* $v \in V_w^-$, *and*
(ii) *if* $v \in P_i$ *with* $|P_i| \geq 2$, *then* $v \in V_w^0$.

Equivalently, for $v \in P_i$ *with* $|P_i| = 1$, *we have* $\gamma^*(H\backslash_w\{v\}) = r - 1$, *while for* $v \in P_i$ *with* $|P_i| \geq 2$, *we have* $\gamma^*(H\backslash_w\{v\}) = r$.

Proof. First note that $\gamma^*(H) = r$. Indeed, any set formed by choosing one vertex from each of *r* parts is a star-dominating set of *H*, so $\gamma^*(H) \leq r$. Conversely, any set T *with* $|T| \leq r - 1$ leaves at least one part without a representative, and a vertex from other parts cannot be star-dominated by *T*, so $\gamma^*(H) \geq r$.

Suppose first that $|P_i| = 1$ say $P_i = \{v\}$. In *H*, every edge contains v, since each edge must take one vertex from every part. After weak deletion of *v*, each edge loses *v* and becomes an $(r-1)$ −subset consisting of one vertex from each of the remaining parts. Thus $H\backslash_w\{v\}$ is isomorphic to a complete $(r-1)$-partite $(r-1)$-uniform hypergraph on the classes $\{P_j : j \neq i\}$. For such a hypergraph, a star dominating set requires at least one vertex from each of the $(r-1)$ parts, and taking exactly one from each part suffices. Hence $\gamma^*(H\backslash_w\{v\}) = r - 1$. Since $\gamma^*(H) = r$, it follows that $v \in V_w^-$.

Next suppose $|P_i| \geq 2$ and $v \in P_i$. we can see that $H' = H\backslash_w\{v\}$ is again a complete *r* partite *r* uniform hypergraph on the same r parts (one part having size reduced by one). So $\gamma^*(H') = r = \gamma^*(H)$. Consequently $v \in V_w^0$.

□

Theorem 2.7. *In a hypergraph H, if a vertex* $v \in V_w^-$, *then for some* γ^* *set S of H,* $v \in S$ *and for all e containing* v, $e_v \cap (V\backslash S) \neq \phi$.

Proof. Let $\gamma^*(H) = k$ and $H' = H\backslash_w\{v\}$. By hypothesis $v \in V_w^-$, and hence $\gamma^*(H') = k - 1$. Let *T* be a minimum star dominating set of H'. Then clearly $S = T \cup \{v\}$ is a star dominating set of *H* and $|S| = |T| + 1 = k$ and hence *S* is a γ^*−set of *H* containing *v*. Now

Received: July 16, 2025

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

suppose there exists an edge $e \in \mathcal{E}$ such that $u \in e$ and $e \subseteq S$. Then $e \setminus \{v\} \subseteq S \setminus \{v\} = T$ which implies v is star dominated by T and hence T is a star dominating set of H of size $k-1$, contradicting the fact that $\gamma^*(H) = k$. Hence all edges containing v intersect with $V \setminus S$. □

Theorem 2.8. *In a hypergraph* $H = (V, \mathcal{E})$ *if* $\min_{e \in \mathcal{E}} |e| \geq 3$, *then* $V_w^+ = \phi$.

Proof. Assume the contrary, $V_w^+ \neq \phi$. If $v \in V_w^+$, then v belongs to every γ^* set of *H*. Let *S* be a γ^**set* of *H*. Then each $v \in S$, satisfies one of the following.

(i) If $e \in \mathcal{E}$ and $v \in e$, then $e_v \cap (V - S) \neq \phi$

(ii) There exists a vertex $u \in V - S$ such that if e is any edge in $\mathcal{E}$ with $u \in e$ and $e - \{u\} \subseteq S$, then $v \in e$.

If v satisfy the condition (i), we can see that no other vertex is star dominated using v, consequently $S \setminus \{v\}$ is a star dominating set of $H' = H \setminus_w \{v\}$, contradicting the assumption that $v \in V_w^+$. Now suppose v satisfies condition (*ii*). Then $e' = e \setminus \{v\}$ is an edge in H' with $|e'| \geq 2$. Also $e' \setminus \{u\} \subseteq S' = S \setminus \{v\}$. Hence u is star dominated by S' in H' and consequently S' is a star dominating set of H' contradicting $v \in V_w^+$. □

Corollary 2.9. *If* $v \in V^+$, *then* v *belongs to an edge with cardinality 2.*

Theorem 2.10. *If* $u \in V^+$ *and* $v \in V^-$, *then* $\{u, v\}$ *is not a hyperedge.*

Proof. Suppose $u \in V^+, v \in V^-$ and $\{u, v\}$is a hyperedge. Since $u \in V^+, u$ is in every γ^* set of *H*. Let S_v be the star dominating set of $H \setminus_w \{v\}$ with cardinality $\gamma^*(H) - 1$. If $u \in S_v$ then S_v dominates *H* also, contradicting γ^* is the domination number of *H*. If $u \notin S_v$ then $S_v \cup \{v\}$ is a domination set of H which doesn't contain u, a contradiction to the fact that u belongs to every γ set of H. □

We now investigate the impact of strong vertex deletion on the star domination number of hypergraphs, focusing on how the removal of vertices influences the minimum cardinality of a star dominating set. The following characterization of V^- remains valid under both weak and strong deletion of a vertex. *Observation 2.4* remains valid under strong deletion as well. Similarly, the corresponding result for *r*-uniform, *k*-partite hypergraphs also holds.

Theorem 2.11. *In the complete r-uniform hypergraph* K_n^r *with* $n \geq r+1$ *and* $r \geq 2$, *every vertex belongs to* V_s^0. *In other words, for any vertex* $v \in V(K_n^r)$,

$$\gamma^*(K_n^r \setminus_s \{v\}) = \gamma^*(K_n^r) = r - 1.$$

Proof. We know that $\gamma^*(K_n^r) = r - 1$ for every $n \geq r$.

Now fix $v \in V(K_n^r)$ and perform strong deletion. In a complete *r*-uniform hypergraph, removing *v* and all edges incident to *v* leaves the complete *r* uniform hypergraph on the remaining $n - 1$ vertices:

$$K_n^r \setminus_s \{v\} = K_{n-1}^r.$$

Received: July 16, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Since $n-1 \geq r$, we have $\gamma^*(K_{n-1}^r) = r-1$. Consequently,

$$\gamma^*(K_n^r \setminus_s \{v\}) = \gamma^*(K_{n-1}^r) = r-1 = \gamma^*(K_n^r).$$

And so γ^* is unchanged by strong deletion of any vertex and every vertex lies in V_s^0.

□

Remark 2.12. *If $n = r$, then strong deletion produces a hypergraph with no edges, so γ^* (as defined for hypergraphs with nonempty edge sets) is not applicable. This justifies the assumption $n \geq r+1$ in the theorem.*

Theorem 2.13. *Let H be the complete r-partite, r-uniform hypergraph with partite classes $P_1, P_2, \dots P_r$, and assume $|P_i| \geq 2$ for every i. Then every vertex of H belongs to V_s^0. Equivalently, for any $v \in V(H)$,*

$$\gamma^*(H \setminus_s \{v\}) = \gamma^*(H) = r.$$

Proof. Fix $v \in P_t$ and form $H' := H \setminus_s \{v\})$ by strongly deleting v (removing v and all edges that contain v). Since $|P_t| \geq 2$, the part P_t remains nonempty in H', and therefore H' is again a complete *r*-partite, *r*-uniform hypergraph. Applying the same argument in *Theorem 2.6* to H' yields $\gamma^*(H') = r$. Hence $\gamma^*(H \setminus_s \{\mathrm{v}\}) = \gamma^*(H) = r$,

so γ^* is unchanged by strong deletion of any vertex and every vertex lies in V_s^0.

□

Remark 2.14. *The hypothesis $|P_i| \geq 2$ for all i in the theorem is essential. If some part P_t has size 1, then the strong deletion of the unique vertex of P_t destroys every edge of the complete r-partite, r-uniform hypergraph.*

Theorem 2.15. *Let $H = (V, \mathcal{E})$ be a hypergraph and $v \in V$. Then $v \in V_s^-$ if and only if the following condition hold: v belongs to some γ^* set S of H, for any edge e_v containing v, $e_v \cap V \setminus S \neq \phi$. Moreover $u \in e_v \cap V \setminus S$, then there exists an edge $e_u \in \mathcal{E}$ containing u such that $e_u \setminus \{u\}$ is a non-empty subset of $S \setminus \{v\}$*

Proof. Suppose *v* satisfies the given condition. The first part of the condition ensures that *v* is dominated only by itself and by no other vertices of *S*. The second part ensures that no vertex of *V\S* is dominated by v. Therefore, $S \setminus \{v\}$ is a dominating set of $H \setminus_s v$, which implies that $v \in V_s^-$.

Conversely, suppose $v \in V_s^-$ and *D* be a γ^* set of $H \setminus_s \{v\}$. Then $S = D \cup \{v\}$ is a γ^* set of *H*. If $e_v \cap V \setminus S$ is empty for any edge e_v containing *v* then *D* will be a star dominating set of *H*, a contradiction. □

Theorem 2.16. $v \in V_s^+$ *of a hypergraph H if and only if v satisfies the following;*

a) v is not an isolate and v belongs to every γ^* set of H and

b) There exist at least two vertices $v_1, v_2 \in V \setminus S$ such that $v_i \in e_i$ and $e_i \setminus \{v_i\} \subseteq S, i = 1,2$ and $v \in e_1 \cap e_2$

Received: July 16, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Proof. Suppose $v \in V_s^+(H)$. If v is an isolated vertex, then clearly $v \in V_s^-(H)$.

Next, assume that v does not belong to every γ^* −set. Let S_1 be a star dominating set of H such that $v \notin S_1$. Then S_1 also star dominates $H\backslash_s\{v\}$ which implies $v \in V_s^+(H)$, a contradiction.

Now suppose there exists exactly one vertex $v_1 \in V \backslash S$ such that, whenever $v_1 \in e_1$ and $e_1\backslash\{v_1\} \subseteq S$, we have $v \in e_1$. Under strong deletion of v, the edge e_1 is removed, and hence $S\backslash\{v\}$ fails to dominate v_1. However, the set $(S\backslash \{v\}) \cup \{v_1\}$forms a star dominating set of $H \backslash_s \{v\}$, again a contradiction.

Finally, suppose there is no vertex as described in condition (b). Then $S\backslash\{v\}$ remains a star dominating set of $H \backslash_s \{v\}$, which again implies $v \notin V_s^+(H)$, a contradiction.

The other implication is clear. □

Theorem 2.17. *For any hypertree H with order* $n \geq 2$ *there exists a vertex* $v \in V$ *such that* $\gamma^*(H \backslash_s\{x\}) = \gamma^*(H)$

Proof. We know that for any hypertree there exists at least two vertices such that each of them is either transversal or hyperpendant. Transversal vertices have the property that on weak deletion of the edge containing only that vertex it becomes a pendant vertex and so weak deletion of that edge will not affect the domination number of *H*. So, without loss of generality, we consider *H* as a hypertree with at least two pendant vertices.

Case 1: Suppose *H* is A hypertree and x is a hyperpendant vertex in *H*. If all of its twin vertex has degree 1, then this hypertree has a single edge. If we strongly delete x, then $H \backslash_s \{x\})$ becomes a hypergraph with $n-1$ isolated vertices. Then $\gamma^*(H \backslash_s \{x\}) = \gamma^*(H) = n-1$. So $x \in V_s^0$.

Case 2: Suppose that x has at least one twin vertex that has degree greater than or equal to 2. Let it be \`y'. If $\mathcal{E}(x) = \mathcal{E}(y)$, then either x or y belong to every γ^* *set* but not both. Without loss of generality assume that $x \in \gamma^*-$ *set*. Strong deletion of *y* deletes all edges containing x also. So $\gamma^*(H \backslash_s \{x\}) = \gamma^*(H)$ and $\gamma^*(H \backslash_s \{y\}) = \gamma^*(H)$. That is $x, y \in V_s^0$. If for every twin vertex y of x, $\mathcal{E}(x) \subset \mathcal{E}(y)$, then $\gamma^*(H \backslash_s \{x\}) = \gamma^*(H)$ □

3 Conclusion and scope

In this work, we established general lower and upper bounds for the star domination number $\gamma^*(H)$ and investigated the effect of vertex deletion— both strong and weak—on $\gamma^*(H)$. Our study highlights how vertex removal influences the minimum size of a star dominating set in hypergraphs, thereby offering new perspectives on the stability of this parameter under structural changes. The results presented here form a foundation for further study, with possible extensions including the effect of other hypergraph operations such as edge contraction, vertex addition, or hypergraph products. Future directions also include exploring algorithmic techniques for the efficient computation of $\gamma^*(H)$ in specific classes of

Received: July 16, 2025

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

hypergraphs. Such developments will broaden the theoretical scope of domination in hypergraphs while enhancing its relevance to applications in network design, optimization, and resilience analysis.

References

[1] P.M Divya, T.V Ramakrishnan, S. Arumugam, A new look at the concept of domination in hypergraphs. *Electronic Journal of Graph theory and Applications* **12**(2) (2024), 181-188

[2] G. Chartrand and L. Lesniak, *Graphs and Digraphs*, CRC Press (2005).

[3] Vitaly. I. Voloshin, *Introduction to Graph and Hypergraph Theory*, Nova Science Publishers,Inc. New york.

[4] C.Berge, *Graphs and Hypergraphs*, North Holland, Amsterdam (1973).

[5] C. Berge, *Hypergraphs*, North Holland, Amsterdam, (1989).

[6] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*, Marcell Dekker (1998).

[7] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Domination in Graphs Advanced Topics*, Marcell Dekker (1998).

[8] B.D. Acharya, Domination in Hypergraphs, *AKCE Int. J. Combin.*, **4** (2007), 117–126.

[9] B.D. Acharya,*Domination in Hypergraphs II, New Directions* in: Proc. Int. Conf-ICDM 2008, Mysore, India, 1–16.

[10] M.A. Henning and C. Lowenstein, Hypergraphs with Large Domination Number and with Edge Sizes at least Three, *Discrete Appl. Math.*, **160** (2012), 1757–1765.

[11] C. Bujt´as, M.A. Henning, Zs.Tuza, Transversals and Domination in Uniform Hypergraphs, *European J. Combin.*, **33** (2012), 62–71.

[12] E.J.Cockayne, S.T. Hedetniemi and D.J. Miller, Properties of Hereditary Hypergraphs and Middle graphs, *Canad. Math. Bull.*, **21** (1978), 461–468 .

Received: July 16, 2025