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A NEW VIEW ON NEUTROSOPHIC Z- MATRICES AND DECISION MAKING

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Abstract

Zadeh developed the concept of Z-number to solve the problems of fuzziness and partial trustworthiness in information. The Z-numbers set has been expanded to include a neutrosophic Z-numbers set (NZNs). This is a powerful model that can describe both the constraints and the reliability of assessment data for unknown information. The goal of this paper was to find different kinds of modal operators like necessity operation, falsity operation, transitive closure, c-transitive closure, nS and S^n operator over the proposed neutrosophic Z-matrix (NZM) with certain characteristics. A new algorithm in decision-making was also suggested for NZM in the choice matrix for both non-normalized and normalized forms. A reduction procedure for dimensionality reduction was also incorporated to get a better accuracy of decision-making in NZM. At last, a numerical example was provided to illustrate the algorithm's capability in decision-making strategy for optimal decision.

Keywords: Neutrosophic Z-set, Neutrosophic Z-matrix , Dimensionality reduction, Decision making .

1 Introduction

The idea of Z-numbers was made by Zadeh [1], who uses a pair of fuzzy numbers to show the constraint and reliability of assessment data. A possibility distribution over probability distributions is presented by Yager [2] for the z-valuation of z-numbers. The Dempster-Shafter theory and z-numbers were further modified by Kang et al. [12] to identify uncertainty in environmental assessment. Li et al. [11] then proposed the fundamental idea of relative entropy in probability theory as well as the idea of relative entropy of z-numbers.

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Kang et al. [10] established a technique of changing z-numbers to classical fuzzy numbers according to fuzzy set. A novel approach of ranking z-numbers based on the concept of value and ambiguity to find decision-making was proposed by Rituparna Chutia in 2020 [9]. For decision-making, Nik et al. [13] discovered the synergic rank of fuzzy z-numbers depending on distance.

The neutrosophic set addresses the intricacies of processing imprecision, ambiguity, and uncertainty in data [4]. For a simplified neutrosophic collection, the multi-criteria decision-making technique (MCDM) was discovered [3]. Some decomposition operators for neutrosophic fuzzy matrices were created by Murugadas et al. [5]. The features of a unique kind of square neutrosophic fuzzy matrix were discovered by Dhar et al. [6].

A new image thresholding approach for neutrosophic utilizing a similarity score and an image segmentation algorithm for neutrosophic in similarity clustering was covered by Guo et al. [7, 8]. Xu et al. [15] created a sample selection model for large-scale data processing in dimensionality reduction, whereas Su et al. [14] explored the concept of dimensionality reduction for multi-dimensional sequences.

In order to convert a higher-dimensional data collection into a lower-dimensional data set, dimensionality reduction is essential. Guleria and Bajaj [16] started a novel process with the objective to reduce dimensionality for decision-making in neutrosophic soft matrices. Later, a methodology for Pythagorean fuzzy soft matrices in the dimensionality reduction process and similarity measure for decision-making in MCDM was extended [18, 17].

Shigui Du given ideas about neutrosophic Z-numbers (NZN) to convey unclear and conflicting information using dependability measures [19]. Three-order pairs of neutrosophic z-numbers are used to illustrate the constraint and reliability of evaluation information over falsity, indeterminacy, and truth. Shigui Du has investigated a novel aggregation operator in NZN set for MCDM problems. Ye et al. [20] showed a solution for MCDM and adjusted the distance measure for NZN. Additionally, Ye et al. [22] talked about a linguistic NZN for weighted aggregation operator in MCDM, and Yong et al. [21] disclosed a magnificent application of trapezoidal NZN for MCDM.

In this work, basic definitions about the proposed work are included in section 2 . The new class of modal operators that deal with transitive closure and c-transitive closure is then covered in Section 3, along with certain fundamental definitions and characteristics. The properties and their operating laws have then been thoroughly examined in Section 4. Section 5 then includes a decision-making algorithm that uses a reduction process and choice matrix. Additionally, the suggested methodology's applicability to the decision-making process is illustrated through an example. To enhance readability, a comparative analysis was also provided. In section six, we finally wrapped up the paper.

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2 Preliminaries

Some essential definitions are given.

2.1 Definition

Let X be a universe set then a Neutrosophic Z-number set (NZNs) in a universe set X is defined in the following for $S_Z = (< \mathrm{x}, \mathrm{T(V,R)(x)}, \mathrm{I(V,R)(x)}, \mathrm{F(V,R)(x)} > x \in \mathrm{X})$ where $\mathrm{T(V,R)(x)} = (T_V(\mathrm{x}), T_R(\mathrm{x}))$, $\mathrm{I(V,R)(x)} = (I_V(\mathrm{x}), I_R(\mathrm{x}))$, $\mathrm{F\,(V,R)(x)} = (F_V(\mathrm{x}), F_R(\mathrm{x})): \mathrm{X} \to [0,1]^2$ are the order pairs of neutrosophic values for truthfulness, indeterminacy, and falsehood; the first component consists of the neutrosophic values in a universe set X, and the second component consists of neutrosophic reliability measures, with the rule of $0 \leq T_V(\mathrm{x}) + I_V(\mathrm{x}) + F_V(\mathrm{x}) \leq 3$ and $0 \leq T_R(\mathrm{x}) + I_R(\mathrm{x}) + F_R(\mathrm{x}) \leq 3$

2.2 Definition

Let X be a universe set and F be a set of parameters. Consider a non empty set $S_Z, S_Z \in F$. Let P(X) is the collection of all neutrosophic z- number sets of X. The set (E, S_Z) is termed to be neutrosophic z- number sets (NZNs) over X, where $E : S_Z \to P(X)$. Hereafter we simply consider S as neutrosophic z-matrices (NZMs) over X instead of (E, S_Z).

2.3 Definition

Let $S_A = (< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >)$ and

$S_B = (< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >)$be two NZMs of order $m \times n$

1.$S_A \subseteq S_B \Leftrightarrow T^A_{V_{ij}} \leq T^B_{V_{ij}}, T^B_{V_{ij}} \leq T^B_{V_{ij}}, I^A_{V_{ij}} \geq I^B_{V_{ij}}, I^A_{R_{ij}} \geq I^B_{R_{ij}}, F^A_{V_{ij}} \geq F^B_{V_{ij}}, F^A_{R_{ij}} \geq F^B_{R_{ij}}$

2.$S_A = S_B \Leftrightarrow S_B \subseteq S_A \ and \ S_A \subseteq S_B$

2.4 Definition

Let $S_A = (< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >)$ and

$S_B = (< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >)$be two NZMs of order $m \times n$, the component wise addition and multiplication is

$$S_A \oplus S_B = (< \left(T^A_{V_{ij}} + T^B_{V_{ij}} - T^A_{V_{ij}}.T^B_{V_{ij}}, T^A_{R_{ij}} + T^B_{R_{ij}} - T^A_{R_{ij}}\,T^B_{R_{ij}}\right),$$
$$\left(I^A_{V_{ij}}.I^B_{V_{ij}}, I^A_{R_{ij}}.I^B_{R_{ij}}\right), \left(F^A_{V_{ij}}.F^B_{V_{ij}}, F^A_{R_{ij}}.F^B_{R_{ij}}\right) >)$$

$$S_A \otimes S_B = (< \left(T^A_{V_{ij}}.T^B_{V_{ij}}, T^A_{R_{ij}}.T^B_{R_{ij}}\right), \left(I^A_{V_{ij}} + I^B_{V_{ij}} - I^A_{V_{ij}}.I^B_{V_{ij}}, I^A_{R_{ij}} + I^B_{R_{ij}} - I^A_{R_{ij}}.I^B_{R_{ij}}\right),$$
$$\left(F^A_{V_{ij}} + F^B_{V_{ij}} - F^A_{V_{ij}}.F^B_{V_{ij}}, F^A_{R_{ij}} + F^B_{R_{ij}} - F^A_{R_{ij}}.F^B_{R_{ij}}\right) >)$$

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2.5 Definition

Let $S_A = (< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >)$ and

$S_B = (< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >)$be two NZMs of order $m \times n$, then

the union and intersection is defined as $S_A \cup S_B = (< (T^A_{V_{ij}} \vee T^B_{V_{ij}}), (T^A_{R_{ij}} \vee T^B_{R_{ij}}), (I^A_{V_{ij}} \wedge I^B_{V_{ij}}), (I^A_{R_{ij}} \wedge I^B_{R_{ij}}), (F^A_{V_{ij}} \wedge F^B_{V_{ij}}), (F^A_{R_{ij}} \wedge F^B_{R_{ij}} >)$

$S_A \cap S_B = (< (T^A_{V_{ij}} \wedge T^B_{V_{ij}}), (T^A_{R_{ij}} \wedge T^B_{R_{ij}}), (I^A_{V_{ij}} \vee I^B_{V_{ij}}), (I^A_{R_{ij}} \vee I^B_{R_{ij}}), (F^A_{V_{ij}} \vee F^B_{V_{ij}}), (F^A_{R_{ij}} \vee F^B_{R_{ij}} >)$

2.6 Definition

Let $S_A = (< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >)$ be NZMs of order $m \times n$, then the complement is denoted as

$$S_A{}^c = (< \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right), \left(1 - I^A_{V_{ij}}, 1 - I^A_{R_{ij}}\right), \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right) >)$$

2.7 Definition

The set of all NZMs of order $m \times n$ an NZNs , S_A in X denoted NZM_{mn} or $NZM_{m\times n}$.If m=n then it is denoted by NZM_n. The zero matrix $O_n = [< (0,0), (1,1), (1,1) >]$ and the identity matrix I_n of order $n \times n$ is defined as I_n whose principal diagonal elements are all $J = [< (1,1), (0,0), (0,0) >]$ and other element $\phi = [< (0,0), (1,1), (1,1) >]$

2.8 Definition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{n\times p}$ then the composition of S_A and S_B is defined as

$$S_A o S_B = (< (\sum_{i=1}^{n} \left(T^A_{V_{ik}} \wedge T^B_{V_{kj}}\right), (\sum_{i=1}^{n} \left(T^A_{R_{ik}} \wedge T^B_{R_{kj}}\right)), (\prod_{i=1}^{n} (I^A_{V_{ik}} \vee I^B_{V_{kj}})), \prod_{i=1}^{n} (I^A_{R_{ik}} \vee I^B_{R_{kj}})),$$
$$(\prod_{i=1}^{n} (F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\prod_{i=1}^{n} (F^A_{R_{ik}} \vee F^B_{R_{kj}})) >)$$

Equivalently we can write,

$$S_A o S_B = (< (\bigcup_{i=1}^{n} \left(T^A_{V_{ik}} \wedge T^B_{V_{kj}}\right)), (\bigcup_{i=1}^{n} \left(T^A_{R_{ik}} \wedge T^B_{R_{kj}}\right)), (\bigwedge_{i=1}^{n} (I^A_{V_{ik}} \vee I^B_{V_{kj}})),$$
$$(\bigwedge_{i=1}^{n} (I^A_{R_{ik}} \vee I^B_{R_{kj}})), (\bigwedge_{i=1}^{n} (F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\bigwedge_{i=1}^{n} (F^A_{R_{ik}} \vee F^B_{R_{kj}})) >)$$

If the number of S_A columns equals the number of rows S_B, then the product is defined.This multiplication procedure is called as max-min composition operator. Consequently, $S_A o S_B$ and are considered conformable for multiplication, rather than using $S_A o S_B$ we use $S_A S_B$where

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$\sum_{i=1}^{n}\left(T_{V_{ik}}^{A} \wedge T_{V_{kj}}^{B}\right)$means max- min operation and $\prod_{i=1}^{n}(I_{V_{ik}}^{A} \vee I_{V_{kj}}^{B}))$means min-max operation.

2.9 Definition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{n\times p}$ then the composition of S_A and S_B is defined as $S_A \lozenge S_B = (\wedge_k \ (S_A \vee S_B) >)$.This is called min-max composition operator.

3 Modal operators on NZMs

In this section, we are going to discuss the relationship of different modal operators with some results. The transitive closure and C-transitive closure definitions were defined with some algebraic properties.

A NZM in S_A is neutrosophic z-equivalence matrix if it satisfy reflexivity, symmetric and transitivity.

For an $S_A \in NZM_{n\times n}$,we define

- S_A is Reflexive if and only if $S_A \geq I_n$
- S_A is Symmetric if and only if $S_A = S_A^T$
- S_A is Idempotent if and only if $S_A = S_A^2$
- S_A is Transitive if and only if $S_A^2 \geq S_A$
- S_A is C-Transitive if and only if $S_A = S_A^{[2]}$

3.1 Definition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{n\times p}$ then

$$S_A \leftarrow S_B = ((\wedge_k < (T_{V_{ik}}^{A}, T_{R_{ik}}^{A}), (I_{V_{ik}}^{A}, I_{R_{ik}}^{A}), (F_{V_{ik}}^{A}, F_{R_{ik}}^{A}) >) \longleftarrow$$
$$<(T_{V_{ij}}^{B}, T_{R_{ij}}^{B}), (I_{V_{ij}}^{B}, I_{R_{ij}}^{B}), (F_{V_{ij}}^{B}, F_{R_{ij}}^{B}))$$

$$S_A \rightarrow S_B = ((\wedge_k < (T_{V_{ik}}^{A}, T_{R_{ik}}^{A}), (I_{V_{ik}}^{A}, I_{R_{ik}}^{A}), (F_{V_{ik}}^{A}, F_{R_{ik}}^{A}) >) \rightarrow$$
$$(<(T_{V_{ij}}^{B}, T_{R_{ij}}^{B}), (I_{V_{ij}}^{B}, I_{R_{ij}}^{B}), (F_{V_{ij}}^{B}, F_{R_{ij}}^{B}) >)$$

3.2 Definition

Let $S_A = (< (T_{V_{ij}}^{A}, T_{R_{ij}}^{A}), (I_{V_{ij}}^{A}, I_{R_{ij}}^{A}), (F_{V_{ij}}^{A}, F_{R_{ij}}^{A}) >)$ be two NZMs of order $m \times n$,

$WS_A = (< (T_{V_{ij}}^{A}, T_{R_{ij}}^{A}), (I_{V_{ij}}^{A}, I_{R_{ij}}^{A}), (1 - T_{V_{ij}}^{A}, 1 - T_{R_{ij}}^{A}) >)$ and

$\lozenge\ S_A = (< (1 - F_{V_{ij}}^{A}, 1 - F_{R_{ij}}^{A}), (I_{V_{ij}}^{A}, I_{R_{ij}}^{A}), (F_{V_{ij}}^{A}, F_{R_{ij}}^{A}) >)$

3.3 Proposition

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For any two $NZM_{m\times n}$ of S_A and S_B , $W(S_A \leftarrow S_B) = W(S_A \leftarrow WS_B)$

Proof:

Given, $W(S_A \leftarrow S_B) = W(S_A \leftarrow WS_B)$ --------------(1)

If $S_A \geq S_B$, then $W(S_A \leftarrow S_B) = W(<(1,1),(0,0),(0,0)>)$

$= (<(1,1),(0,0),(0,0)>)$ -------(2)

Since $S_A \geq S_B \Longrightarrow T^A_{V_{ij}} \geq T^B_{V_{ij}}, T^B_{V_{ij}} \leq T^B_{V_{ij}}, I^A_{V_{ij}} \leq I^B_{V_{ij}}, I^A_{R_{ij}} \leq I^B_{R_{ij}}, F^A_{V_{ij}} \leq F^B_{V_{ij}}, F^A_{R_{ij}} \leq F^B_{R_{ij}}$

$\therefore 1 - T^A_{V_{ij}} \leq 1 - T^B_{V_{ij}}, 1 - T^A_{R_{ij}} \leq 1 - T^B_{R_{ij}}$ and

$\left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right)>\right)$

$\geq (<\left(1 - F^B_{V_{ij}}, 1 - F^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(1 - T^B_{V_{ij}}, 1 - T^B_{R_{ij}}\right)>)$

So $S_A \geq WS_B$. Thus

$W\ S_A \leftarrow W\ S_B) = (<(1,1),(0,0),(0,0)>)$ -------(3)

From (2) & (3) ,(1) holds

If $S_A < S_B$ then

$W\ (S_A \leftarrow S_B) = W\ S_A = (\left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right)>\right)$ ----(4)

$W\ (S_A \leftarrow WS_B) =$

$(\left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right)>\right) \leftarrow (\left(<\left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(1 - T^B_{V_{ij}}, 1 - T^B_{R_{ij}}\right)>\right)$

$=\left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right)>\right)$ ----------(5)

From (4) & (5), (1) holds

Hence the proof.

3.4 Proposition

For any two $NZM_{m\times n}$ of S_A and S_B , $\Diamond(S_A \leftarrow S_B) = \Diamond(S_A \leftarrow \Diamond S_B)$

Proof:

Given, $\Diamond(S_A \leftarrow S_B) = \Diamond(S_A \leftarrow \Diamond S_B)$ ------(6)

If $S_A \geq S_B$, then $\Diamond(S_A \leftarrow S_B) = \Diamond(<(1,1),(0,0),(0,0)>)$

$= (<(1,1),(0,0),(0,0)>)$ -------(7)

Since $S_A \geq S_B \Longrightarrow T^A_{V_{ij}} \geq T^B_{V_{ij}}, T^B_{R_{ij}} \geq T^B_{R_{ij}}, I^A_{V_{ij}} \leq I^B_{V_{ij}}, I^A_{R_{ij}} \leq I^B_{R_{ij}}, F^A_{V_{ij}} \leq F^B_{V_{ij}}, F^A_{R_{ij}} \leq F^B_{R_{ij}}$

$\therefore 1 - T^A_{V_{ij}} \leq 1 - T^B_{V_{ij}}, 1 - T^A_{R_{ij}} \leq 1 - T^B_{R_{ij}}$ and

$\left(<\left(1 - F^A_{V_{ij}}, 1 - F^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right)>\right)$

$\geq (<\left(1 - F^B_{V_{ij}}, 1 - F^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right)>)$

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So $\Diamond S_A \geq \Diamond S_B$. Thus

$\Diamond S_A \leftarrow \Diamond S_B = (<(1,1),(0,0),(0,0)>)$ -------(8)

From (7) & (8) ,(6) holds

If $S_A < S_B$ then

$$\Diamond S_A \leftarrow \Diamond S_B = \Diamond S_A = \left(<\left(1-F^A_{V_{ij}}, 1-F^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right)>\right) \leftarrow (<\left(1-F^B_{V_{ij}}, 1-F^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right)>) \quad \text{----- (9)}$$

$$\Diamond (S_A \leftarrow \Diamond S_B) = \left(<\left(1-F^A_{V_{ij}}, 1-F^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right)>\right) \quad \text{--------(10)}$$

From (9) & (10), (6) holds

Hence the proof.

3.5 Proposition

$S_A$is reflexive NZM iff $WS_A$is reflexive NZM .

Proof:

S_A is reflexive $\Leftrightarrow S_A \geq I_n$

$$\Leftrightarrow \left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right)>\right) \geq (<\left(J^T_{V_{ij}}, J^T_{R_{ij}}\right), \left(J^I_{V_{ij}}, J^I_{R_{ij}}\right), \left(J^F_{V_{ij}}, J^F_{R_{ij}}\right)>) \text{ for all i,j}$$

$$\Leftrightarrow \left(<\left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1-T^A_{V_{ij}}, 1-T^A_{R_{ij}}\right)>\right) \geq (<\left(J^T_{V_{ij}}, J^T_{R_{ij}}\right), \left(J^I_{V_{ij}}, J^I_{R_{ij}}\right), \left(1-J^T_{V_{ij}}, 1-J^T_{R_{ij}}\right)>) \text{ for all i,j}$$

$\Leftrightarrow WS_A \geq I \Leftrightarrow WS_A$ is reflexive.

Hence the proof.

3.6 Proposition

S_A is reflexive NZM iff $\Diamond S_A$is reflexive NZM .

Similar to the above proof.

3.7 Proposition

S_A is reflexive NZM iff $W\, {S_A}^c$ is reflexive NZM .

Proof:

We know that if S_A is reflexive matrix if and only if ${S_A}^c$is reflexive and $W\, {S_A}^c$

Hence the proof

Note:

$\Diamond {S_A}^c$ is reflexive NZM if and only S_A is reflexive NZM .

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3.8 Proposition

S_A is symmetric NZM iff $W\ S_A$is symmetric matrix and $W\ {S_A}^c$ is also symmetric NZM

Proof:

S_A is symmetric matrix

$\Leftrightarrow \left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >\right)$

$= \left(< \left(T^A_{V_{ji}}, T^A_{R_{ji}}\right), \left(I^A_{V_{ji}}, I^A_{R_{ji}}\right), \left(F^A_{V_{ji}}, F^A_{R_{ij}}\right) >\right)$ for all i and j

$\Leftrightarrow \left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right) >\right)$

$= \left(< \left(T^A_{V_{ji}}, T^A_{R_{ji}}\right), \left(I^A_{V_{ji}}, I^A_{R_{ji}}\right), \left(1 - T^A_{V_{ji}}, 1 - T^A_{R_{ij}}\right) >\right)$ for all i and j

$\Leftrightarrow W\ S_A = (W\ S_A)^T$

Thus S_A is symmetric NZM if and only if $W\ S_A$is symmetric NZM .

Similarly, we can prove the next part.

Hence the proof

3.9 Proposition

S_A is symmetric NZM iff $\lozenge\ S_A$is symmetric NZM .

It can be proved easily

3.10 Proposition

S_A is transitive NZM iff $W\ S_A$is transitive NZM

Proof:

S_A is transitive $\Leftrightarrow\ S_A \geq S_A^2$

$$\Leftrightarrow \left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >\right) \geq$$

$$< (\sum_{i=1}^{n} \left(T^A_{V_{ik}}.T^A_{V_{kj}}\right), (\sum_{i=1}^{n} \left(T^A_{R_{ik}}.T^A_{R_{kj}}\right)), (\prod_{i=1}^{n} (I^A_{V_{ik}} + I^A_{V_{kj}})), (\prod_{i=1}^{n} (I^A_{R_{ik}} + I^A_{R_{kj}})), (\prod_{i=1}^{n} (F^A_{V_{ik}} + F^A_{V_{kj}})), (\prod_{i=1}^{n} (F^A_{R_{ik}} + F^A_{R_{kj}})) >)\ \ \forall\, i, j$$

$\Leftrightarrow T^A_{V_{ij}} \geq \sum_{i=1}^{n} \left(T^A_{V_{ik}}.T^A_{V_{kj}}\right)$, $T^A_{R_{ij}} \geq \sum_{i=1}^{n} \left(T^A_{R_{ik}}.T^A_{R_{kj}}\right)$, $I^A_{V_{ij}} \leq \sum_{i=1}^{n} \left(I^A_{V_{ik}} + I^A_{V_{kj}}\right)$,

$I^A_{R_{ij}} \leq \sum_{i=1}^{n} \left(I^A_{R_{ik}} + I^A_{R_{kj}}\right)$, $F^A_{V_{ij}} \leq \sum_{i=1}^{n} \left(F^A_{V_{ik}} + F^A_{V_{kj}}\right)$ and $F^A_{R_{ij}} \leq \sum_{i=1}^{n} \left(F^A_{R_{ij}} + F^A_{R_{ij}}\right)$

Then

$$\Leftrightarrow T^A_{V_{ij}} \geq \sum_{i=1}^{n} \left(T^A_{V_{ik}}.T^A_{V_{kj}}\right)$$

$$\Leftrightarrow 1 - T^A_{V_{ij}} \leq 1 - \sum_{i=1}^{n} \left(T^A_{V_{ik}}.T^A_{V_{kj}}\right)$$

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$\Leftrightarrow < (T^A_{V_{ij}}, 1 - T^A_{V_{ij}}) >$

Hence the proof.

3.11 Proposition

S_A is transitive NZM iff $\Diamond\, S_A$ is transitive NZM .

It can be proved easily.

3.12 Proposition:

S_A is idempotent NZM if and only if $W\, S_A$ is idempotent NZM .

Proof:

Since S_A is idempotent

$$\Leftrightarrow S_A = S_A^2$$

$$\Leftrightarrow \left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >\right)$$

$$= (< (\sum_{i=1}^{n} \left(T^A_{V_{ik}}. T^A_{V_{kj}}\right), (\sum_{i=1}^{n} \left(T^A_{R_{ik}}. T^A_{R_{kj}}\right)), (\prod_{i=1}^{n} (I^A_{V_{ik}} + I^A_{V_{kj}})), (\prod_{i=1}^{n} (I^A_{R_{ik}} + I^A_{R_{kj}})),$$

$$\left(1 - \sum_{i=1}^{n} \left(T^A_{V_{ik}}. T^A_{V_{kj}}\right)\right), (1 - \sum_{i=1}^{n} \left(T^A_{R_{ik}}. T^A_{R_{kj}}\right)) >) \quad \forall\, i, j$$

$$\Leftrightarrow \left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(1 - T^A_{V_{ij}}, 1 - T^A_{R_{ij}}\right) >\right) =$$

$$(< (\sum_{i=1}^{n} \left(T^A_{V_{ik}}. T^A_{V_{kj}}\right), (\sum_{i=1}^{n} \left(T^A_{R_{ik}}. T^A_{R_{kj}}\right)), (\prod_{i=1}^{n} (I^A_{V_{ik}} + I^A_{V_{kj}})), (\prod_{i=1}^{n} (I^A_{V_{ik}} + I^A_{V_{kj}})), (\prod_{i=1}^{n} \left(1 - T^A_{V_{ik}}\right). (1 - T^A_{V_{kj}})), (\prod_{i=1}^{n} \left(1 - T^A_{R_{ik}}\right). (1 - T^A_{R_{kj}})) >)$$

$$\Leftrightarrow W\, S_A = (W\, S_A)^2$$

Hence the proof.

3.13 Proposition

S_A is idempotent NZM if and only if $\Diamond\, S_A$ is idempotent NZM .

It can be derived easily

Note:

If S_A is an neutrosophic z- equivalent matrix then WS_A and $\Diamond\, S_A$ are also neutrosophic z-equivalent matrices.

3.14 Definition

Let $S_A \in$ NZM then the transitive closure and C-transitive closure of S_A is defined by

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$S_A^\infty = S_A \vee S_A^2 \vee S_A^3 \vee \ldots \ldots . . \vee S_A^n$

And $S_{A_\infty} = S_A^c \wedge (S_A^c)^{[2]} \wedge (S_A^c)^{[3]} \wedge \ldots \ldots \wedge (S_A^c)^{[n]}$

3.15 Proposition

For $S_A \in \mathrm{NZM}_{n \times n}$ then $S_{A_\infty} = (S_A^\infty)^c$.

Proof:

$(S_A^\infty)^c = (S_A \vee S_A^2 \vee S_A^3 \vee \ldots \ldots . . \vee S_A^n)^c$

$\quad = (S_A^c \wedge (S_A^2)^c \wedge (S_A^3)^c \wedge \ldots \ldots . . \wedge (S_A^n)^c)$

First we prove

$(S_A^2)^c = (S_A^c)^{[2]}$

We know that

$\Leftrightarrow S_A = S_A^2$

$\Leftrightarrow (S_A^2)^c = (< \left(\prod_{i=1}^n (F_{V_{ik}}^A + F_{V_{kj}}^A)\right), (\prod_{i=1}^n (F_{R_{ik}}^A + F_{R_{kj}}^A)), 1 - \prod_{i=1}^n (I_{V_{ik}}^A + I_{V_{kj}}^A)), 1 - \prod_{i=1}^n (I_{R_{ik}}^A + I_{R_{kj}}^A)), \left(\sum_{i=1}^n \left(T_{V_{ik}}^A . T_{V_{kj}}^A\right), \sum_{i=1}^n \left(T_{R_{ik}}^A . T_{R_{kj}}^A\right)\right) >)$ …..(11)

Also, ${S_A}^c = (< \left(F_{V_{ij}}^A, F_{R_{ij}}^A\right), \left(1 - I_{V_{ij}}^A, 1 - I_{R_{ij}}^A\right), \left(T_{V_{ij}}^A, T_{R_{ij}}^A\right) >)$

Then

$(S_A^c)^{[2]} = (< \left(\prod_{i=1}^n (F_{V_{ik}}^A + F_{V_{kj}}^A)\right), (\prod_{i=1}^n (F_{R_{ik}}^A + F_{R_{kj}}^A)), 1 - \prod_{i=1}^n (I_{V_{ik}}^A + I_{V_{kj}}^A)), 1 - \prod_{i=1}^n (I_{R_{ik}}^A + I_{R_{kj}}^A)), \left(\sum_{i=1}^n \left(T_{V_{ik}}^A . T_{V_{kj}}^A\right), \sum_{i=1}^n \left(T_{R_{ik}}^A . T_{R_{kj}}^A\right)\right) >)$ …………..(12)

Thus by (11) and (12)

$(S_A^2)^c = (S_A^c)^{[2]}$

Generalizing we get

$(S_A^n)^c = (S_A^c)^{[n]}$

$\therefore$ By definition

$(S_A^\infty)^c = (S_A \vee S_A^2 \vee S_A^3 \vee \ldots \ldots . . \vee S_A^n)^c$

$\quad = (S_A^c \wedge (S_A^2)^c \wedge (S_A^3)^c \wedge \ldots \ldots . . \wedge (S_A^n)^c)$

$\quad = ((S_A^c) \wedge (S_A^c)^{[2]} \wedge (S_A^c)^{[3]} \wedge \ldots \ldots . . \wedge \qquad = (S_A^c)^\infty$

Hence the proof.

3.16 Proposition

S_A is transitive if and only if ${S_A}^c$ is C- transitive and so $W\ {S_A}^c$ is transitive.

It can be proved using definition

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3.17 Proposition

Let$S_A, S_B \in NZM_n$ be a reflexive of NZM. Then the following holds

I. $S_A \vee S_B$ is reflexive

ii. $S_A \wedge S_B$ is reflexive if and only if S_B is reflexive.

Pro

(i) It can be proved using definition of reflexive.

(ii) If S_B is reflexive, then

$\left(< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >\right) \neq <(1,1),(0,0),(0,0)>$ for atleast one i

$\left(< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >\right) < (<(1,1),(0,0),(0,0)>)$

$\left(< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >\right) \wedge \left(< \left(T^B_{V_{ij}}, T^B_{R_{ij}}\right), \left(I^B_{V_{ij}}, I^B_{R_{ij}}\right), \left(F^B_{V_{ij}}, F^B_{R_{ij}}\right) >\right) << (1,1),(0,0),(0,0) >)$

$\therefore$ condition S_B is reflexive is necessary.

Sufficient part can be proved easily.

Hence the proof.

3.18 Proposition

If $S_A, S_B \in NZM_n$ where S_A, S_B are reflexive, symmetric and transitive and $S_A \leq S_B$then ${S_A}^{\infty} \leq S_B$

Proof:

Let S_A, S_B are reflexive, symmetric and transitive and $S_A \leq S_B$, then

$$S_A \circ S_B = (<(\sum_{i=1}^{n}\left(T^A_{V_{ij}}.T^B_{ij}\right), (\sum_{i=1}^{n}\left(T^A_{R_{ii}}.T^B_{R_{ij}}\right)), (\prod_{i=1}^{n}(I^A_{V_{ij}} + I^B_{V_{ij}})), (\prod_{i=1}^{n}(I^A_{R_{ij}} + I^B_{R_{ij}})), (\prod_{i=1}^{n}(F^A_{V_{ij}} + F^B_{V_{ij}})), (\prod_{i=1}^{n}(F^A_{R_{ij}} + F^B_{R_{ij}})) >) \quad \forall\, i,j$$

And each

$$(\sum_{k=1}^{n}\left(T^A_{V_{ik}}.T^B_{V_{kj}}\right), (\sum_{k=1}^{n}\left(T^A_{R_{ik}}.T^B_{R_{kj}}\right)), (\prod_{k=1}^{n}(I^A_{V_{ik}} + I^B_{V_{kj}})), (\prod_{k=1}^{n}(I^A_{R_{ik}} + I^B_{R_{kj}})), (\prod_{k=1}^{n}(F^A_{V_{ik}} + F^B_{V_{kj}})), (\prod_{k=1}^{n}(F^A_{R_{ik}} + F^B_{R_{kj}})) >)$$

$$= \begin{cases} <(1,1),(0,0),(0,0)> & if\ i = j \\ S_B & if\ i \neq j \end{cases}$$

thus $S_A S_B = S_B \Rightarrow S_A S_A \leq S_A S_B = S_B$

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That is ${S_A}^2 \leq S_B$, Computing same way we get

${S_A}^3 \leq S_B, \ldots\ldots.. S_A^k \leq S_B$

And also, $S_A \vee S_A^2 \vee S_A^3 \vee \ldots..\vee S_A^n \leq S_B$

And hence ${S_A}^\infty \leq S_B$

Hence the proof

3.19 Proposition

If S_A^∞ is the transitive closure of S_A then the transitive closure of WS_A is WS_A^∞

Proof

Now,

$$\begin{aligned} W\,S_A^\infty &= W[S_A \vee S_A^2 \vee S_A^3 \vee \ldots\ldots.. \vee S_A^n] \\ &= WS_A \vee W\,S_A^2 \vee WS_A^3 \vee \ldots\ldots.. \vee WS_A^n \\ &= WS_A \vee (WS_A)^2 \vee (WS_A)^3 \vee \ldots\ldots.. \vee (WS_A)^n \\ &= (WS_A)^\infty \end{aligned}$$

Hence the proof

Note:

The following results are true for transitive closure

- $WS_{A_\infty} = (WS_A)_\infty$
- $\Diamond\, S_A^\infty = (\Diamond S_A)^\infty$
- $\Diamond\, S_{A_\infty} = (\Diamond S_A)_\infty$

4 More properties on nS and S^n operator

The nS and S^n operator is defined where n is a scalar in NZM. Some properties and results for the neutrosophic z-matrix are investigated using the definition.

4.1 Definition

Let $S_A = (< \left(T_{V_{ij}}^A, T_{R_{ij}}^A\right), \left(I_{V_{ij}}^A, I_{R_{ij}}^A\right), \left(F_{V_{ij}}^A, F_{R_{ij}}^A\right) >)$ be NZMs of order $m \times n$,

Here n > 0 is a scalar then

$$n(S_A) = \left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^n\right), (\left(I_{V_{ij}}{}^A\right)^n, \left(I_{R_{ij}}{}^A\right)^n), \left(\left(F_{V_{ij}}{}^A\right)^n, \left(F_{R_{ij}}{}^A\right)^n\right) >\right)$$

$$(S_A)^n = \left(< \left(\left(T_{V_{ij}}{}^A\right)^n\right), \left(\left(T_{R_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - I_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - I_{V_{ij}}{}^A\right)^n\right)\right), \left(\left(1 - \left(1 - F_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - F_{R_{ij}}{}^A\right)^n\right)\right) >)$$

4.2 Proposition

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Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{m\times n}$ if n >0 are integer then

i) $n(S_A \oplus S_B) = nS_A \oplus nS_B$

ii) $n(S_A \otimes S_B) = nS_A \otimes nS_B$

Proof:

i)Let $n(S_A \oplus S_B) =$

$$n(<\left(T^A_{V_{ij}} + T^B_{V_{ij}} - T^A_{V_{ij}}.T^B_{V_{ij}}, T^A_{R_{ij}} + T^B_{R_{ij}} - T^A_{R_{ij}}\,T^B_{R_{ij}}\right), \left(I^A_{V_{ij}}.I^B_{V_{ij}}, I^A_{R_{ij}}.I^B_{R_{ij}}\right), \left(F^A_{V_{ij}}.F^B_{V_{ij}}, F^A_{R_{ij}}.F^B_{R_{ij}}\right) >)$$

$$=(< (1 - [(1 - T^A_{V_{ij}}) + (1 - T^B_{V_{ij}}) - (1 - T^A_{V_{ij}}).(1 - T^B_{V_{ij}})]^n, \left(1 - [(1 - T^A_{R_{ij}}) + (1 - T^B_{R_{ij}}) - (1 - T^A_{R_{ij}}).(1 - T^B_{R_{ij}})]^n, \left(I^A_{V_{ij}}.I^B_{V_{ij}}, I^A_{R_{ij}}.I^B_{R_{ij}}\right)^n, \left(F^A_{V_{ij}}.F^B_{V_{ij}}, F^A_{R_{ij}}.F^B_{R_{ij}}\right)^n >\right) \quad \ldots\ldots\ldots\ldots(13)$$

$n(S_A) \oplus n(S_B) =$

$$\left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^n\right), (\left(I_{V_{ij}}{}^A\right)^n, \left(I_{R_{ij}}{}^A\right)^n), \quad \left(\left(F_{V_{ij}}{}^A\right)^n, \left(F_{R_{ij}}{}^A\right)^n\right) >\right) \oplus$$

$$\left(< \left(1 - \left(1 - T_{V_{ij}}{}^B\right)^n\right), \left(1 - \left(1 - T_{R_{ij}}{}^B\right)^n\right), (\left(I_{V_{ij}}{}^B\right)^n, \left(I_{R_{ij}}{}^B\right)^n), \left(\left(F_{V_{ij}}{}^B\right)^n, \left(F_{R_{ij}}{}^B\right)^n\right) >\right)$$

$= n(S_A \oplus S_B)$

second part can be proved obviously.

Hence the proof.

4.3 Proposition

For any $S_A \in \mathrm{NZM}_{m\times n}$ if n_1, n_2 >0 are integers then

i.) $n_1 S_A \oplus n_2 S_A = (n_1 + n_2) S_A$

ii.) $S_A^{n_1} \otimes S_A^{n_2} = S_A^{(n_1+n_2)}$

Proof:

i) $n_1 S_A \oplus n_2 S_A = \left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^{n_1}\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^{n_1}\right), (\left(I_{V_{ij}}{}^A\right)^{n_1}, \left(I_{R_{ij}}{}^A\right)^{n_1}),\right.$

$\left(\left(F_{V_{ij}}{}^A\right)^{n_1}, \left(F_{R_{ij}}{}^A\right)^{n_1}\right) >) \oplus \left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^{n_2}\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^{n_2}\right), (\left(I_{V_{ij}}{}^A\right)^{n_2}, \left(I_{R_{ij}}{}^A\right)^{n_2}),\right.$

$\left(\left(F_{V_{ij}}{}^A\right)^{n_2}, \left(F_{R_{ij}}{}^A\right)^{n_2}\right) >)$

Then,

$$=\left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^{(n_1+n_2)}\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^{(n_1+n_2)}\right), (\left(I_{V_{ij}}{}^A\right)^{(n_1+n_2)}, \left(I_{R_{ij}}{}^A\right)^{(n_1+n_2)}),\right.$$

$$\left(\left(F_{V_{ij}}{}^A\right)^{(n_1+n_2)}, \left(F_{R_{ij}}{}^A\right)^{(n_1+n_2)}\right) >)$$

$$=\left(< \left(T_{V_{ij}}{}^A)^{(n_1+n_2)}\right), \left(\left(T_{R_{ij}}{}^A\right)^{(n_1+n_2)}\right), (\left(I_{V_{ij}}{}^A\right)^{(n_1+n_2)}, \left(I_{R_{ij}}{}^A\right)^{(n_1+n_2)}),\right.$$

$$\left(\left(F_{V_{ij}}{}^A\right)^{(n_1+n_2)}, \left(F_{R_{ij}}{}^A\right)^{(n_1+n_2)}\right) >)$$

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$= (n_1 + n_2)S_A$

second part can be proved obviously.

Hence the proof.

4.4 Proposition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{m\times n}$ if n >0 are integer then

i) $n(S_A \cup S_B) = nS_A \cup nS_B$

ii) $n(S_A \cap S_B) = nS_A \cap nS_B$

The above property easily holds using union and intersection definition.

4.5 Proposition

Let S_A be a $NZM_{m\times n}$ if n >0 are integer then

i .) $(S_A^c)^n = (nS_A)^c$

ii.)$n(S_A^c) = (S_A^n)^c$

Proof:

i.) $(S_A^c)^n = (<\left(F_{V_{ij}}^A, F_{R_{ij}}^A\right), \left(1 - I_{V_{ij}}^A, 1 - I_{R_{ij}}^A\right), \left(T_{V_{ij}}^A, T_{R_{ij}}^A\right) >)^n$

$= (<\left(F_{V_{ij}}^A, F_{R_{ij}}^A\right)^n, \left(1 - I_{V_{ij}}^A, 1 - I_{R_{ij}}^A\right)^n, \left(1 - \left(1 - T_{V_{ij}}^A\right)^n, (1 - (1 - T_{R_{ij}}^A\right)^n) >)$

…………(14)

$n(S_A) = \left(< \left(1 - \left(1 - T_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^n\right), (\left(I_{V_{ij}}{}^A\right)^n, \left(I_{R_{ij}}{}^A\right)^n),\right.$
$\left.\left(\left(F_{V_{ij}}{}^A\right)^n, \left(F_{R_{ij}}{}^A\right)^n\right) >\right.$

$(nS_A)^c = \left(< \left(\left(F_{V_{ij}}{}^A\right)^n, \left(F_{R_{ij}}{}^A\right)^n\right), (\left(I_{V_{ij}}{}^A\right)^n, \left(I_{R_{ij}}{}^A\right)^n),\right.$
$\left.\left(1 - \left(1 - T_{V_{ij}}{}^A\right)^n\right), \left(1 - \left(1 - T_{R_{ij}}{}^A\right)^n\right) >\right.$ …………(15)

From (14) and (15) holds. Similarly we can prove next part.

Hence the proof.

5 Applications of NZM for Decision Making Process

5.1 Choice Matrix for a NZM:

We attempt to use a choice matrix to forecast improved decision-making. Two different kinds of algorithms in a choice matrix will be covered in this part, along with some fundamental calculation formulas.Let $S_{CHM} \in NZM_{m\times n}$ then the choice matrix is represented as

$$S_{CHM} = \begin{cases} < (1,1), (0,0), (0,0) > & if\ i\ and\ j\ both\ parameters\ are\ the\ choice\ paramters \\ < (0,0), (1,1), (1,1) > & if\ atleast\ one\ \ parameter\ is\ not\ a\ choice\ paramters \end{cases}$$

…….(16)

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Arithmetic mean $A.M\left(H_j\right) = \sum_{j=1}^{m}(<(\frac{T_{V_j}}{m}, \frac{T_{R_j}}{m}) + \left(\frac{I_{V_j}}{m}, \frac{I_{R_j}}{m}\right) + \left(\frac{F_{V_j}}{m}, \frac{F_{R_j}}{m}\right) >)$

…….(17)

Accuracy value

$A.V\left(H_i\right) = \frac{1}{6n}\sum_{i=1}^{n}(< T_{V_i} + T_{R_i} + \left(1 - I_{V_i}\right) + \left(1 - I_{R_i}\right) + F_{V_i} + F_{R_i} >)$…..(18)

Cardinal value $C(H_i) = \sum_{i=1}^{n}(A.V(H_i))^2$ ……..(19)

Type I : Algorithm for non-normalized choice matrix :

1. Construct a NZM from the NZN set.
2. The choice matrix of NZM were created according to the standard requirement of an decision maker.
3. Using max-min form calculate $H * S_{CHM}$
4. Find the accuracy value of the matrix using formula by row wise.
5. Select the greatest accuracy value of the given NZM.

Type II : Algorithm for normalized choice matrix :

1. Construct a NZM from the NZN set.
2. The choice matrix of NZM were created according to the standard requirement of decision maker.
3. Find average mean $A.M(H_j)$ and calculate accuracy value for each mean $A.V\ (H_i)$
   Now find cardinal value $C\ (H_i)$ of the set using accuracy value.
4. Multiply cardinal value with NZM using scalar multiplication $N_A = C\ (H_i) * H$
   where N_A is a normalized form of H.
5. Using max-min form calculate $N_A * S_{CHM}$
6. Find the accuracy value of the matrix using formula.
7. Select the greatest accuracy value of the given NZM.

5.2 Case study

Let $E = \{E_1, E_2, E_3, E_4\}$ be a neutrosophic z-set .Assume there are four houses where $E = \{E_1, E_2, E_3, E_4\}$ whose credit are assessed by means of component $F = \{F_1, F_2, F_3\}$ where F_1= degree of elegance of house, F_2 = the degree of furniture and F_3 = degree of cost for enhancement. Suppose a person select the top-notch house considering the capabilities.

An example for decision-making problem has been implemented step by step using the proposed technique.

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Example 1:

A neutrosophic z- set is given by

$H\ (F_1) = H_1 = \{\ \big(E_1, (0.4,0.2), (0.6,0.5), (0.4,0.3)\big), \big(E_2, (0.8,0.8), (0.5,0.3), (0.6,0.3)\big),$
$\big(E_3, (0.9,0.7), (0.6,0.3), (0.5,0.3)\big), (E_4, (0.5,0.3), (0.6,0.2), (0.6,0.3))\}$

$H\ (F_2) = H_2 = \{\ \big(E_1, (0.3,0.6), (0.5,0.8), (0.7,0.8)\big), \big(E_2, (0.4,0.5), (0.6,0.7), (0.8,0.3)\big),$
$\big(E_3, (0.6,0.7), (0.8,0.5), (0.7,0.6)\big), (E_4, (0.1,0.2), (0.2,0.3), (0.3,0.1))\}$

$H\ (F_3) = H_2 = \{\ \big(E_1, (0.4,0.2), (0.3,0.2), (0.3,0.2)\big), \big(E_2, (0.5,0.5), (0.3,0.2), (0.3,0.1)\big),$
$\big(E_3, (0.6,0.7), (0.3,0.1), (0.3,0.1)\big), (E_4, (0.5,0.3), (0.3,0.4), (0.2,0.1))\}$

5.3 Type I : Using non-normalized choice matrix

The matrix representation for the above is given as

$$H = \begin{bmatrix} <(0.4,0.2),(0.6,0.5),(0.4,0.5)> & <(0.3,0.6),(0.5,0.8),(0.7,0.8)> & <(0.4,0.2),(0.3,0.2),(0.3,0.2)> \\ <(0.8,0.8),(0.5,0.3),(0.6,0.3)> & <(0.4,0.5),(0.6,0.7),(0.8,0.8)> & <(0.5,0.5),(0.7,0.2),(0.3,0.1)> \\ <(0.9,0.7),(0.6,0.8),(0.5,0.3)> & <(0.6,0.7),(0.8,0.5),(0.7,0.6)> & <(0.6,0.7),(0.3,0.1),(0.3,0.1)> \\ <(0.5,0.3),(0.6,0.2),(0.6,0.3)> & <(0.1,0.2),(0.2,0.3),(0.3,0.1)> & <(0.5,0.3),(0.3,0.4),(0.2,0.1)> \end{bmatrix}$$

We choose a choice matrix as $NZM_{3\times4}$ since H is $NZM_{4\times3}$

$$S_{CHM} = \begin{bmatrix} <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \end{bmatrix}$$

Use max-min form to calculate $H * S_{CHM}$

$$H * S_{CHM} = \begin{bmatrix} <(0.4,0.6),(0.3,0.2),(0.3,0.2)> & <(0.4,0.6),(0.3,0.2),(0.3,0.2)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.8,0.8),(0.3,0.2),(0.3,0.1)> & <(0.8,0.8),(0.3,0.2),(0.3,0.1)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.9,0.7),(0.3,0.1),(0.3,0.1)> & <(0.9,0.7),(0.3,0.1),(0.3,0.1)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.5,0.3),(0.6,0.2),(0.6,0.2)> & <(0.5,0.3),(0.6,0.2),(0.6,0.2)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \end{bmatrix}$$

Using Accuracy formula (18) find

$$A.V(H_i) = \begin{bmatrix} E_1 & 0.4166 \\ E_2 & 0.4583 \\ E_3 & 0.4666 \\ E_4 & 0.40833 \end{bmatrix}$$

Here E_3 gains highest accuracy value of 0.4666.Then E_3 is the best house to be selected.

5.4 Type II : Using normalized choice matrix (Example 1)

The matrix representation for the above is given as

$$H = \begin{bmatrix} <(0.4,0.2),(0.6,0.5),(0.4,0.5)> & <(0.3,0.6),(0.5,0.8),(0.7,0.8)> & <(0.4,0.2),(0.3,0.2),(0.3,0.2)> \\ <(0.8,0.8),(0.5,0.3),(0.6,0.3)> & <(0.4,0.5),(0.6,0.7),(0.8,0.8)> & <(0.5,0.5),(0.7,0.2),(0.3,0.1)> \\ <(0.9,0.7),(0.6,0.8),(0.5,0.3)> & <(0.6,0.7),(0.8,0.5),(0.7,0.6)> & <(0.6,0.7),(0.3,0.1),(0.3,0.1)> \\ <(0.5,0.3),(0.6,0.2),(0.6,0.3)> & <(0.1,0.2),(0.2,0.3),(0.3,0.1)> & <(0.5,0.3),(0.3,0.4),(0.2,0.1)> \end{bmatrix}$$

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We choose a choice matrix as $NZM_{3\times 4}$ since H is $NZM_{4\times 3}$

$$S_{CHM} = \begin{bmatrix} <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(1,1),(0,0),(0,0)> & <(1,1),(0,0),(0,0)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \end{bmatrix}$$

Using the above H matrix representation calculate arithmetic mean

$A.M\ (H_1) = <(0.65,0.5),(0.575,0.45),(0.6,0.35)>$

$A.M\ (H_2) = <(0.35,0.5),(0.525,0.575),(0.625,0.575)>$

$A.M\ (H_3) = <(0.5,0.425),(0.4,0.225),(0.275,0.125)>$

Calculate accuracy value for each mean $A.V\ (H_i)$

$$A.V\ (H_1) = \frac{1}{6}\,(0.65 + 0.5 + (1.0575) + (1 - 0.45) + 0.6 + 0.35) = 0.8$$

Proceeding in this way, we get

$A.V\ (H_2) = 0.49166$ and $A.V\ (H_3) = 0.45$

Then find cardinal value $C\ (H_i)$

$C(H_i) = 0.8^2 + 049166^2 + 0.45^2 = 0.3614$

Calculate normalized value $N_A = C\ (H_i) * H$. Here $C\ (H_i)$ is a scalar value so we use scalar multiplication

$$N_A = \begin{bmatrix} <(0.169,0.077),(0.831,0.778),(0.718,0.778)> & <(0.121,0.282),(0.778,0.923),(0.879,0.923)> & <(0.168,0.077),(0.647,0.559),(0647,0.559)> \\ <(0.441,0.441),(0.778,0.647),(0.836,0.647)> & <(0.169,0.223),(0.831,0.879),(0.923,0.923)> & <(0.223,0.223),(0.879,0.559),(0647,0.435)> \\ <(0.565,0.353),(0.831,0.923),(0.778,0.647)> & <(0.282,0.353),(0.923,0.778),(0.879,0.831)> & <(0.282,0.353),(0.647,0.435),(0647,0.435)> \\ <(0.221,0.121),(0.831,0.559),(0.6831,0.647)> & <(0.031,0.077),(0.559,0.647),(0647,0.435)> & <(0.221,0.12),(0.647,0.718),(0.559,0.435)> \end{bmatrix}$$

Use max-min form to calculate $N_A * S_{CHM}$

$$N_A * S_{CHM} = \begin{bmatrix} <(0.169,0.282),(0.647,0.2559),(0.647,0.559)> & <(0.169,0.282),(0.647,0.2559),(0.647,0.559)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.441,0.441),(0.778,0.559),(0.647,0.435)> & <(0.441,0.441),(0.778,0.559),(0.647,0.435)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.565,0.353),(0.647,0.435),(0.647,0.435)> & <(0.565,0.353),(0.647,0.435),(0.647,0.435)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \\ <(0.221,0.121),(0.559,0.559),(0.559,0.437)> & <(0.221,0.121),(0.559,0.559),(0.559,0.437)> & <(0,0),(1,1),(1,1)> & <(0,0),(1,1),(1,1)> \end{bmatrix}$$

Using Accuracy formula (18) find

$$A.V(H_i) = \begin{bmatrix} E_1 & 0.3709 \\ E_2 & 0.386 \\ E_3 & 0.410 \\ E_4 & 0.352 \end{bmatrix}$$

Here E_3 gains highest accuracy value of 0.410. Then E_3 is the best house to be selected.

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5.6 An application to Dimensionality Reduction using NZM Matrices:

The dimensionality reduction technique is essential for decision-making to be simple and broadly applicable. We propose a technique to calculate the threshold value for the data in the form of a neutrosophic z-matrix. Additionally, we introduce a new approach for the dimensionality reduction process that makes use of both the parameter-oriented and object-oriented neutrosophic z-matrix. Furthermore,

Object oriented NZM with respect to parameter is

$O_p = [< (\sum_q \frac{T_{V_q}}{|\mathrm{F}|}, \sum_q \frac{T_{R_q}}{|\mathrm{F}|}, \sum_q \frac{I_{V_q}}{|\mathrm{F}|}, \sum_q \frac{I_{R_q}}{|\mathrm{F}|}, \sum_q \frac{F_{V_q}}{|\mathrm{F}|}, \sum_q \frac{F_{R_q}}{|\mathrm{F}|})>]$, and q varies from 1 to n , p varies from 1 to m …………………(20)

Where |.| denotes the cardinality of the system.

Parameter oriented NZM with respect to object is

$F_q = [< (\sum_p \frac{T_{V_p}}{|\mathrm{E}|}, \sum_p \frac{T_{R_p}}{|\mathrm{E}|}, \sum_p \frac{I_{V_p}}{|\mathrm{E}|}, \sum_p \frac{I_{R_p}}{|\mathrm{E}|}, \sum_p \frac{F_{V_p}}{|\mathrm{E}|}, \sum_p \frac{F_{R_p}}{|\mathrm{E}|})>]$ and q varies from 1 to n , p varies from 1 to m ………………(21)

5.7 Score Matrix

$$SC(M) = SC(M_{pq}) = \frac{\left[2 + T_{V_{pq}}.T_{R_{pq}} - I_{V_{pq}}.I_{R_{pq}} - F_{V_{pq}}.F_{R_{pq}}\right]}{3} \quad \forall \quad p \,\&\, q \qquad \text{………..(22)}$$

5.8 Threshold Value

$$SC(T) = S(T_{pq}) = \frac{\left[2 + T_{V_{pq}}{}^{M}.T_{R_{pq}}{}^{M} - I_{V_{pq}}{}^{M}.I_{R_{pq}}{}^{M} - F_{V_{pq}}{}^{M}.F_{R_{pq}}{}^{M}\right]}{3} \quad \forall \quad p \,\&\, q \qquad \text{……..(23)}$$

Then

$$T = [< (\sum_{pq} \frac{T_{V_{pq}}}{|\mathrm{E} \times \mathrm{F}|}, \sum_{pq} \frac{T_{R_{pq}}}{|\mathrm{E} \times \mathrm{F}|}, \sum_{pq} \frac{I_{V_{pq}}}{|\mathrm{E} \times \mathrm{F}|}, \sum_{pq} \frac{I_{R_{pq}}}{|\mathrm{E} \times \mathrm{F}|}, \sum_{pq} \frac{F_{V_{pq}}}{|\mathrm{E} \times \mathrm{F}|}, \sum_{pq} \frac{F_{R_{pq}}}{|\mathrm{E} \times \mathrm{F}|})>]$$

$where$ q varies from 1 to n , p varies from 1 to m ……………(24)

5.9 Algorithm for Dimensionality Reduction using NZM

1. Construct the neutrosophic z-matrix of (F, S_z).

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2.Determine the parameter-oriented matrix for the parameter F_q and the object-oriented matrix for the object O_p. Next, use the equation to calculate the score matrix.

3.Using the suggested equation, determine the NZM's threshold value and threshold element.

4. Delete the objects for which $SC(O_p) < SC(T)$ and those parameters for which $SC(\ F_q) > SC(T)$.

5. The new NZM with higher value is the desired dimensionality reduction.

5.10 Dimensionality Reduction using NZM (Example 1)

Step 1: Built a NZM from neutrosophic z- set

Step 2: Using the matrix representation for the above calculate F_q and O_p. Then using score value find $SC(\ F_q)$ and $SC(\ O_p)$

$$H = \begin{bmatrix} <(0.4,0.2),(0.6,0.5),(0.4,0.5)> & <(0.3,0.6),(0.5,0.8),(0.7,0.8)> & <(0.4,0.2),(0.3,0.2),(0.3,0.2)> \\ <(0.8,0.8),(0.5,0.3),(0.6,0.3)> & <(0.4,0.5),(0.6,0.7),(0.8,0.8)> & <(0.5,0.5),(0.7,0.2),(0.3,0.1)> \\ <(0.9,0.7),(0.6,0.8),(0.5,0.3)> & <(0.6,0.7),(0.8,0.5),(0.7,0.6)> & <(0.6,0.7),(0.3,0.1),(0.3,0.1)> \\ <(0.5,0.3),(0.6,0.2),(0.6,0.3)> & <(0.1,0.2),(0.2,0.3),(0.3,0.1)> & <(0.5,0.3),(0.3,0.4),(0.2,0.1)> \end{bmatrix}$$

$$SC\ (F_q) = (<(0.6514, 0.530625, 0.73054>)$$

and

$$H = \begin{bmatrix} <(0.4,0.2),(0.6,0.5),(0.4,0.5)> & <(0.3,0.6),(0.5,0.8),(0.7,0.8)> & <(0.4,0.2),(0.3,0.2),(0.3,0.2)> \\ <(0.8,0.8),(0.5,0.3),(0.6,0.3)> & <(0.4,0.5),(0.6,0.7),(0.8,0.8)> & <(0.5,0.5),(0.7,0.2),(0.3,0.1)> \\ <(0.9,0.7),(0.6,0.8),(0.5,0.3)> & <(0.6,0.7),(0.8,0.5),(0.7,0.6)> & <(0.6,0.7),(0.3,0.1),(0.3,0.1)> \\ <(0.5,0.3),(0.6,0.2),(0.6,0.3)> & <(0.1,0.2),(0.2,0.3),(0.3,0.1)> & <(0.5,0.3),(0.3,0.4),(0.2,0.1)> \end{bmatrix}$$

$$S(O_p) = \begin{bmatrix} 0.55195 \\ 0.673774 \\ 0.7179 \\ 0.6422 \end{bmatrix}$$

Step 3: Evaluate threshold value using definition 5.2

T= <(0.475,0.475), (0.466,0.3666),(0.4175,0.308)> using score value find SC (T)

Then SC (T) = <0.63625>

Step 4:

Delete the objects for which $SC(O_p) < SC(T)$and those parameters for which $SC(\ F_q) > SC(T)$.. Here we remove E_1, F_1 and F_3. The resultant matrices is

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$$F_2 = \begin{bmatrix} E_2 & 0.673774 \\ E_3 & 0.7179 \\ E_4 & 0.6422 \end{bmatrix}$$

Finally, here E_3 is 0.7179 is the highest value so it is awesome to choose a house by the greater dynamic.

5.11 Comparative Analysis:

The comparative remarks from the proposed technique for decision-making.

- ✧ Several scholars have addressed numerous reduction algorithm concerns when discussing the decision-making problem.
- ✧ Using both normalized and non-normalized forms, we get the matrix accuracy value. The matrix greatest value will indicate the optimal approach for making decisions.
- ✧ The new dimensionality reduction process is put forth, and we establish a benchmark value by determining the score value of the threshold value. We compute the most approximate value using the threshold value.
- ✧ The suggested methods are more reliable in identifying the right choice. All the techniques have claimed E_3 as best selection of house by dynamic.

6 Conclusion

First, this paper established the decomposition operators of NZM and provided some basic definitions of neutrosophic z-matrices. The characteristics and certain findings of NZM have been suggested. Additionally, we introduced the idea of scalar multiplication, reviewed its power value in NZM, and analyzed some of its features. Furthermore, we develop a decision strategy technique that uses choice matrices for both normalized and non-normalized forms. A novel approach with a dimensionality reduction algorithm was then put forth. In order to have a more comprehensive understanding of the suggested algorithms, we conclude by comparing the outcomes via a numerical example. In future, we can extend idempotent NZM to k-idempotent NZM and some properties can be extended for modal operators. The dimensionality reduction can be done for further cases like intrusion detection, pattern recognition, medical diagnostics, robotics selection for MCDM.

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