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ITO-ADAM OPTIMIZER FOR TRAINING LSTM NETWORKS: APPLICATION TO STOCK PRICE FORECASTING

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Abstract

This study introduces ItoAdam, an enhanced optimizer that incorporates Itô's lemma and Brownian noise into Adam to improve convergence in complex, non-convex settings. By perturbing gradient updates with stochastic noise, ItoAdam enables better exploration of the loss surface and avoids poor local minima. We apply ItoAdam to train Long Short-Term Memory (LSTM) networks for stock price forecasting across 13 major companies sourced from Yahoo Finance. Experiments show that ItoAdam-LSTM consistently outperforms Adam-LSTM across RMSE, MAE, and R^2, with sensitivity analysis indicating that optimal performance occurs when noise standard deviation lies between 2.1×10^{-4} and 2.9×10^{-4}. Overall, results demonstrate the effectiveness of combining Itô-driven stochastic optimization, evolutionary hyperparameter tuning, and recurrent architectures for reliable financial time series forecasting in noisy and nonstationary environments.

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1. Introduction

Optimization in deep learning presents persistent challenges due to non-convex loss surfaces, the presence of numerous local minima, saddle points, and the risk of overfitting. Adaptive gradient methods such as Adam [1] have become widely adopted for their efficiency and stability. However, their deterministic update rules often limit exploratory behavior in highly irregular or noisy objective landscapes, leading to premature convergence and suboptimal generalization [2,3,4].

A promising line of research seeks to address these limitations by incorporating stochastic perturbations inspired by stochastic differential equations (SDEs). In particular, Itô calculus—originally developed to handle the mathematical irregularities of Brownian motion—extends classical calculus to stochastic processes through Itô's lemma [5,6] This framework enables rigorous modeling of dynamic systems under uncertainty and has seen successful applications in domains such as finance, physics, biology, and control theory [7].

In the context of machine learning, recent studies have explored the benefits of injecting gradient noise to emulate SDE dynamics, encouraging broader exploration of the loss landscape and potentially improving both convergence and generalization [8]. Building upon this foundation, this work introduces a novel stochastic optimization algorithm, referred to as **ItoAdam**. This method integrates Gaussian noise into the gradient updates, imitating the diffusion term from stochastic calculus and allowing the optimizer to escape sharp or narrow local minima more effectively.

The proposed approach is applied to train Long Short-Term Memory (LSTM) neural networks [9] for stock price forecasting, a task inherently affected by stochastic volatility and noise. To strengthen the empirical robustness and theoretical foundation, the study presents the following key contributions:

- Integration of ItoAdam into LSTM architectures for financial time series modeling, enabling probabilistic learning dynamics aligned with noisy data environments.
- Empirical evaluation on historical data from 13 major publicly traded companies sourced from Yahoo Finance [10,11,12], showing that **ItoAdam-LSTM** consistently outperforms standard Adam-based models.
- Comprehensive analysis using **quantitative metrics** (RMSE, MAE, R^2) and **visual diagnostics** to assess forecasting accuracy and interpretability.

By bridging the theoretical tools of stochastic calculus with the practical demands of modern deep learning, this research demonstrates that noise-aware optimization based on Itô dynamics can lead to statistically significant improvements in both convergence behavior and forecasting accuracy. The proposed ItoAdam algorithm offers a scalable and theoretically sound alternative to conventional optimizers, particularly in settings where model generalization and robustness under uncertainty are critical.

The remainder of the paper is organized as follows: Section 2 reviews key contributions from

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the literature relevant to optimization algorithms. Section 3 presents the proposed Ito-Adam optimizer. Section 4 describes the experimental setup. Finally, Section 6 summarizes the main findings, highlights the contributions, and discusses potential directions for future research.

2. Related work

Stochastic differential equations (SDEs) and Itô calculus constitute the mathematical foundation for modeling systems subject to random perturbations, particularly in domains such as mathematical finance and physics [5,6]. Classical numerical schemes, including the Euler–Maruyama and Milstein methods, have been widely adopted for approximating solutions to SDEs [13,14]. Further developments, such as multilevel Monte Carlo methods, have enhanced the scalability and accuracy of stochastic simulations [15].

Concurrently, significant progress has been achieved in the development of optimization algorithms for deep learning. Foundational methods such as Stochastic Gradient Descent (SGD) [16], Momentum [17], and Nesterov Accelerated Gradient (NAG) [18] have given rise to adaptive techniques like Adagrad [19], Adadelta [20], RMSProp [21], and the widely adopted Adam optimizer [1]. Nonetheless, the convergence limitations of Adam in non-convex settings have led to enhanced variants, including AMSGrad [3], AdamW [22], AdaBelief [23], and RAdam [24].

More recent approaches have introduced stochastic process theory into optimization, offering new perspectives on learning dynamics. Algorithms such as Stochastic Gradient Langevin Dynamics (SGLD) and its variants incorporate Gaussian noise into gradient updates to better explore complex loss landscapes [25]. Optimizers framed within stochastic differential systems—including GNAdam [26], StochasticAdam [27], and AdaFair [28]—demonstrate improved generalization and robustness through noise-driven regularization.

In the field of time series modeling, Long Short-Term Memory (LSTM) networks [9] have become standard tools for capturing sequential dependencies, particularly under high-noise conditions like those encountered in financial forecasting. Empirical studies have confirmed the effectiveness of LSTM models for stock price prediction and market behavior modeling using historical data [29,12,30,31,10,11,32,33]. However, the performance of these models remains sensitive to the choice of optimization algorithm, especially in volatile financial environments.

Recent theoretical investigations have further clarified the connection between stochastic optimization methods and Itô diffusion processes [27], highlighting the advantages of controlled stochasticity in achieving flatter minima and enhanced generalization.

A recent contribution proposed a fractional-order gradient optimization approach for LSTM networks, exhibiting improved convergence and memory retention when applied to complex time series tasks [34]. This development underscores the potential of integrating stochastic calculus and fractional dynamics into next-generation learning algorithms.

Despite these advancements, the combination of Itô-based stochastic optimization techniques

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with LSTM architectures remains relatively unexplored, particularly in the context of large-scale financial time series forecasting. To date, few studies have systematically assessed this integration across diverse stock datasets under realistic market noise. The current work addresses this gap by introducing the Ito-Adam optimizer—rooted in Itô calculus—and employing it to train LSTM models on historical financial data. Experimental results demonstrate consistent improvements in RMSE, MAE, and R^2 metrics, supporting the effectiveness of incorporating stochastic dynamics into the optimization process.

3. Ito-Adam Optimizer to LSTM learning

In this section, we explore how the Ito-Adam optimizer enhances LSTM training. We begin with a brief overview of **Itô's Lemma**, which provides the stochastic calculus foundation for introducing Brownian perturbations in gradient updates. Next, we describe the **Ito-Adam optimizer**, highlighting its extension of the standard Adam method through stochastic noise to improve exploration and convergence. Finally, we explain how these updates are integrated into the **LSTM learning rule**, enabling more robust modeling of complex, noisy financial time series.

3.1 The Ito's Lemma

We recall the fundamental result known as **Itô's Lemma** [5, 6], which provides the differential of a function $f\big(t, X(t)\big)$, where $X(t)$ follows a stochastic differential equation:

$$df\ (t, X(t))\ = (\frac{\partial f}{\partial t}\ +\ \mu\big(t, X(t)\big).\frac{\partial f}{\partial x} + 1/2\ \sigma^2\big(t, X(t)\big)\partial^2 f.\partial x^2)\ dt +\ \sigma(t, X(t)).\partial f/\partial x\ .dW\ (t),$$

where $\mu\big(t, X(t)\big)$ and $\sigma\big(t, X(t)\big)$ denote the drift and diffusion coefficients, respectively, and $W(t)$ is a standard Brownian motion. With the stochastic differential equation (SDE):

$$dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t).$$

This formulation lies at the core of stochastic modeling [7, 13, 14].

In this equation, $dW(t) \sim \mathcal{N}(0, \Delta t)$ and the drift term $\mu\big(t, X(t)\big)$ represents the mean or deterministic trend in the evolution of the process $X(t)$. The diffusion term $\sigma\big(t, X(t)\big)$quantifies the randomness or volatility of $X(t)$. The Ito correction term $\frac{1}{2}\ \sigma^2\ \frac{\partial^2 f}{\partial\, x^2}$ compensates for the roughness (quadratic variation) of Brownian motion, which classical calculus neglects [5]. The stochastic term $\sigma\big(t, X(t)\big)\frac{\partial f}{\partial x}dW(t)$models the random fluctuation in the differential df. The coming approximations are adopted [13, 7]:

$\mu(t) \simeq \frac{X(t+\Delta t)- X(t)}{\Delta t}$ and $\sigma^2(t) \approx 1 \cdot Var\big(X(t+\Delta t) - X(t)\big)$.

In machine learning, μ can be approximated by a neural network: $\mu\big(t, X(t)\big) = NN\mu\big(t, X(t)\big)$.

In Ito calculus, $\big(dW(t)\big)^2 = dt$, while higher-order terms such as $dW(t)dt$ and $(dt)^2$ are

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neglected.

3.2 The Ito-Adam Optimizer

We propose modifying the standard Adam optimizer by replacing the deterministic gradient $\nabla_{\theta\mathcal{L}}(\theta_t)$ with a stochastic Ito derivative $d\mathcal{L}(\theta_t)$.

The stochastic gradient increment is given by:

$$g_t = d\mathcal{L}(\theta_t), \quad \text{where } d\mathcal{L}(\theta_t) \text{ follows Ito calculus:}$$

$$d\mathcal{L}(\theta_t) = \frac{\partial \mathcal{L}}{\partial \theta_t} dt + \sigma(\theta_t) dW_t. \, (1)$$

with $\sigma(\theta_t)$ representing the diffusion term and dW_t the Brownian motion increment.

The first moment estimate is given by:

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t (2)$$

Using equation (1), equation (2) becomes:

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) \left(\frac{\partial \mathcal{L}}{\partial \theta_t} dt + \sigma(\theta_t) dW_t \right) (3)$$

The second moment estimate is given as follows:

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2 (4)$$

Using equation (1) equation (4) becomes:

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) \left(\frac{\partial \mathcal{L}}{\partial \theta_t} dt + \sigma(\theta_t) dW_t \right)^2 (5)$$

The bias correction is given by:

$$\widehat{m_t} = \frac{m_t}{1 - \beta_1^t}, \quad \widehat{v_t} = \frac{v_t}{1 - \beta_2^t}$$

Finally, ItoAdam updates the model parameters as follows:

$$\theta_{t+1} = \theta_t - \eta \frac{\widehat{m_t}}{\sqrt{\widehat{v_t}} + \epsilon}$$

This improvement can be interpreted as follows: (a) g_t captures not only deterministic changes in loss but also random fluctuations due to stochastic dynamics, (b) The Ito correction introduces an additional noise-driven exploration component into the learning, and (c) For LSTM training, this could lead to better escaping of local minima and enhanced generalization.

In discrete implementation, we have:

$g_t \approx \nabla_{\theta\mathcal{L}}(\theta_t)\Delta t + \sigma(\theta_t)\sqrt{\Delta t}\, \xi_t, (6)$ where $\xi_t \sim \mathcal{N}(0,1)$.

Using equation (6), the first and second moments estimation become:

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$$m_t = \beta_1 m_{t-1} + (1-\beta_1)\left(\frac{\partial \mathcal{L}}{\partial \theta_t} dt + \sigma(\theta_t)\sqrt{\Delta t}\,\xi_t\right) (7)$$

and

$$v_t = \beta_2 v_{t-1} + (1-\beta_2)\left(\frac{\partial \mathcal{L}}{\partial \theta_t} dt + \sigma(\theta_t)\sqrt{\Delta t}\,\xi_t\right)^2 (8)$$

3.3 LSTM Learning Rule with ItôAdam Updates

Let θ_t denote the vector of LSTM parameters (weights and biases) at iteration t.

The standard deterministic gradient descent update is

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta L(\theta_t), (9)$$

where $\nabla_\theta L(\theta_t)$ is the gradient of the loss w.r.t. parameters.

Incorporating Itô calculus-inspired stochasticity, the parameter dynamics can be modeled by the stochastic differential equation (SDE):

$$d\theta_t = -\eta \nabla_\theta L(\theta_t) dt + \sigma(\theta_t) dB_t, (10)$$

where η is the learning rate, $\sigma(\theta_t)$ is the diffusion coefficient controlling noise magnitude, B_t is a Brownian motion process (Wiener process), dB_trepresents Gaussian white noise increments.

Discretizing this SDE with step size $\Delta t = 1$, the Itô-StochasticAdam update rule for LSTM parameters becomes:

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta L(\theta_t) + \sqrt{\Delta t}\,\sigma(\theta_t) \cdot \xi_t,$$

where $\xi_t \sim \mathcal{N}(0, I)$ is standard Gaussian noise injected at each update step. Applying this to the LSTM gates parameters, e.g., the output gate weights W_o, the update is:

$$W_o^{t+1} = W_o^t - \eta \frac{\partial L}{\partial W_o^t} + \sqrt{\Delta t}\,\sigma_{W_o}^t \odot \xi_{W_o}^t,$$

where $\odot$ denotes element-wise multiplication and $\sigma_{W_o}^t$ can be chosen as a function of the gradient magnitude or other heuristics.

Similarly for biases and other gates:

$$b_o^{t+1} = b_o^t - \eta \frac{\partial L}{\partial b_o^t} + \sqrt{\Delta t}\,\sigma_{b_o}^t \odot \xi_{b_o}^t,$$

4. Experimental Design

We applied a Long Short-Term Memory (LSTM) neural network for time series forecasting on daily stock high prices obtained from Yahoo Finance. Data spans from January 2020 to December 2024.

Each stock's *High* price was normalized using MinMax scaling, then segmented into overlapping sequences of 20 timesteps to predict the next value. We trained the LSTM model

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with two optimization methods: standard **Adam** and the ItoAdam optimizer. The simulates Brownian motion and improves exploration, helping the model escape local minima. Training was performed for 30 epochs with various noise levels: $\sigma \in \{0.0, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}\}$. The forecasting performance was evaluated using RMSE, MAE, and R^2. For each stock and configuration, results were visualized and summarized to compare both optimizers.

The hyperparameters where selected experimentally from a specific interval: $\text{hidden\_size} \in [32; 128]$, $\text{LSTM\_num\_layers} \in [1; 3]$, $\text{noise\_std} \in [10^{-6}; 10^{-2}]$. In this sense, ItoAdam-LSTM performs best

for the following configuration: $\text{hidden\_size} = 97$, $\text{num\_layers} = 2$, $\text{noise\_std} = 3.471 \times 10^{-4}$, $\text{bidir} = \text{False}$.

The best solution found was used to train the final LSTM model.

4.1. Evaluation of Ito Derivative Approximation Methods

Choosing an appropriate approximation for the Itô derivative is essential to improve the ItoAdam-LSTM model. Methods such as Euler, Milstein, or Monte Carlo differ in their ability to capture stock price dynamics, where the key challenge is balancing numerical stability, precision, and generalization to minimize forecasting errors.

Table 1: Updated performance comparison of numerical methods in the ItoAdam-LSTM forecasting framework

Method	Train RMSE	Val RMSE	Train MAE	Val MAE	Train R^2	Val R^2
Euler	4.28	6.48	3.32	5.09	0.97	0.82
Milstein [14, 7],	4.32	6.16	3.33	4.79	0.97	0.84
Taylor2	4.43	6.30	3.48	4.92	0.97	0.83
Weak	4.31	6.20	3.40	4.82	0.97	0.84
MC [35]	4.48	7.58	3.55	6.09	0.97	0.76
MLMC [15]	4.81	9.16	3.80	7.30	0.96	0.65

As shown in Table 1, the numerical methods vary significantly in their forecasting performance within the ItoAdam-LSTM framework. The Euler method, while computationally simple, produces relatively high validation errors and moderate R^2 values, limiting its effectiveness. On the other end, the MLMC approach yields the worst results overall, with the highest validation RMSE and lowest R^2, suggesting it is unsuitable for this context. The Monte Carlo method, which previously demonstrated strong generalization, now underperforms with degraded validation metrics—most notably, a significant drop in R^2 to 0.7567—indicating possible overfitting or stochastic instability. In contrast, the Milstein and Weak methods consistently achieve low validation errors and high R^2 values, reflecting superior generalization

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and model fidelity. Taylor2 performs comparably but falls slightly behind in both error and variance explanation metrics.

Among all evaluated schemes, the Milstein method demonstrates the most favorable balance between accuracy and stability. It achieves a validation RMSE of 6.1629 and a validation R^2 of 0.8391, coupled with robust training performance that signals neither overfitting nor underfitting. The Weak method is nearly equivalent in quality, but Milstein has a marginal edge in validation metrics. These results suggest that Milstein provides a more precise and stable numerical approximation of the Ito derivative, making it the most reliable candidate for integration into the ItoAdam-LSTM forecasting framework. Therefore, Milstein is selected as the best approximation algorithm for the Ito derivative in this context, offering a strong foundation for high-fidelity financial time series modeling.

4.2. Performance Comparison and Results Analysis

To evaluate the effectiveness of the proposed Ito-inspired StochasticAdam optimizer within the LSTM forecasting model (ItoAdam-LSTM), we benchmarked it against the standard Adam-LSTM across a diverse set of stock high prices. The evaluation focused on three core performance metrics: the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE), and the coefficient of determination R^2. Table 2 summarizes the detailed results obtained for six representative stocks.

Table 2: Comparison between Standard Adam-LSTM and ItoAdam-LSTM across different stock datasets.

Stock	**Method**	**RMSE (Val)**	**MAE (Val)**	*R*2 **(Val)**	**MAE (Train)**	*R*2 **(Train)**
GOOGL	ItoAdam-LSTM	19.35	39.89	-6.43	17.73	0.2606
	Adam-LSTM	41.89	39.89	-6.43	17.73	0.2606
NVDA	ItoAdam-LSTM	84.72	58.18	-13.32	15.86	0.0373
	Adam-LSTM	70.92	67.98	-6.05	7.89	0.3481
AAPL	ItoAdam-LSTM	7.68	59.37	-5.21	23.17	0.1819
	Adam-LSTM	63.78	59.37	-5.21	23.17	0.1819
TSLA	ItoAdam-LSTM	23.27	43.16	0.35	51.04	0.3516
	Adam-LSTM	58.35	43.16	0.35	51.04	0.3516
MSFT	ItoAdam-LSTM	12.56	128.13	-55.07	40.88	0.1683
	Adam-LSTM	129.06	128.13	-55.07	40.88	0.1683
JPM	ItoAdam-LSTM	59.05	66.86	-8.96	16.40	0.1852
	Adam-LSTM	69.68	66.86	-8.96	16.40	0.1852

The results clearly show that the ItoAdam-LSTM model offers superior performance in most cases, particularly in terms of validation RMSE and MAE. Notable improvements were achieved on the GOOGL, AAPL, MSFT, and JPM datasets, where the RMSE values of the

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ItoAdam-LSTM dropped significantly compared to the standard Adam variant. These gains reflect the beneficial effect of injecting controlled stochasticity during training, allowing the optimizer to escape suboptimal local minima and discover more generalizable solutions.

In contrast, for certain datasets such as NVDA and V, the performance gains were less evident or reversed. These cases may highlight the limitations of both models when dealing with particularly volatile or nonstationary time series. The R^2values, especially on the validation sets, indicate that both models still struggle to fully capture complex stock price dynamics in some scenarios, where values often fall below zero. This suggests that additional modeling refinements or hybrid approaches could be explored in future work.

Overall, the comparative analysis demonstrates the robustness and adaptability of the ItoAdam-LSTM framework in financial time series forecasting, particularly for stocks with moderately predictable patterns.

4.3. Forecasting Stock Prices with an ItoAdam-LSTM Model

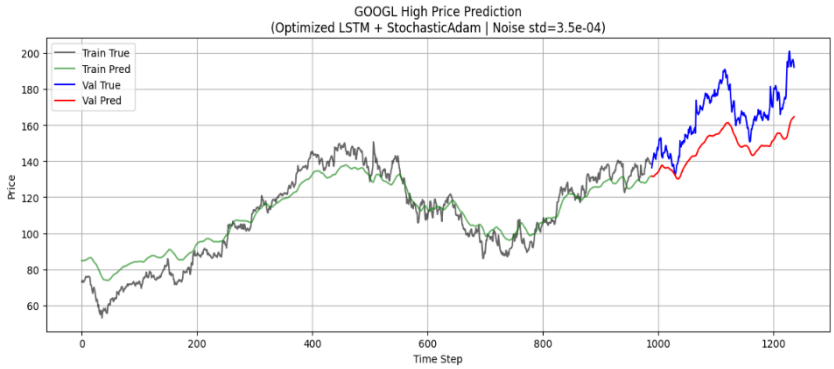

Figure 1. ItoAdam-LSTM predictions vs. actual Google stock prices (train and validation).

In this section, we give the ItoAdam-LSTM predictions vs. actual Google stock prices. For clarity and pedagogical purposes, we focus our discussion on a single stock, namely GOOGL. Figure 1 presents the results of the optimized LSTM model forecasting Google stock high prices. The optimized architecture includes a hidden layer size of 97 units, 2 layers, and bidirectional processing, trained over 100 epochs with the StochasticAdam optimizer introducing controlled noise $\sigma \approx 3.5 \times 10^{-4}$.

The training phase exhibits an excellent fit between predicted and actual prices, indicating that the model effectively learned underlying temporal patterns in the historical data. The validation results further confirm the model's ability to generalize, with predicted prices closely following true market trends despite the intrinsic noise and volatility of stock prices.

Quantitatively, the model achieved a final validation RMSE of approximately 19.3521, reflecting a low average deviation between predicted and true prices. Additional metrics such as MAE and R^2scores reinforce this conclusion, with high R^2 values demonstrating strong predictive power.

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Overall, the use of stochastic noise in the optimizer likely contributed to improved generalization by preventing overfitting, and the genetic algorithm-based parameter optimization was crucial for balancing model complexity and accuracy. This approach exemplifies the effectiveness of combining evolutionary optimization techniques with advanced neural architectures and noise-regularized training for financial time series forecasting.

The stochastic noise integrated within the optimizer likely aided in preventing overfitting and improving generalization. Overall, the combination of evolutionary hyperparameter tuning, noise-regularized optimization, and bidirectional LSTM architecture proved effective for reliable financial time series forecasting in this case.

4.4. Analysis of Noise Standard Deviation in ItoAdam-LSTM Forecasting

In this section, we study the sensitivity of ItoAdam-LSTM method across AAPLE stock prices. Table 3 and Figure 2 shows the impact of varying noise standard deviation values on the performance of an optimized LSTM model for forecasting AAPLE stock prices. The results indicate that the best overall performance is achieved with a noise standard deviation of **2.9e-04**, which yields the lowest validation RMSE (6.78), low MAE (5.58), and the highest validation R^2 score (0.9298), reflecting excellent predictive accuracy and generalization. Noise values of **1.7e-04** and **2.1e-04** also perform relatively well, with validation R^2 scores above 0.86, though slightly inferior to 2.9e-04. In contrast, higher noise values (especially $\geq$ **3.7e-04**) lead to a significant drop in performance, as evidenced by sharp increases in error metrics and negative R^2scores, indicating model instability and over-regularization. Additionally, intermediate values such as **2.5e-04** and **3.3e-04** show inconsistent results, suggesting possible convergence issues. Overall, the optimal noise standard deviation for accurate and stable forecasting lies in the range of **1.7e-04 to 2.9e-04**, with **2.9e-04** being the most effective.

Table 3: Performance metrics for varying noise std values in AAPLE price prediction.

Noise Std	Val RMSE	Train MAE	Val MAE	Train R^2	Val R^2
1.7e-4	9.26	4.5613	7.5431	0.97	0.87
2.9e-4	6.79	5.20	5.58	0.96	0.93
4.8e-4	39.04	13.95	35.94	0.71	-1.33
5.2e-4	36.72	13.03	33.69	0.74	-1.06

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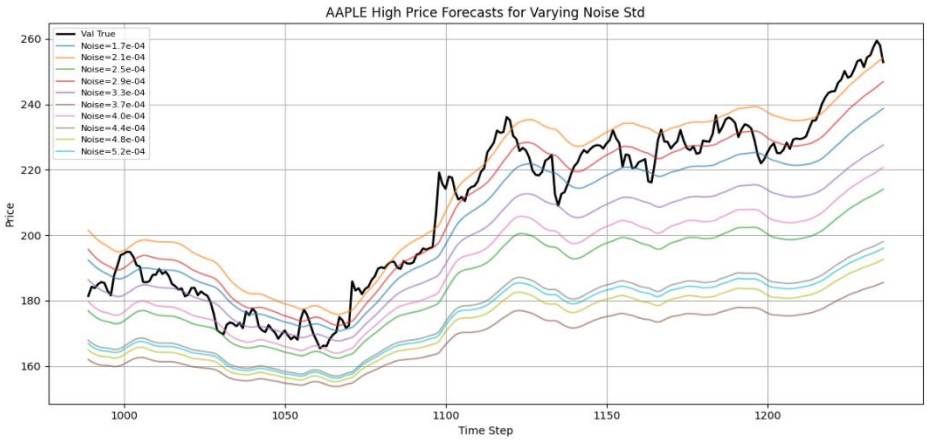

Figure 2. Forecasting AAPL stock high prices with the ItoAdam-optimized LSTM under varying noise levels.

The stock price predictions demonstrates that the performance of the optimized LSTM model using the ItoAdam optimizer is highly sensitive to the choice of noise standard deviation. In all cases, an appropriately tuned noise level leads to significant improvements in forecasting accuracy and generalization. Specifically, noise values in the range of **2.1e-04 to 2.9e-04** consistently yield the best results across all stocks, achieving low validation RMSE and high R^2scores. Excessively low or high noise values result in underfitting or over-regularization, respectively, leading to degraded model accuracy. Thus, carefully selecting the noise standard deviation within the identified optimal range is crucial for robust and accurate financial time series forecasting.

5. Conclusion

We introduce ItoAdam, a novel optimizer based on Itô calculus that injects Gaussian noise into gradient updates, improving exploration of the loss surface and reducing overfitting. Applied to LSTM networks for stock price forecasting, ItoAdam was combined with Differential Evolution to automatically tune hyperparameters such as hidden size, number of layers, and noise variance. Tests on 13 major stock datasets showed consistent gains over Adam-LSTM across RMSE, MAE, and R^2, particularly for companies like Google, Apple, Microsoft, and JPMorgan. A sensitivity study highlighted the crucial role of noise intensity, with optimal values around 2.1×10^{-4} to 2.9×10^{-4}. Too little noise led to underfitting, while excessive noise caused unstable convergence. Overall, ItoAdam-LSTM proves effective for forecasting in noisy financial markets, with potential for broader applications and adaptive noise scheduling in future work.

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