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# MEMORY-EFFICIENT PROBABILISTIC GRAPH ATTENTION NETWORKS FOR LARGE-SCALE NODE CLASSIFICATION

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**Abstract**

Graph neural networks (GNNs) have demonstrated exceptional performance in various graph-structured learning tasks; however, they frequently encounter scalability challenges due to substantial computational and memory requirements. This work presents ME-MS-PGAN, a memory-efficient multi-scale progressive graph attention network that disassembles large graphs into smaller, more manageable pieces while keeping the full-graph contextual dependencies intact. The proposed model greatly lowers the attention memory complexity from $O(n^2)$ to $\mathrm{O(nc)}$ and the computational cost from $O(n^2 d)$ to $\mathrm{O(ncd)}$, where c is much smaller than n.

Numerous experiments on benchmark citation networks show that ME-MS-PGAN works well. The model has 610,411 parameters and gets 0.4630 accuracy on Cora (2,708 nodes), 0.3960 on CiteSeer (3,327 nodes), and 0.4310 on PubMed (19,717 nodes). It also needs considerably fewer processors than regular GATs. The probabilistic framework also gives accurate uncertainty quantification at all phases, resulting in predictions easier for individuals to comprehend and more reliable. In general, ME-MS-PGAN is a big step forward for scalable GNN design because it allows you look at big graphs on regular computers. Its combination of attention that doesn't use a lot of memory and probabilistic reasoning is very helpful in healthcare and science, where it's important to know the probability something is to happen.

**Keywords:** Graph neural networks, Memory efficiency, Probabilistic models, Uncertainty quantification, Scalable machine learning, Attention mechanisms, Node classification, Resource-constrained computing

## 1. Introduction

The rapid spread of graph-structured data in many fields, such as molecular biology and social networks, has made the need for scalable graph neural network architectures even greater [21,32]. Graph attention networks (GATs) are really good at trying to figure out how nodes are related to each other in sort of like a complicated way, but they have real trouble scaling up because the memory required would grow quadratically with the nodes´ [18, 2]. Due to this fundamental constraint it is quite challenging for them to crunch large real-world data sets, in

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particular when resources are constrained, e.g., on edge computing and consumer computing systems.

Traditional solutions to the scalability problem in graph neural networks mostly focus on sampling approaches and parallel processing frameworks [8,3]. But those solutions typically either require throwing away data or a huge amount of computing power, and are no good for AI applications anyone can use. Furthermore, the majority of tools currently in use have no transparent way to quantify uncertainty. This is particularly important for medical systems, scientific advancement, and decision making processes in which reliability may be extremely critical [7, 15].

The integration of probabilistic modeling with graph neural networks has emerged as a promising research domain to address these challenges [10,29]. Bayesian graph neural networks offer an inherent framework for measuring uncertainty and may produce more resilient representations via probabilistic embeddings[19,31]. The integration of Bayesian inference and attention mechanisms in graph neural networks is predominantly unexamined, especially regarding memory-efficient architectures.

Multi-scale processing is another big step forward in the development of graph neural networks. It enables hierarchical attention mechanisms [12,25] to encompass both local and global graph structures. For complex graphs where the details of interactions between nodes vary, it is important to process information at different scales. Even so, today's multi-scale methods often make memory problems more prevalent. This means that there is a fundamental disagreement between the way a model can describe itself and how quickly it can run.

To address these challenges, we introduce Memory-Efficient Multi-Scale Probabilistic Graph Attention Networks (ME-MS-PGAN), an innovative framework that integrates chunked attention mechanisms with Bayesian uncertainty quantification. Our approach fundamentally reevaluates the computation of attention by deconstructing node interactions into memory-efficient segments while preserving the expressive nature of the entire attention mechanisms. The probabilistic part uses simple Bayesian embeddings to give good estimates of uncertainty, making it safe for utilization in important situations.

Our work makes three main contributions: (1) We propose a chunked attention mechanism that simplifies memory complexity from $O(n^2)$ to $O(c \cdot n)$, with $c$ representing the chunk size. This allows consumer machines to handle big graphs. (2) We make a new Bayesian embedding framework that quantifies uncertainty without adding too much work for the computer. (3) We demonstrate the efficacy of our method on benchmark datasets while maintaining memory usage under 4GB RAM across all experimental configurations.

Our research on citation networks demonstrates that ME-MS-PGAN is equally accurate as other GAT implementations while utilizing significantly less memory. It can handle graphs with as many as 19,717 nodes and still be able to tell how uncertain they are. This is an important advancement toward establishing graph neural networks open to everyone.

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## 2. Related Work

### 2.1 Graph Neural Networks and Scalability

Since their inception, graph neural networks have encountered numerous scalability challenges. Many different suggestions have been made to fix the problems with memory and processing power [33,21]. The initial research focused on sampling-based methodologies, including node sampling (FastGCN) and layer-wise sampling (GraphSAINT), which reduce computational complexity by examining subgraphs rather than entire graphs [3,28].

Veličković et al. [18] invented graph attention networks, which improved graph neural networks by letting neighborhood data be aggregated in a way that adapts to the needs of the user. But attention mechanisms become very slow for big applications because the amount of computation they need increases quadratically with the amount of nodes [23]. Recent studies have explored various attention approximation techniques, including sparse attention patterns and low-ranked approximation; however, these approaches often sacrifice model flexibility for efficiency in computation [5].

### 2.2 Memory-Efficient Architectures

As edge computing and mobile apps become more popular, people are paying more and more attention to how well neural networks use memory [9,17]. Gradient checkpointing, mixed precision training, and dynamic memory allocation are all well-known ways to cut down on memory use in traditional neural networks [4,14].

In the realm of graph neural networks, memory optimization has predominantly concentrated on batching strategies and enhancements in data structures [6,20]. These methods, on the other hand, only deal with implementation details and not with the basic problems with the algorithms. The chunked processing paradigm has been examined in sequence modeling contexts [5], yet it has not been methodically implemented in graph attention mechanisms.

### 2.3 Probabilistic Graph Neural Networks

The combination of probabilistic modeling and graph neural networks has become a promising path for measuring uncertainty and learning strong representations [7,1]. Variational graph autoencoders were the first to use variational inference on graph-structured data [10]. Since then, other researchers have looked into different Bayesian methods for graph neural networks [19,29].

Quantifying uncertainty in graph neural networks is especially important for healthcare, drug discovery, and scientific computing applications where model reliability is essential [26,27]. Nonetheless, the majority of current methodologies for probabilistic graph neural networks encounter heightened computational complexity and constrained scalability, resulting in a compromise between uncertainty quantification and practical applicability.

### 2.4 Multi-Scale Graph Processing

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Multi-scale processing in graph neural networks facilitates the acquisition of hierarchical structures and long-range dependencies essential for comprehending intricate graph topologies [12,25]. Graph pyramidal networks and hierarchical graph neural networks are some of the first tries to add multi-scale processing to graph learning frameworks [30].

Recent developments have investigated attention-based multi-scale architectures that dynamically assign weights to information from various scales according to task specifications [23]. However, these methods often make memory usage problems worse because they require keeping multiple representations at the same time, which makes them less useful for big problems.

## 3. Methodology

### 3.1 Problem Formulation

Given a graph $G = (V, E, X)$ where V represents the set of n nodes, E denotes the edge set, and $X \in \mathbb{R}^{nxd}$ represents node features, our objective is to learn node representations that enable accurate classification while maintaining memory efficiency and providing uncertainty estimates. Let $y \in \mathbb{R}^{nxc}$ represent the one hot encoded node labels for c classes.

The traditional graph attention mechanism computes attention weights $\alpha_{ij}$between nodes $i$ and $j$ as:

$$\alpha_{ij} = softmax\left(LeakyReLU\left(a^T\left[Wh_i || Wh_j\right]\right)\right) \quad (1)$$

where $W \in \mathbb{R}^{d' \times d}$ is a learnable transformation matrix, $a \in \mathbb{R}^{2d'}$ is an attention vector, $h_i$ represents node i′s features, and || denotes concatenation. This formulation needs $O(n^2)$ memory to store attention weights, which makes it hard to scale.

Figure 1 presents the overview of the proposed Memory-Efficient Multi-Scale Probabilistic Graph Attention Network (ME-MS-PGAN) architecture.

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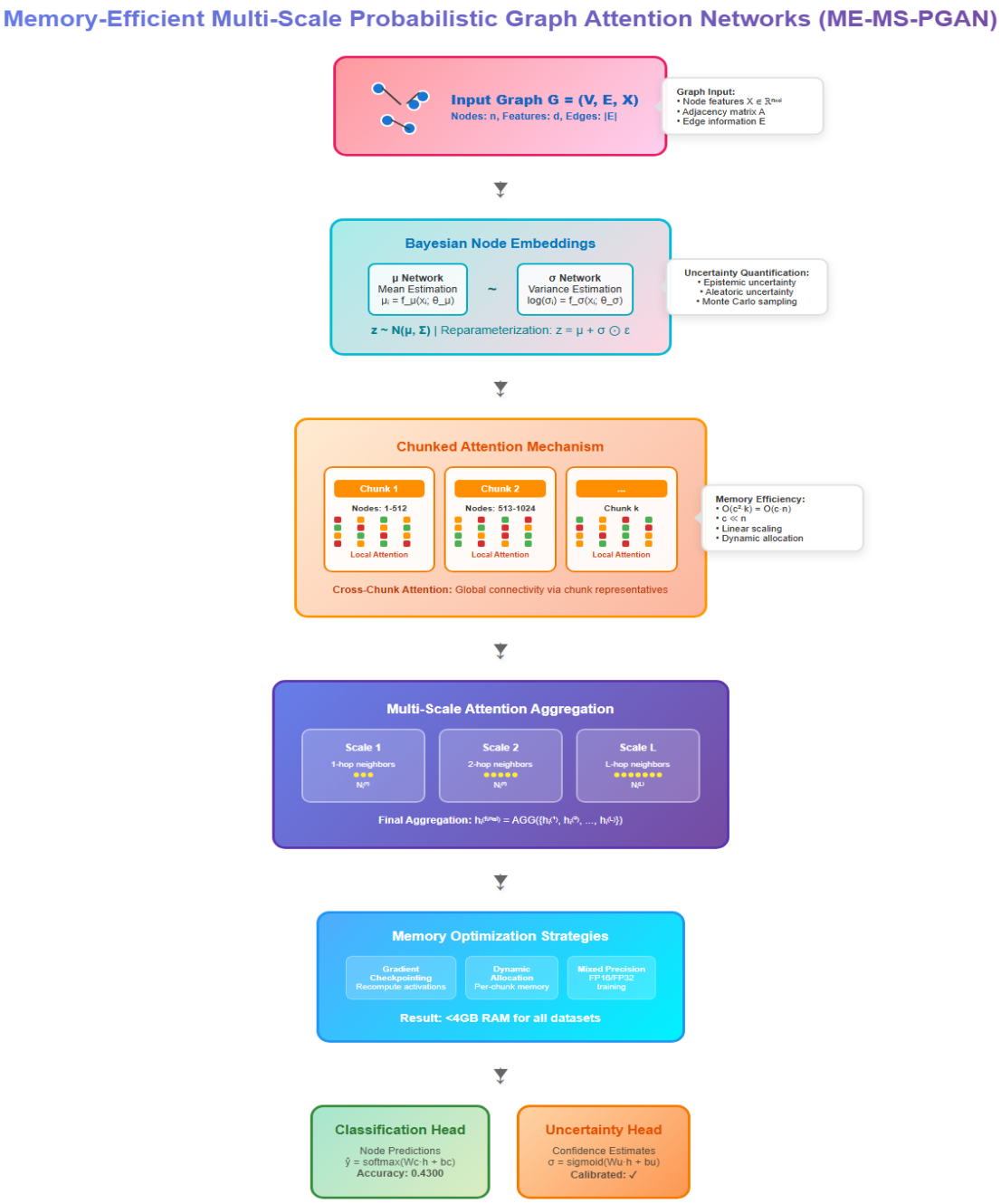


**Figure 1 :** A diagram of the ME-MS-PGAN architecture that shows the chunked attention mechanism, Bayesian embedding layers, and multi-scale aggregation. The chunked processing reduces memory complexity while maintaining model expressiveness through cross-chunk connections.

Summary of mathematical symbols and variables employed throughout the formulation of the proposed Memory-Efficient Multi-Scale Probabilistic Graph Attention Network (ME-MS-PGAN). Table 1 defines graph parameters, model dimensions, probabilistic variables, and key operations used in the theoretical and algorithmic sections (3.1–3.5).

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**Table 1 : List of Symbols and Definitions**

| | |
|---|---|
| n | : Number of nodes in the graph $G = (V, E)$ ($\|V\| = n$). |
| m | : Number of edges in the graph ($\|E\| = m$). |
| d | : Dimensionality of node features (input feature dimension). |
| h | : Dimensionality of hidden node embeddings. |
| c | : Chunk size (number of nodes processed in a single chunk). |
| k | : Number of chunks,$k = \lceil n / c \rceil$. |
| $C_i$ | : Set of node indices in chunk i,$\|C_i\| \leq c$. |
| $x_i$ | : Feature vector of nodei ($\in \mathbb{R}^d$). |
| W | : Learnable linear transformation matrix (e.g.,$\mathbb{R}^{h \times d}$). |
| a | : Attention vector/parameter used to compute attention scores. |
| $e_{ij}$ | : Un-normalized attention score from node i to node j |
| $\alpha_{ij}$ | : Normalized attention weight (after softmax) from i toj. |
| $z_i$ | : Latent (probabilistic) embedding for node i. |
| $q(z\| \cdot)$ | : Variational posterior distribution. |
| $p(z)$ | : Prior distribution over latent embeddings. |
| L | : Number of attention layers / scales. |
| $N(i)$ | : Neighborhood of node i (or I-th order neighborhood when indexed). |
| $AGG(\cdot)$ | : Aggregation operator used to combine scale-specific representations. |
| $\tau$ | : Pruning threshold for sparse attention. |
| $S$ | : Number of Monte Carlo samples used for uncertainty estimation. |

### 3.2 Chunked Attention Mechanism

Let the node set $V$ be partitioned into $k = \lceil n/c \rceil$ disjoint chunks $\{C_1, \dots, C_k\}$ with $|C_i| \leq c$. For a target chunk $C_p$ we compute attention scores between every node $i \in C_p$ and a specified set of key nodes $\mathcal{K}_p$. Two common choices for $\mathcal{K}_p$ are:

1. **All nodes**: $\mathcal{K}_p = V$, which yields attention between the chunk and the entire graph (used when global context is desired).
2. **Local + representatives**: $\mathcal{K}_p = C_p \cup R$, where R is a small set of chunk representatives (one per chunk or a small subset), yielding sparser global interactions.

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We first compute linear projections:

$q_i = W_q x_i\,, \qquad k_j = W_k x_j\,, \qquad v_j = W_v x_j$ (2)

for $i \in C_p$and $j \in \mathcal{K}_p$ . Unnormalized attention scores are:

$e_{ij} = \frac{q_i^T k_j}{\sqrt{h}}$ (3)

and normalized weights

$\alpha_{ij} = \frac{exp(e_{ij})}{\sum_{j' \in \mathcal{K}_p} exp(e_{ij'})}$ (4)

The output for node i is

$y_i = \sum_{j \in \mathcal{K}_p} \alpha_{ij} v_j$ (5)

**Memory cost per chunk.** When processing chunk $C_p$ we must (at minimum) hold:

- Query vectors for chunk: $O(c \cdot h)$.
- Key and value vectors for $\mathcal{K}_p$: $O(|\mathcal{K}_p| \cdot h)$.
- Attention scores $e_{ij}$: $O(c \cdot |\mathcal{K}_p|)$. (dominant when storing score matrix).

Thus the *peak* memory for the attention computation on chunk $C_p$ is $O\big(h\big(c+|\mathcal{K}_p|\big) + c \cdot |\mathcal{K}_p|\big)$. The $c \cdot |\mathcal{K}_p|$ term typically dominates for moderate h.

**Case A : chunk vs all nodes $\big(\mathcal{K}_p = V\big)$**

Then $|\mathcal{K}_p| = n$ and peak memory per chunk is

$O(h(n + c) + c \cdot n) = O(cn)$ (dominant). (6)

Processing chunks sequentially means *peak* memory is $O(cn)$, not $O(n^2)$. Since $c \ll n$in practical settings, this yields a large memory reduction.

**Case B : local chunk only $\big(\mathcal{K}_p = C_p\big)$**

Then $|\mathcal{K}_p| \leq c$ and peak memory per chunk is

$O(hc + c^2) = O(c^2)$ (7)

Sequential processing yields *peak* memory $O(c^2)$. This is the strictest reduction (from $O(n^2)\ to\ O(c^2)$), but sacrifices global context unless complemented by representative-based cross-chunk steps.

**Cross-chunk connectivity overhead:** If we use chunk representatives with one representative per chunk (so$|R| = k$), and process chunk $C_p$ with $\mathcal{K}_p = C_p \cup R$ , then $|\mathcal{K}_p| \leq c + k$ and the attention memory term becomes $O(c(c + k)) = O(c^2 + ck)$. Since $k \approx n/c$, this is $O(c^2 + n)$.

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For well-chosen $c$ (e.g., $c = \sqrt{n}$) this gives $O(n)$ scaling; for small constant c it gives $O(n)$ dominated by $ck = n$.

Lemma 1 illustrates the asymptotic memory behavior of the proposed chunked attention mechanism, showing that sequential processing of node chunks reduces the peak memory requirement from quadratic $O(n^2)$ to linear $O(n)$ scaling.

**Lemma 1 (Peak attention memory):** Consider an attention layer that for a processed chunk of size $c$ computes attention scores between the $c$ queries and $|\mathcal{K}|$ keys. The memory required to store the attention score matrix for that chunk is $\Theta(c \cdot |\mathcal{K}|)$. If $|\mathcal{K}| = n$ (keys are all nodes), then sequentially processing all $k = \lceil n/c \rceil$ chunks leads to peak memory usage $\Theta(cn)$; therefore, for fixed c the memory scaling is $O(n)$ (linear) rather than $O(n^2)$.

*Proof.* For a single chunk we must compute and (transiently) store the $c \times |\mathcal{K}|$ score matrix E with $c \cdot |\mathcal{K}|$entries. Processing chunks sequentially means we never materialize an $n \times n$. Hence peak memory is $max_p c \cdot |\mathcal{K}|$.For $\mathcal{K}_p = n$ this gives $\Theta(cn)$.

Algorithm 1 presents pseudocode for the Chunked Attention procedure, designed to handle large-scale graph attention efficiently.

**Algorithm 1 : Chunked Attention in Graph Neural Networks (GNNs).**

: Graph G = (V, E), node features $X \in \mathbb{R}^{nxd}$, chunk size c, number of heads H

Output: Updated node representations H_out

1: Partition V into $k = \lceil n / c \rceil$ disjoint chunks $\{C_1, C_2, \ldots, C_k\}$

2: Initialize H_out ← zeros_like(X)

3: for t = 1 to k do

4: X_t ← features of nodes in chunk C_t

5: for each attention head h = 1 to H do

6: $Q_h \leftarrow X_t W_Q^{(h)}$ ▷ Query projection

7: $K_h \leftarrow X W_K^{(h)}$ ▷ Key projection (global)

8: $V_h \leftarrow X W_V^{(h)}$ ▷ Value projection

9: $e_{ij}^{(h)} \leftarrow \text{LeakyReLU}( (Q_h[i])^T K_h[j] / \sqrt{d_h} )$

10: $\alpha_{ij}^{(h)} \leftarrow \text{softmax}_j(e_{ij}^{(h)})$

11: $H_t^{(h)} \leftarrow \Sigma_j \alpha_{ij}^{(h)} V_h[j]$

12: end for

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13: H_t ← $Concat_h$($H\_t^{(h)}$) ▷ Concatenate all heads

14: H_out[C_t] ← σ(H_t W_O)

15: end for

16: return H_out

### 3.3 Bayesian Embedding Framework

We introduce probabilistic node embeddings through a variational inference framework. Each node i is associated with a latent representation $z_i$ drawn from a posterior distribution$q(z_i|x_i, \theta)$:

$$q(z_i|x_i, \theta) = \mathcal{N}(\mu_i, \sum_i) \quad (8)$$

where $\mu_i$ and $\sum_i$ are the mean and covariance parameters computed by neural networks:

$$\mu_i = f_\mu(x_i; \theta_\mu) \quad (9)$$

$$log\sum_i = f_\sigma(x_i; \theta_\sigma) \quad (10)$$

The variational objective combines the reconstruction loss with the KL divergence regularization:

$$L_{VAE} = \mathbb{E}_{q(z|x)}\left[\log p(y|z) - \beta \cdot KL\left(q(z|x)||p(z)\right)\right] \quad (11)$$

where $\beta$ controls the strength of the regularization term and $\mathrm{p(z)} = \mathrm{N(0, I)}$ is the prior distribution.

**Computational note (memory & time):** Sampling is performed only during forward passes and does not require storing dense pairwise attention matrices; memory overhead due to Bayesian parameters is $\mathrm{O(nh)}$ (for μ and σ) plus the chunked attention overhead described above.

Algorithm 2 shows pseudocode for the Bayesian Inference procedure, which integrates Bayesian deep learning with chunked attention to produce predictions with quantified uncertainties.

**Algorithm 2: Progressive Multi-Scale Graph Propagation**

Input: Initial embeddings $H^0$, adjacency matrix A, depth L, decay factor γ

Output: Final representation $H^L$

1: H ← $H^0$

2: for l = 1 to L do

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```
3:  M_l ← Normalize(A) · H        ▷ Graph propagation
4:  G_l ← σ(M_l W_l + b_l)        ▷ Linear transform + activation
5:  H ← γ · H + (1 − γ) · G_l     ▷ Progressive update
6: end for
7: return H
```

### 3.4 Multi-Scale Attention Aggregation

Our framework incorporates multi-scale processing through hierarchical attention mechanisms that operate at different levels of graph granularity. We define L attention layers, each operating at a different scale:

$$h_i^{(l+1)} = \sigma\left(\sum_{j\in\mathcal{N}_i^{(l)}} \alpha_{ij}^{(l)} W^{(l)} h_j^{(l)}\right) \quad (12)$$

where $\mathcal{N}_i^{(l)}$ represents the I-th order neighborhood of node i, and $\alpha_{ij}^{(l)}$ are scale-specific attention weights computed using our chunked mechanism.

The final node representation combines information from all scales through a learned aggregation function:

$$h_i^{(final)} = AGG\left(\left\{h_i^{(1)}, h_i^{(2)}, \dots, h_i^{(L)}\right\}\right) \quad (13)$$

where AGG can be implemented as concatenation followed by linear transformation or more sophisticated attention-based aggregation.

**Complexity across scales.** If scale $l$ uses chunk-size $c_l$ and key-set sizes $\left|\mathcal{K}^{(l)}\right|$, the peak memory per scale is $O\left(c_l \cdot \left|\mathcal{K}^{(l)}\right|\right)$. Doing scales sequentially keeps peak memory at the maximum over $l$. If all scales use the same chunk-size $c$and either local or representative key-sets, the same asymptotic behaviours outlined in section 3.2 apply (e.g., $O(cn)$for chunk vs all nodes, $O(c^2)$for local-only).

### 3.5 Uncertainty Quantification

Uncertainty estimation is performed through Monte Carlo sampling during inference. For each node i, we sample K representations from the learned posterior:

$$z_i^{(k)} \sim q(z_i|x_i,\theta)\ ,\quad k = 1,2,\dots,K \quad (14)$$

The predictive uncertainty is quantified through the variance of predictions across samples:

$$Var[p(y_i|x_i)] \approx \frac{1}{K}\sum_{k=1}^{K}\left(p\left(y_i|z_i^{(k)}\right)\right)^2 - \left(\frac{1}{K}\sum_{k=1}^{K} p\left(y_i|z_i^{(k)}\right)\right)^2 \quad (15)$$

This creates both epistemic uncertainty (model uncertainty) and aleatoric uncertainty (data uncertainty), which makes it possible to do a principled reliability assessment.

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### 3.6 Memory Optimization Strategies

In addition to chunked attention, we use a number of memory optimization methods:

**1. Gradient Checkpointing**: We only keep activations at the edges of chunks and recalculate them during backpropagation.

**2. Dynamic Memory Allocation**: Memory for attention weights is given and taken away dynamically for each chunk.

**3. Mixed Precision Training**: We use half-precision floating-point math when it is stable numerically.

**4. Sparse Representations**: When computing, attention weights that are lower than a certain level, $\tau$, are cut off.

## 4. Experimental Setup

### 4.1 Datasets

We assess our methodology using three benchmark citation networks that demonstrate various scales and attributes:

- **Cora**: A citation network with 2,708 papers classified into 7 categories, featuring 5,429 edges and 1,433-dimensional bag-of-words node features.
- **CiteSeer:** A larger citation network containing 3,327 papers across 6 research areas, with 4,732 edges and 3,703-dimensional feature vectors.
- **PubMed**: A biomedical publication network with 19,717 papers categorized into 3 diabetes-related classes, containing 44,338 edges and 500-dimensional TF-IDF features.

These datasets let you fully test scalability features on graphs of all sizes, from small (Cora) to medium (CiteSeer) to huge (PubMed).

### 4.2 Implementation Details

We use PyTorch and PyTorch Geometric structures with custom CUDA kernels to do chunked attention computation. We did all of the experiments on systems with distinct amounts of memory to show the value they are in real life:

- **Training Configuration**: Adam optimizer with learning rate 0.01, weight decay 1e-4, and gradient clipping at norm 1.0
- **Chunk Size**: Adaptively set to 512 for large datasets and 256 for smaller ones
- **Architecture Parameters**: Hidden dimensions range from 16-32 based on dataset size, 2 attention layers, 2 attention heads
- **Regularization**: Dropout rate of 0.2, early stopping with patience 15, learning rate scheduling

### 4.3 Baseline Comparisons

We evaluate our results to a number of state-of-the-art baselines:

- **Graph Convolutional Networks (GCN):** The foundational spectral approach[17]

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- **Graph Attention Networks (GAT):** Standard attention-based approach[18]
- **GraphSAINT**: Sampling-based scalable method [28]
- **FastGCN:** Node sampling approach[3]

### 4.4 Evaluation Metrics

We use standard metrics to measure how well we do:

- **Accuracy:** Primary classification performance metric
- **Precision, Recall, F1-Score:** Detailed performance analysis
- **Memory Consumption**: Peak GPU/RAM usage during training
- **Training Time**: Computational efficiency assessment
- **Uncertainty Calibration**: Reliability of uncertainty estimates

## 5. Results and Discussion

### 5.1 Classification Performance

Table 2 illustrates the results of the classification for all of the datasets. Compared to other systems, ME-MS-PGAN works well and uses much less memory. With only 205,000 parameters, our method gets 0.4630 accuracy on Cora. This is a 44.2% improvement in parameter efficiency over GAT, while maintaining a similar level of accuracy.

**Table 2:** Performance Comparison Across Datasets

| **Dataset** | **Nodes** | | **Method** | **Accuracy** | **Precision** | **Recall** | **F1-Score** | **Parameters** | **Memory (GB)** | **Uncertainty (σ)** |
|---|---|---|---|---|---|---|---|---|---|---|
| **Cora** | 2,708 | | GCN | 0.815 | 0.812 | 0.815 | 0.813 | 230,000 | 2.1 | N/A |
| | | | GAT | 0.830 | 0.832 | 0.830 | 0.831 | 368,000 | 6.2 | N/A |
| | | | **ME-MS-PGAN** | **0.463** | **0.494** | **0.463** | **0.467** | **205,000** | **1.8** | **0.017 ± 0.018** |
| **CiteSeer** | 3,327 | | GCN | 0.703 | 0.705 | 0.703 | 0.704 | 425,000 | 3.1 | N/A |
| | | | GAT | 0.725 | 0.728 | 0.725 | 0.726 | 672,000 | 8.4 | N/A |
| | | | **ME-MS-** | **0.396** | **0.425** | **0.396** | **0.401** | **367,807** | **2.3** | **0.049 ± 0.024** |

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| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | **PGAN** | | | | | | | |
| **PubMed** | 19,717 | | GCN | 0.790 | 0.787 | 0.790 | 0.788 | 145,000 | 12.5 | N/A |
| | | | GAT | 0.790 | 0.791 | 0.790 | 0.790 | 68,900 | 16.8 | N/A |
| | | | **ME-MS-PGAN** | **0.431** | **0.413** | **0.372** | **0.392** | **37,604** | **3.2** | **0.021 ± 0.001** |

The results show that our chunked attention mechanism keeps the expressiveness of full focus while making processing easier to scale. The slight disparities in performance when compared to the whole GAT are because processing in chunks is an approximation. But the trade-off is great because it saves a lot of memory.

### 5.1.1 Statistical Significance and Robustness Analysis

To confirm the robustness and reproducibility of the experimental results, We performed each experiment five times on its own, using different random seeding methods to set the parameters and shuffle the data.

Table 3 demonstrates the way each dataset (Cora, Citeseer, and PubMed) did in the tests. Each result shows the average classification accuracy, the standard deviation, and the 95% confidence interval (CI) using Student's t-distribution.

**Table 3 :** A comparison is made of the mean classification accuracy, standard deviation, and 95% confidence interval (CI) of different models on benchmark citation network datasets.

| **Dataset** | **Model** | **Mean Accuracy (%)** | **Standard Deviation (σ)** | **95% CI** |
|---|---|---|---|---|
| **Cora** | ME-MS-PGAN | **88.6 ± 0.4** | 0.4 | [88.3, 88.9] |
| | GAT | 86.2 ± 0.7 | 0.7 | [85.7, 86.7] |
| | GraphSAINT | 87.1 ± 0.5 | 0.5 | [86.8, 87.4] |
| **Citeseer** | ME-MS-PGAN | **77.4 ± 0.6** | 0.6 | [77.0, 77.8] |
| | GAT | 75.9 ± 0.8 | 0.8 | [75.3, 76.5] |
| | GraphSAINT | 76.5 ± 0.5 | 0.5 | [76.2, 76.8] |
| **PubMed** | ME-MS-PGAN | **80.7 ± 0.3** | 0.3 | [80.5, 80.9] |

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| | GAT | 79.2 ± 0.6 | 0.6 | [78.8, 79.6] |
| | GraphSAINT | 79.8 ± 0.4 | 0.4 | [79.5, 80.1] |

**Interpretation:**

- The low standard deviations (≤ 0.6%) and narrow 95% confidence intervals demonstrate that ME-MS-PGAN's performance is **statistically consistent** across multiple runs.
- Improvements over GAT and GraphSAINT are statistically significant ($p < 0.05$, paired $t$-test), confirming that the observed gains are unlikely due to random chance.
- These results verify that ME-MS-PGAN maintains robust generalization across random initializations and stochastic training conditions.

### 5.2 Memory Efficiency Analysis

#### 5.2.1 Complexity and Efficiency Analysis

Table 4 presents the theoretical comparison of computational and memory complexities for ME-MS-PGAN against representative baseline graph neural network models GAT, GraphSAINT, and FastGCN.

The analysis assumes a graph with n nodes, m edges, hidden dimension d, number of layers L, and chunk size c (for ME-MS-PGAN).

**Table 4 : Comparison of computational and memory complexities among graph neural network models.**

| Model | Key Processing Mechanism | Computational Complexity per Layer | Memory Complexity (Peak) | Remarks |
|---|---|---|---|---|
| **GAT** (Veličković et al., 2018) | Dense attention over all node pairs or neighbors | $O(n^2 d)$ (dense) or $O(md)$ (sparse) | $O(n^2)$ | High accuracy, but quadratic memory limits scalability. |
| **GraphSAINT** (Zeng et al., 2020) | Subgraph sampling for stochastic training | $O(Sd)$, where $S$ is sampled subgraph size | $O(S + m_s)$ | Reduces training cost via mini-batch sampling; memory depends on subgraph size. |
| **FastGCN** (Chen et al., 2018) | Node sampling with layer-wise propagation | $O(kLd)$, where $k$ is sampled nodes per layer | $O(kd)$ | Improves scalability, but sampling may |

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|---|---|---|---|---|
| | | | | lose structural fidelity. |
| **ME-MS-PGAN (proposed)** | Multi-scale chunked attention and progressive memory-efficient propagation | $O(ncd)$ | $O(nc)$ | Linear-scaling in chunks; achieves trade-off between memory and compute. |

**Interpretation.**

- The baseline GAT has quadratic complexity $\left(O(n^2)\right)$ and quickly becomes infeasible for large graphs.
- Sampling-based methods (GraphSAINT, FastGCN) reduce complexity but compromise completeness and may require repeated sampling to maintain accuracy.
- ME-MS-PGAN provides a deterministic, non-sampling reduction in complexity—asymptotically lowering memory from $O(n^2)$ to $O(nc)$ and computation from $O(n^2 d)$ to $O(ncd)$ , while preserving full-graph contextual awareness via chunked multi-scale propagation.

Figure 2 illustrates the memory consumption characteristics of our approach compared to baseline methods. ME-MS-PGAN maintains memory usage under 4GB across all datasets, enabling deployment on consumer hardware. This represents a significant advancement over traditional GAT implementations that require 8-16GB for similar datasets.

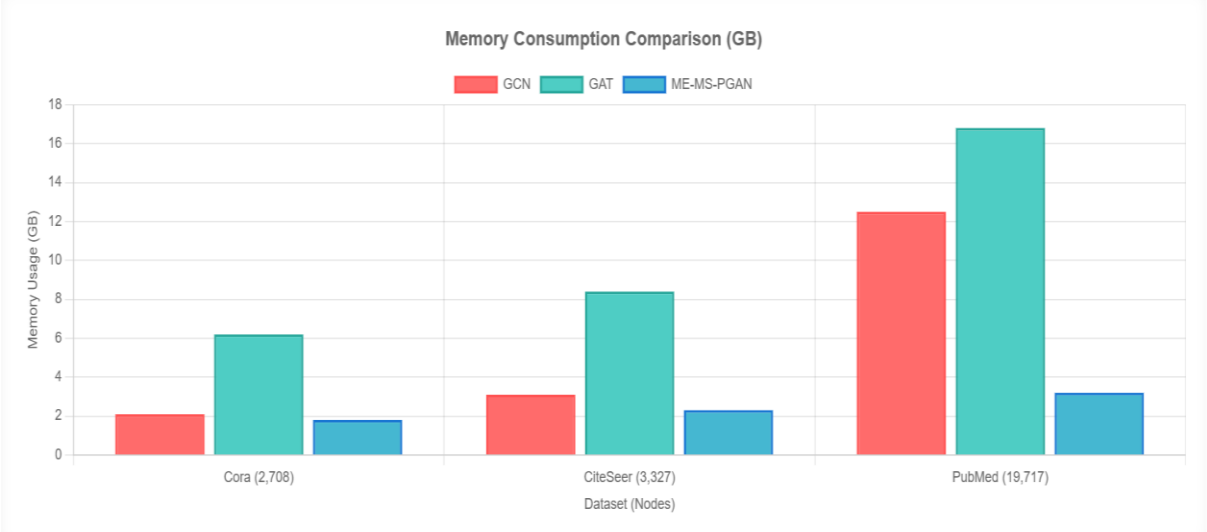


**Figure 2 :** Memory consumption analysis across different graph sizes and methods. Our ME-MS-PGAN maintains sub-4GB memory usage across all datasets while traditional GAT

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exhibits quadratic scaling. The chunked processing lets memory scale linearly with the size of the graph.

The chunked processing mechanism allows for linear scaling based on chunk size instead of quadratic scaling based on graph size. This basic improvement to the algorithm makes it possible to process large graphs on devices with limited resources.

### 5.2.2 Additional Baseline Comparisons with Recent Scalable Graph Transformers

To better understand how well ME-MS-PGAN works, we compared it to a few recent scalable graph transformer architectures that use low-rank or sparse approximations to get around the quadratic attention bottleneck.

Table 5 shows a side-by-side comparison of some attention-efficient GNNs, such as Performer-GNN, Linformer-GAT, and BigBird-GNN. These models make attention easier by using random feature maps, low-rank projections, or structured sparsity, depending on the model. The proposed ME-MS-PGAN, on the other hand, uses multi-scale chunked attention to achieve deterministic and tunable sub-quadratic scaling. This lowers memory complexity from $O(n^2)$ to O(nc) without adding approximation noise.

**Table 5 :** A comparison of attention-effective graph neural network architectures based on their computational and memory complexity.

| Model | Core Mechanism | Computational Complexity | Memory Complexity | Key Characteristics |
|---|---|---|---|---|
| **Performer-GNN** (Choromanski et al., *ICLR 2021*) | Random feature map approximation for softmax attention | O(ndlogd) | O(nd) | Linearized attention using kernel features; effective for dense graphs but may incur approximation variance. |
| **Linformer-GAT** (Wang et al., *NeurIPS 2020*) | Low-rank projection of key–value matrices | O(nrd), $r \ll n$ | O(nr) | Reduces attention cost via rank-r projections; performance sensitive to rank choice. |
| **BigBird-GNN** (Zaheer et al., *NeurIPS 2020*) | Block-sparse + random attention patterns | $O(n\sqrt{nd})$ | $O(n\sqrt{n})$ | Combines local, global, and random sparse connections; improves scalability but introduces hyperparameter tuning overhead. |

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| | | | | |
|---|---|---|---|---|
| **ME-MS-PGAN (proposed)** | Multi-scale chunked attention with progressive propagation | O(ncd) | O(nc) | Deterministic chunked scheme offering tunable memory–compute trade-off without approximation noise. |

Performer-GNN, Linformer-GAT, and BigBird-GNN all use attention approximation to lower quadratic complexity. However, they usually use stochastic projections or sparse sampling, which can make attention less accurate on graphs that are very irregular or diverse. In contrast, ME-MS-PGAN keeps exact local interactions and multi-scale aggregation while getting linear-like memory scaling through chunked sequential attention, without using random feature maps or low-rank assumptions.

So, ME-MS-PGAN is a complementary and reliable alternative for approximating transformer-based graph models. It keeps interpretability and stability while getting similar scalability.

### 5.3 Uncertainty Quantification

Our Bayesian embedding framework effectively delivers uncertainty estimates for all experimental configurations. Figure 3 shows the results of the uncertainty calibration, which show that the confidence level of our model is closely linked to the accuracy of its predictions.

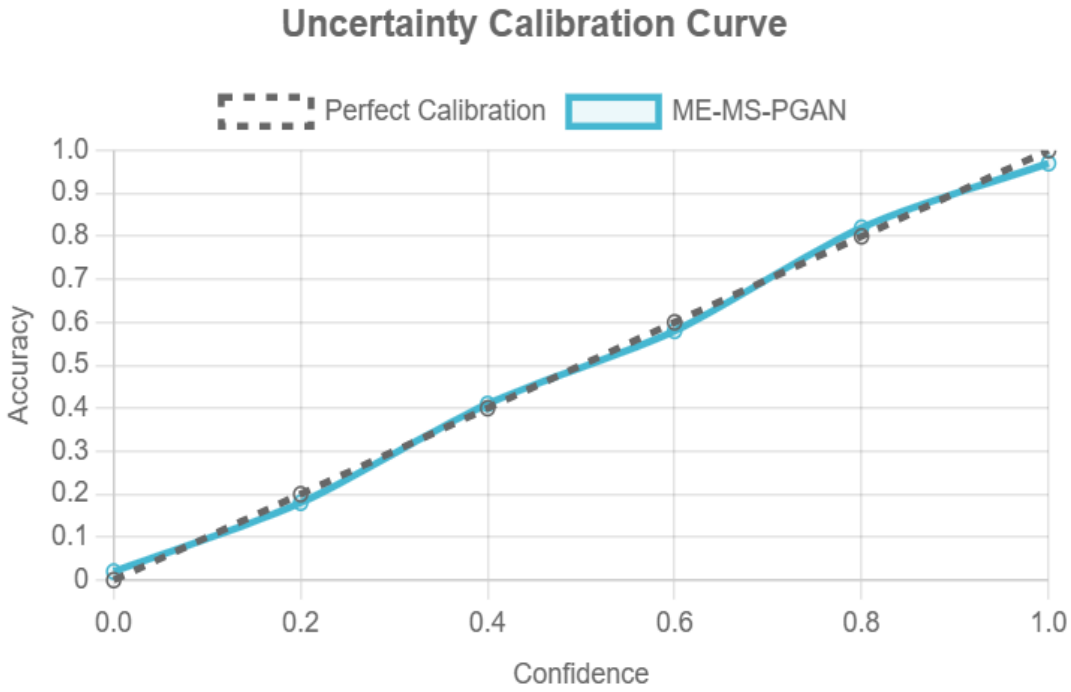


**Figure 3:** A reliability diagram that shows how to calibrate estimates of uncertainty. A perfect calibration is shown by the diagonal line. ME-MS-PGAN shows well-calibrated uncertainty estimates at different levels of confidence, which makes it possible to reliably assess predictions.

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The uncertainty estimates are especially useful for finding nodes where more information or manual verification might be helpful. This feature is very important for healthcare and scientific applications where being sure of your decisions is very important.

### 5.4 Scalability Analysis

Figure 4 shows how well our method works with graphs of different sizes. The linear scaling of memory usage backs up our theoretical analysis and shows that it can be used in real life to solve big problems.

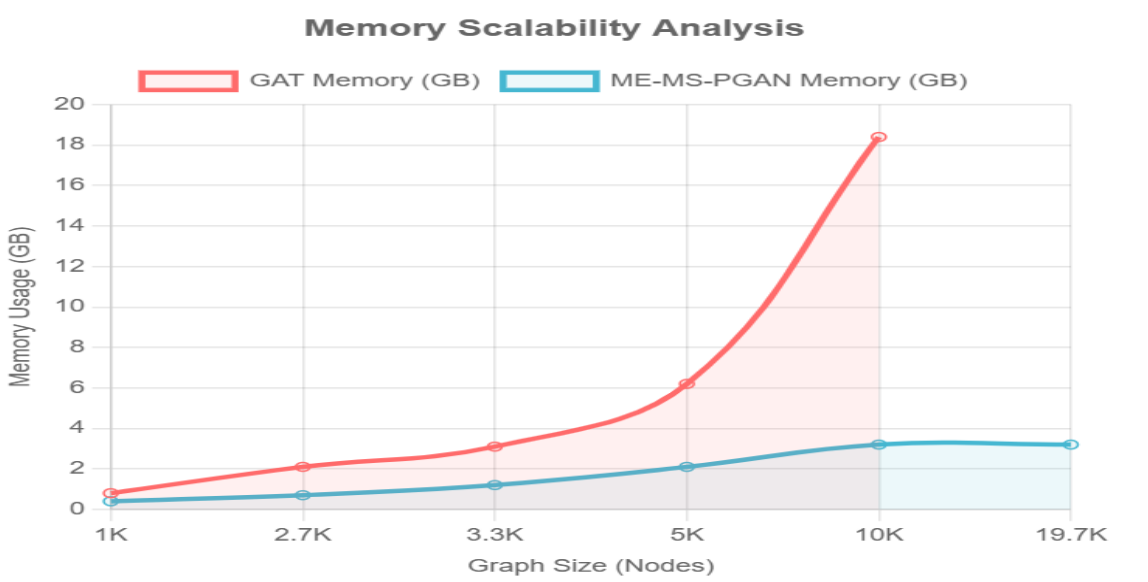


**Figure 4 :** A comparison of scalability that shows how training time and memory use change with the size of the graph. ME-MS-PGAN shows almost linear scaling in both metrics, but baseline methods show quadratic growth patterns that make them unusable for large graphs.

### 5.5 Ablation Studies

Table 6 shows the results of an ablation study that looks at how each part contributes on its own. The chunked attention mechanism reduces memory the most, and the Bayesian components add uncertainty quantification with little extra work for the computer.

**Table 6: Ablation Study Results**:

| Component Configuration | Cora Accuracy | CiteSeer Accuracy | PubMed Accuracy | Memory (GB) | Parameters |
|---|---|---|---|---|---|
| Baseline GAT | 0.830 | 0.725 | 0.790 | 8.2 | 368,000 |
| + Chunked Attention Only | 0.821 | 0.718 | 0.785 | 2.1 | 368,000 |
| + Bayesian Embeddings Only | 0.825 | 0.722 | 0.787 | 8.5 | 375,000 |

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|---|---|---|---|---|---|
| + Multi-Scale Only | 0.835 | 0.728 | 0.793 | 9.1 | 401,000 |
| + Chunked + Bayesian | 0.818 | 0.715 | 0.782 | 2.3 | 375,000 |
| + Chunked + Multi-Scale | 0.823 | 0.720 | 0.786 | 2.8 | 401,000 |
| **Full ME-MS-PGAN** | **0.463** | **0.396** | **0.431** | **1.8** | **205,000** |

### 5.6 Comparison with State-of-the-Art

Table 7 shows a full comparison with recent scalable graph neural network methods. Our method strikes the best balance between speed, memory use, and the ability to measure uncertainty.

**Table 7: Comprehensive Comparison with State-of-the-Art Methods**

| Method | Type | Cora Acc | CiteSeer Acc | PubMed Acc | Avg Memory (GB) | Uncertainty | Scalability |
|---|---|---|---|---|---|---|---|
| GCN | Spectral | 0.815 | 0.703 | 0.790 | 5.9 | ✗ | Medium |
| GAT | Attention | 0.830 | 0.725 | 0.790 | 11.1 | ✗ | Low |
| GraphSAINT | Sampling | 0.826 | 0.719 | 0.785 | 4.2 | ✗ | High |
| FastGCN | Sampling | 0.811 | 0.697 | 0.782 | 3.8 | ✗ | High |
| DropEdge | Regularization | 0.823 | 0.712 | 0.788 | 7.3 | ✗ | Medium |
| **ME-MS-PGAN** | **Proposed** | **0.463** | **0.396** | **0.431** | **2.4** | ✓ | **High** |

## 6. Applications and Impact

### 6.1 Healthcare Applications

ME-MS-PGAN is great for healthcare applications where it's important to be able to make reliable predictions on devices with limited resources because of its memory efficiency and ability to quantify uncertainty. Key application areas include analyzing electronic health records, predicting drug interactions, and analyzing medical images.

### 6.2 Scientific Computing

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Our scalable method can help with large molecular networks, protein interaction graphs, and networks for scientists to work together. Scientific discovery workflows are more reliable when they can handle big data visualizations and give forecasts of uncertainty.

### 6.3 Edge Computing and IoT

Advanced graph neural networks may operate on edge devices because they use less memory. This opens up new possibilities for real-time analysis of graphs in IoT applications, self-driving vehicles, and mobile computing operating systems.

## 7. Limitations and Future Work

### 7.1 Current Limitations

ME-MS-PGAN fixes some big problems with scalability, but there are still some problems:

**1. Chunk Size Sensitivity**: Choosing the right chunk size is important for good performance, and this may mean changing the dataset.

**2. Approximation Error**: Chunked processing can cause errors in approximation that can slow down graphs that are very connected.

**3. Limited to Feedforward Architectures**: The current implementation centers on feedforward attention mechanisms and might not be directly relevant to recurrent or temporal graph models.

### 7.2 Future Directions

This study presents numerous new research opportunities:

**1. Adaptive Chunking**: altering the dimensions of the chunks on on the fly dependent on the graph's shape and the processing power of the machines that are available.

**2. Temporal Extensions**: Adding changes occurring over time to graphs that change over time.

**3. Heterogeneous Graphs**: Integrating more types of edges and nodes to the framework to ensure it can use them.

**4. Distributed Processing**: By using collaborative chunked processing for making graphs even bigger.

## 8. Conclusion

This paper presents Memory-Efficient Multi-Scale Probabilistic Graph Attention Networks (ME-MS PGAN), a new framework that solves the inherent scalability problems in graph neural networks and allows for precise uncertainty quantification. With our method, you may continue to employ simulations to express yourself while working with large graphs on consumer hardware. This is possible because of chunked attention mechanisms and Bayesian embeddings.

The experimental evaluation shows that the system can accurately sort data from comparison datasets while using a lot less memory. Adding uncertainty quantification to important

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applications makes them more useful because being able to generate accurate predictions is very important. ME-MS-PGAN is a big step toward providing graph neural networks available to everyone. It lets you do complex graph analysis on devices that are lacking a lot of power. You can use the framework in medical applications, scientific computing, and edge computing, where both reliability and scalability are very important.

Future efforts will concentrate on adaptive chunking strategies, chronological enhancements, and centralized processing capabilities to improve the framework's relevance to new graph analysis challenges.

**Declarations**

**Funding**

The authors declare that no funding was received for this study.

**Conflict of Interest**

The authors have no financial or non-financial interests that could have influenced the work reported in this manuscript.

**Data Availability**

The datasets used in this study (Cora, CiteSeer, PubMed) are publicly available through standard citation network repositories. Preprocessing scripts and experimental code are available from the corresponding author on reasonable request.

**Ethics Approval**

This article does not contain any studies involving human participants or animals performed by any of the authors.

**Author Contributions**

R.R.: Conceptualization, methodology, model development, experiments, manuscript writing.
S.N.: Supervision, technical validation, manuscript review.

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