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T-B-I OPEN SETS AND ITS PROPERTIES

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Abstract. Ideal and Ideal convergence (I-convergence) through topologic space (X t) is considered one of the major topics in topology, which was initiated in 1933 and continues to the present day, with several authors further developing it. In this paper, we further develop the concept to be defined on an operator topological space and set the study as in the proposed idea of T-I and T-b-I-convergence along with the assumption of T-b-open as well as T-b-open blends.

Keywords. T-I-open set, T-b-I-open set, T-I-convergence, T-b-I-convergence, T-I-sequentially open T-b-I-sequentially open, T-I-sequentially space, and T-b-I-sequentially space.

1. Introduction

The construct ideal was first pronounced by concept of ideal Kuratowski in 1933 [1]. It refers to the collection or group of elements I through X space that it recognizes: (1) I is a non-empty collection of elements, (2) the union of any two sets of I is in it, and (3) the hereditary property under inclusion (i.e., for any subset of a set in I is also in I). The concept of ideal has been utilized in several papers, hand in hand with other notions of general topology [2], [3], [4], [5], [6], [7], [8], [9]. An Example of these notions is what Andrijevi introduced in 1996 about b-open circles in a topologic space. Asub-group A in relation to topologic space can be refered to as b-open if it is moderated within the group of its union of semi-open as well as pre-open subsets [10]. Aysegul and Gulhan (2005) presented and idea regarding b-I-open circles and the continuation through b-I-open sets.

On the other hand, Zhou et al., in early 2020, utilize the concept of ideal I of the natural numbers ℕ and extend it to the notion of I-convergence [3]. Also, Granados, in late 2020, extended Zhou et al.'s work to b-I-convergence on b-open sets [12].

In this paper, we extend the above-mentioned notions by utilizing the idea of operator topologic space and view the T-I-open, T-b-I-open, T-I-convergence, and T-b-I-convergence on the T-open set, as well as their properties. Additionally, we introduce further properties about T-I-sequentially and T-b-I-sequentially.

2. T-b-I- convergence Definitions and Properties

W The paper first mention some important constructs and definitions first introduce some:

Explanation (2.1):[3] A non-trivial ideal $I \subseteq \mathbb{N}$ can be admitted $\Longleftrightarrow$ $\{\{n\}: n \in \mathbb{N}\} \subseteq I.$

Explanation (2.2):[1] assume (X, τ) refers to topologic space the operator T refers to function T : P (x) → $P(x)$ s..t ∀ $A \in \tau, A \subseteq T(A)$. Attention that the triple (X, τ, T) can be called the operator topologic space.

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Explanation (2.3): Assume that L (X, τ, T) can be an operator topological space $A \subset X$ is thought to be T-b-open, and for $x \in A$, there should be $V \in \tau$ in a way that $x \in V \subseteq \mathrm{T}(V) \subseteq A \subseteq cl(int(A)) \cup int(cl(A))$.

Explanation (2.4): Assume L (X, τ, T) can be term as operator topologic $A \subset X$ while $x \in X$.Then set A can be called

1. A b-neighborhood to $x \Leftrightarrow \exists\, B$ a b-open set $\ni x \in B \subset$.
2. A T-neighborhood to $x \Leftrightarrow \exists\, B$ a T-open set $\ni x \in B \subset A$.
3. A T-b-neighborhood to $x \Leftrightarrow \exists\, B$ a T-b-open set $\ni x \in B \subset A$.

Explanation (2.5): Assume $(\mathrm{X}, \tau, \mathrm{T})$ serve the role of operator topologic space and $A \subset X$. Then $the\ A$ can be said to be the T-b-I-open, and if $x \in V \subseteq T(V) \subseteq A \subseteq cl^*(int(A)) \cup int(cl^*(A))$.

Explanation (2.6) : [3] Assume (X, τ) can be a topologic space and $I \subseteq \mathbb{N}$ to serve as ideal. This sequence can be identified $\{x_n\}$, $n \in \mathbb{N}$ as I-convergent through a point x in the classic X, if it happens to exist a neighborhood V of x where $A_v = \{n \in \mathbb{N}: x_n \notin V\} \in I$, and is signified as $I\text{-}\lim_{n\to\infty} x_n = x$.

Explanation (2.7) : [12] Assume (X, τ) can be taken topologic space and $I \subseteq \mathbb{N}$ found to be ideal. This sequence $\{x_n\}$, $n \in \mathbb{N}$ is seen as b-I-convergent through a point x in X, if the existence of b-neighborhood V of x with $A_v = \{n \in \mathbb{N}: x_n \notin V\} \in I$, and is represented as b-I-$\lim_{n\to\infty} x_n = x$.

Explanation (2.8) : $(\mathrm{X}, \tau, \mathrm{T}, I)$ can be found ideal operator topologic space and $I \subseteq \mathbb{N}$. A sequence $\{x_n\}$, $n \in \mathbb{N}$ is considered as T-I-convergent in a point x in X, if it exists a T-neighborhood V of x with $A_v = \{n \in \mathbb{N}: x_n \notin V\} \in I$, and is symbolized as $I\text{-}\lim_{n\to\infty} x_n = x$.

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Explanation (2.9): $(\mathrm{X}, \tau, \mathrm{T}, I))$ is an ideal operator topologic space and $I \subseteq \mathbb{N}$. A classification $\{x_n\}$, $n \in \mathbb{N}$ is considered as T-b-I convergent to a certain point x in X, if it exists a T-b-neighborhood V of x with $A_v = \{n \in \mathbb{N}: x_n \notin V\} \in I$, and is signified as T-b-I-$\lim_{n\to\infty} x_n = x$.

Lemma (2.1): [13] Each T-open category is open set.

Lemma (2.2): [10] each open category is b-open set.

Lemma (2.3): Assume (X, τ, T, I) is an ideal operator topologic space, similarly, the following is considered,

1. Each T-b open set be T-open set.

2. Each T-b -open set be open set.

3. Each T-b -open set is b-open set.

Proof: (1) Assume A be a T-b-open set, then by extension (2. 3.), A will be T-open. Proof of (2) and (3) are insignificant.

Lemma (2.4): Let (X, τ, T, I) is an ideal operator topologic space, then the following are accepted:

1. T-I-convergence suggests I-convergence

2. T-I-convergence infers b-I-convergence

3. T-b-I-convergence indicates T-I-convergence

4. T-b-I-convergence means I-convergence

5. T-b-I-convergence considers b-I-convergence

Proof (1): Assume T-I-convergence, signifies A is a T-open set of (X, τ, T, I), with T-I-convergence series $\{x_n\}$ to indicate x, like that $\{n \in \mathbb{N}: x_n \notin A\} \in I$. by Lemma (2.1) A can be open set. Then $\{x_n\}$ can be I-convergence by Definition (2.6).

International Journal of Applied Mathematics
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Similar proof for (2), (3), (4) and (5) through Lemma (2.2) as well as Lemma (2.3), respectively.

Explanation (2.10): Assume $(\mathrm{X}, \tau, \mathrm{T})$ is an operator topologic space and an ideal $I \subseteq \mathbb{N}$. Then:

1) $B \subset X$ is called T-I-closed if for each sequence $\{x_n\}$ in B with T-I- $\lim_{n\to\infty} x_n = x \in X$. Then $x \in B$.
2) $B \subset X$ is termed T-b-I-closed if for every category $\{x_n\}$ in B with T-b-I-$\lim_{n\to\infty} x_n = x \in X$. Then $x \in B$.
3) $B \subset X$ is called T-b-I-open if $X - B$ T-b-I-closed.

Lemma (2.5): Assume (X, τ, T) is an operator topologic space and an ideal $I \subset .N$ The following then can satisfy

1) Assuming sequence $\{x_n\}$ can be T – I – convergence at point $x \in X$. Then the following sequence via X with $n \in I$ can be T-I-convergence to maintain the same point x.
2) Assuming sequence $\{x_n\}$ can be T-b-I-convergence at point $x \in X$. Then the subsequent sequence in X via $n \in I$ can be T-b-I-convergence at the same point x.

Proof: (1) Assuming $\{y_n\}$ as in the following sequence in X, s.t., $x_n \neq y_n$, $n \in I$. For every T-neighborhood A of x with

$$\{n \in \mathbb{N}: y_n \notin A\} \subseteq \{n \in \mathbb{N}: x_n \neq y_n\} \cup \{n \in \mathbb{N}: x_n \notin A\} \in I$$

$$\Longrightarrow \{n \in \mathbb{N}: y_n \notin A\} \in I$$

As a result, by extension [2.8] $\{y_n\}$ can be T-I-convergence at the same point x Comparable proof for (2) through the use of the meaning (2.9).

International Journal of Applied Mathematics
Volume 38 No. 10s, 2025
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Lemma (2.6): Assume (X, τ, T) is an operator topologic space, as well as an Ideal $I \subseteq J \subset \mathbb{N}$. Then this can be considered to satisfy:

1) Assuming the sequence $\{x_n\}$ can be T-I-convergence at point $x \in X$. At this point, it is T-J- merging to x.
2) Assuming the sequence $\{x_n\}$ can be T-b-I-convergence at a point $x \in X$. At this point, it can be T-b-J- merging to x.

Proof: (1) Assume $\{x_n\}$ can be a T-I-convergence at point $x \in X$. By Extension (2.8), for every T-neighborhood A of x

$$\{n \in \mathbb{N}: x_n \notin A\} \in I \subseteq J$$

Infers $\{n \in \mathbb{N}: x_n \notin A\} \in J$, as a result, $\{x_n\}$ can be T-J- convergence to x Comparable proof for (2) by the use of explanation (2.9).

Lemma (2.7): Assume (X, τ, T)can be an operator topologic space, as well as two Ideals $I \subseteq J \subset \mathbb{N}$. Then below can be satisfied,

1) If $A \subseteq X$ can be T-I-open. Then it can be T-J-open.
2) If $A \subseteq X$ can be T-b-I-open. Then it can be T-b-J-open.

Proof: (1) Supposing $A \subseteq X$ can be T-I-open, consequently $X - A$ T-I-closed, By Explanation (2.11) for e T-I-convergence of sequence $\{x_n\}$ in $X - A$ at point x, Then $x \in X - A$, by Lemma (2.6) (1) $\{x_n\}$ it can also T-J-convergence as at $X - A$ to x infers $X - A$ can be T-J-closed, as a result A can be T-J-open. Comparable proof for (2) through the use of Explanation: (2.10) (1) and through Lemma (2.6) (2).

Consequence (2.7): Assume (X, τ, T) can be an operator topologic space, and the two Ideals $I \subseteq J \subset \mathbb{N}$. Then, thus; T-b-I-sequentially, then T-b-I-is sequential.

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Lemma (2.7): Assume (X, τ, T) can be an operator topologic space, as an Ideals $I \subset \mathbb{N}$, and can be $A \subseteq X$ Then these are equivalent.

1) A can be T-b-I- open
2) For every sequence $\{x_n\}$ in X with can be T-b-I- convergence to sum $x \in A$ where the $\{n \in \mathbb{N}: x_n \in A\} \notin I$
3) For every sequence $\{x_n\}$ in X with can be T-b-I- convergence to sum $x \in A$, where the $\{n \in \mathbb{N}: x_n \in A\} = \omega$.

Proof: (1)⟹ (2): Assume A is a T-b-I-open set as in X. And assume $\{x_n\}$ can be a T-I- convergence in the sequence as in X to a certain point $x \in A$ Let's reflect the contrary supposition, By considering $N_0 = \{n \in \mathbb{N}: x_n \in A\} \in I$, then, thus; $N_0 \neq \mathbb{N}$ and $A \neq X$. Let $x_0 \in X - A$. The sequence can be defined $\{y_n\}$ in X, s.t., $y_n = x_0$ for $n \in N_0$ similar., $y_n = x_n$ and $n \notin N_0$. To Lemma (2.5) [2], the following sequence $\{y_n\}$ can be T-b-I-convergence in x. Observe that $X - A$ can be a T-b-I-closed similar $\{y_n\}$ can be T-b-I-convergence to the sequence as $X - A$ By Extension (2.10)(2) $x \in X - A$ and this can be contradiction!, Thus, $N_0 \in I$.

(1)⟹ (2): It continues from Explanation (2.1) the permissible ideal.

Then to indicate the (3) ⟹ (1) Let presume that A cannot be T-b-I-open thus; $X - A$ can be T-b-I-closed, which suggest that there can be $\{x_n\}$ in $X - A$ can be T-b-I- convergence to $x \in A$ as this can contradicts (3) owing to the fact that $x_n \in A$.

3. T-b-I- successively open as in T-b-I- sequentially in space

Explanation (3.1): Assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space, and to $A \subset X$. Then A said to be T-b-I-successively open ⟺ no sequence as in $X - A$

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has to be T-b-I-limit thus A, that is., a arrangement can hardly be T-b-I-converge out of T-b-I-successfully closed set.

Explanation (3.2): Assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space thus, an ideal $I \subseteq \mathbb{N}$. Now A can be called T-b-I-sequentially space if every T-b-I-closed the set in X can be T-b-closed.

Explanation (3.3): Assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space. thus, in space X can be T-b-I-sequentially, For each set A can be T-b-open $\Leftrightarrow$ it can be T-b-I-sequentially sub-space of X.

Theorem (3.1): Assuming $(\mathrm{X}, \tau, \mathrm{T})$ seems an operator topologic space. Each T-b-I-sequential space can be genetic with regards to this T-b-I-open (T-b-I-closed).

Proof: Assume that X can be a T-b-I-sequential in Space. Assume A can be a T-b-I-open sets as in X It is obvious that A can be T-b-open as in X. As can be shown, that T-b A can be-I-sequential open. Assume V can be T-b-I-open as in A. Then it Claims that V can be T-b-open as in X it can be indicated V can be T-b-I-open as in X: It can be assumed that there can be y in $X - A$ consider an arbitrary $x \in V$ as well as $\{x_n\}$ can be a T-b-I- convergence sequence as in X to a point x. Since, A can be T-b-open in X and $x \in n \ A$ set $\{n \in \mathbb{N}: x_n \notin A\} \in I$. Determined sequence $\{y_n\}$ in X such that $y_n = x_n$ to $n \in A$ and $y_n = y$ for $n \notin A$. According to Lemma (2.5) (2), the set $\{y_n\}$ can be T-b-I-convergence in x. while $\{n \in \mathbb{N}: x_n \notin V\}$=$\{n \in \mathbb{N}: y_n \notin V\}$, Lemma (2.7), V can be T-b-I-open as in X, by Explanation in (3.3) A can be T-b-I-sequential open (T-b-I-sequential sub-space of X).

Likewise, think that X can be T-b-I-sequential Space , Assume that A can be a T-b-I-closed set of X denotes that A can be T-b-closed set as in X. For each T-b-I-closed subdivision $U$of A set in X, U needs to indicate can be T-b-closed set as in X. while X can be a T-b-I-sequential space, by Extension (3.2) it is sufficient to say that U can be T-b-I-closed as in X which states that T-b-closed set. presume that $\{x_n\}$ can be a T-b-I- merging sequence through a specific point $x \in X$ $x_n \in U$. By Explanation (2.10). When A can be T- b-closed, it takes that $x \in A$ and to $x \in U$, as a result, U can be a b-I-closed set of A.

Consequence (3.1): Every T-I-sequential space can be hereditary with regards to T-b-I-open (T-I-closed).

Theorem (3.2): Assume that $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space. T-b-I-sequential spaces are well-maintained by topologic sums.

Proof: Assume $\{Y_{\boldsymbol{\kappa}}\}$ can be a member T-b-I- sequential spaces and where $\check{X}$ can be the topologic sum of this family. Then $\check{X}$can be T-b-I- sequential space. Let assume A can be T-b-I-closed set of $\check{X}$, By Extension of meaning (3.2) it sufficient to indicate that A can be T-b-closed set of $\check{X}$. While, $\{Y_{\boldsymbol{\kappa}}\}$ is a member of a T-b-I- sequential meaning each $\check{X}$ denotes $Y_{\boldsymbol{\kappa}}$ can be T-b- closed set of $\check{X}$ for every κ, so the connection $A \cap Y_{\boldsymbol{\kappa}}$ can be T-b-I- closed set of $\check{X}$. It becomes obvious that $A \cap Y_{\boldsymbol{\kappa}} \subseteq Y_{\boldsymbol{\kappa}}$ can be T-b-I- closed within $Y_{\boldsymbol{\kappa}}$.Assuming ($\{Y_{\boldsymbol{\kappa}}\}$ can be a member of a T-b-I- sequential spaces) we indicate that $A \cap Y_{\boldsymbol{\kappa}}$ can be T-b-closed within $Y_{\boldsymbol{\kappa}}$. Through the meaning of $\check{X}$ denotes $A \cap Y_{\boldsymbol{\kappa}}$ is T-b-closed set within $\check{X}$ and A can be T-b-closed set within $\check{X}$. Therefore, by the meaning of (3.2) the summation of the member of T-b-I-sequential spaces can also be T-b-I-sequential space.

Corollary (3.2): A T-I- sequential spaces are well-preserved by topologic summations.

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Corollary (3.3): Let assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space. Let assume $\{Y_{\kappa}\}$ can be the member of T-b-I-open sets

1) $\bigcup_{\kappa} Y_{\kappa}$ of T-b-I-open sets can also be, T-b-I-open set.
2) $\bigcup_{\kappa} Y_{\kappa}$ of T-I- open sets can also be, T-I-open set.
3) The unlimited many $\bigcap_{\kappa} Y_{\kappa}$ of successively T-b-I- open sets can also be, successively T-b-I-open set.
4) The unlimited many $\bigcap_{\kappa} Y_{\kappa}$ of successively T-I- open sets can also be, successively T-I-open set.

Explanation (3.4): [3] Let assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space as well as an ideal $I \subseteq \mathbb{N}$. A sequence $\{x_n\}$ in X can be I-finally in A if $\exists\ B \in I$ s.t. $\{\forall n \in \mathbb{N} - B: x_n \in\ A\} \in I$.

Proposition (3.1): Let assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space as I can be a maximal ideal $I \subseteq \mathbb{N}$. $A \subseteq X$, A can be T-b-I- open $\Leftrightarrow$ for every $\{x_n\}$ in X, with $x_n \longrightarrow^{TbI} x\ \in A$ is I-finally in A.

Proof: ($\Leftrightarrow$) Let assume A can be T-b-I- open as $\{x_n\}$ can be a T-b-I-convergence sequence to a certain point $x \in A$. $\Leftrightarrow$Since, I can be maximal the ideal there in existence $J = \{n \in \mathbb{N}: x_n\ \notin A\} \in I$, $\Leftrightarrow$ by Lemma (2.7) $\forall n, \{n \in \mathbb{N} - J: x_n\ \in A\}$, $\Leftrightarrow$ therefore in the meaning (3.4), A can be I-eventually in A.

Theorem (3.3): Let assume $(\mathrm{X}, \tau, \mathrm{T})$ can be an operator topologic space, as an ideal $I \subseteq \mathbb{N}$. The connection of the two T-b-I-open sets can be T-b-I-open.

Proof: Let assume A and B can be two T-b-I-open sets as in X then, there is the need to indicate that $A \cap B$ can be T-b-I-open sets as in X.

It is sufficient to indicate that each sequence $\{x_n\}$ in X, with $x_n \longrightarrow^{TbI} x \in A \cap B$ is I-eventually in $A \cap B$, and by Suggestion (3.1) $A \cap B$ can be T-b-I-open sets as in X.

Supposing that J and K can be a sub-set of the ideal I in a way that for every n $\{n \in \mathbb{N} - J: x_n \in A\}$ as well as $\{n \in \mathbb{N} - K: x_n \in B\}$. Subsequently $A \cup B \in I$ denotes that every n $\{n \in \mathbb{N} - (J \cup K): x_n \in A \cap B\}$. Therefore, $\{x_n\}$ can be I-finally in $A \cap B$. By Suggestion (3.1) $A \cap B$ can be T-b-I-open sets in X.

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