International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

ODD SUN-FREE TRIANGULATED GRAPHS ARE *S*-PERFECT

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Abstract

For a graph G with the vertex set $V(G)$ and the edge set $E(G)$ and a star subgraph S of G, let $\alpha S(G)$ be the maximum number of vertices in G such that no two of them are in the same star subgraph S and $\theta S(G)$ be the minimum number of star sub-graph S that cover the vertices of G. A graph G is called S-perfect if for every induced subgraph H of G, $\alpha S(H) = \theta S(H)$. Motivated by perfect graphs discovered by Berge, Ravindra in-troduced S-perfect graphs. In this paper we prove that a trian-gulated graph is S-perfect if and only if G is odd sun-free. This result leads to a conjecture which if proved is a structural char-acterization of S-perfect graphs in terms of forbidden subgraphs.

Math. Subject Classification: 05C17, 05C75.

Keywords and Phrases: S-perfect graphs; Extended sun graph; Sun graph

1 Introduction

All graphs $G = (V, E)$ in this paper are finite and simple with the vertex set $V(G)$ and the edge set $E(G)$. Inspired by perfect graphs due to Berge [1], the concept of F-perfect graphs was introduced by Ravindra in 2011 [2]. For the terminology and notations which are not defined here, we refer the readers to West [3].

A graph G is said to be *triangulated* if G has no induced cycle of length at least 4. If $D \subseteq V(G)$, the subgraph of G induced by D is obtained by deleting the vertices of $V(G) - D$ and is denoted by $G[D]$. Given a graph H, we say that a graph G is H-free, if G does not contain an induced subgraph isomorphic to H. Given a family $\{H1, H2, ...\}$ of graphs, we say that G is $(H1, H2, ...)$-free if G is Hi – free, for every $i \geq 1$.

Received: July 28, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

A *star* is a tree consisting of one vertex adjacent to all other vertices. Note that $K1$ and $K2$ are stars.

A vertex v in G is called an *simplicial vertex* if v belongs to only one maximal clique of G. A clique of G is *free* if it contains at least one simplicial vertex. A triangle is *free triangle* if it has only one simplicial vertex.

Let G be a graph. An S-*cover* of G is a family of stars contained in G such that every vertex of G is in one of the stars. The *S-covering number* of G is the minimum cardinality of a S-cover and is denoted by$\theta S(G)$. A *θS-cover* of G is an S-cover of G containing $\theta S(G)$ stars.

A set $T \subseteq V(G)$ is an *S-independent* set of G if no two vertices of T are contained in the same star in G (equivalently, any two ver-tices in T are at a distance at least 3). The maximum cardinality of an S-independent set is called the *S-independence number* of G and is de-noted by $\alpha S(G)$. An *αS- independent set* of G is a S-independent set with $\alpha S(G)$ vertices.

It is easy to see that the graph parameters $\alpha S(G)$ and $\theta S(G)$ both satisfy the additive property, that is

- $\alpha S(k\ i{=}1 Gi) = k\ i{=}1 \alpha S(Gi)$, and
- $\theta S(k\ i{=}1 Gi) = k\ i{=}1 \theta S(Gi)$.

It is also interesting to observe that the parameters $\alpha(G)$ and $\theta(G)$ of Berge perfect graphs satisfy monotone property, that is if H is an induced subgraph of G, then $\alpha(H) \leq \alpha(G)$ and $\theta(H) \leq \theta(G)$. How-ever, the parameters $\alpha S(G)$ and $\theta S(G)$ do not satisfy monotone prop-erty. That is, if H is an induced subgraph of G, $\alpha S(H)$ (or $\theta S(H)$) may exceed $\alpha S(G)$ (or $\theta S(G)$). For example if $G = K1,n,\ n \geq 2$ with central vertex $v0$, then $\alpha S(K1,n) = 1$ and $\theta S(K1,n) = 1$. However$\alpha S(K1,n - v0) = n$ and $\theta S(K1,n - v0) = n,\ n \geq 2$.

For any graph G, $\theta S(G) \geq \alpha S(G).\theta S(H) \leq \alpha S(H)$, then $\alpha S(H) = \theta S(H)$.

So, if for some graph H,

Now we examine the graphs with the property that $\alpha S(H) = \theta S(H)$ for every induced subgraph H of G and call such graphs S-perfect graphs. We formally present the definition of *S-perfect graphs*.

Definition 1.1. *A graph is called S-perfect if $\theta S(H) = \alpha S(H)$, for every induced subgraph H of G.*

A graph G is said to be *minimal S-imperfect* if (i) $\theta S(G) \neq \alpha S(G)$ and (ii) $G - v$ is S-perfect for every vertex v of G. Every minimal S-imperfect graph is a connected graph, since a graph G is S-perfect if and only if every component of G is S-perfect. If G is not S-perfect, now onwards we may assume that G is minimal S-imperfect.

Since S-perfectness is a hereditary property, one expects a forbidden subgraph characterization for S-perfect graphs. We realize this in this paper and prove a characterization theorem for triangulated S-perfect graphs.

Received: July 28, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The property that the parameters αS and θS do not satisfy monotone property causes some difficulty in some of the results related to their equality.

Before we state the characterization theorem for triangulated S-

perfect graphs, we define *sun graphs*.

Definition 1.2. *Let H be a graph with a Hamiltonian cycle* $C = v1v2 \dots vkv1$. *Let Ai,* $1 \le i \le k$ *be mutually disjoint complete graphs such that* $|Ai| \ge 1$, $\forall i$ *such that* $1 \le i \le k$. *Let H*1 *be a graph constructed from H such that every vertex in Ai is adjacent to vi and vi*+1 *and all uis in Ai are simplicial in H*1 *(i's are taken modulo k). H*1 *is called k-extended sun. H*1 *is an extended odd (even) sun if k is odd (even). If* $|Ai| = 1$, *for all i then H*1 *is k-sun graph. H*1 *is an odd (even) sun if k is odd (even).*

Obviously, a sun is an extended sun. A 3-sun is contained in every *k*-

extendedsun. For example in Figure 1 we see [*{{A*1*}, v*1*, v*2*, v*5*, v*4*, {A*5*}}*]=3-sun.

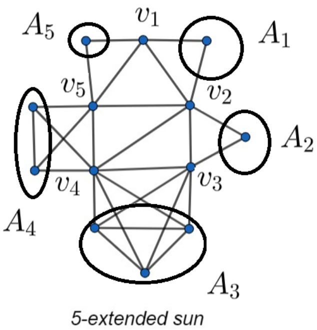

Figure 1: Examples of 5-extended sun and 5-sun graph

The goal of this paper is to prove the following result.

Theorem 2.2 (Characterization Theorem for Triangulated *S*-perfect graphs). A triangulated graph G is S-perfect if and only if G is odd sun-free.

The theorem is a min-max theorem for triangulated S-perfect graphs. For any graph G, the central vertices of stars in S-cover of G is a dominating set of G. The minimum parameter $\theta S(G)$ is essentially the domination number, $\gamma(G)$ of G, which has many applications like surveillance, controlling and monitoring. The maximum parameter $\alpha S(G)$ have been extensively studied in various contexts with different termi-nology. For example, S-independent sets are studied in [4], where an S-independent set in G corresponds to a color class in a $(k, 3)$-coloring of G. A study on characterization of star-perfect graphs, where all the stars in a star-cover of G are essentially induced, is also done in [5] and some properties of star-perfect graphs were studied in [6].

Received: July 28, 2025

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Our characterization of triangulated S-perfect graphs goes parallel to triangulated neighbourhood perfect graphs. The notion of neigh-bourhood number which was introduced by Sampathkumar and Neer-alagi [7]. Lehel and Tuza defined neighbourhood perfect graphs and characterized triangulated neighbourhood perfect graphs [8]. Though seemingly neighbourhood perfect graphs and S-perfect graphs appear different (For example: $C6k$+2 and $C6k$+4, $k \geq 1$ are neighbourhood perfect but not S-perfect and $C6k$+3, $k \geq 1$ are S-perfect but not neigh-bourhood perfect), surprisingly they are same for triangulated graphs (Theorem 2.2).

2 Results and Discussions

We use the following theorem and lemmas in proving the main theorem which characterizes triangulated S-perfect graphs.

Theorem 2.1. *[5] A graph G is star-perfect if and only if G is* ($C3$, $C3k$+1, $C3k$+2)*-free, $k \geq 1$.*

Lemma 2.1. *[5] Let k be any positive integer. Then,*

(i) $\theta(P) = k$ *and* $\alpha(P) = k$.

$sk3$ $sk3$

(ii) Pk is star-perfect.

(iii) The disjoint union of paths is a star-perfect graph.

The proofs of this Lemmas 2.2, 2.3, 2.4 are similar to as shown in [5].

Lemma 2.2. *Any cycle of length* $3k$, $k \geq 1$ *is S-perfect.*

Lemma 2.3. *Any cycle of length* $3k + 1$ *or* $3k + 2$, $k \geq 1$ *is minimal S-imperfect.*

Our next goal is to show that a minimal S-imperfect graph is a block.

Lemma 2.4. *If G is minimal S-imperfect graph, then G is a block.*

Lemma 2.5. *[9,10] Let G be a triangulated graph, then G has a sim-plicial vertex.*

Lemma 2.6. *If G is minimal S-imperfect, then* $\theta S(G) = \alpha S(G) + 1$.

Proof. Since G is triangulated, by Lemma 2.5, G has a simplicial vertex v. If $\alpha S(G) = 1$, then $\theta S(G) \neq 1$ since $\theta S(G) \neq \alpha S(G)$. There-fore $\theta S(G) \geq 2$. We observe that $\alpha S(G - v) = 1$. (A) If not, there exist $v1$, $v2$ in $V(G-v)$ such that $\{v1, v2\}$ is S-independent in $G - v$. Since $\alpha S(G) = 1$, $v \leftrightarrow \{v1, v2\}$. Then $v1 \leftrightarrow v2$, since v is simplicial in G, a contradiction. Since $\theta S(G) \geq 2$, $\theta S(G) \geq \alpha S(G)+1$. Also $\theta S(G) \leq \theta S(G - v) + 1$, since a S-cover of G contains at least$\theta S(G - v) + 1$ stars. Then $\theta S(G) \leq \alpha S(G - v) + 1 = \alpha S(G) + 1$, implying $\theta S(G) = \alpha S(G) + 1$, so the lemma is true if $\alpha S(G) = 1$. If $\alpha S(G) \geq 2$, let $T = \{v1, v2, . . . , vk\}$, $k \geq 2$ be an αS-independent set of $G - v$. If $k = 1$, then $\alpha(G - v) = 1$ and so $\theta S(G - v) = 1$. Then $\theta S(G) \leq \theta S(G - v) + 1 = \alpha S(G - v) + 1 = 2$, by (A). That is $\theta S(G) = \alpha S(G)$, a contradiction to G being minimal S-imperfect. Therefore $k \geq 2$.

Received: July 28, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

If T is not an S-independent set in G, then then there are two ver-tices $v1$, $v2$ in T such that $\{v1, v2\}$ is not S-independent in G. As

argued earlier we have a contradiction. $|T| \leq \alpha S(G)$. By defini-tion of T, $|T| = \alpha S(G-v)$. Therefore $\alpha S(G) \geq \alpha S(G-v)$ Since $\alpha S(G-v) = \theta S(G-v)$ we have $\alpha S(G) \geq \theta S(G-v)$. That is $\alpha S(G) + 1 \geq \theta S(G-v) + 1 \geq \theta S(G)$. Therefore $\theta S(G) = \alpha S(G)$ or $\theta S(G) = \alpha S(G) + 1$. However $\theta S(G) \neq \alpha S(G)$, since G is minimal

S-imperfect, therefore $\theta S(G) = \alpha S(G) + 1$. Hence the lemma.

Lemma 2.7. *If G is a minimal S-imperfect graph, then for u, v* $\in V(G)$*, neither* $N(u) \subseteq N(v)$ *nor* $N(v) \subseteq N(u)$.

Proof. Suppose false, then say $N(u) \subseteq N(v)$. Let $S = \{S1, S2, \ldots Sw, \ldots, Sk\}$ be a θS-cover of G where each star Si is maximal and Sv is a star con-taining v. By Lemma 2.6, the stars in θS-cover of G are $\alpha S(G) + 1$ in number. Since $N(u) \subseteq N(v)$, any star S'containing u is a substar (a subgraph which is a star) of Sv, $\theta S(G-u) \leq \theta S(G)$. We observe that $\theta S(G-u) = \theta S(G)$. If $\theta S(G-u) < \theta S(G)$, then $\theta S(G-u) \leq \theta S(G)-1$, and $G-u$ will be covered by $\theta S(G)-1$ stars. Since $S'$is a substar of Sv, G will also be covered by $\theta S(G)-1$ stars, contradiction to $\theta S(G)$ being minimum. Therefore $\theta S(G-u) = \theta S(G)$.

Let T be an αS-independent set in $G-u$. We observe that T is an αS-independent set in G. If not, u is adjacent to at least two vertices in T in G. Thus v is adjacent to two vertices in T since $N(u) \subseteq N(v)$. But then T is not an S-independent set in $G-u$ as $v \in V(G-u)$, a contradiction. Thus T is an S-independent set in G and $\alpha S(G) \geq |T| = \alpha S(G-u) = \theta S(G-u) = \theta S(G)$. This implies $\alpha S(G) \geq \theta S(G)$, a

contradiction. Hence the lemma.

Lemma 2.8. *If G is a triangulated minimal S-imperfect graph, then G is Hamiltonian.*

Proof. Let $C = v1v2 \ldots vkv1$ be a largest cycle in G. If $V(C) = V(G)$, then the lemma is true.

So let $v \in V(G) - V(C)$. Since G is minimal S imperfect graph, it is a block and hence connected. Let $v \leftrightarrow v1$. Since G is a block, there exists an induced cycle C'containing the edges $vv1$ and $v1v2$.

Since G is triangulated, $C'$is a triangle. This implies that $v \leftrightarrow v2$. Therefore C together with v is a bigger cycle than that of C, a contradiction to the choice of C. Hence such a v does not exist and

therefore G is Hamiltonian.

Lemma 2.9. *Let G be a triangulated S-imperfect graph. Then there exists an extended sun* $G*$*containing G as an induced subgraph.*

Received: July 28, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

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Proof. Since G triangulated, G has a simplicial vertex u, by Lemma 2.5. Let Q be a free clique in G containing u. The number of non-simplicial vertices in Q is at least 2. If not, then the only non-simplicial vertex in Q is a cut-vertex in G, a contradiction to the fact that G is a block, by Lemma 2.4. If the simplicial vertices of G are removed, the

resulting graph H is obviously Hamiltonian. Let $C = v1v2 \ldots vkv1$ be a Hamiltonian cycle in H. Let $A1, A2, \ldots, Al$, $1 \leq l \leq k$ be mutually disjoint complete subgraphs in G such thatfor every $ui \in Ai$, $ui \leftrightarrow \{vi, vi+1\}$ and ui is simplicial in G. If $l = k$, we are done. If $l < k$, let $ul+1$ be a vertex not in G and let $ul+1$ is not adjacent to any simplicial vertex in G. Let $G1$ be the graph formed by G and $ul+1$ such that $ul+1$ is simplicial in $G1$. If $G1$ is an extended sun, then the lemma is true, otherwise we repeat the process to get $G2, G3, \ldots Gm$, where Gm is an extended sun. Considering $Gm = G*$, G is an induced subgraph of $G*$

and hence the lemma is true.

Lemma 2.10. *Let G be a minimal S-imperfect graph. Let u be a vertex not in G. Let G'be a graph formed by G and u such that u is simplicial in G'and u is adjacent to an edge in G which is not adjacent to a simplicial vertex of G. Then* $\alpha S(G') \neq \theta S(G')$.

Proof. On the contrary suppose $\alpha S(G') = \theta S(G')$. This implies that every vertex of an αS-set of $G'$is in exactly one star of θS-cover of G'. Since u is simplicial in G', by nature of G' any S-independent set in G'cannot have more than $\alpha S(G) + 1$ vertices. By Lemma 2.6,$\theta S(G) = \alpha S(G) + 1$. Then $\alpha S(G) + 1 = \theta S(G) \leq \theta S(G') = \alpha S(G')$, since a θS-cover of G'contains a θS-cover of G. If $\alpha S(G)+1 = \alpha S(G')$, then $\theta S(G) = \theta S(G')$. Then there is an S-independent set of G of

size $\alpha S(G) + 1$, a contradiction to the fact that $\alpha S(G)$ is maximum.

Therefore $\alpha S(G') \neq \theta S(G')$.

Lemma 2.11. *Odd sun is not S-perfect.*

Proof. Let G be an odd sun and $U = \{u1, u2, \ldots, uk\}$ be the set

of simplicial vertices and $W = \{v1, v2, \ldots, vk\}$ be the set of non-

simplicial vertices in G. Then $T' = \{u1, u3, \ldots, uk-2\}$ forms an S-

independent set of G and the stars centered at $v1, v3, \ldots, vk$ form an $\alpha S(H*) \neq \theta S(H*)$, hence the lemma.

Since k-odd sun is an induced subgraph of k-extended sun, then the

following lemma is immediate.

Lemma 2.12. *Odd extended sun is not S-perfect.*

Definition 2.1. *Let P be a path. A special path P*is constructed from P such that an edge of P is contained in a free triangle.*

Lemma 2.13. *A special path is S-perfect.*

Received: July 28, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

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Proof. Let $P*$be a special path formed from the path P. If $P*$is $K2$ or $K3$, then obviously P is S-perfect. So let $P* \neq K2$ or $K3$. Then $P*$will have a cut vertex, since every block of $P*$is $K2$ or $K3$, by definition.If $P*$is not S-perfect, let $P*$is minimal S-imperfect. By

Lemma 2.4, $P*$is a block, a contradiction.

Lemma 2.14. *If G is a 3-sun free triangulated even extended sun, then G is S-perfect.*

Proof. Let $U = \{u1, u2, . . . , uk\}$ be the set of simplicial vertices and $W = \{v1, v2, . . . , vk\}$ be the set of non-simplicial vertices in G. Let ui represent the simplicial vertices in Ai. Ai and C have the same mean-ing as in Definition 1.2. If $vivj$ and vi+1vj+2 are alternate edges in C, ui and ui+2 are at a distance 3. Since k is even, $\{u1, u3, . . . , ul, ul+3, . . . \}$ is an S-independent set in G of size$k$ 2. Similarly, the stars at $v1, v3, . . . , vl, vl+3$ is an S-cover of G of size$k$

2. Then $\alpha S(G) \geq k\ 2 \geq \theta S(G)$. This implies that $\alpha S(G) = \theta S(G)$. Every proper induced subgraph of G is a com-plete graph or a union of disjoint paths or special paths. Every complete graph is S-perfect and by Lemma 2.1 and Lemma 2.13, every induced

subgraphs of G is S-perfect.

Lemma 2.15. *Let G be minimal S-imperfect triangulated graph. If for v, u, w* $\in V(G)$ *such that v is a simplicial vertex in G, u, w* $\in N(v)$*, neither* $N(u) \subseteq N(w)$ *nor* $N(w) \subseteq N(u)$*, then G contains 3-sun as an induced subgraph.*

Proof. By Lemma 2.5, G has a simplicial vertex v. If u and w be two vertices in $N(v)$ such that $N(u) \not\subseteq N(w)$ and $N(w) \not\subseteq N(u)$, then there exists vertices $u1 \in N(u)$ and $w1 \in N(w)$ in G such that $u \not\leftrightarrow w1$ and $w \not\leftrightarrow u1$. For vertices $v1$ and $v2$ in G, let $P(v1, v2)$ denote an induced path connecting $v1$ and $v2$ in G. Since G is a block, there exists a path $P(u1, w1)$ connecting $u1$ and $w1$ in G not containing v. Since G is a block we can choose a $P(u1, w1)$ such that $u, w \notin P(u1, w1)$. Let $t1$ be the last vertex in $P(u1, w1)$ such that $u \leftrightarrow t1$. Let $t2$ be the last vertex in $P(u1, w1)$ such that $w \leftrightarrow t2$. We consider the following two cases:

Case 1: $t1 \neq t2$.

Then $ut1 . . . t2wu$ is an induced cycle of length at least 4, a contra-diction to G being triangulated. Therefore Case 1 does not arise at all.

Case 2: $t1 = t2 = t$, say.

Then $t \leftrightarrow \{u, w\}$, and $G[\{u, u1, . . . , t\}]$ will not contain an in-duced cycle of length at least 4, since G is triangulated. There-fore u is adjacent to all the vertices in $P(u1, t)$. If x is the vertex before $t1$ in $P(u1, t)$, then $x \leftrightarrow u, t$ and $x \not\leftrightarrow w$, by the choice of t. Similarly there is a vertex y in $P(w, t)$ such that $y \leftrightarrow \{w, t\}$ and $y \not\leftrightarrow u$. Then $G[\{v, u, w, x, t, y\}]$=3-sun, a contradiction to our assumption. Therefore Case 2 does not arise at all.

Hence the lemma.

Received: July 28, 2025

International Journal of Applied Mathematics

Volume 38 No. 2s, 2025

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Lemma 2.16. *Let G be a minimal S-imperfect graph. Then there exist no three distinct vertices v, u, w* $\in V(G)$ *such that v is simplicial vertex in G and u and w* $\in N(v)$*,* $N(u) \subseteq N(w)$ *or* $N(w) \subseteq N(u)$.

The proof is similar to the proof of Lemma 2.7 (We have to replace v by w in proof of Lemma 2.7).

Theorem 2.2 (**Main Theorem**). *A triangulated graph G is S-perfect if and only if G is odd sun-free.*

Proof. Let G be a triangulated S-perfect graph. Then by Lemma 2.11 G is odd-sun free.

Conversely, let G be an odd sun-free triangulated graph. If G is not S-perfect, without loss of generality, let G be a minimal S-imperfect graph. Then by Lemmas 2.9 and 2.10, there exists an extended sun $G*$ such that G is an induced subgraph of $G*$and $\alpha S(G*) \neq \theta S(G*)$. We may assume that $G*$is minimal S-imperfect. But then $G* = G$, since G is an induced subgraph of $G*$. By Lemma 2.16, G has no three vertices $u, v, w \in V(G)$ such that $N(u) \subseteq N(w)$ or $N(w) \subseteq N(u)$, where v is simplicial vertex in G and $u, w \in N(v)$. Then G contains 3-sun as an induced subgraph by Lemma 2.15. Since $G = G*$, G is an extended sun and contains 3-sun as an induced subgraph, a contradiction to our assumption. By definition of odd extended sun, it contains odd sun as an induced subgraph. Therefore G is 3-sun free even extended sun. Then by Lemma 2.14, G is S-perfect. This completes the proof of the theorem.

3 Future Directions

The main theorem leads to the following conjecture.

Conjecture. A graph is S-perfect if and only if G is $\{C3k+1, C3k+2, k \geq 1$, odd super sun$\}$-free where super sun is defined as follows.

Let H be a graph with a hamiltonian cycle $C = v1v2 \ldots vkv1$. Let Ai, $1 \leq i \leq k$ be mutually disjoint set of vertices not in H such that $|Ai| \geq 1$. Let $H*$be a graph constructed from H such that $H*$has the following properties:

1. the induced subgraph on Ai is a path Pi in $H*$
2. the end vertices of Pi, that is $ui1$ and $ui|Pi|$ are respectively adja-cent to vi and $vi+1$ and the number of vertices in Pi is $3k+1$, $k \geq 1$

Then $H*$is called k-super sun. $H*$is even or odd super sun depending on k being even or odd.

Received: July 28, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

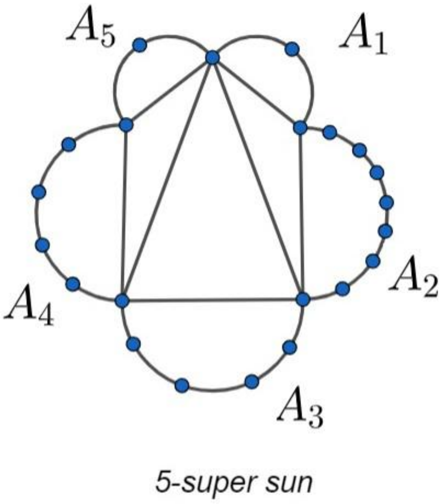

Figure 2: An example of 5-super sun graph

Acknowledgement

The authors profusely thank S. A. Choudum, CHRIST (Deemed to be University) for helpful discussions and critically looking into the entire manuscript.

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