

**NUMERICAL STUDY OF ELECTROMAGNETIC AND
THERMOPHYSICAL EFFECTS ON MICROPOLAR
NANOFLUID FLOW OVER A STRETCHING SHEET**

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Abstract

This study presents a numerical investigation of mixed convection heat and mass transfer in an electrically conducting micropolar nanofluid flow over a stretching surface embedded in a porous medium. The model integrates several significant physical phenomena, including Brownian motion, thermophoretic diffusion, electromagnetic effects from transverse magnetic and electric fields, radiative heat transfer based on the Rosseland approximation, and internal heat generation or absorption. The governing partial differential equations, formulated under boundary layer assumptions, are transformed into a system of ordinary differential equations through similarity transformations. These equations are then solved using the Keller-box method, an implicit finite difference technique known for its stability and accuracy. The impact of various dimensionless parameters, such as velocity ratio, magnetic field intensity, Prandtl number, Schmidt number, and Eckert number, is analysed in detail. Microrotation and flow characteristics are significantly influenced by parameters like wall mass transfer and buoyancy, according to the study, which demonstrates that an increase in the thickness of the magnetic field thickens thermal and velocity boundary layers. The robustness of the proposed model is demonstrated by the strong agreement between the results obtained and the existing literature.

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1. Introduction

The study of nanofluid dynamics has garnered considerable attention in recent years due to its wide-ranging applications in thermal management, biomedical systems, and advanced manufacturing processes. Nanofluids, which are engineered colloidal suspensions of nanoparticles in conventional base fluids, offer enhanced thermal conductivity and heat transfer

properties. When these fluids exhibit microstructural characteristics, such as particle spin and couple stresses, they are modeled as micropolar nanofluids, introducing an additional complexity through microrotation effects. Over-stretching surfaces within porous media, particularly under the influence of external fields like magnetic and electric forces, further increases the complexity of such flows. These configurations are fascinating when designing energy-efficient cooling systems, microfluidic devices, and chemical reactors. To better comprehend and predict the behavior of such complex systems, robust mathematical models incorporating multiple physical effects like Brownian motion, thermophoresis, thermal radiation, and electromagnetic interactions are becoming increasingly important.

Several researchers have contributed significantly to understanding micropolar fluid flow and heat transfer in porous media under physical effects. Vafai and Tien [1] examined the influences of a solid boundary and inertial forces on flow and heat transfer in porous media. Zakaria [2] investigated the impact of a transverse magnetic field on the one-dimensional motion of an electrically conducting micropolar fluid through a porous medium using the Laplace transform technique. Siddheswar and Sri Krishna [3] performed both linear and weakly nonlinear analyses of convection in a micropolar fluid within a high-porosity medium by employing the Brinkman-Eringen momentum equation.

In a subsequent study, Zakaria [4] extended the previous work by further analysing the influences of a transverse magnetic field on the conducting micropolar fluid through a porous medium using the Laplace transforms. Hassanien et al. [5] examined the natural convection of a micropolar fluid over a vertical, permeable, semi-infinite plate embedded in a porous medium and found that micropolarity enhances skin friction while reducing the heat transfer rate relative to Newtonian fluids. Incorporating non-Darcian effects, Abo-Eldahab and El Aziz [6] expanded the classical boundary layer formulation, highlighting the role of inertial resistance in altering the heat transfer profile in porous substrates. Abo-Eldahab and Ghonaim [7] also investigated radiative heat transfer in a micropolar fluid flowing over a stationary horizontal plate within a porous medium, employing the shooting method to solve the governing equations. Later, in 2006, The influence of thermal radiation on boundary layer characteristics in micropolar fluid flows over-stretching surfaces was numerically addressed by Mahmoud et al. [8], particularly under the presence of a transverse magnetic field. El-Kabeir et al. [9] addressed the boundary layer equations for the two-dimensional Hiemenz flow of a micropolar viscoelastic and incompressible fluid, incorporating both a transverse magnetic field and a porous medium. Further, radiative impacts on hydromagnetic convection under slip boundary conditions have been explored in micropolar fluids by Chaudhary and Jain [10], showing that both radiation and magnetic field orientation significantly alter thermal boundary layer growth.

Chaudhary and Jha [11] analysed the unsteady flow of a micropolar fluid through a porous medium adjacent to a semi-infinite vertical plate under slip-flow conditions. Their study incorporated first-order homogeneous chemical reactions and a uniform magnetic field perpendicular to the porous surface. Zueco et al. [12] analyzed fully developed mixed convection heat transfer in a magneto-micropolar fluid through a Darcy–Forchheimer porous

medium, considering the presence of an internal heat source within a vertical circular duct. Pal and Chatterjee [13] investigated the steady, two-dimensional MHD boundary layer flow of a micropolar fluid over a stretching sheet embedded in a non-Darcian porous medium under a uniform magnetic field, accounting for heat and mass transfer. Pandey and Chaube [14] examined the peristaltic flow of a micropolar fluid through a porous channel driven by sinusoidal waves along the channel walls, incorporating MHD effects. Pal and Chatterjee [15] further investigated the MHD boundary layer behavior of an electrically conducting micropolar fluid over a non-isothermal stretching surface, considering a porous medium with spatially varying thermal conductivity and simultaneous heat and mass transfer. Shaw et al. [16] focused on the impact of combined homogeneous and heterogeneous chemical reactions on the steady micropolar fluid flow along stretching or shrinking surfaces within a porous medium. Shit and Roy [17] investigated the influence of slip velocity on the peristaltic transport of a physiological fluid in a porous, non-uniform channel, assuming long wavelengths and low Reynolds number conditions. Doh et al. [18] explored fully developed convective flow and heat transfer characteristics of a micropolar fluid confined between two vertical plates with internal heat generation, utilising the Crank–Nicolson implicit finite difference method to solve the governing equations.

Das and Duari [19] worked on the stagnation point flow of micropolar nanofluid with heat and mass transfer constrained by a vertical elongating sheet, solving the governing equations using the Runge-Kutta Fehlberg technique. Patel et al. [20] examined the magnetohydrodynamic flow behavior of micropolar nanofluids, taking into account the microstructural characteristics of the suspended substructure particles and inertial forces. Yadav and Verma [21] explored the flow dynamics of immiscible Newtonian and micropolar fluids through an inclined channel containing two porous regions with differing permeabilities employing Brinkman's equation. Khan et al. [22] examined the radiative flow of a micropolar nanofluid over an exponentially stretched curved surface, incorporating the effects of viscous dissipation and entropy generation. Usafzai et al. [23] analysed the boundary layer flow of a water-based micropolar nanofluid induced by a bidirectional permeable surface. Walelign and Seid [24] studied the influence of thermophysical properties on micropolar nanofluid transport near an inclined exponentially stretching sheet. The Optimal Homotopy Asymptotic Method (OHAM) was employed to solve the governing nonlinear equations. Additionally, Mahajan and Raj [25] conducted linear and nonlinear stability analyses to assess fluid flow stability in a porous medium subject to uniform internal heat generation.

The novelty of this work lies in its comprehensive modeling and numerical analysis of micropolar nanofluid dynamics over a stretching surface, incorporating a unique integration of advanced physical phenomena that have rarely been addressed simultaneously in previous studies. Specifically, the model accounts for Brownian motion and thermophoresis, electromagnetic forces induced by combined transverse magnetic and electric fields (Lorentz force), radiative heat flux based on the Rosseland approximation, and internal heat generation all within a porous medium described by the Brinkman-Forchheimer extended Darcy model.

Moreover, the inclusion of microrotation dynamics sets this study apart by capturing the angular momentum effects of nanoparticles. Using the Keller-box method to solve the resulting complex, coupled, nonlinear ordinary differential equations add robustness and accuracy to the analysis, providing a unified platform for evaluating a wide range of parameter influences on flow, thermal, and concentration fields in engineering and biomedical applications.

2. Mathematical Formulations

Consider the flow of an incompressible electrically conducting micropolar nanofluid over a stretching surface while considering the presence of porous media with a porosity ϕ and permeability k_1 . The velocity of the sheet is assumed to vary linearly with the distance x from a fixed point on the sheet. The stretching velocity is defined as $u_w = ax$, where a is a positive constant. We investigated the phenomenon of heat and mass transfer in the presence of mixed convectional flow, thermal radiation, heat source/sink, thermophoresis, and Brownian influences. The flow field is subjected to the effect of a transverse magnetic field with a strength of $\vec{B} = (0, B_0, 0)$ and a uniform electric field of $\vec{E} = (0, 0, -E_0)$. It is assumed that the magnetic Reynolds number is very small. The governing equations for the boundary layer problem, with all the specified assumptions, are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{(\mu_{nf} + \kappa)}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho_{nf}} \frac{\partial N}{\partial y} - \frac{\mu_{nf} \phi}{\rho_{nf} k_1} u - \frac{C_b}{\sqrt{k_1}} \phi u^2 \quad (2.2)$$

$$+ g\beta_{T_{nf}}(T - T_\infty) + g\beta_{C_{nf}}(C - C_\infty) + \frac{\sigma_{nf}}{\rho_{nf}}(E_0 B_0 - B_0^2 u)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma_{nf}}{\rho_{nf} j} \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho_{nf} j} \left(2N + \frac{\partial u}{\partial y} \right) \quad (2.3)$$

$$(\rho C_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{2_{nf}} \left(\frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\partial y} + Q^*(T - T_h) + \sigma_{nf}(u B_0 - E_0)^2 \quad (2.4)$$

$$+ (\rho C_p)_{nf} \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right]$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (2.5)$$

subject to boundary conditions,

$$u = u_w = ax, v = v_w, N = -n \frac{\partial u}{\partial y}, T = T_w = T_\infty + A_0 x^2, C = C_w = C_\infty + A_1 x^2 \text{ at } y = 0$$

PST case PST case

(2.6)

$$u = u_\infty = bx, v = 0, N = 0, T = T_\infty, C = C_\infty \text{ at } y = \infty.$$

Where u and v are velocity components in the x and y directions, respectively, dynamic viscosity μ_f and μ_{nf} for fluid and nanofluid, respectively, fluid density ρ_f and nanofluid density ρ_{nf} , microrotation/angular velocity N perpendicular to the xy -plane, microinertia per unit mass j , spin gradient viscosity γ_{nf} , vortex viscosity κ , temperature T , l_1 and l_2 are membrane permeability and slip factor for temperature, respectively, λ is the velocity ratio parameter, and v_w is the velocity of wall mass transfer, where $v_w < 0$ is mass suction and $v_w > 0$ is mass injection. It needs to be noted that n is a constant that lies between 0 and 1. On the surface, the $n = 0$ case suggests the $N = 0$ situation. It describes a dense flow of particles where microelements adjacent to the wall surface are restricted from rotating, a condition commonly known as high microelement concentration. The scenario for $n = \frac{1}{2}$ implies a low concentration of microelements and the vanishing of the stress tensor's anti-symmetric part. In contrast, the case $n = 1$ is utilised to model turbulent boundary layer flows. γ is Spin gradient viscosity, given by

$$\gamma_{nf} = \left(\mu_{nf} + \frac{\kappa}{2}\right)j = \mu_f \left(\frac{\mu_{nf}}{\mu_f} + \frac{N_1}{2}\right)j \quad (2.7)$$

Where $N_1 = \kappa/\mu_f$ is the material parameter. This assumption allows equations to forecast behaviour when microstructure effects are negligible and total spin N reduces to the angular velocity. In the boundary conditions for heat and mass transfer, only the PST (prescribed surface temperature) case is considered to maintain a constant temperature difference between hot and cold fluids.

Substituting,

$$-\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} = u_\infty \frac{du_\infty}{dx} + \frac{\mu_{nf}\phi}{\rho_{nf}k_1} u_\infty + \frac{C_b\phi}{\sqrt{k_1}} u_\infty^2 + B_0^2 u_\infty \quad (2.8)$$

in equation (2.2), we get,

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & u_\infty \frac{du_\infty}{dx} + \frac{(\mu_{nf} + \kappa)}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho_{nf}} \frac{\partial N}{\partial y} - \frac{\mu_{nf}\phi}{\rho_{nf}k_1} (u - u_\infty) \\ & - \frac{C_b}{\sqrt{k_1}} \phi (u^2 - u_\infty^2) + g\beta_{T_{nf}}(T - T_\infty) + g\beta_{C_{nf}}(C - C_\infty) \\ & + \frac{\sigma_{nf}}{\rho_{nf}} (E_0 B_0 - B_0^2 (u - u_\infty)) \end{aligned} \quad (2.9)$$

For radiation, one can use Rosseland's approximation to obtain a relation to radiative heat flux, i.e., $q_r = -\left(\frac{4\sigma}{3k^*}\right) \frac{\partial T^4}{\partial y}$, where σ is the Stefan-Boltzmann constant and k^* is the absorption coefficient. Given a specific temperature variation in the flow, the fourth power of temperature,

T^4 , can be expressed using a Taylor series expansion. In this case, by expanding T^4 around T_∞ and ignoring higher-order terms, one can get, $T^4 = 4T_\infty^3 T - 3T_\infty^4$.

So, the heat transfer equation will be written as,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{2nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^* \rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q^*(T - T_\infty)}{(\rho C_p)_{nf}} + \frac{\sigma_{nf}}{(\rho C_p)_{nf}} (uB_0 - E_0)^2 \quad (2.10)$$

Introducing the following similarity transformation,

$$u = axf'(\eta), \quad v = -(av_f)^{\frac{1}{2}}f(\eta), \quad N = ax\left(\frac{a}{v_f}\right)^{\frac{1}{2}}g(\eta), \quad \frac{T - T_\infty}{T_0 - T_\infty} = \theta, \quad (2.11)$$

$$\frac{C - C_\infty}{C_0 - C_\infty} = \Phi \text{ and } \eta = \left(\frac{a}{v_f}\right)^{\frac{1}{2}}y$$

Applying the boundary conditions given in equation (2.6) to equations (2.1), (2.3), (2.5), (2.8), and (2.9) leads to the following set of ordinary differential equations.

$$f'^2 - ff'' = \lambda^2 + \frac{\rho_f}{\rho_{nf}} \left[\left(\frac{\mu_{nf}}{\mu_f} + N_1 \right) f''' + N_1 g' \right] - \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} Da^{-1} (f' - \lambda) \quad (2.12)$$

$$- \alpha (f'^2 - \lambda^2) + \frac{\beta_{Tnf}}{\beta_{Tf}} \frac{Gr_t}{Re_x^2} \theta + \frac{\beta_{cnf}}{\beta_{cf}} \frac{Gr_c}{Re_x^2} \Phi$$

$$+ \frac{\sigma_{nf}}{\sigma_f} \frac{\rho_f}{\rho_{nf}} M(E - f' + \lambda)$$

$$f'g - fg' = \frac{\rho_f}{\rho_{nf}} \left(\frac{\mu_{nf}}{\mu_f} + \frac{N_1}{2} \right) g'' - \frac{\rho_f}{\rho_{nf}} N_1 (2g + f'') \quad (2.13)$$

$$2f'\theta - f\theta' = \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} \frac{1}{Pr} \left(\frac{4}{3} \frac{1}{Rd} + \frac{k_{2nf}}{k_{2f}} \right) \theta'' + \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} \Pi \theta \quad (2.14)$$

$$+ \frac{\sigma_{nf}}{\sigma_f} \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} MEc [f' - \lambda - E]^2 + Nb\theta'\Phi' + Nt\theta'^2$$

$$Sc(2f'\Phi - f\Phi') = \Phi'' + \frac{Nt}{Nb}\theta'' \quad (2.15)$$

subject to boundary conditions,

$$f'(0) = 1, f(0) = \frac{-v_w}{\sqrt{av_f}} = S, g(0) = -nf''(0), \theta(0) = 1, \Phi(0) = 1 \quad (2.16)$$

$$f'(\eta) \rightarrow \lambda, g(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \Phi(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty$$

$$\text{where, } N_1 = \frac{\kappa}{\mu_f}, Da^{-1} = \frac{\nu_f \phi}{k_1 a}, \alpha = \frac{c_b \phi}{a \sqrt{k_1}} u_w, Gr_t = \frac{g \beta_{tf} (T_w - T_\infty) x^3}{\nu_f^2}, Gr_c = \frac{g \beta_{cf} (C_w - C_\infty) x^3}{\nu_f^2}, Re_x = \frac{u_w x}{\nu_f}, M = \frac{\sigma_f B_0^2}{\rho_f a}, E = \frac{E_0}{B_0 u_w}, j = \frac{\nu_f}{a}, Pr = \frac{\mu_f c_p}{k_{2f}}, Rd = \frac{k^* k_{2f}}{4 \sigma T_\infty^3}, \Pi = \frac{Q^*}{(\rho c_p)_f a}, Ec = \frac{u_w^2}{a c_{pf} (T_w - T_\infty)}, Sc = \frac{\nu_f}{D_B}, Nt = \frac{\tau_{DT} (T_w - T_\infty)}{\nu_f T_\infty} \text{ and } Nb = \frac{\tau_{DB} (C_w - C_\infty)}{\nu_f}.$$

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s$$

$$\frac{k_{2nf}}{k_{2f}} = \frac{k_{2s} + 2k_{2f} - 2\phi (k_{2f} - k_{2s})}{k_{2s} + 2k_{2f} + \phi (k_{2f} - k_{2s})}$$

$$(\rho \beta_t)_{nf} = (1 - \phi) (\rho \beta_t)_f + \phi (\rho \beta_t)_s$$

$$(\rho \beta_c)_{nf} = (1 - \phi) (\rho \beta_c)_f + \phi (\rho \beta_c)_s$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\phi \left(\frac{\sigma_s}{\sigma_f} - 1 \right)}{2 + \frac{\sigma_s}{\sigma_f} - \phi \left(\frac{\sigma_s}{\sigma_f} - 1 \right)}$$

$$Cf_x = \frac{\tau_w}{\rho_f U_w^2}, \quad \tau_w = - \left[(\mu_{nf} + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}$$

$$M_x = \frac{m_w}{\mu_{nf} U_w}, \quad m_w = \left(\mu_{nf} + \frac{\kappa}{2} \right) j \left[\frac{\partial N}{\partial y} \right]_{y=0}$$

$$Nu_x = \frac{x q_w}{k_{2nf} (T_w - T_\infty)}, \quad q_w = - \left(k_{2nf} + \frac{16 \sigma T_\infty^3}{3 k^*} \right) \left[\frac{\partial T}{\partial y} \right]_{y=0}$$

$$Sh_x = \frac{x q_m}{D_B (C_w - C_\infty)}, \quad q_m = - D_B \left[\frac{\partial C}{\partial y} \right]_{y=0}$$

$$Cf_x Re_x^{\frac{1}{2}} = - \left[\frac{\mu_{nf}}{\mu_f} + N_1 (1 - n) \right] f''(0),$$

$$M_x = \left(\frac{\mu_{nf}}{\mu_f} + \frac{N_1}{2} \right) g'(0),$$

$$Nu_x Re_x^{-\frac{1}{2}} = - \left(\frac{k_{2nf}}{k_{2f}} + \frac{4}{3} \frac{1}{Rd} \right) \theta'(0),$$

$$Sh_x Re_x^{-\frac{1}{2}} = - \Phi'(0)$$

In this study, water and kerosene are considered base fluids, while copper is selected as the nanoparticle due to its high thermal conductivity. The relevant thermophysical and electromagnetic properties, such as specific heat capacity, density, thermal conductivity, dynamic viscosity, thermal and solutal expansion coefficients, electrical conductivity, and Prandtl number, are summarized in *Table 1*. These values are used as the essential inputs in the formulation and numerical simulation of the governing equations.

Table 1: Thermophysical and electromagnetic properties of water, kerosene, and Copper Nanoparticles used for micropolar nanofluid analysis

	Water	Kerosene	<i>Cu</i>
C_p	4.1816	2.04	0.385
ρ	1000	800	8850
k_2	0.613	0.149	401
μ	0.001003	0.00164	-
β_t	69	330	17
β_c	217	990	51
σ	5.5×10^{-6}	6×10^{-10}	5.96×10^7
Pr	6.8377	23.004	-

3. Methodology

Dimensionless forms of the governing equations are derived through dimensional analysis to facilitate the analysis of velocity, microrotation, temperature, and concentration profiles. This yields a set of coupled, nonlinear, ordinary differential equations that characterise the behavior of the micropolar nanofluid system under the influence of stretching, porous media, and electromagnetic effects. These equations are then solved numerically using the Keller-box method, a robust and accurate implicit finite difference scheme widely employed for boundary layer equations. This technique uses three main computational strategies: the finite difference method, Newton's method, and the block elimination technique. The system of coupled ODE's is first transformed into a system of first-order equations via the substitutions $f' = u, f'' = u' = v, g' = h, \theta = t, \theta' = t' = s, \Phi = c$ and $\Phi' = c' = p$ applied to equations (2.11)-(2.15). These equations are then discretized using difference approximations over a uniform mesh in the transformed domain. In the second technique, the nonlinear difference equations obtained from the discretization step are linearized by utilizing the iterates. The previous technique involves expressing a set of linear equations in the form of a vector-matrix equation, where the matrix is a block tridiagonal matrix. The solution to this system is obtained by employing the LU decomposition method.

The implementation is carried out by coding in MATLAB, following the procedural framework outlined by Cebeci and Bradshaw. A stable step size $\Delta y = 0.01$ is adopted for spatial

discretization, and a stringent convergence criterion with an error tolerance of 10^{-15} is used to ensure high numerical accuracy in the computed solutions.

4. Results and Discussions

The coupled nonlinear ordinary differential equations for the Micropolar nanofluid model are solved by the implicit finite difference-based Keller-box method. The results of this problem are represented in graphical form. *Table 2* and *Table 3* compare the skin friction coefficient with a few literatures mentioned in the tables for the mass transfer parameter S and velocity ratio parameter λ .

Table 2 Comparison of skin friction coefficient Cf_x for different values of the mass transfer parameter S .

S	Mansur et al. [26]	Mastroberardino [27]	Present results
2	-2.4142	-2.4142	-2.414171906
3	-3.3028	-3.3028	-3.302775638
4	-4.2361	-4.2361	-4.236067977

Table 3 Comparison of skin friction coefficient Cf_x for different values of the velocity ratio parameter λ .

λ	Pal and Chatterjee[15]	Nazar et al. [28]	Mahapatra and Gupta [29]	Present results
0.01	-0.99670	-0.9980		-0.998065406
0.2	-0.91921	-0.9181	-0.9181	-0.918107934
0.5	-0.66732	-0.6673	-0.6673	-0.667265482
1.0	0.0	0.0		4.09E-33
2.0	2.01729	2.0175	2.0175	2.017530801
5.0	11.75029	11.7537		11.7524331
10.0	36.25047	36.2687		36.26026093
20.0	106.4931	106.5744		106.5239427

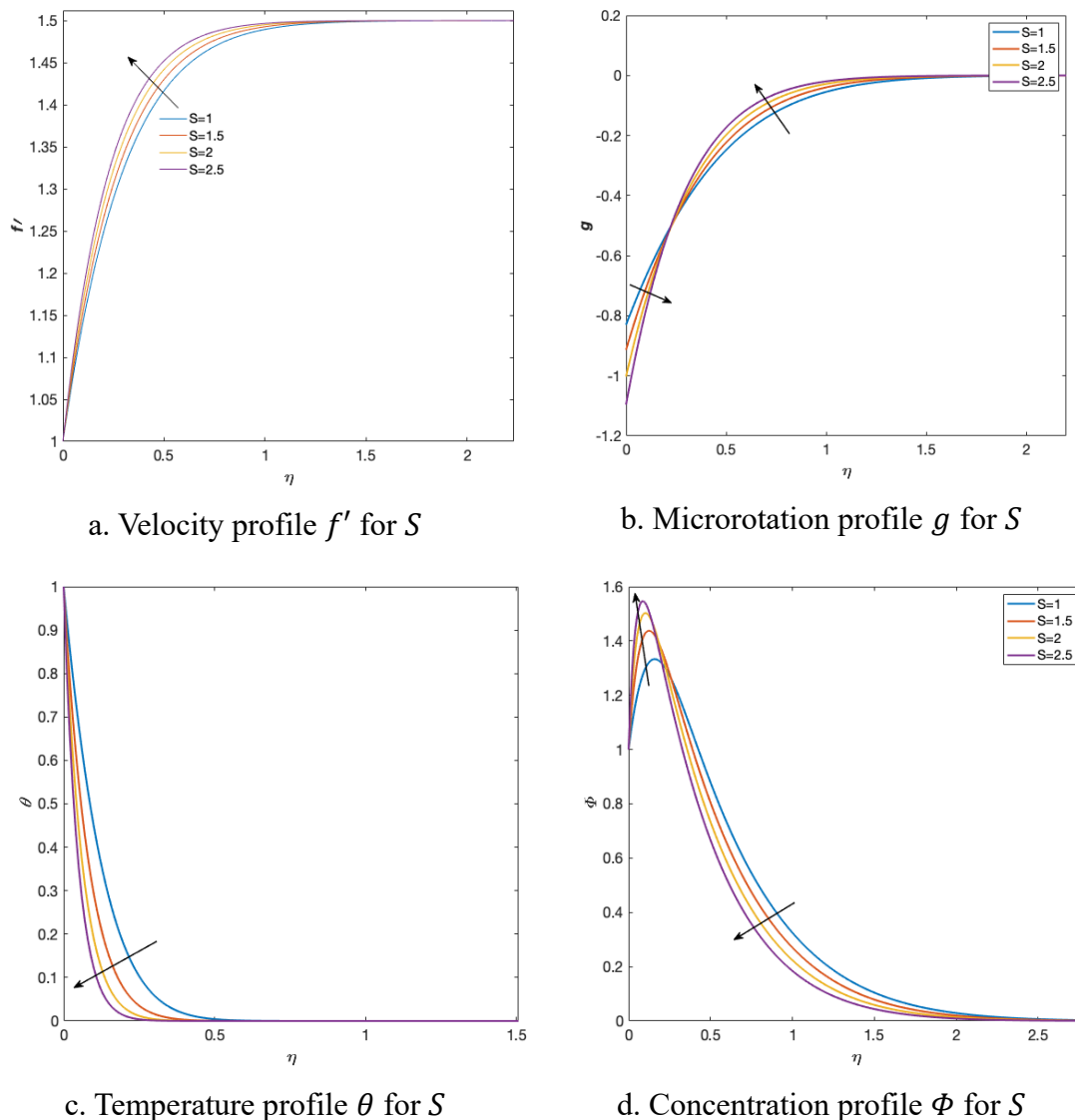
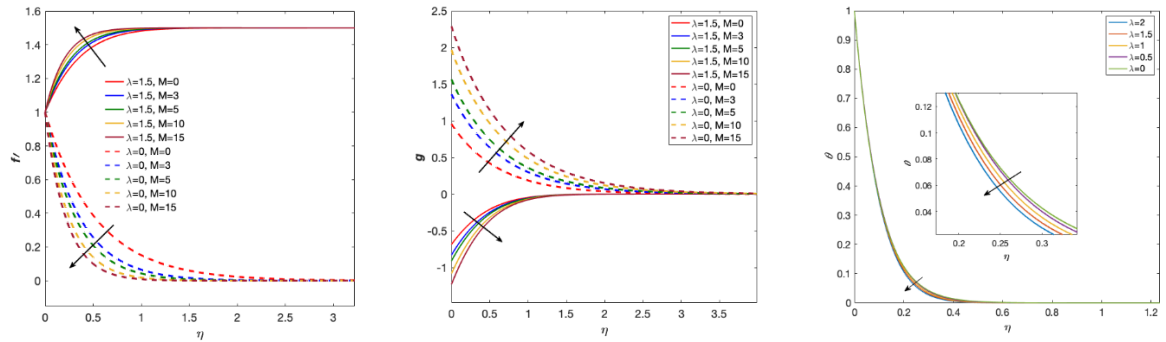


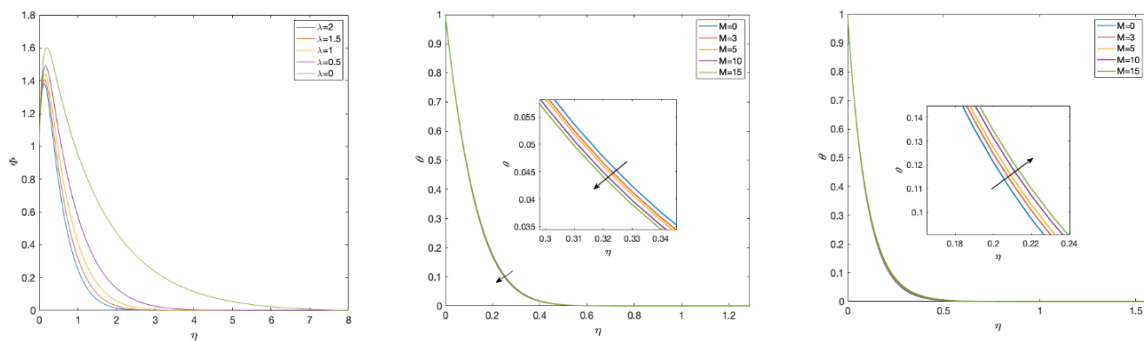
Figure 1: Variation of profiles for wall mass transfer S with $Re = 10$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $\lambda = 1.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

Figure~1 illustrates the variation of different profiles with respect to the wall mass transfer parameter S under the fixed set of values to the other parameters. In Figure 1(a), velocity profile f' increases as wall mass transfer S increases. As S increases, corresponding to stronger wall suction, the velocity profile shows a significant increase near the wall. This is because suction removes low-momentum fluid particles from the boundary layer, allowing high-momentum fluid to replace them, thus increasing velocity and thickening the boundary layer. Figure 1(b) presents the microrotation profile g , which demonstrates a dual nature: initially decreasing near the wall due to suction suppressing the rotational freedom of microelements and then adjusting to conserve angular momentum. In Figure 1(c), temperature profile θ decreases as wall mass transfer S increases. This is attributed to the removal of thermal energy near the surface due to

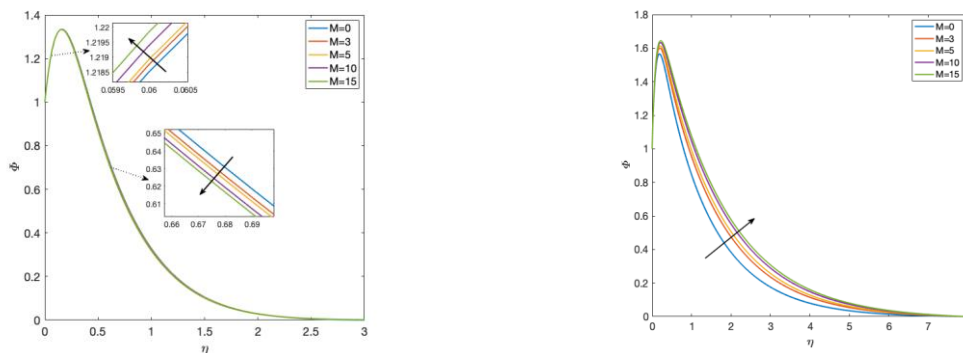
suction, leading to a thinner thermal boundary layer. As shown in Figure 1(d), the concentration profile Φ initially increases near the wall, followed by a decrease with S . This indicates enhanced mass transport at the wall and subsequent dilution due to suction.



a. Velocity profile for λ and M b. Microrotation profile for λ and M c. Temperature profile for λ and M



d. Concentration profile for λ and M e. Temperature profile for M at $\lambda = 1.5$ f. Temperature profile for M at $\lambda = 0$



g. Concentration profile for M at $\lambda = 1.5$ h. Concentration profile for M at $\lambda = 0$

Figure 2: Variation of profiles for the velocity ratio parameter λ and Magnetic parameter M with $S = 1$, $Re = 10$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

Figure~2 presents the variation of profiles with respect to the velocity ratio parameter λ and the magnetic parameter M , while maintaining the fixed values for other parameters. In Figure 2(a), velocity increases with increasing ratio λ . Due to the Lorentz force, two opposite behaviours can be observed for different magnetic parameters at $\lambda < 1$ and $\lambda > 1$. At $\lambda = 1.5$, the velocity profile increases for increasing magnetic parameters. Whereas at $\lambda = 0$, the velocity profile decreases for increasing values of the magnetic parameter M . However, boundary layer thickness increases in both cases, thus conserving the trend. In Figure 2(b), microrotation profile g decreases as the ratio λ increases. The microrotation profile increases at $\lambda < 1$ and increases at $\lambda > 1$ for increasing values of the magnetic parameter M . In Figure 2(c), the temperature profile decreases for increasing values of λ . In Figure 2(d), the concentration profile decreases as λ increases. In Figure 2(e), at $\lambda > 1$, the temperature profile decreases as M increases. In Figure 2(f), at $\lambda = 0$, the temperature profile increases as M increases. As for $\lambda = 1.5$ shown in Figure 2(g), the concentration profile will increase at first near the stretching sheet; then, it starts decreasing as the magnetic parameter increases. In Figure 2(h), at $\lambda = 0$, the concentration profile increases as M increases.

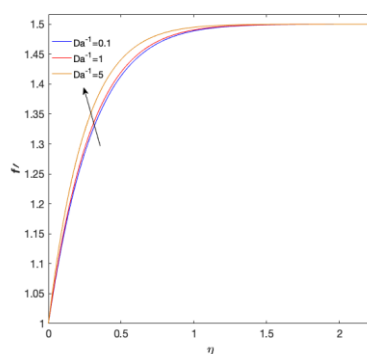
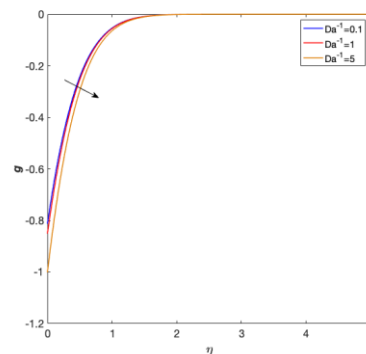
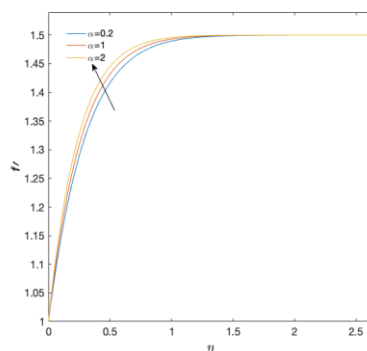
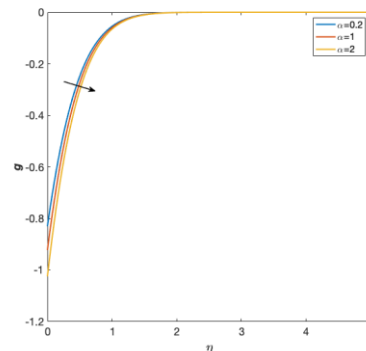
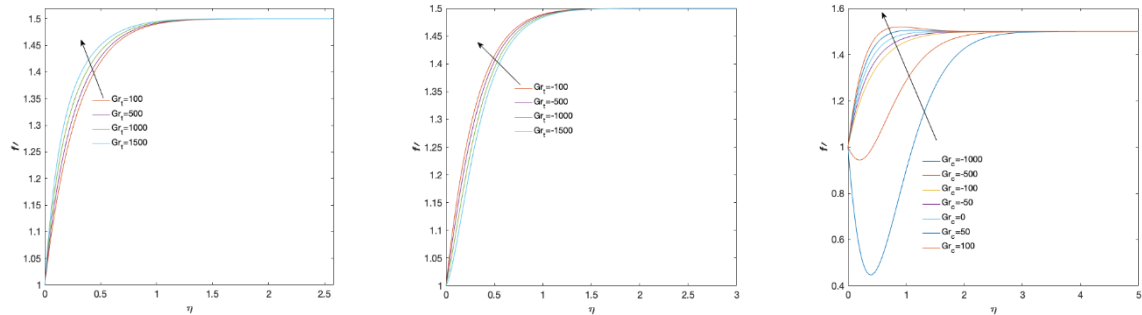

a. Velocity profile f' for Da^{-1}

b. Microrotation profile g for Da^{-1}

c. Velocity profile f' for α

d. Microrotation profile g for α

Figure 3: Variation of profiles for Inverse Darcy number Da^{-1} and local inertia coefficient parameter α with $S = 1$, $Re = 10$, $\lambda = 1.5$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

It can be observed in Figures 3(a) and (c) that as the inverse Darcy number Da^{-1} and local inertia coefficient α increases, the velocity profile increases. Signifying boundary layer

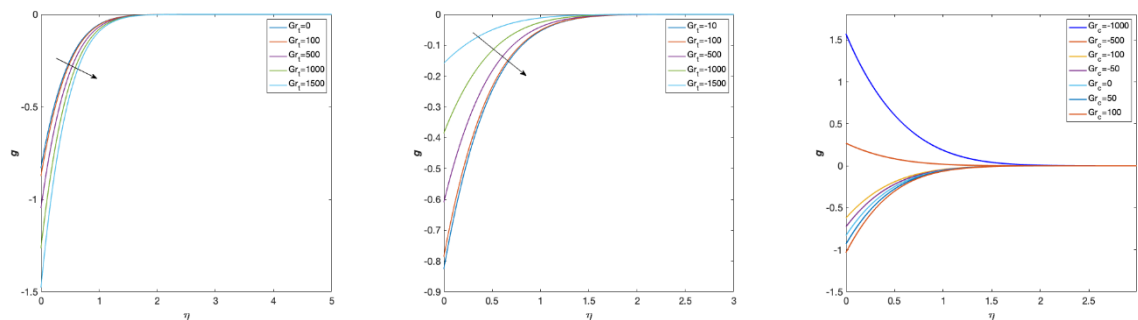
thickness rises for both Da^{-1} and α . It is due to the fact that the inverse Darcy number Da^{-1} and local inertia coefficient α increases the porosity of the porous media, hence increasing the boundary layer thickness. Also, to conserve angular momentum, in Figure 3(b) and (d), the microrotation profile decreases for both Da^{-1} and α . This is due to greater flow resistance or inertial effects that suppress the rotational behaviour of microelements, likely owing to alignment with the main flow or damping by increased shear.



a. Velocity profile f' for Gr_t b. Velocity profile f' for Gr_t c. Velocity profile f' for Gr_c

Figure 4: Variation of velocity profile for Grashof number Gr_t and Gr_c with $S = 1$, $Re = 10$, $\lambda = 1.5$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

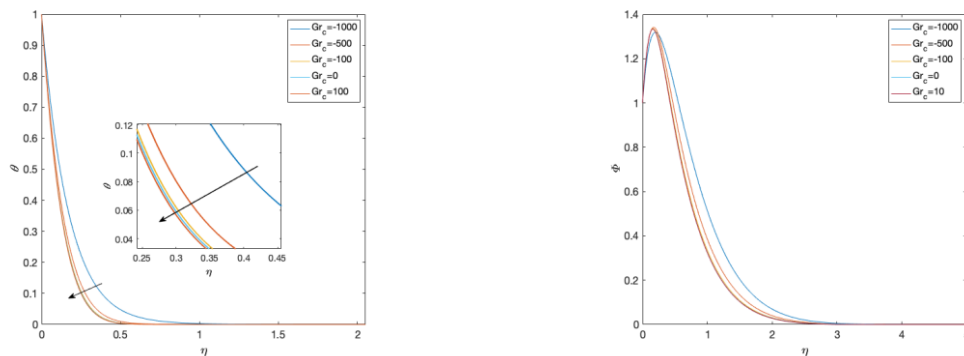
Figure~4 shows the variation of the velocity profile for the mixed convective parameter Gr_t and Gr_c . For assisting the flow, the velocity profile f' increases as Gr_t increases, signifying decreasing boundary layer thickness for increasing temperature buoyancy effects. For opposing flow, the velocity profile decreases as $-Gr_t$ increases. From the graph, it can be observed that boundary layer thickness increases with $-Gr_t$. Velocity profile increases as Gr_c increases from -1000 to 100 . For $Gr_c = -1000$ and $Gr_c = -500$, the velocity profile would overshoot while increasing with Gr_c .



d. Microrotation profile g for Gr_t e. Microrotation profile g for Gr_t f. Microrotation profile g for Gr_c

Figure 5: Variation of Microrotation profile for Grashof number Gr_t and Gr_c with $S = 1$, $Re = 10$, $\lambda = 1.5$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

Figure 5 shows the variation of the microrotation profile for mixed convective parameter Gr_t and Gr_c . Microrotation profile g decreases as the buoyancy force Gr_t increases for assisting flow. This indicates that stronger upward convection aligns the microelements more with the flow, reducing rotational effects. In the case of opposing flow, a reverse trend can be observed. The microrotation profile decreases as Gr_t from -10 to -1500 due to increased shear and rotational disturbances. For $Gr_c = -1000$ and $Gr_c = -500$, the microrotation profile overshoots as it goes on, decreasing for increasing Gr_c .

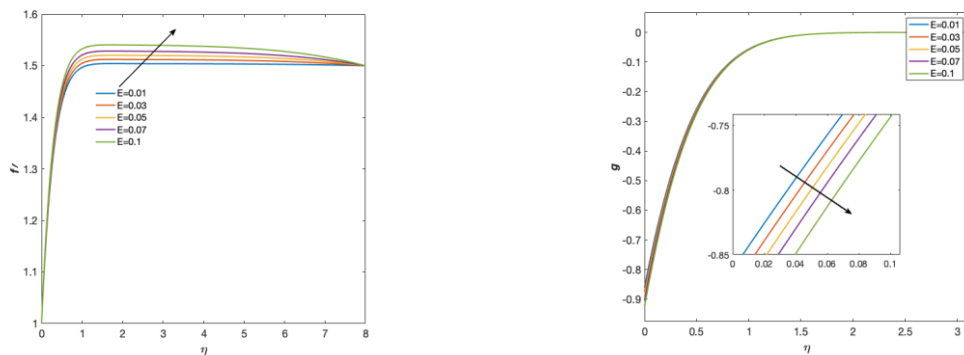


a. Temperature profile for Gr_c

b. Concentration profile for Gr_c

Figure 6: Variation of Temperature and concentration profile for Grashof number Gr_c with $S = 1$, $Re = 10$, $\lambda = 1.5$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

Figure 6 shows the variations of temperature and concentration profile for the mixed convective parameter Gr_c . The temperature profile declines with increasing Gr_c . Near the stretching sheet, the temperature profile increases at first due to enhanced convective heating; then it starts decreasing as it converges to zero for increasing Gr_c . The concentration profile shows increasing behaviour near the stretching sheet due to enhanced solutal diffusion but eventually decreases as solutal buoyancy drives nanoparticles away from the wall.



c. Velocity profile for E

d. Microrotation profile for E

Figure 7: Variation of velocity and microrotation profile for Electric field E with $S = 1$, $Re = 10$, $\lambda = 1.5$, $Da^{-1} = 0.5$, $\alpha = 0.2$, $\phi = 0.01$, $n = 0.5$, $M = 3$, $N_1 = 0.05$, $Pr = 22$, $Rd = 1$, $\Pi = \pi$, $Sc = 0.5$ and $Ec = 0.01$.

Figure~7 shows the variations of velocity and microrotation profile for the Electric field parameter E . Due to the Lorentz force, the electric field increases the magnetic field. Hence, the velocity profile overshoots if the electric field E is applied, implying an increase in boundary layer thickness. The same behaviour can be assumed for $\lambda < 1$. The microrotation profile decreases as the electric field parameter E increases.

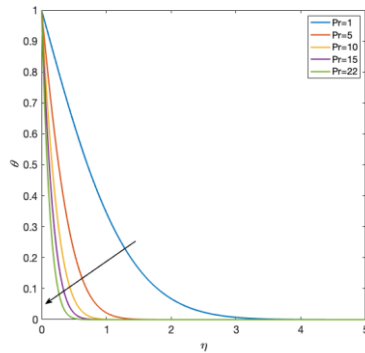
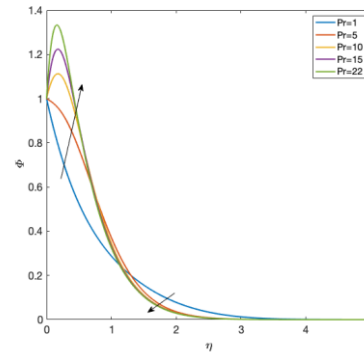
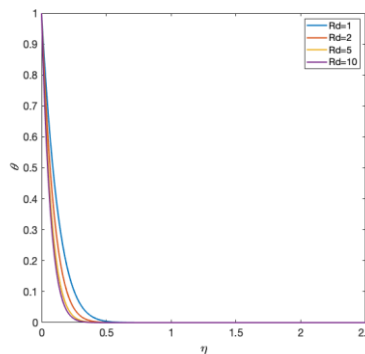
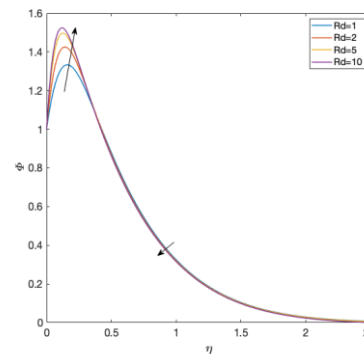

e. Temperature profile θ for Pr

f. Concentration profile Φ for Pr

g. Temperature profile θ for Rd

h. Concentration profile Φ for Rd

Figure 8: Variation of Temperature and concentration profiles for Prandtl number Pr and thermal radiation parameter Rd with $S = 1, Re = 10, \lambda = 1.5, Da^{-1} = 0.5, \alpha = 0.2, \phi = 0.01, n = 0.5, M = 3, N_1 = 0.05, \Pi = \pi, Sc = 0.5$ and $Ec = 0.01$.

Figure~8 shows the variations of temperature and concentration profile for the Prandtl number Pr and thermal radiation parameter Rd . Higher Prandtl number Pr , associated with low thermal diffusivity fluids like kerosene, leads to thinner thermal boundary layers, shown by a steep temperature drop. This happens because heat diffuses slowly in such fluids. The concentration profile near the wall increases due to thermophoretic effects but decreases further out as diffusion becomes less efficient. Similarly, increasing the radiation parameter Rd reduces temperature gradients, thinning the thermal layer. Radiation enhances energy transport, which explains the reduced thermal gradient and lower wall temperature. Concentration follows a similar trend, increasing near the sheet due to thermophoresis and decreasing later due to thermal thinning.

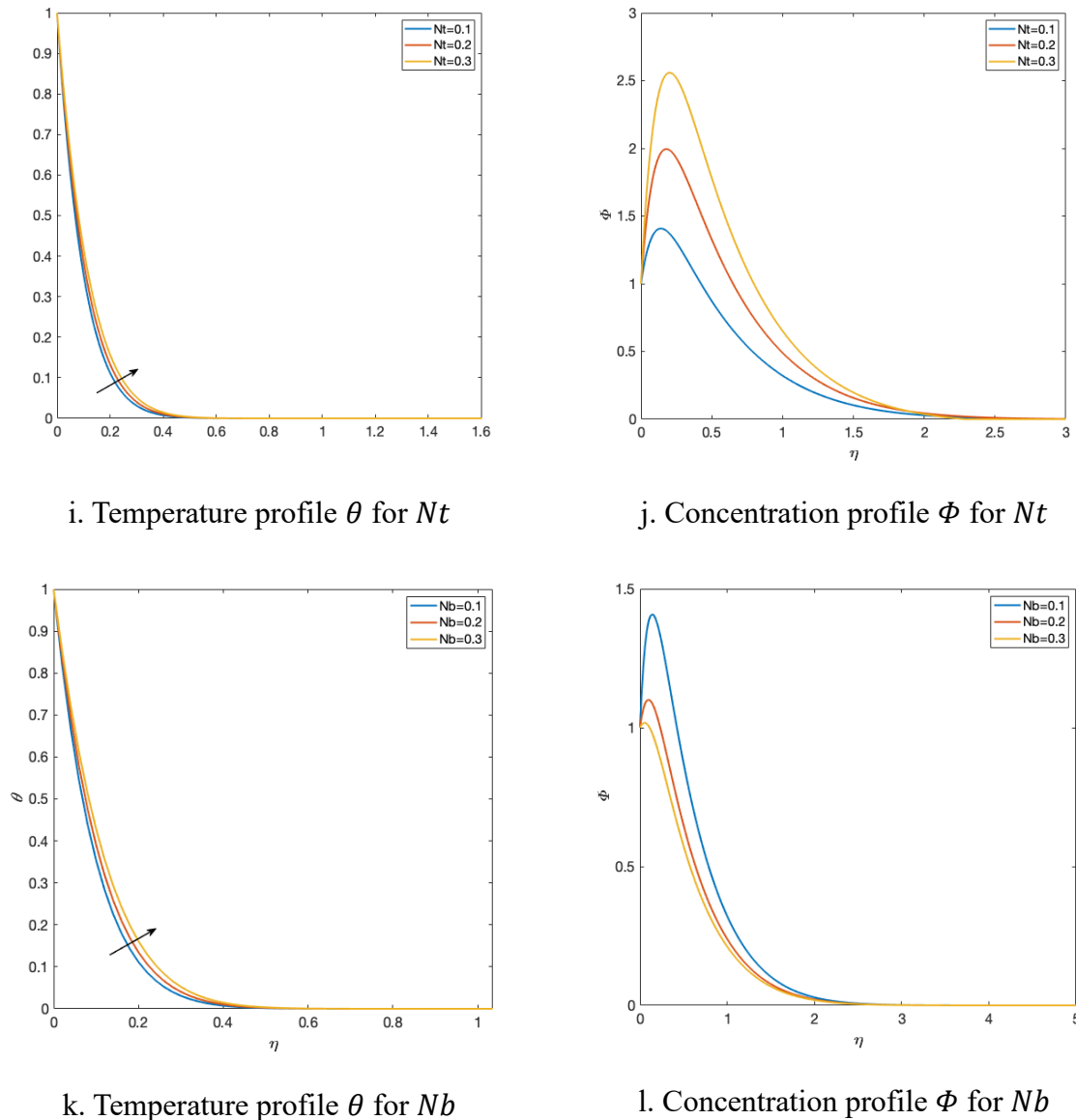


Figure 9: Variation of Temperature and concentration profile for Thermophoresis parameter Nt and Brownian motion parameter Nb with $S = 1, Re = 10, \lambda = 1.5, Da^{-1} = 0.5, \alpha = 0.2, \phi = 0.01, n = 0.5, M = 3, N_1 = 0.05, Rd = 1, Pr = 22, \Pi = \pi, Sc = 0.5$ and $Ec = 0.01$.

Figure~9 shows the variations of temperature and concentration profile for the Thermophoresis parameter Nt and Brownian motion parameter Nb . The thermal boundary layer thickness increases for both parameters Nt and Nb . Whereas the concentration profile increases for Nt and decreases for Nb . Thermophoresis pushes particles from hot to cold regions, increasing concentration further from the wall. Brownian motion, which moves particles from high to low-concentration regions, results in a lower concentration near the wall. This behaviour underscores how nanoparticle migration contributes to heat and mass transport.

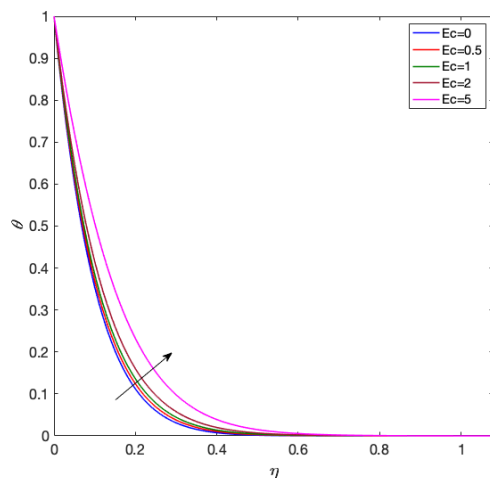
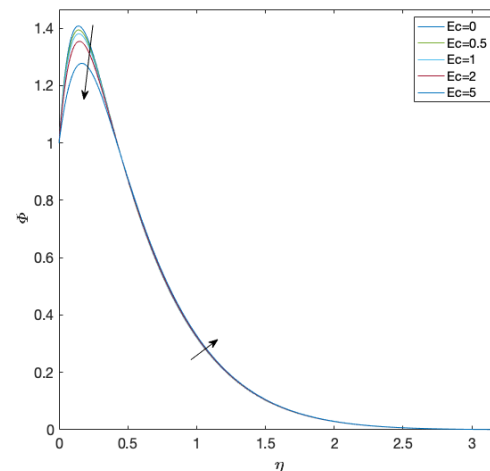
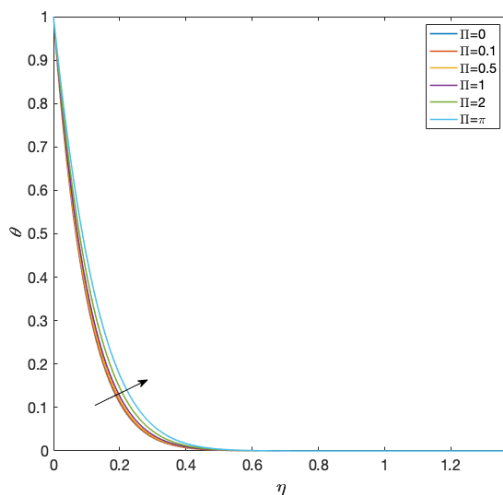
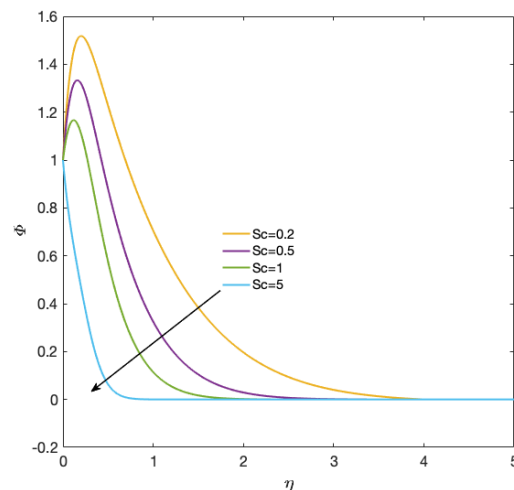
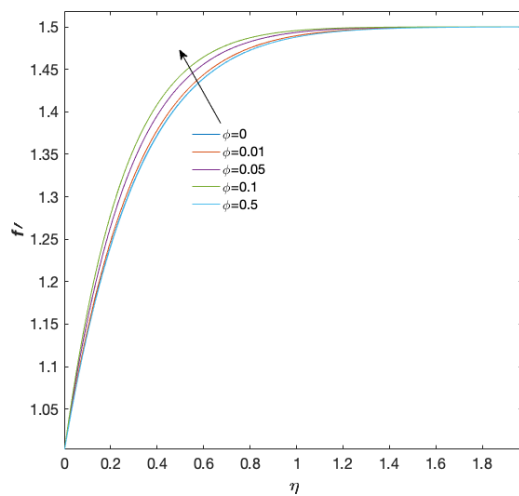
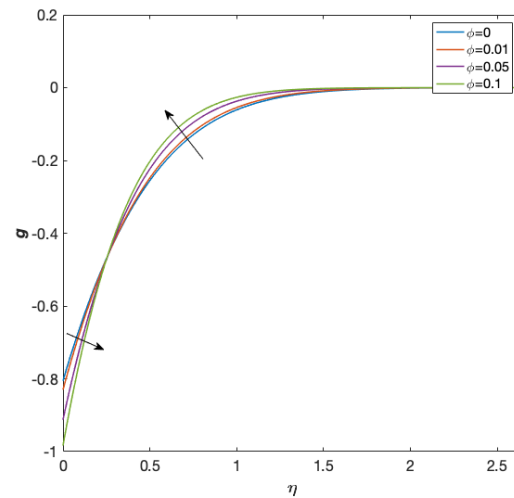

m. Temperature profile θ for Ec

n. Concentration profile Φ for Ec

o. Temperature profile θ for Π

p. Concentration profile Φ for Sc

Figure 10: Variation of Temperature and concentration profile for Eckert number Ec , heat source parameter Π and Schmidt number Sc with $S = 1, Re = 10, \lambda = 1.5, Da^{-1} = 0.5, \alpha = 0.2, \phi = 0.01, n = 0.5, M = 3, N_1 = 0.05, Rd = 1, Pr = 22, \Pi = \pi, Sc = 0.5$ and $Ec = 0.01$.

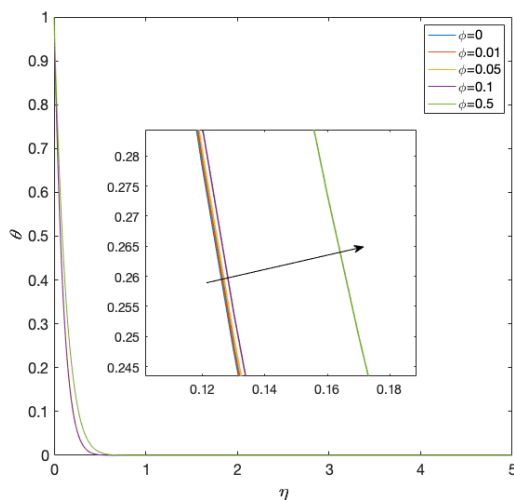
Figure~10 shows the variations of temperature and concentration profile for the Eckert number Ec , heat source parameter Π , and Schmidt number Sc . As the Eckert number Ec increases, the thermal boundary layer becomes thicker due to the enhanced conversion of kinetic energy into internal energy. Near the stretching sheet, the concentration profile decreases at first, and then it increases with the Eckert number Ec . The concentration profile diminishes with an increase in the Schmidt number Sc , while the thermal boundary layer becomes thicker as the heat source parameter Π increases.



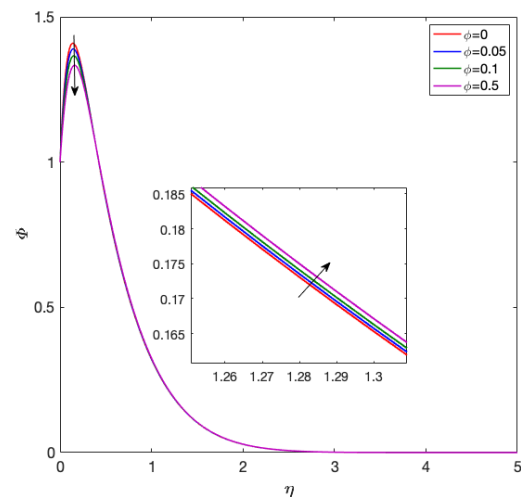
e. Velocity profile f' for ϕ



f. Microrotation profile g for ϕ



g. Temperature profile θ for ϕ



h. Concentration profile Φ for ϕ

Figure 11: Variation of profiles for Nanoparticle volume fraction ϕ with $S = 1, Re = 10, \lambda = 1.5, Da^{-1} = 0.5, \alpha = 0.2, \phi = 0.01, n = 0.5, M = 3, N_1 = 0.05, Rd = 1, Pr = 22, \Pi = \pi, Sc = 0.5$ and $Ec = 0.01$.

Figure~11 shows the variation profiles for the Nanoparticle volume fraction ϕ . The velocity profile is enhanced with increasing volume fraction ϕ . This leads to a rise in velocity due to improved viscosity and density characteristics, resulting in thicker boundary layers. The microrotation profile shows a dual nature, decreasing near the wall due to particle crowding and increasing farther away due to micro-rotational diffusion. Thermal boundary layer thickness increases with increasing volume fraction ϕ . Near the stretching sheet, the concentration profile decreases at first, and then it starts increasing as the volume fraction increases.

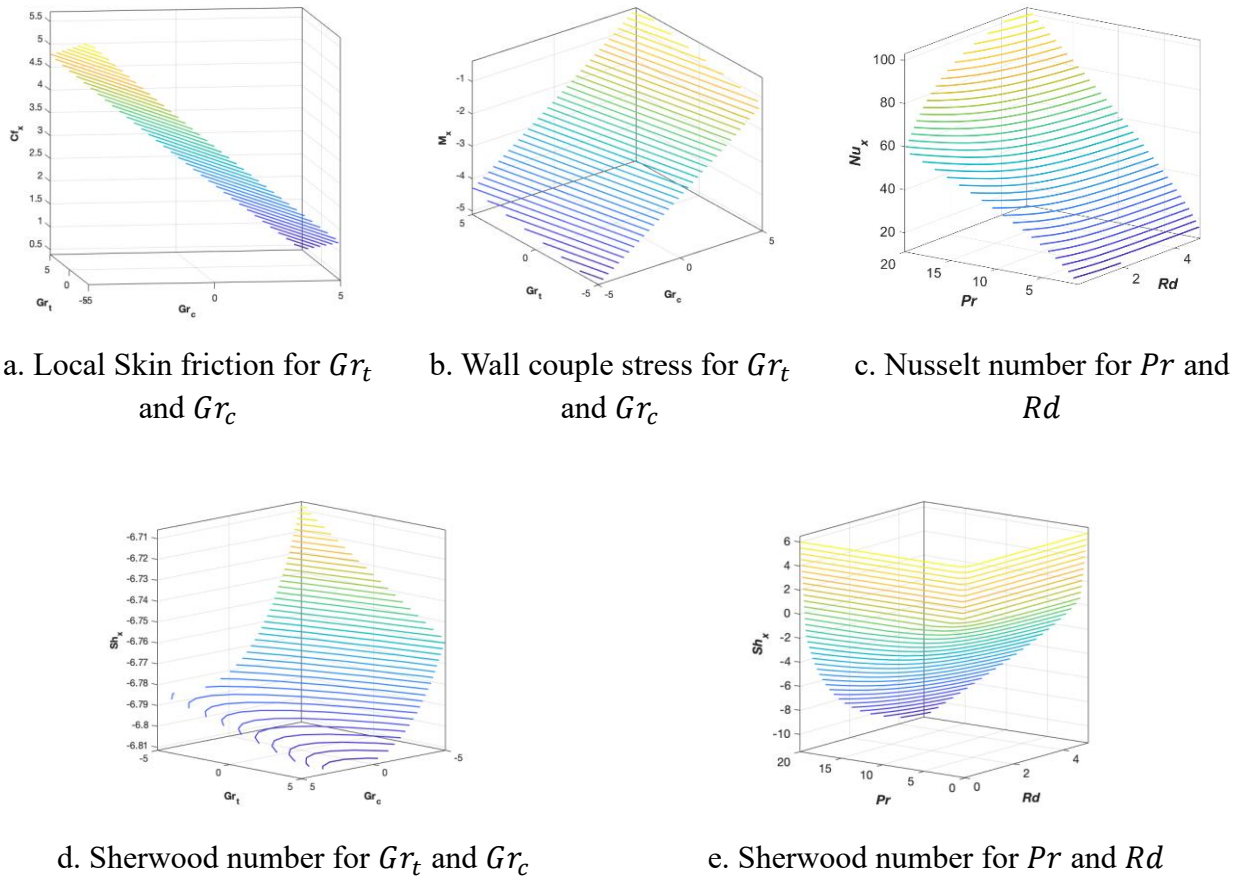


Figure 12: Variation of local skin friction coefficient, wall couple stress, Nusselt number, and Sherwood number with different parameters

Figure~12 provides critical insights into how various thermophysical and flow parameters influence the flow characteristics of a micropolar nanofluid. Figure 12(a) illustrates the variation of the local skin friction coefficient (Cf_x) with respect to the thermal Grashof number Gr_t and solutal Grashof number Gr_c . As both Gr_t and Gr_c increase, there is a noticeable rise in the skin friction coefficient. This trend is attributed to the enhanced buoyant forces, which accelerate the fluid motion near the surface, thereby increasing the velocity gradients and resulting in greater shear stress along the wall. This indicates that both temperature and concentration-induced buoyancy forces play significant roles in modifying the surface shear behaviour in such complex flow systems.

In Figure 12(b), the wall couple stress (M_x) is plotted against Gr_t and Gr_c . The graph shows that with increasing buoyancy forces, the microrotation of fluid elements near the surface intensifies, leading to higher rotational stresses. This increase in wall couple stress underscores the importance of microstructure effects in micropolar fluids, especially under buoyancy-driven flow regimes. The enhancement in microrotation reflects the energy exchange between linear and angular momentum due to the presence of suspended microelements in the nanofluid.

Figure 12(c) shows the behaviour of the Nusselt number (Nu_x) with variations in the Prandtl number (Pr) and thermal radiation parameter (Rd). It is observed that the Nusselt number increases with higher Pr values, indicating that fluids with lower thermal diffusivity facilitate more efficient surface heat transfer due to thinner thermal boundary layers. In contrast, as the thermal radiation parameter Rd increases, the Nusselt number decreases. This is because stronger radiative heat transfer tends to elevate the fluid temperature away from the surface, thereby reducing the temperature gradient at the wall and diminishing the convective heat transfer rate.

Figures 12(d) and 12(e) analyse the Sherwood number (Sh_x), representing the mass transfer rate. In Figure 12(d), Sh_x is shown to increase with Gr_t and Gr_c , suggesting that buoyancy forces enhance convective mass transfer in a manner similar to their influence on momentum and heat transfer. Figure 12(e) demonstrates the effects of Pr and Rd on the Sherwood number. A higher Prandtl number and increased thermal radiation both result in a reduction of Sh_x , due to the thickening of the thermal boundary layer and reduced surface concentration gradients. This inverse behavior emphasizes the coupling between thermal and mass transport phenomena in nanofluid systems and the necessity of considering radiative and diffusive effects in practical applications.

5. Conclusions

Based on the numerical results and analysis, the principal findings of this investigation are summarized below:

- The numerical outcomes derived through the Keller-box method have been validated by comparison with previously published results, showing excellent agreement and thereby confirming the accuracy of the proposed model.
- The combined effects of Lorentz force, viscous force, buoyancy force, and frictional force are found to significantly affect velocity, microrotation, temperature, and concentration profiles.
- Variations in the velocity ratio parameter (λ) cause noticeable changes in profile trends of key flow quantities. However, the trend of increasing boundary layer thickness remains consistent.
- Both velocity and microrotation (angular) boundary layer thicknesses increase with increases in thermal (Gr_t) and solutal (Gr_c) Grashof numbers, which represent buoyancy forces due to temperature and concentration differences.
- Increasing the volume fraction of nanoparticles results in the thickening of both the velocity and angular momentum boundary layers, emphasizing the influence of nanoparticle concentration on improving transport processes.

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