International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Single-Valued Neutrosophic R-Dynamic Edge Colouring of Graphs

A Stanis Arul Mary [1]

[1] Assistant Professor,Department of Mathematics,
Nirmala College for Women Coimbatore,641001,Tamilnadu,India e-mail:
sisterstanis@gmail.com

Abstract

This paper introduces Single-Valued Neutrosophic Edge Colouring (SVNEC) and compares it with R-Dynamic colour- ing. It proposes a novel Single-Valued Neutrosophic R- Dynamic Edge Colouring (SVNREC) model for certain graphs, integrating uncertainty handling with dynamic local colour- ing constraints, opening new dimensions in graph theory and in modelling.

Math. Subject Classification: 03E72, 05C90

Key Words: Single-valued Neutrosophic graph, SVN Edge colouring, SVNR-Dynamic Edge colouring

1 Introduction

In the branch of Mathematics, ”Graph Theory plays a vital role in current real-world applications. Within this field, Graph Colour- ing is a fundamental concept. In 1852, Francis Guthrie introduced the concept of graph colouring while examining how maps could be coloured. He initially put forward the Four Colour Problem, which stated that any planar map could be coloured using only four colours, ensuring that no two neighbouring regions shared the same colour. Although Guthrie’s hypothesis was eventually proven, his early work established the groundwork for the study of graph colouring.In the field of graph theory, edge colouring involves as- signing colours to a graph’s edges so that no two edges that are ad- jacent share the same colour. The chromatic index, also known as the edge chromatic number and represented as $\chi'(G)$, is the small- est number of colours required to achieve a proper edge colouring of the graph G.

1. Proper Edge Colouring: Colouring the edges in such a way that no two adjacent edges share the same colour.
2. Chromatic Index (Edge Chromatic Number): The least num- ber of colours needed for a proper edge colouring of a graph.
3. Adjacent Edges: Two edges are considered adjacent if they have a common vertex.

Received: August 10 2025

International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

4. Vizing's Theorem: For any graph, the chromatic index is ei- ther equal to the maximum degree $\Delta(G)$ or $\Delta(G) + 1$.

Neutrosophic logic is a logical framework designed to handle uncer- tainty, imprecision, and vagueness by incorporating three distinct elements: truth, falsehood, and indeterminacy. It builds upon fuzzy logic and intuitionistic fuzzy logic by explicitly addressing the in- determinacy aspect. This foundational concept also underpins the development of neutrosophic sets and statistics.In practical scenar- ios, single-valued neutrosophic sets are commonly utilized, where the degrees of truth, falsity, and indeterminacy are quantified as single values within the range [0, 1]. Generalization of Neutrosophic sets extend the principles of fuzzy sets and intuitionistic fuzzy sets by explicitly incorporating the indeterminacy factor. The concept of R-Dynamic colouring was introduced by Bruce Montgomery in 2001. He defined it as a proper vertex colouring where each vertex's neighbours must be coloured with at least $\min\{r, \deg(v)\}$ distinct colours. The smallest such k is called the R-Dynamic chromatic number, denoted as $\chi^r(G)$.

2 Preliminaries

Definition 1 Following [1], a Neutrosophic Graph (NG) is defined as

$$\zeta = (V, \lambda, \vartheta),$$

where V is a non-empty set of vertices and $\lambda = (\lambda_t, \lambda_i, \lambda_f) : V \to [0, 1]$, $\vartheta = (\vartheta_t, \vartheta_i, \vartheta_f) : V \times V \to [0, 1]$, such that for all $a, b \in V$:

$\vartheta_t(a, b) \leq \min\{\lambda_t(a), \lambda_t(b)\}$,

$\vartheta_i(a, b) \leq \min\{\lambda_i(a), \lambda_i(b)\}$,

$\vartheta_f(a, b) \leq \min\{\lambda_f(a), \lambda_f(b)\}$.

Here, $\vartheta_t, \vartheta_i, \vartheta_f$ represent the *edge set* in ζ, and $\lambda_t, \lambda_i, \lambda_f$ represent the *vertex set* in ζ.

Definition 2 In [2], a Single-Valued Neutrosophic Graph (SVNG) is defined to satisfy:

$T_Q(a, b) \leq \min\{T_P(a), T_P(b)\}$,

$I_Q(a, b) \leq \min\{I_P(a), I_P(b)\}$,

$F_Q(a, b) \leq \max\{F_P(a), F_P(b)\}$.

Definition 3 The strength of an edge was introduced by Akram [3]. Let $\zeta = (V, \lambda, \vartheta)$ be a Neutrosophic Graph. The strength of an edge (a, b) is denoted by

$$O(a, b) = \quad O_T(a, b), O_I(a, b), O_F(a, b) \ ,$$

Received: August 10 2025

International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

where

$$O_T(a, b) = \frac{\vartheta_T(a, b)}{\min\{\lambda_t(a), \lambda_t(b)\}}$$

$$O_I(a, b) = \frac{\vartheta_I(a, b)}{\min\{\lambda_i(a), \lambda_i(b)\}},$$

$$O_F(a, b) = \frac{\vartheta_F(a, b)}{\min\{\lambda_f(a), \lambda_f(b)\}}.$$

Definition 4 The Strength Score of an Edge in an NG was introduced in [4]. Consider $\zeta = (V, \lambda, \vartheta)$, and let $O(a, b) = \left(O_T(a, b), O_I(a, b), O_F(a, b)\right)$. Then the strength score of an edge (a, b) is defined as:

$$S(a, b) = \frac{2 + O_T(a, b) - O_I(a, b) - O_F(a, b)}{3}.$$

Definition 5 In Neutrosophic Graph Edge (NGE) colouring, two adjacent edges must have different fuzzy colours. Let $\zeta = (V, \lambda, \vartheta)$ and let the basic set of colours be

$C = \{c_1, c_2, \ldots, c_n\}$.

Any edge (a, b) gets its fuzzy colour as

$$(c_k, f(c_k)),$$

where c_k is the basic colour of the edge (a, b) and $f(c_k)$ is the depth of the basic colour, which is equal to the strength score of the edge:

$$f(c_k) = S(a, b).$$

3 Single-Valued Neutrosophic Edge Colour- ing (SVNEC)

3.1 Definition

Let

$\Psi = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}$

Received: August 10 2025

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

be a family of SVN fuzzy sets. This family is called an m-Single- Valued Neutrosophic Edge Colouring (SVNEC) of the SVNG $G = (P, Q)$ if the following conditions hold:

1. For every edge $e \in Q$, there exists at least one colour $\lambda_i \in \Psi$ such that $\vee\lambda_i(e) = Q$. (That is, each edge is assigned a colour from Ψ).
2. The intersection of any two distinct colours is zero, i.e.,
   $\lambda_i \wedge \lambda_j = 0, \quad i \neq j.$
3. For any two edges $e_1 = uv$ and $e_2 = uw$ that share a common vertex:
   $\min\{\lambda_i(T_P(e_1)), \lambda_i(T_P(e_2))\} = 0,$
   $\min\{\lambda_i(I_P(e_1)), \lambda_i(I_P(e_2))\} = 0,$
   $$\max\{\lambda_i(F_P(e_1)), \lambda_i(F_P(e_2))\} = 1, \text{ for all } 1 \leq i \leq k.$$

Here, $T_P(e)$, $I_P(e)$, $F_P(e)$ represent the truth-membership value (TMV), indeterminacy-membership value (IMV), and falsity-membership value (FMV) of edges, respectively.

The smallest k for which such a colouring can be achieved is called the **SVNE chromatic number** of G, denoted by $\chi'_e(G)$.

3.2 Example

Consider the Single-Valued Neutrosophic Graph $G = (P, Q)$ with vertex set $P = \{v_1, v_2, v_3, v_4, v_5\}$ and edge set $Q = \{e_1 = (v_1, v_2), e_2 = (v_2, v_3), e_3 = (v_3, v_4), e_4 = (v_4, v_1), e_5 = (v_2, v_5), e_6 = (v_3, v_5)\}$.

The vertex membership values are listed in Table 1. The corresponding edge membership values are presented in Table 2. Finally, the strength scores of the edges are summarized in Table 3.

Assign the set of colours

$C = \{c_1, c_2, c_3\}$

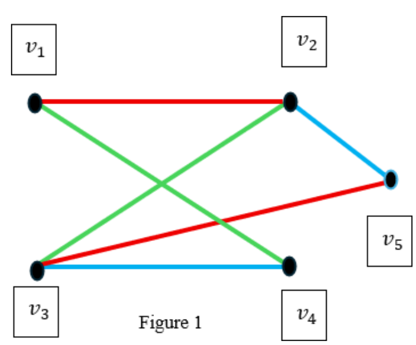

Figure 1

Received: August 10 2025

International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Table 1: Vertex Membership of SVNG of Figure 1

Vertex	λ_T	λ_I	λ_F
v_1	0.80	0.10	0.10
v_2	0.90	0.05	0.05
v_3	0.70	0.20	0.10
v_4	0.60	0.30	0.10
v_5	0.50	0.40	0.20

Table 2: Edge Membership of SVNG of Figure 1

Edge	Endpoints	ϑ_T	ϑ_I	ϑ_F
e_1	(v_1, v_2)	0.70	0.03	0.02
e_2	(v_2, v_3)	0.65	0.02	0.01
e_3	(v_3, v_4)	0.50	0.10	0.05
e_4	(v_4, v_1)	0.45	0.05	0.02
e_5	(v_2, v_5)	0.45	0.02	0.03
e_6	(v_3, v_5)	0.40	0.10	0.05

so that any adjacent edges sharing a vertex have different basic colours. Then:

$$c_1 = \{(v_1, v_2) = 0.625, \ (v_3, v_5) = 0.60\},$$

$$c_2 = \{(v_2, v_3) = 0.7762, \ (v_4, v_1) = 0.6833\},$$

Table 3: Strength Score of Edges of SVNG of Figure 1

Edge	O_T	O_I	O_F	Strength $S(e)$
e_1	0.8750	0.60	0.40	0.6250
e_2	0.9286	0.40	0.20	0.7762
e_3	0.8333	0.50	0.50	0.6111
e_4	0.7500	0.50	0.20	0.6833
e_5	0.90	0.40	0.60	0.6333
e_6	0.80	0.50	0.50	0.60

$c_3 = \{(v_3, v_4) = 0.6111, \ (v_2, v_5) = 0.6333\}$.

Hence, all the vertices have distinct colours on incident edges, and

$$\chi'_e(G) = 3.$$

4 Single-Valued Neutrosophic R-Dynamic Edge

Received: August 10 2025

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Colouring (SVNREC)

4.1 Definition

Let

$\Psi = \{\lambda_1, \lambda_2, \ldots, \lambda_m\}$

be a collection of SVN fuzzy sets. This collection is said to be an m- Single Valued Neutrosophic R-dynamic Edge Colouring (SVNREC) of a graph $G = (P, Q)$ if the following conditions hold:

1. For every edge $e \in Q$, at least one member of Ψ assigns it a colour, i.e.,

$$\bigcup \lambda_i(e) = Q.$$

2. Any two distinct members do not overlap in their assignment on the same edge:

$$\lambda_i \wedge \lambda_j = 0, \quad i \neq j.$$

3. For any pair of edges $e_1, e_2 \in Q$,

$\min\{\lambda_i(T_P(e_1)), \lambda_i(T_P(e_2))\} = 0,$

$\min\{\lambda_i(I_P(e_1)), \lambda_i(I_P(e_2))\} = 0,$

$\max\{\lambda_i(F_P(e_1)), \lambda_i(F_P(e_2))\} = 1,$

for all $1 \leq i \leq k$, where $T_P(e)$, $I_P(e)$, $F_P(e)$ denote the truth, indeterminacy, and falsity membership values of edges respec- tively.

4. If an edge has k adjacent edges, then the set of these adjacent edges must collectively use at least $\min\{R, k\}$ distinct colours from Ψ.

Here, R is a fixed integer satisfying $1 \leq R \leq K$, where K denotes the largest number of adjacent edges to any single edge in G. The smallest integer m for which such a colouring exists is called the SVNR-Dynamic edge chromatic number of G, denoted by $\chi'^{e}(G)$.

4.2 Example

Consider the Single-Valued Neutrosophic Graph

$G = (P, Q), \qquad P = \{v_1, v_2, v_3, v_4\}, \quad Q = \{12, 13, 14, 23, 24, 34\}.$

Vertex membership values:

$$(T_P(v_i), I_P(v_i), F_P(v_i)) = \begin{cases} (0.8, 0.2, 0.1), & i = 1, \\ (0.7, 0.3, 0.2), & i = 2, \\ (0.9, 0.1, 0.05), & i = 3, \\ (0.6, 0.4, 0.3), & i = 4. \end{cases}$$

Received: August 10 2025

Volume 38 No. 9s , 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Edge membership values:

$$(T_Q(v_j, v_k), I_Q(v_j, v_k), F_Q(v_j, v_k)) = \begin{cases} (0.7, 0.2, 0.1), & e = 12, \\ (0.75, 0.2, 0.1), & e = 13, \\ (0.6, 0.3, 0.2), & e = 14, \\ (0.65, 0.3, 0.15), & e = 23, \\ (0.55, 0.35, 0.25), & e = 24, \\ (0.7, 0.25, 0.2), & e = 34. \end{cases}$$

Since K_4 is complete, the maximum number of adjacent edges for any edge is $R = 3$.

Case 1: $1 \leq R \leq 2$

Let $\Psi = \{\lambda_1, \lambda_2, \lambda_3\}$, where assignments satisfy the edge colouring conditions. Then

$$\chi'^e_R(G) = 3 \qquad \text{for } 1 \leq R \leq 2.$$

Case 2: $3 \leq R \leq 4$

Let $\Psi = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ with colouring assignments such that the R-dynamic condition is met. Then

$$\chi'^e_R(G) = 4 \qquad \text{for } 3 \leq R \leq 4.$$

Thus, the conclusion is:

$$\chi'^e_R(G) = \begin{cases} 3, & 1 \leq R \leq 2, \\ 4, & 3 \leq R \leq 4. \end{cases}$$

4.3 Theorem

Let $n \geq 3$ and P_n be a path graph with n vertices and $n-1$ edges. The SVNRDE chromatic number of P_n, denoted $\chi'^e_R(P_n)$, is given by

$$\chi'^e_R(P_n) = \begin{cases} 2, & R = 1, \\ 3, & R = 2. \end{cases}$$

Received: August 10 2025

International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Proof. **Case 1:** $R = 1$. Let $\Psi = \{\lambda_1, \lambda_2\}$ where

$$\lambda_1(e_i) = \begin{cases} (T_Q(e_i), I_Q(e_i), F_Q(e_i)), & i \text{ odd}, \\ (0, 0, 1), & \text{otherwise}, \end{cases}$$

$$\lambda_2(e_i) = \begin{cases} (T_Q(e_i), I_Q(e_i), F_Q(e_i)), & i \text{ even}, \\ (0, 0, 1), & \text{otherwise}. \end{cases}$$

This satisfies the R-dynamic condition, hence $\chi'^{e}_{R}(P_n) = 2$. □

Case 2: $R = 2$. Let $\Psi = \{\lambda_1, \lambda_2, \lambda_3\}$ with assignments based on modulo 3. Each edge satisfies the dynamic condition with $R = 2$, hence $\chi'^{e}_{R}(P_n) = 3$.

4.4 Theorem

Let $n \geq 3$ and C_n be a cycle graph with n vertices and m edges. The SVNRDE chromatic number of C_n, denoted $\chi'^{e}_{R}(C_n)$, is given by

$$\chi'^{e}_{R}(C_n) = \begin{cases} 2, & R = 1,\ n \text{ even}, \\ 3 & R = 1,\ n \text{ odd}, \\ 3, & R = 2,\ n \text{ even}, \\ 4, & R = 2,\ n = 3m, \\ 5, & \text{otherwise}. \end{cases}$$

Proof. **Case 1**: $R = 1$, n even. $\chi'^{e}_{R}(C_n) = 2$.

Case 2: $R = 1$, n odd. $\chi'^{e}_{R}(C_n) = 3$.

Case 3: $R = 2$, n even, divisible by 3, requires 4 colours.

Case 4: Otherwise, $\chi'^{e}_{R}(C_n) = 5$.

International Journal of Applied Mathematics

Volume 38 No. 9s , 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

5 Bistar $B_{(m,n)}$

5.1 Definition

Let $m, n \geq 1$ be integers. The bistar $B_{(m,n)}$ is the tree obtained by taking two stars S_{m+1} and S_{n+1} with centers u and v, respectively.

Thus,

$u = m + n + 2,$ $v = m + n + 1$ $\deg(u) = m + 1,$ $\deg(v) = n + 1$

$$\Delta = \max\{m + 1, n + 1\}$$

5.2 Bistar in SVNR-Dynamic Colours

5.2.1 Theorem

Let $B_{(m,n)}$ and $1 \leq R \leq m$, the Single-valued Neutrosophic R- dynamic Edge chromatic number satisfies

$$\chi_R^{'e}(B_{(m,n)}) = \max\{\Delta, R\}$$

Proof. **Lower Bound Conclusion:** Any proper edge colouring of a graph requires at least Δ distinct basic colours. Hence

$$\chi_R^{'e}(B_{(m,n)}) \geq \Delta$$

The R-dynamic requirement for the central edge uv demands that the set of edges adjacent to it contain at least

$\min\{R, m\}$ distinct colours.

If $R > \Delta$, we need at least R colours overall to supply that many distinct colours among adjacent edges. Therefore,

$$\chi_R^{'e}(B_{(m,n)}) \geq R, \quad \chi_R^{'e}(B_{(m,n)}) \geq \max\{\Delta, R\}$$

Upper Bound Conclusion: Choose k distinct basic colours

$C = \{c_1, c_2, \ldots, c_k\}$.

Because $B_{(m,n)}$ is a tree, it admits a proper edge-colouring using exactly Δ colours. If $k > \Delta$, we can still colour with k colours and optimally recolour some edges to introduce the extra colours.

R-dynamic condition: if $k \geq R$, we can arrange the colours on the edges incident at each center so that the set of incident colours are at least

$\min\{R,$ adjacency count$\}$ distinct colours, $k \geq \Delta$.

Thus, $k = \max\{\Delta, R\}$ colours suffice for proper SVN R-Dynamic edge colouring.

□

Received: August 10 2025

International Journal of Applied Mathematics
Volume 38 No. 9s , 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

6 Conclusion

This new approach satisfies the dynamic colouring constraint pa- rameterized by R. This demonstrates that the required number of colours varies with the parameter R and with the structural properties of the underlying graph such as parity, divisibility, and maximum edge adjacency. This can be extended in developing al- gorithms and applications such as communication networks, social networks, and uncertain transport systems”.

References

[1] M. Akram and M. Sitra, “Novel applications of single-valued Neutrosophic graph Structure in decision-making,” *Journal of Applied Mathematics and Computing*, vol. 56, no. 1-2, pp. 501-532, 2018.

[2] M. Akram and S. Shahzadi, “Neutrosophic soft graphs with application,” *Journal of Intelligent and Fuzzy Systems*, 32(1), 841-858, 2017.

[3] M. Akram, “Single-Valued Neutrosophic Planar Graphs,” *In- ternational Journal of Algebra and Statistics*, vol. 5, pp. 157- 167, 2016.

[4] R. Mahapatra, S. Samanta, and M. Pal, “Edge Colouring of Neutrosophic Graphs and Its Application in Detection of Phishing Website,” *Discrete Dynamics in Nature and Society*, 2022.

[5] A. Vasanthi, N. Mohanapriya, S. Broumi, “Single-valued Neu- trosophic R-Dynamic Vertex colouring of Graphs,” *NSS*, Vol- 48, 2022.

Received: August 10 2025