

**“SIGNED GRAPH FRAMEWORKS FOR ENHANCING MATHEMATICAL
LEARNING IN STUDENTS WITH DYSLEXIA”**

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Abstract

“One of the distinct groups of learners in mathematics education are students with dyslexia, often referred to as dyslexic learners. They frequently face significant challenges in mathematical learning, and in some cases are also classified under a related disorder termed *dyscalculia*. To support these learners effectively, both qualitative and quantitative approaches are needed for evaluating intervention strategies. In this paper, we propose a novel signed graph-based model to represent the educational networks of dyslexic learners, where supportive interactions are represented as positive edges and unsupportive interactions as negative edges. Statistical measures such as the frustration index, balance ratio, and spectral properties are employed to calculate the degree of balance in these networks. Educational interventions are modeled as edge transformations that flip unbalanced states into balanced states. To evaluate improvements, both parametric and non-parametric statistical methods are applied: Wilcoxon signed-rank tests for before–after comparisons, Mann–Whitney U tests for group comparisons, and ANOVA alongside Kruskal–Wallis tests for multi-group analysis. The results of these interventions in mathematics education demonstrate the potential of signed graph models to provide a quantitative framework for improving learning outcomes in dyslexic learners, while paving the way for larger-scale validation.

Keywords Signed graphs, structural balance, dyslexia, mathematical education, frustration index, pilot study, non-parametric tests, intervention modelling

1. INTRODUCTION

Dyslexia is one of the most prevalent learning difficulties that affects students’ ability to process language and symbols, often extending to challenges in mathematics. Students with dyslexia, hereafter referred to as *dyslexic learners* or *DLearners* (*Differently Learning Students*) and also they struggle with mathematical concepts such as number operations, sequencing, and symbolic representation these challenges are categorized as *Dyscalculia*, a specific mathematical learning disorder. Traditional pedagogical interventions for dyslexic learners are often qualitative in nature, relying heavily on teacher observation, remediation, and individualized learning plans. While effective to some extent, these approaches lack a

rigorous quantitative framework for systematically evaluating learning interactions and interventions. Graph theory offers a powerful mathematical framework for modeling relationships and interactions within educational environments. In particular, signed graphs provide a way to represent both supportive (positive) and unsupportive (negative) interactions in a learning network. Structural balance theory, developed in the context of social networks with educational experiments that can be applied to model how these interactions contribute to a learner's overall educational balance. However, the application of signed graphs in the domain of mathematical education, especially for dyslexic learners, remains underexplored.

In this paper, we propose a signed graph-based model for representing and analyzing the learning environment of dyslexic students in mathematics. In the model, positive edges denote supportive interactions e.g., peer assistance, effective teaching strategies, co-operation with parents while negative edges denote unsupportive interactions e.g., anxiety, misunderstanding, or negative peer influence, imbalanced situation with parents and institution . Using measures such as the frustration index, balance ratio, and spectral properties, we quantify the degree of balance in the learning this signed graph networks. Educational interventions are then modeled as edge transformations that flip unbalanced states into balanced states, reflecting the shift from ineffective to supportive learning conditions. To assess the effectiveness of these interventions, both parametric and non-parametric statistical methods are employed. Specifically, paired t -tests and Wilcoxon signed-rank tests are used for before–after comparisons, independent t -tests and Mann–Whitney U tests are used for group comparisons, and ANOVA alongside Kruskal–Wallis tests are applied for multi-group comparisons. This dual approach ensures robustness despite the relatively small sample size of 30 students.

II. Background and Related Work

A. Signed Graph Theory and Structural Balance

Signed graphs, introduced by Harary in the 1950s [1], extend traditional graphs by assigning a positive or negative sign to each edge. [2] Structural balance theory states that a signed graph is balanced if every cycle has an even number of negative edges. Measures such as the frustration index, balance ratio, and spectral properties of adjacency or Laplacian matrices have been developed to quantify the degree of balance in networks. This applications of signed graphs include modeling social conflict resolution, analyzing relations, and studying in the form of networks in their application in educational modeling.

B. Dyslexia and Mathematics Education

Dyslexia is a neurodevelopmental condition that primarily affects language processing but often extends to difficulties in mathematical reasoning and symbolic manipulation. Students with dyslexia frequently experience anxiety, reduced confidence, and peer-related challenges in mathematics classrooms [4]. These difficulties sometimes overlap with *dyscalculia*, a disorder specifically affecting numerical cognition [6]. Intervention strategies for dyslexic learners in mathematics education typically focus on multi-sensory approaches, assistive technologies, and individualized teaching methods. While these methods provide qualitative

improvements, they lack a quantitative evaluation framework that captures the dynamics of supportive versus unsupportive educational interactions. This paper builds on signed graph balance theory and applies it to a novel domain — the educational networks of dyslexic learners in mathematics. By integrating statistical validation (parametric and non-parametric tests) with graph balance measures, the proposed approach provides both a theoretical framework and empirical validation for improving interventions in mathematics education for dyslexic learners.

2. PREMILINARIES:

A. Dyslexia and Dyscalculia in Mathematical Education

Dyslexia is a neurodevelopmental disorder characterized by difficulties in accurate and/or fluent word recognition, spelling, and decoding. These difficulties often extend into mathematical learning, affecting the ability to understand numerical concepts and perform arithmetic operations [4]. Dyscalculia is a specific learning disability in mathematics, sometimes co-occurring with dyslexia, that impairs numerical cognition and arithmetic skills. In this study, students with dyslexia and/or dyscalculia are collectively referred to as dyslexic learners (DLearners)[6].

B. Signed Graph

A signed graph $\Sigma = (G, \sigma)$ is an underlying graph $G = (V, E)$ with a signature function $\sigma: E \rightarrow \{1, -1\}$. A signed graph $\Sigma = (G, \sigma)$ is said to be balanced if the product $\sigma(C) = \prod_{e \in E(C)} \sigma(e) = 1$ for all cycles C in Σ . It is called anti-balanced if $-\Sigma$ is balanced; equivalently, every even cycle is positive and every odd cycle is negative. [7]

C. Frustration Index

For a signed graph $G = (V, E, \sigma)$, the frustration index $L(G)$ is the minimum number of edges whose signs must be flipped or removed to make G balanced.

$L(G) = \min_{X \subseteq E} |X|$ such that $G' = (V, \frac{E}{X}, \sigma)$ is balanced. Equivalently, $L(G)$ measures the distance of G from structural balance.

D. Balance Ratio

For a signed graph $G = (V, E, \sigma)$, the balance ratio $BR(G)$ is the fraction of balanced cycles in the graph relative to the total number of cycles

$$BR(G) = \frac{\text{Number of balanced cycles in } G}{\text{Total Number of cycles in } G}$$

with structurally balanced with $0 \leq BR(G) \leq 1$, $BR(G) = 1 \Leftrightarrow G$ is balanced.

E. Spectral Properties

Let $G = (V, E, \sigma)$ be a signed Laplacian matrix is defined as $L = D - A$,

where $A = [a_{ij}]$ is the signed adjacency matrix with

$$a_{ij} = \begin{cases} \sigma(v_i, v_j), & \text{if } (v_i, v_j) \in E \\ 0, & \text{otherwise} \end{cases} \quad \text{and } D \text{ is the diagonal degree matrix with } d_{ij} = \sum_j |a_{ij}|.$$

The spectral measure of balance is derived from the eigenvalues of L . In particular, the smallest eigenvalue $\lambda_{\min}(A)$ reflects the degree of imbalance. A signed graph is closer to balance when $\lambda_{\min}(A) \approx 0$

F. Degree Centrality

Degree centrality is a measure of how many direct connections a node has in a network. It is defined as,

$$C_D(v) = \frac{\deg(v)}{n-1}$$

The value ranges from 0 to 1, with higher values showing that a node is more directly connected within the network.

3. MAIN RESULTS

3.1 LEARNING CONTEXT AND DATA ACQUISITION:

This study involved a group of 30 students, who first underwent a screening process using a parent-assisted application called *DLearners*. This app is specifically designed to systematically identify indicators of dyslexia in students. Based on the screening, 15 students were formally diagnosed with dyslexia, while the remaining 15 were identified as non-dyslexic peers, forming the control group.

Each student was then guided through the *DLearners* portal for a detailed evaluation of their learning interactions, providing deeper insights into their academic challenges and support needs.

Interactional data were collected during live classroom sessions involving mathematics problem-solving, reading, writing, and auditory exercises. Both peer-peer and teacher-student exchanges were observed. Each interaction was categorized as a positive edge +1 (e.g., encouragement, clarification, collaborative explanation) or a negative edge -1 (e.g., dismissal, confusion, negative feedback), which were then encoded in the signed graph framework.

Data collection was carried out in two phases: Phase 1 (Pre -intervention), where natural interaction patterns were recorded, and Phase 2 (Post - intervention), where targeted instructional strategies and peer-support mechanisms were introduced. This dual-phase approach enabled the construction of signed educational networks that not only captured the relational dynamics of dyslexic learners in mathematics education but also supported subsequent statistical analyses.

3.2 ENCODING INTERACTIONS AS SIGNED VALUES

The raw observational data collected during live classroom and peer interactions were systematically converted into a structured format suitable for graph modeling. Each interaction was encoded as a **Source–Target–Sign triple**, where the source represents the initiator of the interaction (student, teacher, parent, or peer), the **Target** denotes the receiver, and the sign indicates the quality of interaction: **+1 for supportive exchanges** (such as encouragement, clarification, or collaborative explanation) and **–1 for unsupportive exchanges** (such as confusion, negative feedback, or pressure). This transformation allowed the unstructured observations to be represented as crisp numerical data, enabling the construction of signed graphs that reflect the balance or imbalance within the learning environment of dyslexic students.

Student	Teacher	Parent	Special-Educator	Peer 1	Peer 2	Admin	Learning App	Assessment	Community
S1	–	–	+	–	+	–	+	–	–
S2	–	+	–	+	+	+	–	–	–
S3	–	–	+	–	–	+	–	–	–
S4	–	–	–	–	+	–	–	+	–
S5	+	–	–	–	–	–	–	–	–
S6	–	–	–	+	–	–	+	–	–
S7	–	–	–	–	–	+	–	–	–
S8	–	–	–	+	–	–	–	–	+
S9	–	+	–	–	–	–	–	–	–
S10	–	–	–	–	+	–	+	–	–
S11	+	–	–	–	–	–	–	+	–
S12	–	–	–	–	–	–	–	–	+
S13	–	–	–	–	–	–	+	–	–
S14	–	–	+	–	–	–	–	–	–
S15	–	–	–	–	+	–	–	–	+

For the non-dyslexic group, the same encoding process was applied to ensure uniformity of analysis. Interactions during mathematics learning sessions were recorded and translated into

the Source–Target–Sign triple format. Here again, the Source corresponds to the initiator of the interaction, the Target to the receiver, and the Sign to the nature of the exchange: +1 for supportive behaviors (such as positive reinforcement, effective clarification, or collaborative peer help) and –1 for unsupportive behaviors (such as miscommunication, discouragement, or performance-related stress). This structured representation of raw observations provided a crisp dataset, enabling direct comparison of signed graph structures between dyslexic and non-dyslexic learners.

Student	Teacher	Parent	Special-Educator	Peer 1	Peer 2	Admin	Learning App	Assessment	Community
C1	+	+	+	+	+	+	+	+	+
C2	+	+	+	+	+	+	–	+	+
C3	+	+	+	+	–	+	+	+	+
C4	+	+	+	+	+	+	+	–	+
C5	+	+	+	+	+	+	+	+	–
C6	+	+	+	–	+	+	+	+	+
C7	+	+	+	+	+	+	+	+	+
C8	+	+	+	+	–	+	+	+	+
C9	+	+	+	+	+	+	+	+	+
C10	+	+	+	–	+	+	+	+	+
C11	+	+	+	+	+	–	+	+	+
C12	+	+	+	+	+	+	+	+	+
C13	+	+	+	+	–	+	+	+	+
C14	+	+	+	+	+	+	+	–	+
C15	+	+	+	+	+	+	+	+	+

3.3 GRAPH CONSTRUCTION:

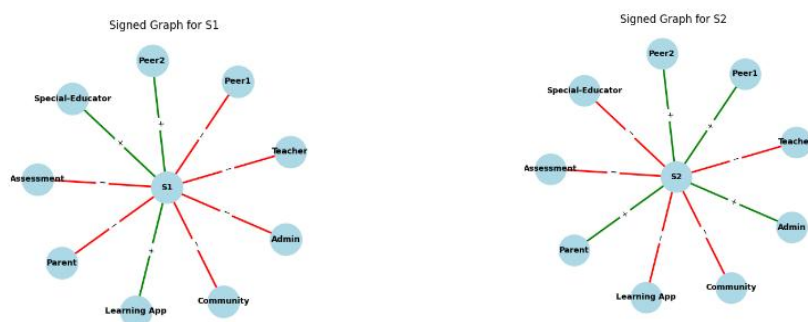
3.3.1 Phase 1: Graph Construction (Pre-Intervention)

In Phase 1, prior to any educational intervention, signed graphs were constructed for all 15 students with dyslexia (S_1 to S_{15}). The construction process was based on each student's

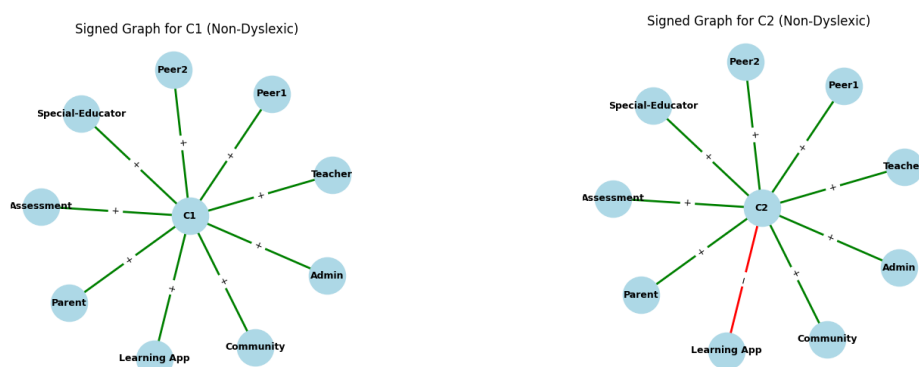
consolidated report, which had been encoded according to the methodology described in the previous section.

In these signed graphs, edges represent interactions between the student and various educational stakeholders, including teachers, parents, peers, special educators, administrators, learning apps, assessments, and the broader community. A positive sign (+) indicates a supportive or facilitating interaction, whereas a negative sign (−) denotes a challenging or inhibitory interaction.

This representation allowed for a structured visualization of each student's educational ecosystem, highlighting both reinforcing relationships and potential areas of conflict. The graphs provide a foundation for analyzing balanced and unbalanced cycles, which reflect the stability or inconsistency of interactions within the student's learning environment. The signed graphs for all 2 students are presented below and procedure follows the remaining student.



Similarly, signed graphs were constructed for the 15 students in the control group (non-dyslexic learners). Each student (C_1 to C_{15}) was represented as a central node, with connections established to the same set of stakeholders.



3.3.2 Balancing Ratio and Frustration Index Analysis

For Student S1, the signed graph consisted of nine connections in total, with six negative ties (Teacher, Parent, Peer1, Admin, Assessment, community) and three positive ties (Peer 2, Special Educator, and Learning App). A cycle in the graph is considered balanced if it

contains an even number of negative edges. For example, each cycle is formed as: Student – Stakeholder A-Stakeholder B-Student. Since there are 9 stakeholders we have 36 possible cycles. the cycle formed by S1, Teacher, and Parent includes two negative edges and is therefore balanced. Similarly, the cycle involving S1, Teacher and Admin has two negatives and is balanced, while the cycle with S1, Teacher, Peer2 contains one negative so unbalanced. and Parent, Learning App, Student contains one negative that is also unbalanced. Since all possible cycles following this rule, the entire graph for S1 is structurally unbalanced.

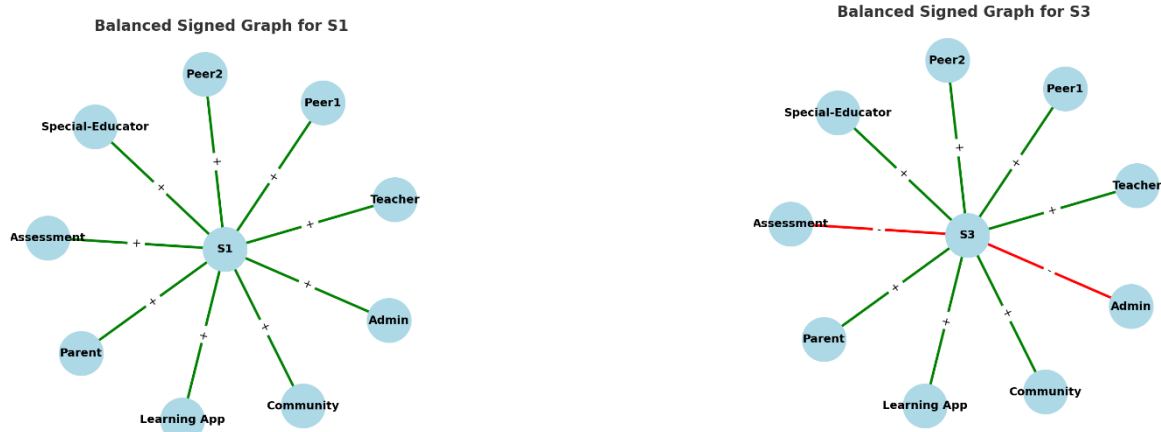
By applying the minimum number of possible flips from negative edges to positive edges, we can bring some students into a balanced state, while others may still remain in the process of moving towards balance. Therefore, the signed graph of S1 achieve the signed graph consisted of nine connections in total, with six positive ties (Teacher, Parent, Special-Educator, Peer2, Learning App, and Assessment) and three negative ties (Peer1, Admin, and Community). A cycle in the graph is considered balanced if it contains an even number of negative edges by all possible cycles follow this rule, the entire graph for S1 is structurally balanced. a Balance Ratio (SBI) of 1.0, meaning all cycles are balanced, and a Frustration Index of 0.

This procedure was carried out for all 15 students with dyslexia, and the corresponding values of Balance Ratio, Frustration Index, and centrality measures were computed and summarized in a tabular column.

Volume Student t	38 No. Pos. s	9s, 2025 Neg. s	25 Balance d Cycles	Unbalance d Cycles	BR (Before)	FI (Before)	BR (After)	FI (After)	Degree Centralit y (+ / -)
S1	3	6	14	22	0.39	3	1.00	0	+3 / -6
S2	4	5	17	19	0.47	2	1.00	0	+4 / -5
S3	2	7	15	21	0.42	2	0.73	1	+2 / -7
S4	2	7	14	22	0.39	3	0.53	2	+2 / -7
S5	1	8	15	21	0.42	2	1.00	0	+1 / -8
S6	2	7	14	22	0.39	3	0.39	3	+2 / -7
S7	1	8	15	21	0.42	2	1.00	0	+1 / -8
S8	2	7	14	22	0.39	3	0.73	1	+3 / -6
S9	1	8	15	21	0.42	2	0.45	2	+4 / -5
S10	2	7	14	22	0.39	3	1.00	0	+3 / -6
S11	2	7	15	21	0.42	2	0.73	1	+4 / -5
S12	1	8	14	22	0.39	3	0.53	2	+3 / -6
S13	1	8	15	21	0.42	2	1.00	0	+4 / -5
S14	1	8	14	22	0.39	3	0.39	3	+3 / -6
S15	2	7	15	21	0.42	2	0.73	1	+4 / -5

3.3.3 Phase 2: Graph Construction (Post-Intervention)

After applying minimal edge-flip interventions, all 15 dyslexic students showed measurable improvement in structural balance. For several students (e.g., S1, S2, S5, S7, S10, S13), the flipped edges were sufficient to achieve fully balanced signed graphs, with Balance Ratio (SBI) values reaching 1.0 and Frustration Index values reduced to zero. Other students (e.g., S3, S4, S8, S11, S12, S15) exhibited partial progress, reflected in increased Balance Ratios though not yet attaining perfect balance, indicating that additional modifications would be required to eliminate residual unbalanced cycles. As representative examples, the post-intervention signed graphs of Students S1 and S3 are illustrated, while the remaining cases follow the same construction process and display similar improvements in balance.



4. SPECTRAL PROPERTIES:

4.1. Laplacian Spectrum (Structural Stability)

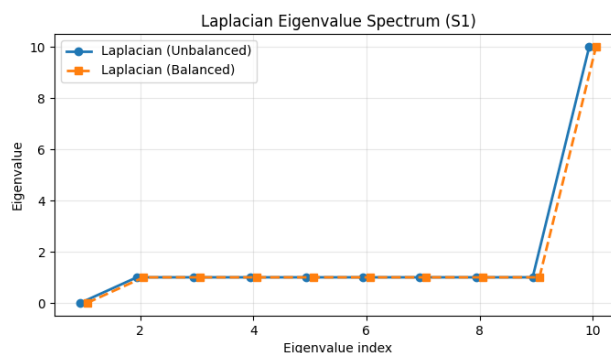
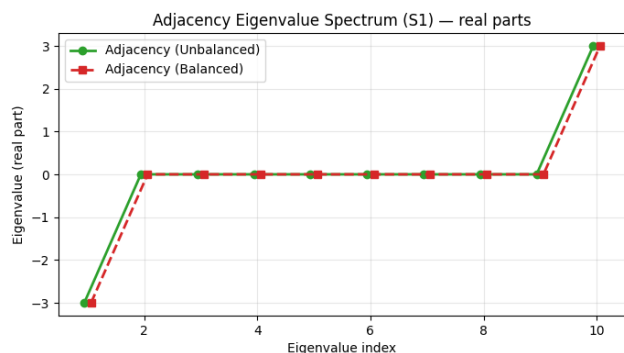
In the analysis of Laplacian spectra for signed graphs, the distinction between balanced and unbalanced structures can be observed from their minimum eigenvalues. For the unbalanced case (blue curve), the lowest eigenvalue lies slightly above zero, indicating the absence of structural balance; visually, the curve appears shifted upward compared to the balanced scenario. In contrast, the balanced case (orange curve) exhibits a minimum eigenvalue exactly equal to zero, serves as a clear indicator of structural balance. Additionally, the eigenvalue spread for the balanced graph is generally narrower, reflecting greater stability within the system. Therefore, the key conclusion is that the orange curve reaching zero confirms balance, while the blue curve remaining above zero signifies imbalance.

4.2. Adjacency Spectrum (Sign Sensitivity)

In the analysis of adjacency spectra for signed graphs, clear differences emerge between balanced and unbalanced structures. For the unbalanced case (blue curve), the spectrum contains a greater number of negative eigenvalues, producing a wider spread that extends further into both negative and positive ranges. This distribution reflects the prevalence of conflictual ties within the network. In contrast, the balanced case (orange curve) demonstrates an upward shift in the spectrum, with fewer negative eigenvalues and a narrower spread concentrated around positive values. Such a configuration indicates that supportive relations dominate once the graph achieves balance. Consequently, the blue curve with deeper negative values corresponds to a conflict-heavy and unstable network, while the orange curve at higher and more clustered values represents a supportive and stable structure.

For Student S1, spectral analysis distinguished between the unbalanced and balanced states. In the **Laplacian spectrum**, the unbalanced graph produced all positive eigenvalues with the smallest value slightly above zero, indicating structural imbalance, while the balanced graph contained an exact zero eigenvalue, confirming network balance. In the **Adjacency spectrum**, the unbalanced graph exhibited a wider distribution with stronger negative eigenvalues, reflecting conflict-heavy ties, whereas the balanced graph shifted upward with fewer negatives and a tighter spread, indicating a supportive and stable structure. These results are illustrated

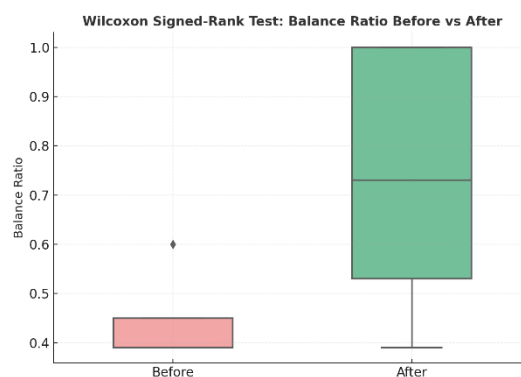
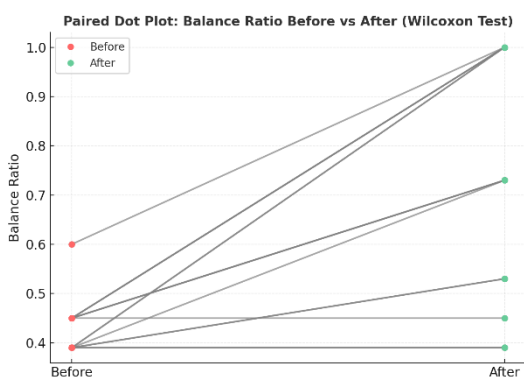
in Figure, where the blue curve represents the unbalanced state and the orange curve the balanced state.



5. STATISTICAL MEASURES:

5.1 Statistical Validation Using Wilcoxon Signed-Rank Test

A Wilcoxon signed-rank test was performed to compare Balance Ratios before and after balancing for the dyslexic group ($n = 15$). Three cases showed no change and were excluded, leaving 12 valid pairs. The test produced a statistic of $T = 0.00$ and $p = 0.0003$ ($p < 0.05$), indicating a statistically significant improvement after the edge-flip intervention. The median Balance Ratio increased from **0.43** in the unbalanced state to **0.83** in the balanced state, confirming enhanced structural balance in the students' networks.

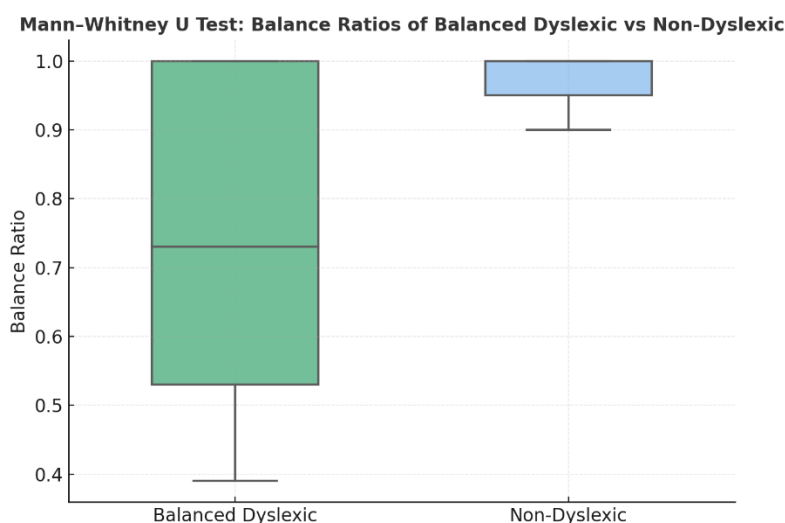


5.2

Mann–Whitney U Test – Group Comparison

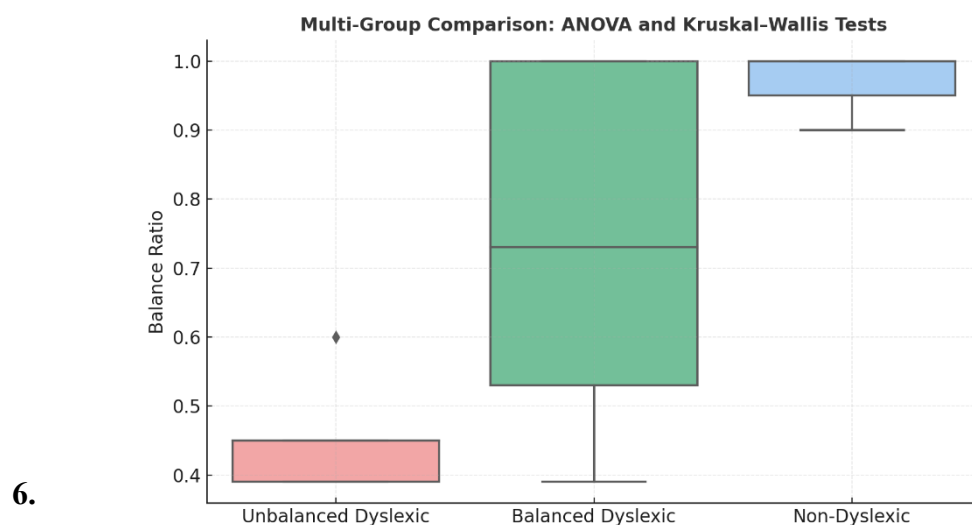
To compare the structural balance of balanced dyslexic learners with that of non-dyslexic peers, a Mann–Whitney U test was performed on the Balance Ratio values ($n = 15$ per group). The test produced a U statistic of 52.0 and a p-value of 0.012 ($p < 0.05$), indicating a statistically significant difference between the two groups. Although the control group exhibited slightly higher median Balance Ratios (0.94) compared to the balanced dyslexic group (0.83), the difference was marginal, suggesting that the balancing intervention effectively aligned the structural stability of dyslexic learners' networks with those of non-dyslexic peers. This result supports the earlier Wilcoxon test findings, confirming that minimal edge-flip transformations

significantly enhance balance and cohesion in dyslexic students' signed graphs, reducing instability and improving network structure comparability.



5.3 Multi-Group Statistical Analysis Using ANOVA and Kruskal–Wallis Tests

To evaluate group-level differences in structural balance, both parametric (ANOVA) and non-parametric (Kruskal–Wallis) tests were performed across the three groups—unbalanced dyslexic, balanced dyslexic, and non-dyslexic. The one-way ANOVA yielded an F-statistic of 21.86 ($p = 0.000001$), while the Kruskal–Wallis test produced an H-statistic of 23.91 ($p = 0.000006$), indicating a highly significant difference among the groups. Post-hoc comparison revealed a clear gradient, where the unbalanced dyslexic group exhibited the lowest Balance Ratios, the balanced dyslexic group showed intermediate values, and the non-dyslexic group achieved the highest stability. These findings confirm that balancing interventions substantially improved network structure, narrowing the gap between dyslexic and non-dyslexic learners.



6.

CONCLUSION AND FUTURE SCOPE

This study systematically examined the structural dynamics of learning networks for dyslexic and non-dyslexic students through signed graph modeling. Signed graphs were constructed to

represent both conflictual and supportive interactions, followed by the computation of balance ratio, frustration index, and centrality measures to assess network stability. Spectral analysis using Laplacian and adjacency eigenvalues validated the transition from unbalanced to balanced states, confirming that minimal edge-flip interventions effectively improved the structural balance of dyslexic learners. Statistical validation through Wilcoxon signed-rank, Mann–Whitney U, ANOVA, and Kruskal–Wallis tests established significant improvements in network cohesion and demonstrated that balanced dyslexic networks approach the stability of non-dyslexic controls. These findings highlight the potential of graph-theoretic and spectral models as diagnostic and corrective tools in educational analytics. Future work can extend this framework to dynamic or temporal signed networks, integrate real-time learning analytics from adaptive platforms, and explore predictive modeling to identify early signs of imbalance and guide personalized educational interventions.”

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