International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

CRITICAL THINKING IN MATHEMATICS TEACHING

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Abstract

This study analyzes the relationship between critical thinking and mathematics education from an epistemological and interpretive perspective. It argues that critical thinking is an essential cognitive operation aimed at the rational evaluation of beliefs, claims, and assumptions, using explicit criteria of justification and evidence. In the educational field, its development transcends the memorization of content, promoting comprehension, argumentation, and informed decision-making. Within this framework, mathematics is presented as a strategic formative field, capable of enhancing cognitive autonomy when approached as a language of reasoning, proof, and refutation.

This documentary and hermeneutic research reconstructed the main theories on critical thinking and mathematics education, highlighting the contributions of authors such as Facione, Ennis, Paul, Elder, Schoenfeld, and Skovsmose. The results show that activities such as reformulating assumptions, evaluating validity, open-ended problems, using counterexamples, and epistemological meta-questions promote intellectual vigilance and rational deliberation. It is concluded that teaching mathematics with a critical approach allows the formation of reflective subjects, ethically responsible and capable of discerning between knowledge and belief, turning the classroom into a space for the autonomous construction of knowledge and the strengthening of rational judgment.

1. Introduction

The development of critical thinking stands as one of the most significant educational objectives in contemporary education, particularly within higher and secondary education contexts. In a social environment marked by an overabundance of information and the need to discern between valid data, opinions, and ideological discourses, the ability to think critically

Received: August 13, 2025

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

emerges as a fundamental requirement for cognitive autonomy and rational judgment. This competence transcends the mere accumulation of knowledge by placing understanding, argumentation, and the justification of statements that constitute scientific knowledge at the core of the educational process. Within this framework, mathematics—traditionally conceived as an exact discipline appears as a privileged space for cultivating the epistemic operations that underpin critical thinking.

Historically, mathematics teaching has been associated with problem-solving and the repetition of algorithmic procedures. Although this mechanistic approach may be effective for developing operational skills, it limits the possibility of questioning the conceptual foundations of mathematical knowledge. Consequently, many students can perform calculations without understanding the reasoning that legitimizes their results, reproducing a form of instrumental learning. In contrast, an approach that fosters critical thinking should conceive mathematics not merely as a set of abstract rules, but as a language of justification, proof, and refutation a field in which every statement must be examined, debated, and rationally demonstrated.

Critical thinking, understood as an epistemological practice of rational evaluation of beliefs and assertions, transforms the educational experience into a reflective process directed toward knowledge construction. In this sense, mathematics teaching can become a laboratory of reasoning, where error, doubt, and argumentation occupy a central place in the development of judgment. Authors such as Skovsmose [1], Boaler [2], and Schoenfeld [3] emphasize that critical mathematics does not aim to reach a single correct answer but rather to establish validity criteria grounded in evidence, promoting in students the capacity to justify and problematize the foundations of their own thinking.

Therefore, this study explores the relationship between critical thinking and mathematics teaching from an epistemological and interpretive perspective, with the purpose of showing how certain pedagogical practices—such as the reformulation of assumptions, the analysis of validity, open-ended problems, and the pedagogy of counterexamples—enable the emergence of autonomous, reflective, and ethically responsible thinking. In this way, it seeks to contribute to a deeper understanding of the role of mathematics as both a cognitive and ethical mediation in the formation of critical judgment.

2. Methodology

This study is framed within the interpretive paradigm, since its purpose is not to measure causal relationships between variables but rather to understand and reconstruct the meanings, epistemological assumptions, and formative logics underlying the connection between mathematics and critical thinking. As Martínez [4] points out, the interpretive paradigm privileges the elucidation of meanings over the verification of laws and assumes that educational phenomena are symbolic constructions rather than neutral facts available for linear measurement.

Within this framework, the adopted approach is qualitative, specifically documentary–hermeneutic in nature, aimed at problematizing texts rather than producing empirical field data. The hermeneutic method allows for the comprehension of discourses by considering their

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Volume 38 No. 9s, 2025

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context of production, theoretical assumptions, boundaries of analysis, and the normative implications they carry concerning critical thinking and mathematics [5]. Hermeneutics is conceived here not merely as the interpretation of content, but as a practice of critical reading that questions both what the discourse asserts and what it systematically leaves unspoken.

The research involved two complementary operations:

a) A review of theories on critical thinking [6]-[7]-[8]-[9], mathematics education [3]-[10]-[11], and socio-critical approaches [1]-[12]-[13].

b) An interpretive state of the art**,** understood not as an inventory of publications but as an analytical reconstruction of how the problem is currently being conceptualized [14]. This process entailed mapping convergences, divergences, tensions, and silences in the literature, particularly in empirical works that study classroom experiences aimed at fostering critical thinking through mathematics.

3. Results

Critical Thinking as an Epistemic Operation

Critical thinking is conceived as an essential cognitive process aimed at structuring, evaluating, and connecting ideas, concepts, and knowledge in a logical and systematic manner. From an epistemological perspective, this type of reasoning enables individuals to adopt a well-founded stance toward a specific issue or situation, reducing the influence of biases or personal judgments. Its etymological origin derives from *pensare* (Latin), "to think," and *krinein* (Greek), "to discern" or "to separate," which refers to the ability to distinguish, judge, and examine reality through rational procedures of understanding [4].

In academic and scientific contexts, critical thinking is characterized as a rational and reflective exercise aimed at the objective analysis of phenomena, differentiating verifiable facts from value judgments. This form of reasoning opposes the uncritical acceptance of socially normalized statements as unquestionable truths or those subject to ethical debate. Critical thinking thus emerges as an indispensable intellectual resource for achieving deeper, justified, and rationally supported understandings of reality [15]-[16].

The literature agrees that critical thinking is not merely a technique for "better reasoning," but a practice of rational evaluation of beliefs, assumptions, and statements according to explicit criteria of justification [7]-[12]-[6]. From this perspective, to think critically means to scrutinize not only conclusions but also the criteria by which those conclusions are deemed valid. In sum, it is a mechanism of error control and epistemological vigilance, not a vague "reflective attitude."

Purpose of Critical Thinking

Critical thinking is understood as an intellectual exercise directed toward the reasoned and well-founded analysis of reality. To fulfill this purpose, the individual must rely on mechanisms of knowledge validation such as systematic observation, empirical experience, source triangulation, and, in more demanding contexts, the application of procedures proper to the

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Volume 38 No. 9s, 2025

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scientific method. This process is not limited to expressing opinions but aims to formulate objective, ethically grounded, and verifiable statements [17].

The exercise of critical thinking involves gathering relevant evidence, contrasting information from reliable sources, and adopting an impartial stance that minimizes the influence of personal or affective biases. In this sense, it constitutes a practice with an ethical dimension, avoiding both paralyzing skepticism and the uncritical acceptance of statements lacking rational foundation [15].

From the fields of pedagogy and philosophy of mind, it is argued that the development of critical thinking requires adopting the intellectual disposition of a rigorous thinker. This entails, among other actions, identifying arguments that support or contradict a given position; recognizing and scrutinizing personal prejudices or those implicit in others' discourse; critically evaluating the reliability, relevance, and sufficiency of the consulted sources; and subjecting both data and positions to systematic procedures of analysis and contrast.

This form of rational organization not only promotes a deep understanding of phenomena in their structural complexity but also contributes to the construction of well-founded personal criteria capable of guiding decision-making in everyday, academic, scientific, or professional contexts. Thus, critical thinking transcends mere intellectual exercise to become a practice of responsible judgment that enables cognitive autonomy and argumentative responsibility in social life [15]-[16].

Main Characteristics of Critical Thinking

Critical thinking is configured as a complex and systematic cognitive process that transcends the mere accumulation of information by seeking to understand and evaluate reality in a reasoned manner. According to specialized literature, this competence is characterized by a set of traits that give it depth and rigor.

First, objectivity stands out, understood as the conscious effort to reach conclusions supported by truthful, verifiable information, free from distortions derived from personal judgments or unfounded beliefs [18]. This objectivity does not imply absolute neutrality but rather a critical exercise of self-reflection that allows individuals to recognize and minimize their own biases during the analytical process [19].

Rigorous observation constitutes the starting point of critical thinking, as it enables the identification of phenomena, problems, or situations worthy of examination. This initial stage goes beyond mere perception of facts, demanding selective and systematic attention that facilitates the formulation of relevant and guiding questions [20].

Next, the capacity for analysis allows one to decompose problems into their constituent elements, identify causes and consequences, and establish logical relationships leading to coherent explanations. This analytical skill fosters a deep understanding of phenomena and the construction of consistent interpretative frameworks [21].

Intellectual inquiry is another indispensable characteristic, as it motivates the continuous search for information and the questioning of assumptions. This exploratory drive strengthens critical

Received: August 13, 2025

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judgment by broadening the horizon of knowledge and encouraging the integration of diverse perspectives [19].

Finally, continuous reflection gives critical thinking a dynamic and dialectical character, in which conclusions are subjected to review, reconsideration, and grounding in solid evidence. This ongoing evaluative exercise prevents dogmatic positions and promotes openness to new interpretations [18].

Taken together, these characteristics demonstrate that critical thinking is not an occasional activity but rather an intellectual habit that demands discipline, a willingness to learn, and an open mind. Its systematic cultivation in higher education is considered essential for training citizens capable of acting autonomously, responsibly, and ethically in increasingly complex and changing environments [19]-[20].

Development of Critical Thinking through Mathematics

The development of critical thinking through mathematics holds a strategic formative value. This discipline can operate either as an extraordinarily fertile intellectual field or, conversely, as an intellectually sterile space, depending on the pedagogical approach with which it is taught. When mathematics is reduced to the mechanical execution of algorithms and obedience to predefined procedures, it produces habituation to cognitive conformity and discourages rational inquiry. In contrast, when mathematics is approached as a language of justification, proof, and refutation, it becomes a setting that demands argumentation, the revision of assumptions, and the examination of assertions—conditions that are essential for critical thinking.

Available empirical evidence shows that the promotion of critical thinking does not depend on the *difficulty* of problems but rather on the epistemic regime under which mathematics is practiced: it only emerges when mathematical statements are treated as examinable, revisable, and debatable assertions, rather than as unquestionable truths or answers to be repeated without challenge [1]-[2]-[3]. Under this premise, mathematics ceases to be an exercise in technical obedience and transforms into a practice of epistemological vigilance with direct effects on the development of

Mathematical Activities that Enable Critical Thinking from an Epistemological Perspective

Specialized literature agrees that the formation of critical thinking through mathematics does not depend on increasing the numerical or technical complexity of exercises, but rather on transforming the epistemological contract of the task—that is, the way in which justification, proof, or truth is defined within the classroom. Among the practices identified with the greatest critical potential are the following:

a) Explicit Reformulation of Assumptions

The activity of identifying the implicit assumptions of a mathematical problem reveals that no statement is epistemologically neutral: every problem presupposes certain conditions of possibility that are not usually stated. Questions such as “What conditions of the world must

Received: August 13, 2025

Volume 38 No. 9s, 2025

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be true for this problem to make sense?" compel students to distinguish between the explicit data and the conceptual structure that makes them intelligible. This operation introduces a form of epistemological deconstruction that shifts the focus from *"how it is calculated"* to *"under what assumptions it is calculable,"* opening a gateway to rational critique [1].

b) Evaluation of Validity, Not Only Solution

Working on validity as an object of analysis—not merely procedural correctness—implies recognizing that in mathematics, it is not enough to reach a result; it is necessary to justify why that result deserves to be accepted as knowledge. Asking *"What would make this solution invalid?"* introduces the logic of falsifiability and boundary conditions, transforming the answer into a statement subject to scrutiny. This operation situates the student within the epistemic regime of proof rather than assent, exercising the distinction between *"solving"* and *"justifying"* [11].

c) Open Problems with Multiple Plausible Solutions

Problems with more than one reasonable solution break the dominant didactic assumption that mathematics is a territory of uniqueness and certainty. When plurality of solutions is admitted, the focus shifts from the final product to the criteria of argumentative legitimacy: the question is no longer *"What is the correct answer?"* but *"Which solution is better justified, and under what criteria?"* Thus, uncertainty becomes an epistemological input rather than a pedagogical flaw, activating rational deliberation [2].

d) The Pedagogy of the Counterexample as Controlled Epistemic Destructiveness

The counterexample is not merely an illustration; it is a tool that destabilizes apparent regularities and compels the revision of a claim's scope. Its formative power lies in showing how a single negative instance can invalidate a generalization. The student thus learns that mathematics is not only the construction of patterns but also the critical vigilance of the robustness of assertions. This practice trains the disposition to avoid naturalizing regularities, incorporating doubt as a methodological principle [12].

e) **Epistemological Metainquiries as Object Displacement**

Metainquiries such as *"What legitimizes this proof?"*, *"What evidence would refute it?"*, or *"Who benefits if this truth prevails?"* shift attention from operation to the epistemic and political architecture of mathematical knowledge. Such questions dismantle claims of neutrality and confront the student with the fact that every "mathematical truth" is also a decision about what counts as evidence and what consequences follow from accepting it. This change in level transforms mathematics into a field of deliberation rather than mere execution [1].

In all five cases, the activity does not consist merely in "doing mathematics" but in interrogating mathematics as knowledge, restoring to the classroom its condition as an epistemic rather than purely procedural space. This shift allows mathematics to function as a device for the formation of critical thinking.

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3. Discussion

Critical thinking, understood as an epistemic operation oriented toward rational discernment and argumentative grounding, represents one of the essential pillars of contemporary scientific knowledge. Based on the results obtained, it becomes evident that its development cannot be reduced to an instrumental cognitive skill or a mere didactic resource; rather, it must be conceived as a permanent intellectual disposition of epistemological vigilance. This perspective aligns with Facione's [7] notion of "thinking about thinking," according to which the subject not only analyzes external objects but also examines the logical and conceptual structures that underpin their own beliefs.

In the educational context, critical thinking gains relevance as a form of cognitive emancipation in response to the tendency toward uncritical reproduction of knowledge. Traditional teaching, centered on memorization and the mechanical repetition of procedures, often inhibits autonomous reflection. Conversely, education oriented toward critical judgment seeks to ensure that students understand that every claim requires rational justification and that the validity of knowledge derives not from the authority that articulates it but from the coherence of the arguments and the evidence that supports them [6]-[8].

The articulation between critical thinking and mathematics, as addressed in the results, is particularly significant. When mathematics is confined to the execution of algorithms, it becomes a domain of intellectual obedience. However, when it is approached as a language of reasoning, justification, and refutation, it transforms into a privileged field for exercising epistemic rationality. This shift in focus implies moving from the learning of answers to the comprehension of the foundations that legitimize those answers, thereby situating mathematical practice within the realm of argumentation rather than reproduction.

The discussion on mathematical activities that foster critical thinking deepens this idea. Reformulating assumptions, evaluating validity, working with open problems, or introducing counterexamples are not merely pedagogical strategies; they are epistemological procedures that teach students to interrogate knowledge and recognize its constructed, revisable, and conditioned nature. Such practices promote a cognitive shift from passive acceptance to rational deliberation. By making visible that every problem contains presuppositions, every solution demands justification, and every assertion can be falsified, a culture of questioning is built—one that constitutes the very core of critical thinking.

Furthermore, the ethical dimension of critical thinking must not be overlooked. The ability to reason rationally entails a responsibility toward truth and a commitment to cognitive justice, as critical judgment avoids both dogmatism and naive relativism. Teaching that fosters critique does not seek to deny truth but to make it an object of ongoing justification, acknowledging its provisional and improvable character.

In conclusion, the discussion affirms that the development of critical thinking through mathematics constitutes a privileged form of epistemological literacy. It is not only about learning to calculate or demonstrate but about learning to think in a grounded, analytical, and argumentative way. This approach transforms the teaching of mathematics—and, by extension,

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Volume 38 No. 9s, 2025

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all knowledge—into an exercise in intellectual freedom, where learning to reason also means learning to discern, to doubt methodically, and to construct meaningful knowledge.

5. Conclusions

The analysis carried out allows us to conclude that critical thinking, conceived as an epistemic operation, constitutes a fundamental competence for scientific, ethical, and civic formation. It is not limited to a cognitive skill but is expressed as a reflective habit oriented toward rational justification, the evaluation of evidence, and informed decision-making. Its consistent practice enables individuals to distinguish between knowledge and opinion, evidence and belief, and epistemic authority and ideological dogmatism.

The study confirms that the teaching of mathematics, when approached from a critical perspective**,** can become a privileged setting for cultivating this competence. When mathematics is treated as a language of argumentation rather than a mere set of algorithms, it offers a space for analyzing assumptions, validating arguments, and formulating judgments based on evidence. Activities such as reformulating statements**,** evaluating validity**,** working with open-ended problems**,** and using counterexamples act as catalysts for the development of epistemological awareness, thereby transforming educational practice.

Furthermore, it is concluded that critical thinking is not an endpoint but a continuous process of self-reflection and revision**.** Its purpose is not to produce absolute certainties but to strengthen the ability to question, contrast, and sustain ideas through verifiable arguments. In this sense, critical thinking is directly associated with cognitive autonomy, understood as the ability to think independently without abandoning the rigor of evidence.

Finally, the research demonstrates that the development of critical thinking in higher education requires moving beyond pedagogical models centered on one-way knowledge transmission and adopting methodologies that promote deliberation, argumentation, and the problematization of knowledge**.** Encouraging reasoned doubt, the confrontation of ideas, and reflection on the foundations of knowledge means forming individuals capable of resisting manipulation, recognizing the complexity of phenomena, and acting ethically in the face of contemporary intellectual challenges.

In conclusion, critical thinking, as both an epistemic and ethical practice, emerges as an instrument of intellectual emancipation, enabling individuals to participate actively in the construction of a more rational, just, and democratic society, where truth is not imposed but argued and substantiated through evidence.

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Received: August 13, 2025