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GRUNDY EDGE COLORING OF SHADOW GRAPH OF SOME GRAPHS

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Abstract

An edge coloring of a graph G refers to the process of assigning colors to the edges such that any two edges sharing a common vertex receive different colors. The *Grundy edge chromatic number*, denoted by Γ'(G), is defined as the largest number of colors that can be used by a greedy edge-coloring procedure across all permutations of the edges. This work focuses on computing Γ'(G), for the shadow graphs derived from families of graphs, including star graphs, double star graphs, comb graphs, fan graphs, and ladder graphs.

Keywords: Grundy chromatic number, Edge Grundy Chromatic number, shadow graph, Path, Cycle, Sunlet, Tadpole graph.

2020 MSC: 05C15, 05C38, 05C76, 05C78.

1. Introduction

Graph theory is a fundamental area within discrete mathematics that explores the interconnections between pairs of objects, typically modelled using vertices and edges. These structures are used to represent various real-world systems, such as transportation networks, computer architectures, and social relationships. A prominent line of study within this discipline involves graph coloring an assignment of labels or colors to elements of a graph, subject to specific rules.

One of the most widely studied coloring problems is vertex coloring, where each vertex of a graph is assigned a color such that no two adjacent vertices share the same color. The least number of distinct colors required to accomplish this for a graph G is known as its *chromatic number*, denoted by χ(G). This concept has widespread applications, including conflict-free scheduling, assigning frequencies in mobile networks, and optimizing memory allocation in compilers [3].

Beyond the optimal coloring represented by the chromatic number, another related notion is the *Grundy chromatic number*, denoted by Γ(G). This parameter stems from a greedy coloring algorithm applied to vertices. In this process, the vertices of the graph are ordered

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arbitrarily as $v_1, v_2, \ldots, v_n$ and each vertex v_i is colored using the smallest natural number not already assigned to its adjacent predecessors in the sequence. The largest color that appears during the coloring defines the Grundy number for that ordering, and Γ(G) is the maximum such value over all possible vertex orderings [1, 2, 3, 5]. It is important to note that the inequality $\chi(G) \leq \Gamma(G)$ always holds, which signifies that greedy methods may use more colors than necessary in unfavorable orderings.

The greedy coloring approach and the concept of the Grundy number are not restricted to vertices alone; they extend naturally to edge coloring. In this variant, the objective is to assign colors to edges such that adjacent edges (those sharing a common vertex) receive different colors. The *edge Grundy chromatic number*, symbolized as $\Gamma'(G)$, is defined analogously. Edges are considered in a particular sequence, and each is colored with the smallest color that does not conflict with colors assigned to previously processed adjacent edges. As with the vertex-based version, $\Gamma'(G)$ represents the worst-case color usage over all possible edge orderings.

Edge Grundy coloring holds practical relevance in areas involving sequential resource distribution. For example, in network communications, each edge might represent a data link requiring a unique channel among its neighbors to prevent interference. Similarly, in transportation planning, edges may represent routes that must be timed or assigned without overlap at shared junctions [9].

Studying the Grundy chromatic number and its edge counterpart sheds light on the performance of greedy algorithms, particularly in scenarios where inputs must be processed sequentially. These parameters help quantify the potential overhead introduced by greedy strategies when compared to optimal methods. Moreover, they are valuable in evaluating algorithmic behavior in dynamic settings, where decisions must be made without full knowledge of the system, reinforcing the importance of understanding these upper bounds in both theoretical and applied contexts.

2. Preliminaries

This section introduces specific graph structures pertinent to our study of chromatic and Grundy chromatic numbers.

- **Shadow Graph**: Given a connected graph G, the *shadow graph* [7] $D_2(G)$ is constructed by taking two copies of G, say G and G′, and connecting each vertex v in G to the neighbors of its corresponding vertex v′ in G′.

- **Star Graph**: A graph G on n vertices is said to be *star* [6], a vertex in V(G) is adjacent to the remaining vertices of G.

- **Double Star Graph**: A *double star* [4] S(n,m) is formed by joining the centers of two-star graphs $K_{1,n}$ and $K_{1,m}$ with an edge, where $n \geq m \geq 0$.

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- **Comb Graph:** A *comb* graph [6] is created by attaching a pendant edge to each vertex of a path graph P_n. It is denoted as $P_n \odot K_1$.

- **Fan Graph**: A *fan graph* [8] $F_{m, n}$ is defined as the join of a complete graph K_m and a path graph P_n, denoted as $K_m + P_n$. When m = 1, it corresponds to the standard fan graph; m = 2 yields the double fan.

- **Ladder Graph**: A *ladder graph* [8] L_n consists of two parallel path graphs P_n connected by a series of rungs, forming a structure resembling a ladder. It can be represented as the Cartesian product $P_2 \square P_n$.

These graph classes exhibit diverse structural properties that significantly influence their chromatic and Grundy chromatic numbers. Subsequent sections will delve deeper into these influences.

3. Results

Theorem 3.1. For any positive integer $n \geq 2$, the Grundy edge chromatic number of shadow graph of star graph satisfies

$$\Gamma' (D_2(K_{1, n})) = 2n + 1.$$

Proof. Let the vertex set of $D_2(K_{1, n})$ be given by

$$V (D_2(K_{1, n})) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_0, v_0\},$$

and the edge set by

$$E (D_2(K_{1, n})) = \{u_0u_i : 1 \leq i \leq n\} \cup \{v_0v_i : 1 \leq i \leq n\} \cup \{v_0u_i : 1 \leq i \leq n\} \cup \{u_0v_i : 1 \leq i \leq n\}.$$

We define the following edge color classes:

- $E_k = \{u_0u_k\}$ for $1 \leq k \leq n$,
- $E_p = \{u_0v_{(p \bmod n)}\}$ for $n + 1 \leq p \leq 2n$,
- $E_{2n+1} = \{v_0v_n\}$,
- $E_q = \{v_0v_q\}$ for $1 \leq q \leq n - 1$,
- $E_s = \{v_0u_{((s \bmod n) + 1)}\}$ for $n \leq s \leq 2n - 1$.

By inspecting the indices (suffixes) of the edges in each E_i and E_j with $i \neq j$, we observe that

$$E_i \cap E_j = \phi \text{ for all } i \neq j.$$

Hence, the sets $E_1, E_2, \ldots, E_{2n+1}$ are pairwise disjoint, and their union gives the full edge set:

$$\bigcup_{i=1}^{2n+1} E_i = E (D_2(K_{1, n})).$$

Therefore, $D_2(K_1, n)$ admits a Grundy edge coloring with 2n + 1 colors. Furthermore, no additional color can be used since there is no edge in the graph adjacent to 2n + 1 edges

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colored with distinct colors. Thus, adding a (2n + 2)-th color is not possible. So, the Grundy edge chromatic number satisfies

$$\Gamma'(D_2(K_{1,n})) = 2n + 1.$$

Additionally, the maximum degree of a vertex in $D_2(K_{1,n})$ is 2n, and the maximum degree in the line graph $L(D_2(K_{1,n}))$ is also 2n. Therefore,

$$\Gamma'(D_2(K_{1,n})) \leq \Delta(L(D_2(K_{1,n}))) + 1 = 2n + 1.$$

Combining both bounds, we conclude that

$$\Gamma'(D_2(K_{1,n})) = 2n + 1, \text{ for all integers } n \geq 2.$$

■

Theorem 3.2. For any positive integers m, n ≥ 2 with m ≥ n the Grundy edge chromatic number of shadow graph of double star graph satisfies

$$\Gamma'(D_2(S(m, n))) = m + n + 2.$$

Proof. Let the vertex set of $D_2(S(m, n))$ be

$$V(D_2(S(m,n))) = \{u_i : 1 \leq i \leq m\} \cup \{v_i : 1 \leq i \leq n\} \cup \{x_i : 1 \leq i \leq m\} \cup \{y_i : 1 \leq i \leq n\} \cup \{u_0, v_0, x_0, y_0\}.$$

The edge set is given by

$$E(D_2(S(m,n))) = \{u_0u_i : 1 \leq i \leq m\} \cup \{v_0v_i : 1 \leq i \leq n\} \cup \{u_0y_i : 1 \leq i \leq n\} \cup \{v_0x_i : 1 \leq i \leq m\} \cup \{x_0v_i : 1 \leq i \leq n\} \cup \{y_0u_i : 1 \leq i \leq m\} \cup \{u_0v_0, x_0y_0\},$$

We define the following edge color classes:

- $E_k = \{u_0u_k\}$ for $1 \leq k \leq m$,
- $E_p = \{u_0y_{(p \bmod m)}\}$ for $m + 1 \leq p \leq m + n - 1$,
- $E_{m+n+1} = \{y_0v_n, u_0v_0\}$,
- $E_{m+n+2} = \{x_0y_0\}$,
- $E_{m+n} = \{x_0x_1\}$,
- $E_r = \{y_0y_r\}$ for $1 \leq r \leq n - 1$,
- $E_s = \{y_0x_{s \bmod (n-1)}\}$ for $n \leq s \leq m + n - 1$,
- $E_c = \{x_0x_{c+1}\}$ for $1 \leq c \leq m - 1$,
- $E_d = \{v_0x_d\}$ for $1 \leq d \leq m$,
- $E_l = \{x_0y_{l \bmod (m-1)}\}$ for $m \leq l \leq m + n + 1$,
- $E_e = \{v_0v_{e \bmod (m+1)}\}$ for $m + 1 \leq e \leq m + n$.

By inspecting the suffixes of the edges in E_i and E_j for $i \neq j$, we observe that

$$E_i \cap E_j = \phi \text{ for all } i \neq j.$$

Hence, the edge classes $E_1, E_2, \ldots, E_{m+n+2}$ are pairwise disjoint and

$$\bigcup_{i=1}^{m+n+2} E_i = E\,(D_2(S\,(m, n))).$$

Thus, $D_2(S\,(m, n))$ is Grundy edge colorable using $m + n + 2$ colors. Suppose we try to color the graph with $m + n + 3$ colors. There is no edge adjacent to $m + n + 2$ distinct colors, so no such coloring is possible.

Hence,

$$\Gamma'(D_2(S(m, n))) = m + n + 2.$$

Also, the maximum degree of the line graph is $\Delta(L(D_2(S(m,n)))) = m + n + 1$, and so

$$\Gamma'(D_2(S(m,n))) \leq \Delta(L(D_2(S(m,n)))) + 1 = m + n + 2$$

Combining both bounds, we conclude:

$$\Gamma'(D_2(S(m,n))) = m + n + 2, \text{ for all integers } m, n \geq 2 \text{ with } m \geq n.$$

■

Theorem 3.3. For any positive integer $n \geq 5$, we have

$$\Gamma'\,(D_2(P_n \odot K_1)) = 9.$$

Proof. Let

$$V(D_2(P_n \odot K_1)) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\} \cup \{x_i : 1 \leq i \leq n\} \cup \{yi : 1 \leq i \leq n\},$$

The edge set is given by:

$$E(D_2(P_n \odot K_1)) = \{u_iu_{i+1} : 1 \leq i \leq n\text{-}1\} \cup \{u_ix_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n\text{ -}1\}$$
$$\cup \{v_iy_i : 1 \leq i \leq n\} \cup \{x_iv_i : 1 \leq i \leq n\} \cup \{u_iy_i : 1 \leq i \leq n\}$$
$$\cup \{u_iv_{i+1} : 1 \leq i \leq n\text{-}1\} \cup \{u_{i+1}v_i : 1 \leq i \leq n\text{ -}1\}.$$

Case (i): $n = 5$

$E_1 = \{x_1v_1, u_1v_2, u_4v_3\} \cup \{x_{i+3}v_{i+3} : 1 \leq i \leq 2\} \cup \{x_{i+1}\, v_{i+1} : 1 \leq i \leq 2\}$,

$E_2 = \{v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4, u_5x_5, v_5x_5\}$,

$E_3 = \{u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5\}$,

$E_5 = \{u_2v_3, v_1v_2, u_4x_4, u_5v_4\}$,

$E_6 = \{u_2u_3, v_3y_3\}$,

$E_7 = \{u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

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Case (ii): n = 6

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3\}$ U $\{x_{i+1}\, u_{i+1} : 1 \le i \le 2\}$ U $\{x_{i+3}v_{i+3} : 1 \le i \le 2\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4. u_5x_5, v_5x_5\}$,

$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

Case (iii): n = 7

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3, x_7v_7, u_7y_7\}$ U $\{x_{i+1}\, u_{i+1} : 1 \le i \le 2\}$ U $\{x_{i+3}v_{i+3} : 1 \le i \le 2\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4. u_5x_5, v_5x_5, u_7x_7, v_7y_7\}$,

$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5, u_7v_6\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5, u_6u_7, v_6v_7\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3, u_6v_7\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

Case (iv): n = 8

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3, x_7v_7, u_7y_7, x_8v_8\}$ U $\{x_{i+1}u_{i+1}: 1 \le i \le 2\}$ U $\{x_{i+3}v_{i+3}: 1 \le i \le 2\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4. u_5x_5, v_5x_5, u_7x_7, v_7y_7, u_8x_8, v_8y_8\}$,

$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5, u_7v_6, u_8y_8, v_7v_8\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5, u_6u_7, v_6v_7\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4, u_7v_8, u_8v_7\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3, u_6v_7, u_7u_8\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

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$E_9 = \{u_3u_4\}$.

Case (v): n = 9

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3, x_7v_7, u_7y_7, x_8v_8, x_9v_9, u_8u_9\}$

$\cup \{x_{i+1}u_{i+1} : 1 \le i \le 2\} \cup \{x_{i+3}v_{i+3} : 1 \le I \le 2\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4. u_5x_5, v_5x_5, u_7x_7, v_7y_7, u_8x_8, v_8y_8, u_9x_9, v_9y_9\}$,

$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5, u_7v_6, u_8y_8, v_7v_8, u_9y_9\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5, u_6u_7, v_6v_7, u_8v_9, u_9v_8\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4, u_7v_8, u_8v_7\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3, u_6v_7, u_7u_8, v_8v_9\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

Case (vi): n = 10

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3, x_7v_7, u_7y_7, x_8v_8, x_9v_9, u_8u_9, x_{10}v_{10}, u_{10}y_{10}\}$

$\cup \{x_{i+1}u_{i+1}: 1 \le i \le 2\} \cup \{x_{i+3}v_{i+3} : 1 \le i \le 2\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4, u_5x_5, v_5x_5, u_7x_7, v_7y_7, u_8x_8, v_8y_8, u_9x_9, v_9y_9,$

$u_{10}x_{10}, v_{10}y_{10}\}$,

$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5, u_7v_6, u_8y_8, v_7v_8, u_9y_9, v_9v_{10}\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5, u_6u_7, v_6v_7, u_8v_9, u_9v_8\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4, u_7v_8, u_8v_7, u_9v_{10}, u_{10}v_9\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3, u_6v_7, u_7u_8, v_8v_9, u_9u_{10}\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

Case (vii): $n \ge 11$

$E_1 = \{u_5u_6, x_6v_6, x_1v_1, u_1v_2, u_4v_3, x_7v_7, u_7y_7, x_8v_8, x_9v_9, u_8u_9, x_{10}v_{10}, u_{10}y_{10}\} \cup \{x_{i+1}u_{i+1}:1\le i\le 2\}$

$\cup \{x_{i+3}v_{i+3} : 1 \le i \le 2\} \cup \{x_{i+10}v_{i+10} : 1 \le i \le n - 10\} \cup \{u_{i+10}y_{i+10} : 1 \le i \le n - 10\}$,

$E_2 = \{u_6x_6, v_6y_6, v_1y_1, x_1v_1, u_1u_2, u_3y_3, u_4y_4, v_3v_4, u_5x_5, v_5x_5, u_7x_7, v_7y_7, u_8x_8, v_8y_8, u_9x_9, v_9y_9,$

$u_{10}x_{10}, v_{10}y_{10}\} \cup \{u_{i+10}x_{i+10} : 1 \le i \le n - 10\} \cup \{v_{i+10}y_{i+10} : 1 \le i \le n - 10\}$,

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$E_3 = \{u_6y_6, u_1y_1, u_2v_1, v_2y_2, x_3v_3, u_4v_5, v_4y_4, u_5y_5, u_7v_6, u_8y_8, v_7v_8, u_9y_9, v_9v_{10}\}$

$\cup \{u_{i+9}v_{i+10} : 1 \leq i \leq n - 10\}$,

$E_4 = \{u_1x_1, u_2y_2, v_2v_3, u_4u_5, v_4v_5, u_6u_7, v_6v_7, u_8v_9, u_9v_8\} \cup \{u_{2i-1+9}u_{2i+9} : 1 \leq i \leq \left\lceil\frac{n}{2}\right\rceil - 5\}$

$\cup \{v_{2i-1+9}v_{2i+9} : 1 \leq i \leq \left\lceil\frac{n}{2}\right\rceil - 5\}$,

$E_5 = \{u_6v_5, u_2v_3, v_1v_2, u_4x_4, u_5v_4, u_7v_8, u_8v_7, u_9v_{10}, u_{10}v_9\} \cup \{u_{i+10}v_{i+9} : 1 \leq i \leq n - 10\}$,

$E_6 = \{u_5v_6, u_2u_3, v_3y_3, u_6v_7, u_7u_8, v_8v_9, u_9u_{10}\} \cup \{u_{2i+9}u_{2i+1+9} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 5\}$

$\cup \{v_{2i+9}v_{2i+1+9} : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 5\}$,

$E_7 = \{v_5v_6, u_3v_2\}$,

$E_8 = \{u_3v_4\}$,

$E_9 = \{u_3u_4\}$.

In all cases, by observing the suffixes of the edges of E_i and E_j ($i \neq j$), it is inferred that there are no common edges between E_i and E_j, i.e.,

$$E_i \cap E_j = \phi \text{ for } i \neq j,$$

Clearly, the sets E_i are pairwise disjoint, and

$$\bigcup_{i=1}^{9} E_i = E\,(D_2(P_n \odot K_1)).$$

Hence, $D_2(P_n \odot K_1)$ is Grundy edge colorable with 9 colors.

Even if one tries to assign a 10th color, there does not exist a vertex incident with 9 different colors (i.e., no edge with 9 differently colored adjacent edges), so no edge can be assigned the 10th Grundy color in $D_2(P_n \odot K_1)$.

Therefore,

$$\Gamma'(D_2(P_n \odot K_1)) = 9 \text{ for } n \geq 5.$$

■

Theorem 3.4. For any positive integer $n \geq 3$, we have:

$$\Gamma'(D_2(F_{1,n})) = \begin{cases} 2n+2, & if\ n = 3 \\ 2n+1, & if\ n \geq 4 \end{cases}$$

Proof. Let

$$V\,(D_2(F_{1,n})) = \{u_0, v_0\} \cup \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\},$$

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and

$$E(D_2(F_{1,n})) = \{u_iu_{(i+1) \bmod n} : 1 \le i \le n\} \cup \{v_iv_{(i+1) \bmod n} : 1 \le i \le n\} \cup \{u_iv_{(i+1) \bmod n} : 1 \le i \le n\} \cup \{u_{(i+1) \bmod n}v_i : 1 \le i \le n\}$$

Case (i): n = 3.

We define the edge color classes $E_1, E_2, \ldots, E_8$ as follows:

$E_1 = \{u_0u_1, v_0v_1, u_0v_2, v_2v_3\}$,

$E_2 = \{u_1v_2, u_0u_2, v_0v_3\}$,

$E_3 = \{u_0u_3, u_3v_2, u_2v_1, v_0u_1\}$,

$E_4 = \{u_0v_1, u_1u_2, v_0u_3\}$,

$E_5 = \{u_0v_3, v_1v_2, v_0u_2\}$,

$E_6 = \{u_0v_2, u_2v_3\}$,

$E_7 = \{v_0v_2\}$,

$E_8 = \{u_3v_2\}$.

Each edge in E_i is distinct and pairwise disjoint from edges in E_j for $i \ne j$. Hence, the coloring is a valid Grundy edge coloring using 2n + 2 = 8 colors. Since no edge can be colored with 2n + 3 = 9 under the Grundy condition, we conclude:

$$\Gamma'(D_2(F_{1,3})) = 8 = 2n + 2.$$

Case (ii): n = 4.

Define sets similarly for E_k, E_p, E_q, E_r, E_s as:

$E_k = \{u_0u_k\}$ for $1 \le k \le n$, $E_p = \{u_0v_{p \bmod n}\}$ for $n + 1 \le p \le 2n$, $E_q = \{v_0v_q\}$ for $1 \le q \le 2$,

$E_r = \{v_0v_{r+1}\}$ for $3 \le r \le n - 1$, $E_s = \{v_0u_{s \bmod (n-1)}\}$ for $n \le s \le 2n - 1$.

$E_1 = \{u_2u_3, v_2v_3\}$,

$E_2 = \{u_3u_4, v_3v_4\}$,

$E_3 = \{u_1u_2, v_1v_2, u_4v_3\}$,

$E_4 = \{u_2v_1, u_3v_2\}$,

$E_5 = \{u_1v_2, u_3v_4\}$

$E_6 = \{u_2v_3\}$.

These sets are pairwise disjoint and form a Grundy edge coloring with 2n + 1 = 9 colors.

Case (iii): n = 5

$E_k = \{u_0u_k\}$ for $1 \le k \le n$,

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$E_p = \{u_0v_{(p \bmod n)}\}$ for $n + 1 \leq p \leq 2n$,

$E_{2n+1} = \{v_0v_3\}$,

$E_q = \{v_0v_q\}$ for $1 \leq q \leq 2$,

$E_r = \{v_0v_{r+1}\}$ for $3 \leq r \leq n - 1$,

$E_s = \{v_0u_{(s \bmod (n-1))}\}$ for $n \leq s \leq 2n - 1$,

$E_1 = \{u_2u_3, v_2v_3, u_4u_5, v_4v_5\}$,

$E_2 = \{u_3u_4, v_3v_4\}$,

$E_3 = \{u_1u_2, v_1v_2, u_4v_3\}$,

$E_4 = \{u_1v_2, u_3v_4, u_2v_1\}$,

$E_5 = \{u_2v_3, u_3v_2, u_4v_5\}$,

$E_6 = \{u_5v_4\}$.

Case (iv): $n \geq 6$

$E_k = \{u_0u_k\}$ for $1 \leq k \leq n$,

$E_p = \{u_0v_{(p \bmod n)}\}$ for $n + 1 \leq p \leq 2n$,

$E_{2n+1} = \{v_0v_3\}$,

$E_q = \{v_0v_q\}$ for $1 \leq q \leq 2$,

$E_r = \{v_0v_{r+1}\}$ for $3 \leq r \leq n - 1$,

$E_s = \{v_0u_{(s \bmod (n-1)}\}$ for $n \leq s \leq 2n - 1$,

$E_1 = \{u_2u_3, v_2v_3, u_4u_5, v_4v_5\} \cup \bigcup_{i=1}^{\lceil n/2 \rceil - 3}\{u_{2i+4}u_{2i+5}\} \cup \bigcup_{i=1}^{\lceil n|2 \rceil - 3}\{v_{2i+4}v_{2i+5}\}$,

$E_2 = \{u_3u_4, v_3v_4\} \cup \bigcup_{i=1}^{\left\lfloor \frac{n}{2} \right\rfloor - 2}\{u_{2i+3}u_{2i+4}\} \cup \bigcup_{i=1}^{\left\lfloor \frac{n}{2} \right\rfloor - 2}\{v_{2i+3}v_{2i+4}\}$,

$E_3 = \{u_1u_2, v_1v_2, u_4v_3\} \cup \bigcup_{i=1}^{\left\lfloor \frac{n}{2} \right\rfloor - 2}\{u_{2i+3}v_{2i+4}\} \cup \bigcup_{i=1}^{\left\lfloor \frac{n}{2} \right\rfloor - 2}\{u_{2i+4}v_{2i+3}\}$,

$E_4 = \{u_1v_2, u_3v_4, u_2v_1\} \cup \bigcup_{i=1}^{\lceil n|2 \rceil - 3}\{u_{2i+4}v_{2i+5}\} \cup \bigcup_{i=1}^{\lceil n|2 \rceil - 3}\{u_{2i+5}v_{2i+4}\}$,

$E_5 = \{u_2v_3, u_3v_2, u_4v_5\}$,

$E_6 = \{u_5v_4\}$.

In cases (ii), (iii) and (iv) by observing the subscripts of the edges in E_i and E_j for $i \neq j$, we note that there are no common edges between E_i and E_j. That is,

$$E_i \cap E_j = \phi \text{ for } i \neq j.$$

Clearly, the Ei’s are pairwise disjoint, and

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$$\bigcup_{i=1}^{2n+1} E_i = E(D_2(F_{1,n})).$$

Hence, the graph $D_2(F_{1,n})$ is Grundy edge colorable with 2n + 1 colors.

Suppose we try to assign color 2n + 2. There is no edge that can be assigned this color without violating the Grundy edge coloring condition in $D_2(F_{1,n})$. Thus, we conclude:

$$\Gamma'(D_2(F_{1,n})) = 2n + 1.$$

Since $\Delta(L(D_2(F_{1,n}))) = 2n + 4$, we have:

$$\Delta(E(D_2(F_{1,n}))) + 1 = 2n + 5 \geq \Gamma'(D_2(F_{1,n})) \geq \chi'(D_2(F_{1,n})).$$

Therefore,

$$\Gamma'(D_2(F_{1,n})) = 2n + 1 \text{ for all } n \geq 4.$$

■

Theorem 3.5. For any positive integer $n \geq 2$, we have:

$$\Gamma'(D2(Ln)) = \begin{cases} 6, & if\ n = 2, \\ 8, & if\ n = 3, \\ 9, & if\ n = 4, \\ 10, & if\ n \geq 5. \end{cases}$$

Proof. Let

$V(D_2(L_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\} \cup \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\}$,

$E(D_2(L_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_ix_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{v_iy_i : 1 \leq i \leq n\}$

$\cup \{x_iv_i : 1 \leq i \leq n\} \cup \{u_iy_i : 1 \leq i \leq n\} \cup \{u_iv_i+1 : 1 \leq i \leq n - 1\} \cup$

$\{u_{i+1}v_i : 1 \leq i \leq n-1\} \cup \{x_ix_{i+1} : 1 \leq i \leq n - 1\} \cup \{y_iy_i+1 : 1 \leq i \leq n - 1\}$

$\cup \{x_iy_{i+1} : 1 \leq I \leq n - 1\} \cup \{x_{i+1}y_i : 1 \leq i \leq n - 1\} \cup \{u_1u_n, v_1v_n, u_1v_n, u_nv_1\}$.

Case (i): n = 2

$E_1 = \{u_2u_1, v_2v_1, u_2v_3, u_3v_2\}$,

$E_2 = \{u_1v_2, u_2v_1, u_3v_4\}$,

$E_3 = \{u_1u_2, v_1v_2, u_3v_2\}$,

$E_4 = \{u_1u_2, v_2v_3\}$,

$E_5 = \{u_2v_1, u_4v_3\}$,

$E_6 = \{u_2u_3\}$.

By observing the suffixes of the edges of E_i and E_j for $i \neq j$, it is inferred that there are no common edges between E_i and E_j, i.e., $E_i \cap E_j = \phi$ for $i \neq j$. Clearly, the E_i are pairwise disjoint and

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$$\bigcup_{i=1}^{6} E_i = E(D_2(L_2)).$$

Hence, $D_2(L_2)$ is Grundy edge colorable with 6 colors. If color 7 is attempted, there is no edge with 6 adjacent colors, so it is not feasible. Therefore,

$$\Gamma'(D_2(L_2)) = 6.$$

Case (ii): n = 3

$E_1 = \{x_1v_1, u_1v_2, u_2x_2, y_2y_3, x_2v_3\}$,

$E_2 = \{u_1u_2, x_2v_2, y_1y_2, u_3x_3, v_3y_3\}$,

$E_3 = \{u_1y_1, u_2v_1, x_2y_3, u_3v_2, x_3y_2\}$,

$E_4 = \{u_1x_1, v_1y_1, v_2v_3, u_2u_3, x_2x_3\}$,

$E_5 = \{x_1x_2, u_2v_3, v_1v_2, u_3y_3\}$,

$E_6 = \{x_2y_1, u_2y_2\}$,

$E_7 = \{v_2y_2\}$,

$E_8 = \{x_1y_2\}$.

Again, by observing the suffixes of edges in E_i and E_j for $i \neq j$, there are no overlapping edges: $E_i \cap E_j = \phi$. Clearly,

$$\bigcup_{i=1}^{8} E_i = E(D_2(L_3)),$$

So $D_2(L_3)$ is Grundy edge colorable with 8 colors. No additional edge has 8 distinct adjacent colors, so color 9 is not possible. Therefore,

$$\Gamma'(D_2(L_3)) = 8.$$

Case (iii): n = 4

$E_1 = \{u_1x_1, v_1y_1, x_2x_3, y_2y_3, u_2u_3, x_4v_4, u_4v_3\}$,

$E_2 = \{x_1v_1, u_1y_1, u_2x_2, x_3y_2, u_3y_3, v_3v_4, u_4y_4\}$,

$E_3 = \{u_1u_2, v_1v_2, x_2y_3, x_3v_3, u_3v_4, u_4x_4\}$,

$E_4 = \{x_1y_2, u_1v_2, x_2y_1, u_2v_3, x_3y_4, u_3u_4, x_4y_3\}$,

$E_5 = \{x_1x_2, y_1y_2, u_2v_1, u_3x_3, v_3y_3, v_4y_4\}$,

$E_6 = \{u_2y_2, u_3v_2, x_3x_4, y_3y_4\}$,

$E_7 = \{x_2v_2\}$,

$E_8 = \{v_2y_2\}$,

$E_9 = \{v_2v_3\}$

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By observing the edges in E_i and E_j for $i \neq j$, it is inferred that $E_i \cap E_j = \phi$ for all $i \neq j$. Clearly, the Ei's are pairwise disjoint and cover all edges of $D_2(L_n)$, i.e., $\bigcup_{i=1}^{9} E_i = E(D_2(L_n))$. Thus, $D_2(L_n)$ is Grundy edge colorable with 9 colors.

If one attempts to use a 10th color, there will not exist a vertex incident to edges colored with all 9 previous colors, violating the Grundy condition. Hence, confirming that $\Gamma'(D_2(L_n)) = 9$ for n = 4.

Case (iv): n = 5

$E_1 = \{x_1v_1, y_1y_2, u_1v_2, u_2u_3, x_2y_3, x_3x_4, v_3v_4, u_4u_5, x_5y_4, v_5y_5\}$,

$E_2 = \{u_1y_1, x_1y_2, u_2x_2, u_3v_2, y_2y_3, x_3y_4, x_4v_4, u_4v_5, u_5y_5\}$,

$E_3 = \{v_1y_1, x_1x_2, u_1u_2, v_2y_2, u_3x_3, x_4y_3, u_4y_4, x_5v_5, u_5v_4\}$,

$E_4 = \{u_1x_1, x_2v_2, u_2v_1, x_3y_2, u_3y_3, u_4x_4, v_4v_5, y_4y_5, u_5x_5\}$,

$E_5 = \{u_2y_2, v_1v_2, x_2x_3, y_3y_4, u_3u_4, x_4x_5\}$,

$E_6 = \{x_2y_1, u_3v_4, u_4v_3, x_4y_5\}$,

$E_7 = \{u_2v_3, v_4y_4\}$,

$E_8 = \{x_3v_3\}$,

$E_9 = \{v_2v_3\}$,

$E_{10} = \{v_3y_3\}$.

Again, all sets E_i are pairwise disjoint and collectively cover all edges of $D_2(L_n)$, i.e.,

$$\bigcup_{i=1}^{10} E_i = E(D_2(L_n)).$$

So, $D_2(L_n)$ is Grundy edge colorable with 10 colors.

Since no vertex is incident to edges with 10 distinct colors, one cannot add a color 11 under Grundy conditions. Hence, confirming $\Gamma'(D_2(L_n)) = 10$ for n = 5.

Case (v): n = 6

$E_1 = \{x_1v_1, y_1y_2, u_1v_2, u_2u_3, x_2y_3, x_3x_4, v_3v_4, u_4u_5, x_5y_4, v_5y_5, u_6y_6, x_6v_6\}$,

$E_2 = \{u_1y_1, x_1y_2, u_2x_2, u_3v_2, y_2y_3, x_3y_4, x_4v_4, u_4v_5, u_5y_5, x_5x_6, v_6y_6\}$,

$E_3 = \{v_1y_1, x_1x_2, u_1u_2, v_2y_2, u_3x_3, x_4y_3, u_4y_4, x_5v_5, u_5v_4, x_6y_5\}$,

$E_4 = \{u_1x_1, x_2v_2, u_2v_1, x_3y_2, u_3y_3, u_4x_4, v_4v_5, y_4y_5, u_5x_5, u_6x_6\}$,

$E_5 = \{u_2y_2, v_1v_2, x_2x_3, y_3y_4, u_3u_4, x_4x_5, u_5u_6, v_5v_6, y_5y_6\}$,

$E_6 = \{x_2y_1, u_3v_4, u_4v_3, x_4y_5, x_5y_6, u_5v_6, u_6v_5\}$,

$E_7 = \{u_2v_3, v_4y_4\}$,

$E_8 = \{x_3v_3\}$,

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$E_9 = \{v_2v_3\}$,

$E_{10} = \{v_3y_3\}$.

Case (vi): n = 7

$E_1 = \{x_1v_1, y_1y_2, u_1v_2, u_2u_3, x_2y_3, x_3x_4, v_3v_4, u_4u_5, x_5y_4, v_5y_5, u_6y_6, x_6v_6, u_7x_7, v_7y_7\}$,

$E_2 = \{u_1y_1, x_1y_2, u_2x_2, u_3v_2, y_2y_3, x_3y_4, x_4v_4, u_4v_5, u_5y_5, x_5x_6, v_6y_6, u_6u_7, x_7v_7\}$,

$E_3 = \{v_1y_1, x_1x_2, u_1u_2, v_2y_2, u_3x_3, x_4y_3, u_4y_4, x_5v_5, u_5v_4, x_6y_5, u_6v_7, x_7y_6, u_7v_6\}$,

$E_4 = \{u_1x_1, x_2v_2, u_2v_1, x_3y_2, u_3y_3, u_4x_4, v_4v_5, y_4y_5, u_5x_5, u_6x_6, v_6v_7, y_6y_7\}$,

$E_5 = \{u_2y_2, v_1v_2, x_2x_3, y_3y_4, u_3u_4, x_4x_5, u_5u_6, v_5v_6, y_5y_6, x_6x_7, u_7y_7\}$,

$E_6 = \{x_2y_1, u_3v_4, u_4v_3, x_4y_5, x_5y_6, u_5v_6, u_6v_5, x_6y_7\}$,

$E_7 = \{u_2v_3, v_4y_4\}$,

$E_8 = \{x_3v_3\}$,

$E_9 = \{v_2v_3\}$,

$E_{10} = \{v_3y_3\}$.

Case(vii): n = 8

$E_1 = \{x_1v_1, y_1y_2, u_1v_2, u_2u_3, x_2y_3, x_3x_4, v_3v_4, u_4u_5, x_5y_4, v_5y_5, u_6y_6, x_6v_6, u_7x_7, v_7y_7, u_8x_8, v_8y_8\}$,

$E_2 = \{u_1y_1, x_1y_2, u_2x_2, u_3v_2, y_2y_3, x_3y_4, x_4v_4, u_4v_5, u_5y_5, x_5x_6, v_6y_6, u_6u_7, x_7v_7, x_8v_8, u_8y_8\}$,

$E_3 = \{v_1y_1, x_1x_2, u_1u_2, v_2y_2, u_3x_3, x_4y_3, u_4y_4, x_5v_5, u_5v_4, x_6y_5, u_6v_7, x_7y_6, u_7v_6, x_8y_7\}$,

$E_4 = \{u_1x_1, x_2v_2, u_2v_1, x_3y_2, u_3y_3, u_4x_4, v_4v_5, y_4y_5, u_5x_5, u_6x_6, v_6v_7, y_6y_7, x_7x_8, u_7v_8\}$,

$E_5 = \{u_2y_2, v_1v_2, x_2x_3, y_3y_4, u_3u_4, x_4x_5, u_5u_6, v_5v_6, y_5y_6, x_6x_7, u_7y_7, u_8v_7\}$,

$E_6 = \{x_2y_1, u_3v_4, u_4v_3, x_4y_5, x_5y_6, u_5v_6, u_6v_5, x_6y_7, x_7y_8, u_7u_8, v_7v_8\}$,

$E_7 = \{u_2v_3, v_4y_4, y_7y_8\}$,

$E_8 = \{x_3v_3\}$,

$E_9 = \{v_2v_3\}$,

$E_{10} = \{v_3y_3\}$.

Case(viii): Let $n \geq 9$.

$E_1 = \{x_1v_1, y_1y_2, u_1v_2, u_2u_3, x_2y_3, x_3x_4, v_3v_4, u_4u_5, x_5y_4, v_5y_5, u_6y_6, x_6v_6, u_7x_7, v_7y_7, u_8x_8, v_8y_8\}$

$\quad \cup \bigcup_{i=1}^{n-8}\{u_{i+8}x_{i+8}\} \cup \bigcup_{i=1}^{n-8}\{v_{i+8}y_{i+8}\}$,

$E_2 = \{u_1y_1, x_1y_2, u_2x_2, u_3v_2, y_2y_3, x_3y_4, x_4v_4, u_4v_5, u_5y_5, x_5x_6, v_6y_6, u_6u_7, x_7v_7, x_8v_8, u_8y_8\}$

$\quad \cup \bigcup_{i=1}^{n-8}\{x_{i+8}v_{i+8}\} \cup \bigcup_{i=1}^{n-8}\{u_{i+8}y_{i+8}\}$,

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$E_3 = \{v_1y_1, x_1x_2, u_1u_2, v_2y_2, u_3x_3, x_4y_3, u_4y_4, x_5v_5, u_5v_4, x_6y_5, u_6v_7, x_7y_6, u_7v_6, x_8y_7\}$

$\cup \bigcup_{i=1}^{n-8}\{x_{i+8}y_{i+7}\} \cup \bigcup_{i=1}^{\lceil n|2\rceil-4}\{u_{2i+6}u_{2i+7}\} \cup \bigcup_{i=1}^{\lceil n|2\rceil-4}\{v_{2i+6}v_{2i+7}\}$,

$E_4 = \{u_1x_1, x_2v_2, u_2v_1, x_3y_2, u_3y_3, u_4x_4, v_4v_5, y_4y_5, u_5x_5, u_6x_6, v_6v_7, y_6y_7, x_7x_8, u_7v_8\}$

$\cup \bigcup_{i=1}^{n-8}\{u_{i+7}v_{i+8}\} \cup \bigcup_{i=1}^{\lceil n|2\rceil-4}\{y_{2i+6}y_{2i+7}\} \cup \bigcup_{i=1}^{\lfloor \frac{n}{2}\rfloor-4}\{x_{2i+7}x_{2i+8}\}$ for $n = 2q+8$, $q\geq 1$,

$E_5 = \{u_2y_2, v_1v_2, x_2x_3, y_3y_4, u_3u_4, x_4x_5, u_5u_6, v_5v_6, y_5y_6, x_6x_7, u_7y_7, u_8v_7\} \cup \{\bigcup_{i=1}^{n-8}. u_{i+8}v_{i+7}\}$

$\cup \bigcup_{i=1}^{\lceil n|2\rceil-4}\{x_{2i+6}x_{2i+7}\} \cup \bigcup_{i=1}^{\lfloor \frac{n}{2}\rfloor-4}\{y_{2i+7}y_{2i+8}\}$ for $n = 2q+8$, $q\geq 1$,

$E_6 = \{x_2y_1, u_3v_4, u_4v_3, x_4y_5, x_5y_6, u_5v_6, u_6v_5, x_6y_7, x_7y_8, u_7u_8, v_7v_8\} \cup \bigcup_{i=1}^{n-8}\{x_{i+7}y_{i+8}\}$

$\cup \bigcup_{i=1}^{\lfloor \frac{n}{2}\rfloor-4}\{u_{2i+7}u_{2i+8}\} \cup \bigcup_{i=1}^{\lfloor \frac{n}{2}\rfloor-4}\{v_{2i+7}v_{2i+8}\}$ for $n = 2q+8$, $q\geq 1$,

$E_7 = \{u_2v_3, v_4y_4, y_7y_8\}$,

$E_8 = \{x_3v_3\}$,

$E_9 = \{v_2v_3\}$,

$E_{10} = \{v_3y_3\}$.

In cases (iv) to (viii), by observing the suffixes of the edges in E_i and E_j ($i \neq j$), we infer that there are no common edges between E_i and E_j ; i.e., $E_i \cap E_j = \phi$ for $i \neq j$. Clearly, the sets E_i are pairwise disjoint and

$$\bigcup_{i=1}^{10} E_i = E(D_2(L_n)).$$

Hence, $D_2(L_n)$ is Grundy edge colorable with 10 colors. If we attempt to assign an 11th color, we find that there is no adjacent edge with Grundy index 10 in the given graph $D_2(L_n)$).

Therefore, for all $n \geq 5$, we have:

$$\Gamma' (D_2(L_n)) = 10.$$

4. Conclusion

In this study, we have determined the Grundy edge chromatic number Γ' (G) for the shadow graphs derived from various graph classes, including star, double star, comb, fan and ladder graphs. These results enhance our understanding of greedy edge-coloring algorithms and their performance on specific graph structures. The findings contribute to the broader field of graph theory by providing insights into the coloring properties of shadow graphs associated with these well-known graph families.

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