International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

GENERATING TRIANGULAR GRACEFUL TREES FROM CATTERPILLARS BY RECURRING REPEATED ACCESSORY

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ABSTRACT

A graph G with a line that is 1-1 g:V(G)→w (w∈W) and the property that the resulting line labels are likewise different is considered to have a graceful labeling. The labels |g(u)-g(w)| are assigned to the line occurrences with the nodes u and w. A graph is considered graceful if it allows for a graceful labeling; the triangular graph of a complete graph is equal to $\frac{a(a+1)}{2}$, where an is a positive integer. This study presents an iterative augmentation methodology for combining graceful triangle trees, which is inspired by K. M. Koh's [7] method, which uses well-known graceful trees to obtain larger triangular graceful trees. demonstrate the gracefulness of the repeating repeated added tree $M_x = M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$, for all x > 0, where $M_0$is a foundation tree chosen as a caterpillar and An accessory tree identified as a caterpillar is$M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$ is the term for the tree obtained by combining a replica of $M^{\mathcal{B}_{-1+x}}$ at every node in M_{-1+x} for x > 0 that has a degree greater than or equal to two. Consequently, for all repeatedly added caterpillar trees M_x for x > 0, the triangular elegant tree is accurate.

Keywords: Graceful tree, Graceful tree conjecture, Graph labeling, Triangular gracefulTree.

1. Introduction:

A graph labeling is a sequence of obligations to the nodes, lines, or both of the graphs, and the associated subject of certain constraints derived from a theoretical or applied

Received: August 07, 2025

problem. Ringel proposed the conjecture that the graph term K_{2a+1} decayed into 2a+1, akin to a tree with a line, at the Smolenice symposium in 1963. In 1965 A. Kotzig [1]inference a particular idea of Ringel that known to be complete graph K_{2a+1} be able to cyclicallydecayed$2a+1$ replication of any tree by$'a'$ lines. This research of endeavor to settle these two inferenceA. Rosa[2]. Initiateda hierarchical series of an evaluationτ, γ, ϑ and ω-assessment of the graph and it used an evaluationmeans to studythe cyclic decay of complete graph. After thatS. W. Golomb [10] called the ϑevaluation is 'graceful labeling' this word isgenerally used. A graceful labeling of a graph G with $a$lines are one - one$g: V(G) \rightarrow w$ ($w \in W$ - whole number) with the property that the consequential line labels are also different, where lineoccurrence to the nodes u and w is assigned the label $|g(u) - g(w)|$.

The term "graceful graph" refers to a graph that has been labeled gracefully. In his seminal work, A. Rosa [2] shown that a tree M with lines and graceful labeling can regularly decay into a 2a+1 replication of the tree, as can the entire graph 〖 K〗_(2a+1). The renowned Ringel, A. Kotzig[1], argues that every tree is graceful as a result of these results. Another name for this reasoning is graceful tree inference. M. Horton demonstrates that graceful tree inference is accurate for some trees with at least 29 nodes [8]. There are a few distinct tree classes that resemble caterpillars [2], diameter five trees [9], banana trees [3], and [4]. Every single one is shown to be graceful.

2. Structure of $M_x$where $x > 0$.

Assume for the moment that $M_0$is a caterpillar known as the base caterpillar. Let M_0 be the total number of nodes that are allowed to belong to M_0. Assume that $M^{\mathcal{B}_0}$is the first accessory caterpillar, wherein one of the interior nodes, w^*, is designated as an accessory node. Consider m_0 copies of $M^{\mathcal{B}_0}$ each consequent to a unique interior node M_0. Then join the node w^* of the copy of $M^{\mathcal{B}_0}$ have unique interior node of M_0. Then the consequentialtree $M_0 \oplus M^{\mathcal{B}_0}$, for a simplesystemindicated it by $M_1$and it is also called the first recursive accessory tree. That is $M_1 = M_0 \oplus M^{\mathcal{B}_0}$. In common, for $x > 1$, the x^{th}recursive accessory tree M_x is created recursively as $M_x = M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$. More specifically for $x > 1$,we think the$(x-1)^{th}$ recursive accessory tree M_{-1+x} and identify the number of inside nodes in M_{-1+x}. If the number of interior nodes of M_{-1+x} is t_{-1+x}, then we take t_{-1+x}replicationof any caterpillar $M^{\mathcal{B}_{x-1}}$ called the $(x-1)^{th}$accessory caterpillar, with any interior node w^*selected as the accessory node of $M^{\mathcal{B}_{x-1}}$, where every copyconsequent to a distinctiveinterior node of M_{x-1}. The consequential tree is $M_x = M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$. In the below lemma differentiate a distinctiveseries of interior nodes of the treeM_x, for that ideathe research tries to usenext terminology.

There are precisely two penultimate nodes in a caterpillar (excluding star). The tail and head of the caterpillar are the names given to two of the penultimate nodes, respectively. The caterpillar's unique greatest path can thus be described as follows: $a_0, a_1, a_2, \dots \dots .\ a_l, a_{l+1}$, where $a_l$represents the head and a_l represents the tail.

Received: August 07, 2025

International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

In a caterpillar (exclude star) there are accurately two penultimate nodes of the tree. One of the penultimate nodes is called the tail of the caterpillar and the other penultimate node is n called the head of the caterpillar. Then the distinctgreatest path of the caterpillar candescribe as $a_{0,}\ a_1,\ a_2, \ldots\ldots.\ a_l, a_{l+1}$, with a_l is the head and a_1 is the tail.

LEMMA 2.1

For each $x > 0$in the x^{th}recursive attachment treeM_x, a distinct sequence that consists of every internal node ofM_x.

The research tries to prove by method of induction on x.

Put$x = 1$, Consider tree M_1. By the definition of $M_1, M_1 = M_0 \oplus M^{\mathcal{B}_0}$, here M_0 and $M^{\mathcal{B}_0}$ are caterpillars. It is clear that in M_0 we can all the times identify a sequence of interior nodes beginningall the interior nodes connect with the tail passing and ends with the head of M_0(greatest path of M_0). This sequence byP_0, i.e) $P_0 = w_1, w_2, \ldots\ldots w_{m_0}$ where w_{m_0} and w_1 are the head and tail respectively of M_0, andv_x, $1 < x \leq M_0 - 1$ are the left overinterior nodes of M_0. By the structure of M_1, the accessory node w^* of every copy of $M^{\mathcal{B}_0}$ is attached to its relatedinterior node of M_0.

For our convenient, indicate the replica of $M^{\mathcal{B}_0}$connectedat the node w_y in P_0 as $M_y^{\mathcal{B}_0}$,for $0 < y \leq m_0$. Since every$M_y^{\mathcal{B}_0}$, $0 < y \leq m_0$ are caterpillars and by the structure of $M_1 = M_0 \oplus M^{\mathcal{B}_0}$ it follows that everyinterior nodes of M_1, precisely the interior nodes of $M_y^{\mathcal{B}_0}$, $0 < y \leq m_0$. Name the interior nodes of every connectedreplica of $M^{\mathcal{B}_0}$starting in the sort with the tail and concluding with the head, then first we name the interior nodes $M_1^{\mathcal{B}_0}$, as$w_1, w_2, \ldots\ldots w_{t_0}$ in the sort, where w_{t_0} is the head and w_1 is the tail of $M_1^{\mathcal{B}_0}$, where $t_0$indicates the number of interior node in $M^{\mathcal{B}_0}$. Name the interior nodes of $M_2^{\mathcal{B}_0}$ as $w_{2t_0}, w_{2t_{0-1}}, w_{t_0}, \ldots\ldots w_{t_{0+1}}$ in the order where $w_{t_{0+1}}$is head and w_{2t_0} is the tail of $M_2^{\mathcal{B}_0}$.Accordingly, for$y = 3,4,5, \ldots\ldots z_0$, when y is odd name the interior nodes of $M_y^{\mathcal{B}_0}$ as $w_{(y-1)t_{0+1}}, w_{(y-1)t_{0+2}}, w_{t_0}, \ldots\ldots w_{yt_0}$ in the sort, where $w_{(y-1)t_{0+1}}$ is the tail and w_{yt_0} is the head of the $M_y^{\mathcal{B}_0}$; and when y is even name the interior nodes of $M_y^{\mathcal{B}_0}$ as $w_{yt_0}, w_{yt_{0-1}}, w_{yt_{0-2}}, \ldots.\ w_{(y-1)t_{0+1}}$, where $w_{(y-1)t_{0+1}}$ is the head and $w_{yt_0}$is the tail of the $M_y^{\mathcal{B}_0}$. After that consider the sequence of interior nodes of the replication $M_y^{\mathcal{B}_0}$, for $0 < y \leq m_0$in the increasing order of the index y& indicate theseries byP_1. Therefore, the series of interior nodes P_1 in M_1 is $w_1, w_2, \ldots\ldots w_{t_0}, w_{t_{0+1}}, w_{2t_0}, w_{2t_{0-1}}, w_{t_0}, \ldots w_{t_{0+1}}, \ldots w_{(y-1)t_{0+1}}, w_{(y-1)t_{0+2}}, w_{t_0}, \ldots\ldots w_{z_0t_0}$. For our convenience we allow $z_0t_0 = z_1$. Since every$M_y^{\mathcal{B}_0}$ is a caterpillar, the seriesP_1 is the

Received: August 07, 2025

International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

distinctive in M_1 ending with the head or tail and beginning with the tail w_{z_1} of $M_{z_0}^{\mathcal{B}_0}$ depend on $z_0$is even or odd.

By the induction suppose that in M_{l-1}, then there exist a distinctiveseries of nodes P_{l-1} containing all the interior nodes M_{l-1}, beginning with the tail of $M_1^{\mathcal{B}_{l-2}}$ and ending with the tail or head of $M_{z_{l-2}}^{\mathcal{B}_{l-2}}$ depend on $z_{l-2}$is even or odd. The research tries toagree to the series$P_{l-1} = w_1, w_2, \ldots\ldots w_{z_{k-1}}$ where $w_1$the tail of is$M_1^{\mathcal{B}_{l-2}}$ and $w_{z_{l-1}}$ the tail or head of $M_{z_{l-2}}^{\mathcal{B}_{l-2}}$ depending $z_{l-2}$is even or odd.

To finish the orientation, researchthink the tree $M_l = M_{l-1} \oplus M^{\mathcal{B}_{l-1}}$ by the structure of M_l, the accessory node of everyreplica of $M^{\mathcal{B}_{l-1}}$ is connected to its equivalentinterior node of M_{l-1}more accurately, to the nodes in the seriesP_{l-1} . Therelated to the base case, we indicate the copy of $M^{\mathcal{B}_{l-1}}$connected at the node w_y in P_{l-1} as $M_y^{\mathcal{B}_{l-1}}$ for $0 < y \leq z_{l-1.}$ By induction hypothesis, there exist a distinctiveseries of the nodes in M_{l-1}includes all its interior nodes, as $M_y^{\mathcal{B}_{l-1}}$, $0 < y \leq z_{l-1}$are caterpillars, it follows from the structure of $M_l = M_{l-1} \oplus M^{\mathcal{B}_{l-1}}$,aninterior nodes of M_l are accurately the interior nodes $M_y^{\mathcal{B}_{l-1}}$, $0 < y \leq z_{l-1}$. Name the interior nodes of everyconnected replica of $M^{\mathcal{B}_{l-1}}$ beginning from the tail and endby head as they show in the greatest path of $M^{\mathcal{B}_{x-1}}$. After that, name the interior nodes of $M_1^{\mathcal{B}_{l-1}}$ as $w_1, w_2, \ldots\ldots w_{t_{l-1}}$,where$w_{t_{l-1}}$is head and w_1 is tail of $M_1^{\mathcal{B}_{l-1}}$, where t_{l-1}indicates the number of interior nodes of $M^{\mathcal{B}_{l-1}}$. Name the interior nodes of $M_2^{\mathcal{B}_{l-1}}$ as $w_{2t_{l-1}}, w_{2t_{l-1}-1}, \ldots\ldots w_{t_{l-1}+1}$, where $w_{t_{l-1}+1}$ is the head and $w_{2t_{l-1}}$ is the tail of $M_2^{\mathcal{B}_{l-1}}$. Thus for $y = 3,4,5, \ldots z_{l-1}$, when y is odd, name the interior nodes of $M_y^{\mathcal{B}_{l-1}}$ as $w_{(y-1)t_{l-1}+1} w_{(y-1)t_{l-1}+2}, \ldots\ldots w_{yt_{l-1}}$, where $w_{yt_{l-1}}$, is the head and $w_{(y-1)t_{l-1}+1}$ is the tail of $M_y^{\mathcal{B}_{l-1}}$; and when y is even, name the interior nodes of $M_y^{\mathcal{B}_{l-1}}$ as $w_{yt_{l-1}} w_{yt_{l-1}-1} \ldots\ldots w_{(y-1)t_{l-1}+2}$ where $w_{(y-1)t_{l-1}+2}$ is the head and $w_{yt_{l-1}}$is the tail of $M_y^{\mathcal{B}_{l-1}}$.

Next, consider the sequence of interior nodes of the replication of $M_y^{\mathcal{B}_{l-1}}$,for $0 < y \leq z_{l-1}$ in the increasing order of the index y and indicate this seriescalledP_l.
Therefore P_l is the series$(w_1, w_2, \ldots\ldots w_{t_{l-1}}, \ w_{t_{l-1}+1}, w_{2t_{l-1}}, \ldots . w_{(y-1)t_{l-1}+1}, \ldots . w_{z_{l-1}t_{l-1}})$,

Containingfor all the interior node of M_l , here w_1 is tail of $M_1^{\mathcal{B}_{l-1}}$. The research allows $(z_{l-1})(t_{l-1}) = m_k$. Afterward w_{zl} is head or tail of $M_{z_{l-1}}^{\mathcal{B}_{l-1}}$ depends on (z_{l-1}) is even or odd. As each and every$M_y^{\mathcal{B}_{l-1}}$ is a caterpillar, the real series P_l is the distinctiveseries in P_l, ending with the head or tail w_{z_l} of $M_{z_{l-1}}^{\mathcal{B}_{l-1}}$ and beginning with the tail w_1 of $M_1^{\mathcal{B}_{l-1}}$depending z_{l-1} is odd or even, containing all the interior node of M_l. Hence prove induction.

Received: August 07, 2025

International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

THEOREM 2.1

For $x > 0$, the tree $M_x = M_{-1+x} \oplus M^{\mathcal{B}_x}$ is Triangular graceful.

Proof:

For $x > 0$, consider the tree $M_x = M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$. By the above lemma, each M_x has a distinctivesequence P_x, consisting of all the interior nodes of M_x, for $x > 0$, that is $P_x = (w_1, w_2, \ldots\ldots w_{z_x})$, where $w_{y,} 0 < y \leq m_x$ are the interior nodes of M_x.

For an interior nodes of w in M_x, for $x > 0$.Let $D_1(w)$ represent the collection of all pendent nodes that are close to node w.

Classify the sequence $\left(T_{w_y}\right) = \left(w_y, D_1(w_{y+1})\right)$, where $D_1(w_{y+1})$is arranged as a series, for each $w_y$inP_x and for y, $1 \leq y \leq m_x - 1$ Examine the next two novel node sequences:

$A_1 = \left(T_{w_1}, T_{w_3}, T_{w_5}, \ldots\ldots\ldots T_{2\left\lfloor\frac{z_x}{2}\right\rfloor - 1}\right)$ and$A_2 = \left(D_{1(w_1)}, T_{(w_2)}, T_{(w_4)}, \ldots\ldots\ldots T_{2\left\lfloor\frac{z_x - 1}{2}\right\rfloor}\right)$.

For convenience, the modified sequence of A_1, is also indicated by A_1, when $z_x$is odd. If $z_x$is odd, insert the node (w_{z_x}) known to be the ending of the series A_1, or else place the node (w_{z_x}) at the end of the sequence A_2. On the other hand, A_2 notes the modified sequence of A_2 when z_x is even. Given a sequence A_1, $T_{w_y} = \left(w_y, D_1(w_{y+1})\right)$, every node v in A_1 has neighbors in A_2. Similarly, for any node v in a sequence A_2, every one of its neighbours is in A_1 For the bipartite network M_x, $x > 0$, the partition of (A_1, A_2) therefore defines a bipartition.
The researchindicate the nodes of A_1 as $(a_1, a_2, a_3, \ldots\ldots a_r)$, and the nodes of A_2as $(\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots\ldots \mathscr{b}_o)$ .Note that $r + o = D + 1$.

The nodes $(\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots\ldots \mathscr{b}_o)$of A_2 and the nodes $(a_1, a_2, a_3, \ldots\ldots a_r)$ ofA_1should be assigned the labels $D, D-1, D-(o-1)$ accordingly. This assignment is clearly one-on-one and on-to. Consequently, we observe that the node labels differ and are associated with 0,1,2,....D. We will now refer to a node by its label in order to determine the distinctness of line labels. We arrange the occurrence line at the node r-y for all y,y=1,2,3,….r in a such that the other end nodes of the incidence line at r-y emerge in ascending order. The line labels of the aforementioned set of lines then create a sequence that increases monotonically, and the labels collected over the set {n∈N} makes it evident that the tree x>0,M_x is Triangular graceful.

Received: August 07, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

3. GENERATING DIFFERENT TRIANGULAR GRACEFUL TREES $M_x, x > 0$.

Burzio and Ferarese introduced the δ+1 structure in [7] to construct unique triangular graceful trees from accepted graceful trees. This δ+1 structure stimulates the generation of unique triangular graceful trees from the graceful trees M_x, where x>0, by modifying specific lines of M_x.

Consider the tree M_x for some $x,\ x > 0$ and its graceful labeling $g: V(M_x) \to \{a\}$, where $a \in W$ (W – whole number) that is defined as in the preceding theorem's proof. Examinethat, the tree M_x, it's got from M_{-1+x} by combining the replica$M_y^{\mathcal{B}_{-1+x}}$of $M^{\mathcal{B}_{-1+x}}$, $1 \le y \le m_{-1+x}$ with its consequent interior nodew_y in the sequence $P_{m-1} = (w_1, w_2, w_3, \dots . w_{m-1})$ of nodes of M_{x-1}.

For every $y,\ 1 \le y \le m_{-1+x} - 1$ and y odd, consider the series of interior nodes$u_1^y, u_2^y, u_3^y \dots \dots u_{l_1^{y-1}}^y$ of $M_y^{\mathcal{B}_{-1+x}}$ which begins from the node adjacent to accessorynodew^* of $M_y^{\mathcal{B}_{x-1}}$ and concludes in the order of $M_y^{\mathcal{B}_{-1+x}}$, and considers the sequence of internal nodes as well $u_1^{y+1}, u_2^{y+1}, u_3^{y+1} \dots \dots u_{l_1^{y-1}}^{y+1}$ of $M_{y+1}^{\mathcal{B}_{-1+x}}$ which begins from the node adjacent to accessorynode w^* of $M_{y+1}^{\mathcal{B}_{-1+x}}$ and ends with the head of $M_{y+1}^{\mathcal{B}_{-1+x}}$ in the order. In the same way when y is even, consider the series of insidenodes$u_1^y, u_2^y, u_3^y \dots \dots u_{l_2^{y-1}}^y$ of $M_y^{\mathcal{B}_{-1+x}}$ which begins from the node adjacent to accessorynodew^* of $M_y^{\mathcal{B}_{-1+x}}$ and end with the tail of $M_y^{\mathcal{B}_{-1+x}}$ in the order, and also reflect on the series of interiornodes$u_1^{y+1}, u_2^{y+1}, u_3^{y+1} \dots \dots u_{l_2^{y-1}}^{y+1}$ of $M_{y+1}^{\mathcal{B}_{-1+x}}$ which begins from the node adjacent to accessorynode w^* of $M_{y+1}^{\mathcal{B}_{x-1}}$ and ends with the tail of $M_{y+1}^{\mathcal{B}_{-1+x}}$ in the order.

Since y is odd, let $l = l_1^{-1+x}$ instead than $l = l_2^{-1+x}$, since y is even. Then, according to the definition of $M_x$in $M_x$where$y = 1,2,3, \dots . . m_{-1+x} - 1$, the nodes (u_d^y, u_d^{y+1}) in u_d^y of $M_y^{\mathcal{B}_{-1+x}}$and the node $M_y^{\mathcal{B}_{-1+x}}$ of $M_{y+1}^{\mathcal{B}_{-1+x}}$ are non-adjacent.

$$\left| g(u_{d)}^y - g(u_d^{y+1}) \right| = |\, g(w_*^y) - g(w_*^{y+1})|$$

For every $y,\ 1 \le y \le m_{-1+x} - 1$, except when $y \equiv 0(mods_{(x-2)})$, for $d, 1 \le d \le l$, here w_*^y and w_*^{y+1} is denote the accessory nodes of $M_y^{\mathcal{B}_{-1+x}}$ and $M_{y+1}^{\mathcal{B}_{-1+x}}$.

Subsequently, if we delete the line $(w_*^y,\ w_*^{y+1})$ in M_x and add any one of the non adjacent couple of nodes $(u_d^y,\ w_d^{y+1})$ in M_x, for some $d, 1 \le d \le l$, then the realconsequential graph, denoted M_x is linked& also non cyclic. AccordinglyM_x is tree.

Moreover, the node labels in the tree $\breve{M}_x$ stay the same as they do in the tree $\breve{M}_x$. As a result, for $N = |E(\breve{M}_x)|$, all of the node labels of $\breve{M}_x$are distinct and series across {0,1,2,3…..N}.

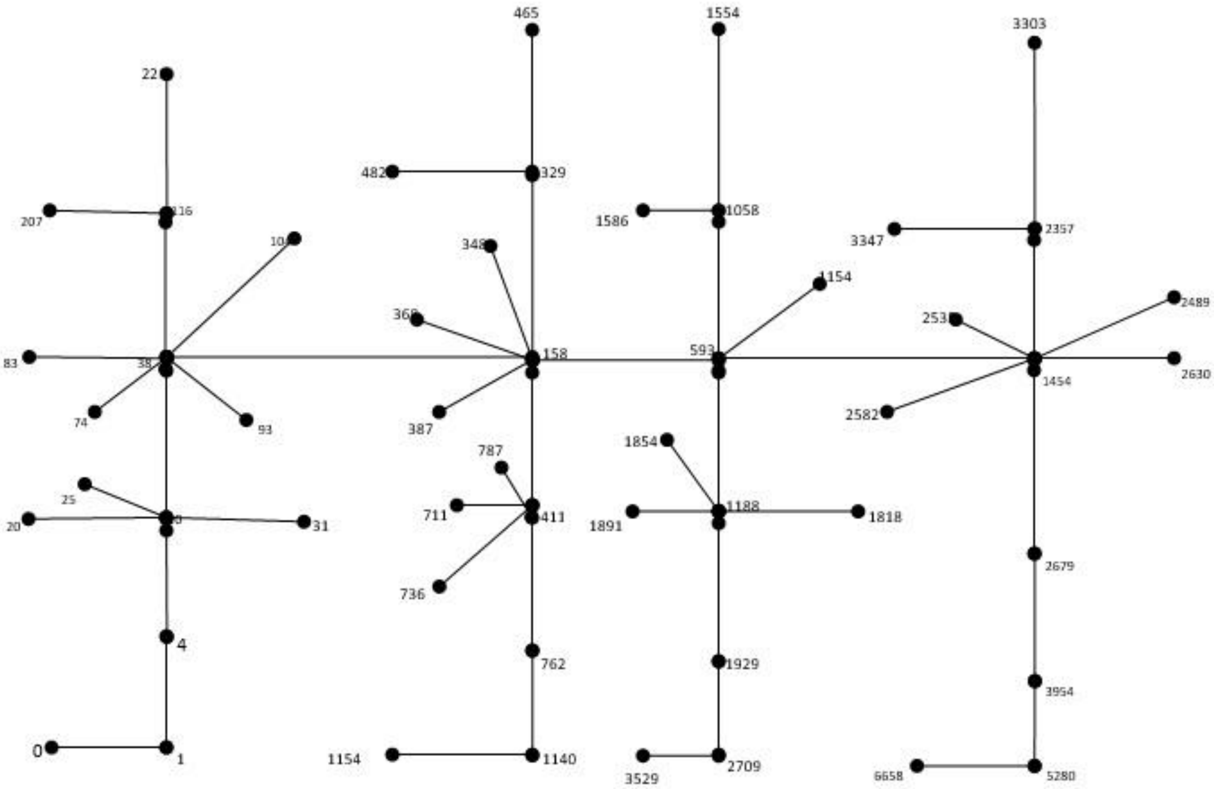
Received: August 07, 2025

International Journal of Applied Mathematics

Volume 38 No. 9s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The definition tree $\widetilde{M_x}$ indicates that [5] all of the line labels in $\widetilde{M_x}$ are distinct and series over {0,1,2,3…..N}. As a result, the tree $\widetilde{M_x}$ is similarly graceful in a triangle.



Conclusion:

This work presents the introduction of triangular elegant labeling. The caterpillar graph is Triangular graceful labeling graph and also sub division of path, comb, bi-star, coconut tree $aK_{1,3}$ are studied, and we have exposed that the recursive accessory tree $M_x = M_{-1+x} \oplus M^{\mathcal{B}_{-1+x}}$ triangular graceful tree, for $x > 0$.This study provides fresh findings from the graph labeling theory.

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Received: August 07, 2025