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Analysis of the Sum Connectivity Index for Graphs Derived from a Special Class of Monogenic Semigroups

Rajadurai R[1] , Sheeja G[1*]

[1] Department of Mathematics, College of Engineering and Technology
SRM Institute of Science and Technology
Kattankulthur – 603 203, Tamil Nadu, India.

*Corresponding author: sheejag@srmist.edu.in

Abstract

This study investigates the properties based on the sum connectivity index, a special class of graph concerning a specific category of monogenic semigroup graph. Moreover, an efficient algorithm has been implemented using Python to compute the vertex degrees relevant to the sum connectivity index of undirected graphs. The main goal of the research is to establish the SCI metrics and emphasize their application to this distinct class of graphs. Through a comprehensive analysis, this paper illustrates the fundamental characteristics of SCI concerning the structural properties of the monogenic semigroup graph. Moreover, the study delves into the implication and significance of SCI in modeling and analyzing connectivity within this graph category. Finally, we also compute the proposed problem based on an algorithm of the topological index with detailed examples.

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Key Words and Phrases: Algorithm of topological index, Monogenic Semigroups, Sum Connectivity Index (SCI).

1 Introduction

The idea of monogenic semigroup graphs(MSG) was motivated by the study of zero divisor graphs. Initially, our exploration will center on zero divisor graphs before delving into the realm of MSG, as outlined in references [1], [2], [3], [17]. The researcher [21],[22] have conducted investigation on zero divisor graphs, encompassing both commutative and non-commutative contexts. The adjacency rule among vertices has been applied in this study. Building upon the research outlined in [21],[22] a new graph has been established, establishing a relationship with the MSG defined in [19]. A necessary and sufficient condition for establishing a connection between two distinct vertices x^i and x^j within S_M is expressed as $i + j > n$ (where $1 \leqslant i, j \leqslant n$). Our comprehensive exploration of the MSG persists in subsequent works, specifically detailed in [5], [6], [7], [13],[14].

The significance of topological indices spans across various scientific disciplines, offering invaluable insights. These indices serve as crucial metrics connecting structural properties and the chemical makeup of compounds. Within molecular graphs, where atoms form vertices and bonds represent edges, numerous graph indices have emerged, originating from modifications made to molecules. Graph representations are essential for extracting relevant data concerning the fundamental nature of molecular structures.

These indices encompass values linked to the structural properties and configurations of molecules. Numerous graph indices have been specifically designated for molecular graphs, originating from alterations in molecules involving atoms, vertices, and bonds forming edges. In the realm of chemistry, graphs serve as indispensable tools for gathering data concerning the essence of chemical compounds. Within chemical graph theory, topological indices find

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widespread application in the analysis of chemical structures (refer to [32]). The Sombor index, a molecular structure descriptor based on vertex degrees and introduced by Gutman [23], establishes both lower and upper bounds. In recent studies, Rajadurai and Sheeja [9], [10] explored the SCI by utilizing cartesian, strong product, and special graphs to determine the vertex degrees within monogenic semigroup graphs. Recent research has extensively explored the practical applications of this index in chemistry (see [15], [16], [18], [26], [31]). Additionally, studies focusing on determining lower and upper bounds for the Sombor index of a graph can be found in references such as [20], [24], [27], [30] and other related sources.

In our investigation, we illustrate G as the set of a graph, where V(G) represents the vertex set of G, and E(G) signifies the edge set of G. The Sombor index, as originally defined by Gutman in [23], is described below. Subsequently, Kulli introduced the Banhatti-Sombor indices in [20], presenting specific formulas applicable to certain nano-structures. In [28], [29] the author delineated an equation for the Sombor index and an equation for the Nirmala index of a graph within a monogenic semigroup, respectively. The primary focus of this paper is to compute the sum connectivity index of a graph within a monogenic semigroup.

$$SCI(\mathcal{G}_{\mathcal{MS}}) = \sum_{\mathfrak{u}\mathfrak{v}\in E(\mathcal{G}_{\mathcal{MS}})} \Big[\mathfrak{d}(\mathfrak{i}) + \mathfrak{d}(\mathfrak{j})\Big]^{-\frac{1}{2}}. \tag{1}$$

Let us consider SCI of undirected graph and which is expressed as: [8, 12]

In this way, for a positive integer n, we have

$$\lfloor \frac{\kappa}{2} \rfloor = \begin{cases} \frac{\kappa}{2} & \text{if } \kappa \text{ is even} \\ \frac{\kappa-1}{2} & \text{if } \kappa \text{ is odd.} \end{cases} \tag{2}$$

2 Novelty of the study

The reviewed literature indicates that limited focus has been placed on the SCI of monogenic semigroup graphs. Consequently,

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- This study aims to address a gap in the existing literature by analyzing the SCI and the special graph associated with monogenic semigroups. Our investigation is centered on these structural properties as derived from monogenic semigroup graphs.

- We examine the connectivity properties of these graphs, with particular emphasis on the SCI, which measures the sharp variation in connectivity between pairs of vertices.

- In addition, numerical techniques and computational tools are employed to calculate the index of the specialized graph. The analysis highlights the relationship between the algebraic features of MSG and the connectivity attributes of their associated graphs.

- Our results advance the fields of graph theory and algebraic semigroup theory by offering new insights into the relationship between algebraic structures and graph connectivity.

This work aims to bridge the identified research gap by offering fresh perspectives on monogenic semigroups. We present significant numerical finding related to the study of the sum connectivity index and the special graphs associated with these structures.

This study aims to derive the precise equation for the sum connectivity index of a graph within a monogenic semigroup.

3 Basic concepts

An algorithm introduced in [5] was developed to simplify the computation of vertex neighborhoods. In this study, we utilize that algorithm to calculate the initial sum connectivity index of graphs arising from monogenic semigroups. The notations, $\{x^{\kappa}, x^{\kappa-1}, x^{\kappa-2}, ..., x^{1}\}$, represent the elements of the vertex set.

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Definition 3.1. [11] Let $\mathcal{M}_{\mathcal{SG}} = a, a^2, a^3, \ldots, a^\kappa$ represent a monogenic semigroup containing a zero element. The corresponding graph has a vertex set consisting of the non-zero elements of SA. An edge exists between two vertices a^i and a^j if and only if the sum of their exponents, $i + j$, is greater than or equal to κ.

Lemma 3.2. *Suppose $d_1, d_2, d_3, ..., d_\kappa$ represent the degrees of vertices $\{x^1, x^2, x^3, ..., x^\kappa\}$ in a monogenic semigroup graph $(\Gamma(S_M))$, respectively. in this case, the sequence of degrees satisfies the conditions: $d_1 = 1, d_2 = 2, ..., d_{\lfloor \frac{\kappa}{2} \rfloor} = \lfloor \frac{\kappa}{2} \rfloor, d_{\lfloor \frac{\kappa}{2} \rfloor + 1} = \lfloor \frac{\kappa}{2} \rfloor, d_{\lfloor \frac{\kappa}{2} \rfloor + 2} = \lfloor \frac{\kappa}{2} \rfloor + 1, ..., d_\kappa = \kappa - 1$.*

Remark 3.3. *When examining the reiterated elements within the preceding lemma, as outlined below $d_{\lfloor \frac{\kappa}{2} \rfloor} = \lfloor \frac{\kappa}{2} \rfloor = d_{\lfloor \frac{\kappa}{2} \rfloor + 1}$*

The degree of d_κ is represented as $\kappa - 1$, considering that the total number of vertices is κ.

4 An algorithm

Algorithm for computing the sum connectivity index

Input: A simple undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
Output: SCI of the graph

Algorithm 1 Compute sum connectivity index

```
Initialize SCI ← 0
for each edge (u, v) ∈ E do
    Compute d_u ← degree(u)
    Compute d_v ← degree(v)
    SCI ← SCI + 1/√(d_u + d_v)
end for
return SCI
```

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Mathematical Expression:

$$SCI(\mathcal{G}_{\mathcal{MS}}) = \sum_{\mathfrak{uv} \in E(\mathcal{G}_{\mathcal{MS}})} \Big[d(\mathfrak{i}) + d(\mathfrak{j}) \Big]^{-\frac{1}{2}}$$

Python code for sum connectivity index

Listing 1: Python code to compute the Sum Connectivity Index

```
import networkx as nx
import math

def sum_connectivity_index(graph):
sci = 0
for u, v in graph.edges():
degree_u = graph.degree[u]
degree_v = graph.degree[v]
sci += 1 / math.sqrt(degree_u + degree_v)
return sci

# Example Usage
G = nx.Graph()
G.add_edges_from([(1, 2), (2, 3), (3, 4), (4, 1), (2, 4)])
# Example Graph
sci_value = sum_connectivity_index(G)
print("Sum_Connectivity_Index:", sci_value)
```

In the case where κ is an odd integer:
Step 1 - I_κ: The vertex x^κ is adjacent to each vertex x^{i_1}, where $1 \leq i_1 \leq \kappa - 1$.
Step 2 - $I_{\kappa-1}$:vertex $x^{\kappa-1}$ is adjacent to each vertex x^{i_2}, where $2 \leq i^2 \leq \kappa - 2$.
Step 3 - $I_{\kappa-2}$:vertex $x^{\kappa-2}$ is adjacent to each vertex x^{i_3}, where $3 \leq i^3 \leq \kappa - 3$.
Proceeding with the algorithm as outlined leads to the following

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conclusion.

In the case where κ is an even integer:
Step 4 - $I_{\frac{\kappa}{2}+2}$: The vertex $x^{\frac{\kappa}{2}+2}$ is connected not only to $x^{\frac{\kappa}{2}-1}$, $x^{\frac{\kappa}{2}}$, and $x^{\frac{\kappa}{2}+1}$, but also to the vertices $x^{\kappa}, x^{\kappa-1}, x^{\kappa-2}, \ldots, x^{\frac{\kappa}{2}+3}$.
Step 5 - $I_{\frac{\kappa}{2}+1}$:The vertex $x^{\frac{\kappa}{2}+1}$ is connected not only to $x^{\frac{\kappa}{2}}$ but also to the vertices $x^{\kappa}, x^{\kappa-1}, x^{\kappa-2}, ..., x^{\frac{\kappa}{2}+2}$.
If κ is an odd number:
Step 6 - $I_{\frac{\kappa+1}{2}}$:The vertex $x^{\frac{\kappa+1}{2}+2}$ is connected not only to $x^{\frac{\kappa+1}{2}-2}, x^{\frac{\kappa+1}{2}-1}$ and $x^{\frac{\kappa+1}{2}+1}$ but also to the vertex
$x^{\kappa}, x^{\kappa-1}, x^{\kappa-2}, ..., x^{\frac{\kappa+1}{2}+3}$.
Step 7 - $I_{\frac{\kappa+1}{2}+1}$:The vertex $x^{\frac{\kappa+1}{2}+1}$ is connected not only to $x^{\frac{\kappa+1}{2}-1}$ and $x^{\frac{\kappa+1}{2}}$ but also to the vertex $x^{\kappa-1}, x^{\kappa}$.
Extensive research on degree sequences can be found in [4] and [19], along with additional references cited therein. The proof of the following lemma, originally presented in [19], is also incorporated within the algorithm described in [5].

5 SCI Over a Monogenic Semigroup Graph

In this section, we derive a precise equation for the initial sum connectivity index pertaining to a graph within a monogenic semigroup.

Theorem 5.1. *Let S_M as a monogenic semigroup. The initial sum connectivity index of $\Gamma(S_M)$ is given by*
$SCI(\Gamma(S_M))$ =

$$\begin{cases} \sum_{z=1}^{\frac{\kappa}{2}-1} \sum_{i=z}^{\kappa-z-1} \left[(\kappa-z)+i\right]^{-\frac{1}{2}} + \sum_{z=1}^{\frac{\kappa}{2}-1} \left[(\kappa-z)+\frac{\kappa}{2}\right]^{-\frac{1}{2}} & \text{if } \kappa \text{ is even} \\ \sum_{z=1}^{\frac{\kappa-1}{2}} \sum_{i=z}^{\kappa-z-1} \left[(\kappa-z)+i\right]^{-\frac{1}{2}} + \sum_{z=1}^{\frac{\kappa-1}{2}} \left[(\kappa-z)+\frac{\kappa}{2}\right]^{-\frac{1}{2}} & \text{if } \kappa \text{ is odd.} \end{cases}$$

Proof. The intention is to systematically formulate $\mathrm{SCI}(\Gamma(S_M))$ based on the summation of degrees. In our computation, we will

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employ the provided algorithm, which employs a systematic approach to summing up the degrees of vertices through equation addition. Additionally, we require equations (3.2, 2) and remark 3.3 for reference in our calculations.
If n is an odd number:

$$\begin{aligned}
&[SCI](\Gamma(S_M)) \\
&= \left[\mathfrak{d}_\kappa + \mathfrak{d}_1\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_\kappa + \mathfrak{d}_2\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_\kappa + \mathfrak{d}_3\right]^{-\frac{1}{2}} \\
&+ \left[\mathfrak{d}_\kappa + \mathfrak{d}_4\right]^{-\frac{1}{2}} + ... + \left[\mathfrak{d}_\kappa + \mathfrak{d}_{\kappa-2}\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_\kappa + \mathfrak{d}_{\kappa-1}\right]^{-\frac{1}{2}} \\
&+ \left[\mathfrak{d}_{\kappa-1} + \mathfrak{d}_2\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_{\kappa-1} + \mathfrak{d}_3\right]^{-\frac{1}{2}} + ... \\
&+ \left[\mathfrak{d}_{\kappa-1} + \mathfrak{d}_{\kappa-2}\right]^{-\frac{1}{2}} + ... + \left[\mathfrak{d}_{\frac{\kappa+1}{2}+2} + \mathfrak{d}_{\frac{\kappa+1}{2}-2}\right]^{-\frac{1}{2}} \\
&+ \left[\mathfrak{d}_{\frac{\kappa+1}{2}+2} + \mathfrak{d}_{\frac{\kappa+1}{2}-1}\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_{\frac{\kappa+1}{2}+2} + \mathfrak{d}_{\frac{\kappa+1}{2}}\right]^{-\frac{1}{2}} \\
&+ \left[\mathfrak{d}_{\frac{\kappa+1}{2}+2} + \mathfrak{d}_{\frac{\kappa+1}{2}+1}\right]^{-\frac{1}{2}} + \left[\mathfrak{d}_{\frac{\kappa+1}{2}+1} + \mathfrak{d}_{\frac{\kappa+1}{2}-1}\right]^{-\frac{1}{2}} \\
&+ \left[\mathfrak{d}_{\frac{\kappa+1}{2}+1} + \mathfrak{d}_{\frac{\kappa+1}{2}}\right]^{-\frac{1}{2}}.
\end{aligned}$$

As a result, the sum connectivity index of $\Gamma(S_M)$ is derived as the subsequent summation.

$$\begin{aligned}
&[SCI](\Gamma(S_M)) \\
&= \sum_{ij\in E(G)} \left[\mathfrak{d}_i + \mathfrak{d}_j\right]^{\frac{1}{2}} = [SCI]_1 + [SCI]_{\kappa-1} \\
&+ [SCI]_{\kappa-2} + ... + [SCI]_{\frac{\kappa+1}{2}+2} + [SCI]_{\frac{\kappa+1}{2}+1}.
\end{aligned}$$

Each sum connectivity index within the provided sum will be computed individually. Furthermore, we take into account the equation

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$\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$ as specified in 5.2 when n is an odd number.

$$
\begin{aligned}
&[SCI]_{\kappa}\\
&\quad = \big[(\kappa-1)+1\big]^{-\frac{1}{2}} + \big[(\kappa-1)+2\big]^{-\frac{1}{2}} + \big[(\kappa-1)+3\big]^{-\frac{1}{2}} + \dots\\
&\quad + \big[(\kappa-1)+\lfloor\frac{\kappa}{2}\rfloor\big]^{-\frac{1}{2}} + \dots + \big[(\kappa-1)+(\kappa-2)\big]^{-\frac{1}{2}}\\
&\quad + \big[(\kappa-1)+\lfloor\frac{\kappa}{2}\rfloor\big]^{-\frac{1}{2}}\\
&\quad = \sum_{i=1}^{\kappa-2}\big[(\kappa-i)+i\big]^{-\frac{1}{2}} + \big[(\kappa-1)+\frac{\kappa-1}{2}\big]^{-\frac{1}{2}}
\end{aligned}
$$

If we implement an identical procedure as used in $[SCI]_{\kappa}$ fro the application to $[SCI]_{\kappa-1}, \dots, [SCI]_{\frac{\kappa+1}{2}+2}$, $[SCI]_{\frac{\kappa+1}{2}+1}$

$$
\begin{aligned}
[SCI]_{\kappa-1} &= \sum_{i=2}^{\kappa-3}\big[(\kappa-i)+i\big]^{-\frac{1}{2}} + \big[(\kappa-2)+\frac{\kappa-1}{2}\big]^{-\frac{1}{2}},\\
[SCI]_{\frac{\kappa+1}{2}+2} &= \big[(\frac{\kappa+3}{2})+(\frac{\kappa-3}{2})\big]^{-\frac{1}{2}} + \big[(\frac{\kappa+3}{2})+(\frac{\kappa-1}{2})\big]^{-\frac{1}{2}}\\
&\quad + \big[(\frac{\kappa+3}{2})+(\frac{\kappa-1}{2})\big]^{-\frac{1}{2}} + \big[(\frac{\kappa+3}{2})+(\frac{\kappa+1}{2})\big]^{-\frac{1}{2}}.
\end{aligned}
$$

So,

$$
[SCI]_{\frac{\kappa+1}{2}+1} = \big[(\frac{\kappa+1}{2})+(\frac{\kappa-1}{2})\big]^{-\frac{1}{2}} + \big[(\frac{\kappa+1}{2})+(\frac{\kappa-1}{2})\big]^{-\frac{1}{2}}.
$$

Hence,

$$
\begin{aligned}
&[SCI]_{\kappa} + [SCI]_{\kappa-1} + \dots + [SCI]_{\frac{\kappa+1}{2}+2} + [SCI]_{\frac{\kappa+1}{2}+1}\\
&\quad = \sum_{z=1}^{\frac{\kappa-1}{2}}\sum_{i=z}^{\kappa-z-1}\big[(\kappa-z)+i\big]^{-\frac{1}{2}} + \sum_{z=1}^{\frac{\kappa-1}{2}}\big[(\kappa-z)+(\frac{\kappa}{2})\big]^{-\frac{1}{2}}.
\end{aligned}
$$

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If κ is an even number, $[SCI]_\kappa$ can be expressed as

$$\begin{aligned}
[SCI]_\kappa &= \left[(\kappa-1)+1\right]^{-\frac{1}{2}} + \left[(\kappa-1)+2\right]^{-\frac{1}{2}} + \left[(\kappa-1)+3\right]^{-\frac{1}{2}} + ... \\
&+ \left[(\kappa-1)+\lfloor\frac{\kappa}{2}\rfloor\right]^{-\frac{1}{2}} + ... + \left[(\kappa-1)+(\kappa-2)\right]^{-\frac{1}{2}} \\
&+ \left[(\kappa-1)+\lfloor\frac{\kappa}{2}\rfloor\right]^{-\frac{1}{2}} \\
&= \sum_{i=1}^{\kappa-2} \left[(\kappa-i)+i\right]^{-\frac{1}{2}} + \left[(\kappa-1)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}}.
\end{aligned}$$

If we implement an identical procedure as used in $[SCI]_\kappa$ fro the application to $[SCI]_{\kappa-1}$,...,$[SCI]_{\frac{\kappa+1}{2}+2}$, $[SCI]_{\frac{\kappa+1}{2}+1}$

$$[SCI]_{\kappa-1} = \sum_{i=2}^{\kappa-3} \left[(\kappa-i)+i\right]^{-\frac{1}{2}} + \left[(\kappa-2)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}},$$

$$\begin{aligned}
&[SCI]_{\frac{\kappa+1}{2}+2} \\
&\quad= \left[(\frac{\kappa}{2}+3)+(\frac{\kappa}{2}-3)\right]^{-\frac{1}{2}} + \left[(\frac{\kappa}{2}+3)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}} \\
&\quad+ \left[(\frac{\kappa}{2}+3)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}} + \left[(\frac{\kappa}{2}+3)+(\frac{\kappa}{2}+1)\right]^{-\frac{1}{2}},
\end{aligned}$$

So

$$[SCI]_{\frac{\kappa+1}{2}+1} = \left[(\frac{\kappa}{2}+1)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}} + \left[(\frac{\kappa}{2}+1)+(\frac{\kappa}{2}-1)\right]^{-\frac{1}{2}}.$$

Hence,

$$\begin{aligned}
&[SCI]_\kappa + [SCI]_{\kappa-1} + ... + [SCI]_{\frac{\kappa+1}{2}+2} + [SCI]_{\frac{\kappa+1}{2}+1} \\
&= \sum_{z=1}^{\frac{\kappa}{2}-1} \sum_{i=z}^{\kappa-z-1} \left[(\kappa-z)+i\right]^{-\frac{1}{2}} + \sum_{z=1}^{\frac{\kappa}{2}} \left[(\kappa-z)+(\frac{\kappa}{2})\right]^{-\frac{1}{2}}
\end{aligned}$$

Example 5.2 shows how to compute the SCI of $\Gamma(S_{M_4})$ and $\Gamma(S_{M_6})$ (see Figure 1 and 2) , illustrating the primary theorem. □

From the above two cases, equation shows the desired result. Hence, the proof is complete.

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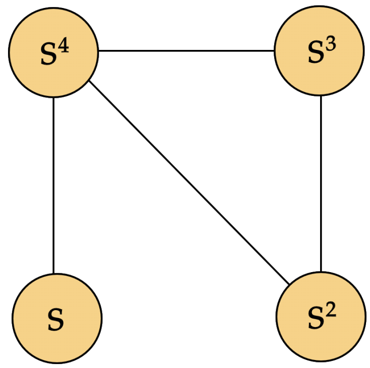

Figure 1: $S_{\mathcal{M}_4}$ Monogenic Semigroup Graph

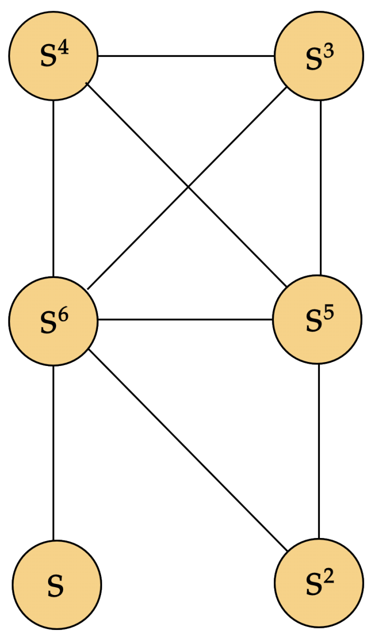

Figure 2: $S_{\mathcal{M}_6}$ Monogenic Semigroup Graph

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Example 5.2. *Let be a monogenic semigroup that is given below.*

$$S_{\mathcal{M}_6} = \{x, x^2, x^3, x^4, x^5, x^6\}$$

$SCI(\Gamma(S_M))$

$$= \sum_{z=1}^{2}\sum_{i=z}^{5-z}\big[(6-z)+i\big]^{-\frac{1}{2}} + \sum_{z=1}^{3}\big[(6-z)+3\big]^{-\frac{1}{2}}$$
$$= \big[1+5\big]^{-\frac{1}{2}} + \big[2+5\big]^{-\frac{1}{2}} + 2\big[3+5\big]^{-\frac{1}{2}} + \big[4+5\big]^{-\frac{1}{2}}$$
$$+ \big[2+4\big]^{-\frac{1}{2}} + 2\big[3+4\big]^{-\frac{1}{2}} + \big[3+3\big]^{-\frac{1}{2}}.$$

6 Conclusions

This study closely examines the properties of a particular class of monogenic semigroup graphs' sum connectivity index (SCI). These characteristics are crucial to graph theory and abstract algebra. The main goal is to define and clarify the SCI metrics, focusing on how this specific class of graphs can use them. This paper thoroughly analyzes the relationship between the structure of MSG and the fundamental properties of SCI. Furthermore, the study explores the ramifications and importance of SCI in modeling and assessing connectivity in this class of graphs. The study concludes with a part that computes the specified issue using a topological index technique, with specific examples for clarity and presentation.

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