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BINARY $g^{\#}$-CLOSED SETS IN BINARY TOPOLOGICAL SPACES

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Abstract.

The new class of sets we present in this study is called $bg^{\#}$-closed sets in binary topological spaces. The class of bg-closed sets and the class of b-closed sets are in between each other. Their importance in the study of binary topology will be demonstrated by our examination of their characteristics and examples. We will also highlight the distinctive features of $bg^{\#}$-closed sets by analyzing their links with other well-known set classes.

INTRODUCTION AND PRELIMINARIES

In [6], Nithyanantha Jothi et al. introduced topology between two sets and specified some of their features. Binary continuous maps and b-open and b-closed forms of sets have been presented and investigated by a number of researchers in their generalizations, such as (See [1-5] and [7-12]). This study presents a novel type of sets called $bg^{\#}$-closed sets in binary topological spaces. This kind is situated between the b-closed set class and the bg-closed set class. We will explore the properties of these $bg^{\#}$-closed sets and establish their significance in the broader context of topology. By doing so, we hope to provide insights into how these sets interact with existing topological concepts and contribute to our understanding of binary topological structures.

A binary topological space $(X, Y, \mathcal{M})$ will henceforth be termed $\mathcal{B}^{\mathcal{M}}$.

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Key words and phrases. $g^{\#}$-closed set, $bg^{\#}$-closed set and $bg^{\#}_{\alpha}$-closed set.

2 Binary $g^{\#}$-closed sets

Definition 2.1 *A subset of $\mathcal{B}^{\mathcal{M}}$ is denoted by (S_1, S_2). Then, a binary $g^{\#}$-closed set (in a word $bg^{\#}$-closed) is defined as (S_1, S_2) if $cl_b(S_1, S_2) \subseteq (H_1, H_2)$ in (X, Y) whenever $(S_1, S_2) \subseteq (H_1, H_2)$ and (H_1, H_2) is $b\alpha g$-open.*

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Definition 2.2 *A subset of $\mathcal{B}^{\mathcal{M}}$ is denoted by (S_1, S_2). Then, a binary $g_\alpha^\#$-closed set (in a word $bg_\alpha^\#$-closed) is defined as (S_1, S_2) if $\alpha cl_b(S_1, S_2) \subseteq (H_1, H_2)$ in (X, Y) whenever $(S_1, S_2) \subseteq (H_1, H_2)$ and (H_1, H_2) is $b\alpha g$-open.*

$bg^\#$-open (resp. $bg_\alpha^\#$-open) is the complement of a $bg^\#$-closed (resp. a $bg_\alpha^\#$-closed).

Proposition 2.3 *In $\mathcal{B}^{\mathcal{M}}$ has $bg^\#$-closed sets for b-closed set.*

Proof. (I_1, I_2) is b-closed set in (X, Y) and (H_1, H_2) is $b\alpha g$-open set $\subseteq (I_1, I_2)$, then $(H_1, H_2) \supseteq (I_1, I_2) = b\text{-}cl(I_1, I_2)$. Hence (I_1, I_2) is $bg^\#$-closed.

Remark 2.4 *As indicated by the next example, contrary statements of Proposition 2.3 need not always be true.*

Example 2.5 $X = \{h_1, h_2\}$, $Y = \{a_1, a_2\}$ *and* $\mathcal{M} = \{(\phi, \phi), (\phi, \{a_2\}), (\{h_1\}, \{a_1\}), (\{h_1\}, Y), (X, Y)\}$. *Then* $(\phi, \phi), (\{h_2\}, \phi), (\{h_2\}, \{a_2\}), (X, \{a_1\}), (X, Y)$ *are called b-closed. Then $bg^\#$-closed sets are* $(\phi, \phi), (\{h_2\}, \phi), (\{h_2\}, \{a_1\}), (\{h_2\}, \{a_2\}), (\{h_2\}, Y), (X, \phi), (X, \{a_1\}), (X, \{a_2\}), (X, Y)$. *Here, $(I_1, I_2) = \{\{h_2\}, \{a_1\}\}$ is $bg^\#$-closed set but not b-closed in $\mathcal{B}^{\mathcal{M}}$.*

Proposition 2.6 *Let $(I_1, I_2) \subseteq (X, Y)$ in $\mathcal{B}^{\mathcal{M}}$, any $bg^\#$-closed set is $bg_\alpha^\#$-closed(resp. bg-closed, $bg^\star$-closed, bsg-closed, $b\alpha g$-closed, bgs-closed, $bgsp$-closed).*

Proof. 1. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any $b\alpha g$-open set containing (I_1, I_2), then $(H_1, H_2) \supseteq cl_b(I_1, I_2) \supseteq \alpha\text{-}cl_b(I_1, I_2)$. (I_1, I_2) hence is $bg_\alpha^\#$-closed.

2. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any b-open set containing (I_1, I_2), since every b-open set is $b\alpha g$-open, we have $(H_1, H_2) \supseteq cl_b(I_1, I_2)$. (I_1, I_2) hence is bg-closed.

3. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any bg-open set containing (I_1, I_2), then $(H_1, H_2) \supseteq cl_b(I_1, I_2)$. We know that every bg-open set is $b\alpha g$-open. (I_1, I_2) hence is $bg^\star$-closed in (X, Y).

4. Suppose that $(I_1, I_2) \subseteq (H_1, H_2)$ and (H_1, H_2) is bs-open in (X, Y). Since every bs-open set is $b\alpha g$-open and (I_1, I_2) is $bg^\#$-closed, therefore $scl_b(I_1, I_2) \subseteq cl_b(I_1, I_2) \subseteq (H_1, H_2)$. (I_1, I_2) hence is bsg-closed in (X, Y).

5. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any b-open set containing (I_1, I_2), since every b-open set is $b\alpha g$-open, we have $(H_1, H_2) \supseteq cl_b(I_1, I_2) \supseteq \alpha cl_b(I_1, I_2)$. (I_1, I_2) hence is $b\alpha g$-closed in (X, Y).

6. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any b-open set containing (I_1, I_2), since every b-open set is $b\alpha g$-open, we have $(H_1, H_2) \supseteq cl_b(I_1, I_2) \supseteq scl_b(I_1, I_2)$. (I_1, I_2) hence is bgs-closed in (X, Y).

7. (I_1, I_2) is a $bg^\#$-closed subset of (X, Y) and (H_1, H_2) is any b-open set containing (I_1, I_2), every b-open set is $b\alpha g$-open, we have $(H_1, H_2) \supseteq cl_b(I_1, I_2) \supseteq spcl_b(I_1, I_2)$. (I_1, I_2)

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hence is $bgsp$-closed in (X, Y).

Remark 2.7 *The example that follows shows that the converse of Proposition 2.6 does not necessarily have to be true.*

Example 2.8 1. Let $X = \{a_1, a_2, a_3\}$, $Y = \{m, n\}$ and $\mathcal{M} = \{\phi, \phi), (\{a_1\}, \{m\}), (\{a_2\}, \phi), (\{a_2\}, \{n\}), (\{a_1, a_2\}, \{m\}), (\{a_1, a_2\}, Y), (X, Y)\}$. Then $(\phi, \phi), (\{a_3\}, \phi), (\{a_3\}, \{m\}), (\{a_3\}, \{n\}), (\{a_3\}, Y), (\{a_1, a_3\}, \phi), (\{a_1, a_3\}, \{m\}), (\{a_1, a_3\}, \{n\}),$ $(\{a_1, a_3\}, Y), (\{a_2, a_3\}, \phi), (\{a_2, a_3\}, \{m\}), (\{a_2, a_3\}, \{n\}), (\{a_2, a_3\}, Y), (X, \phi), (X, \{m\}), (X, \{n\}),$ (X, Y) are bg-closed sets and $(\phi, \phi), (\{a_3\}, \phi), (\{a_3\}, \{m\}), (\{a_3\}, \{n\}), (\{a_3\}, Y), (\{a_1, a_3\}, \phi), (\{a_1, a_3\}, \{m\}), (\{a_1, a_3\}, \{n\}),$ $(\{a_1, a_3\}, Y), (\{a_2, a_3\}, \phi), (\{a_2, a_3\}, \{m\}), (\{a_2, a_3\}, \{n\}), (\{a_2, a_3\}, Y), (X, \{n\}), (X, Y)$ are $bg^{\#}$-closed sets. Then the subset $(I_1, I_2) = (X, \{m\})$ is bg-closed but not $bg^{\#}$-closed set in $\mathcal{B}^{\mathcal{M}}$.

2. $X = \{a_1, a_2\}$, $Y = \{m, n\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{m\}), (\{a_1\}, \{m\}), (X, Y)\}$. Then $(\phi, \phi), (\{a_2\}, \{n\}), (X, \{n\}), (X, Y)$ are $bg^{\#}$-closed sets. Then $bg^{\#}_{\alpha}$-closed sets are $(\phi, \phi), (\phi, \{n\}), (\{a_1\}, \phi), (\{a_1\}, \{n\}), (\{a_2\}, \phi), (\{a_2\}, \{n\}), (X, \phi), (X, \{n\}), (X, Y)$. Then the subset $(I_1, I_2) = (\phi, \{n\})$ is $bg^{\#}_{\alpha}$-closed but not $bg^{\#}$-closed set in $\mathcal{B}^{\mathcal{M}}$.

3. $X = \{a_1, a_2\}$, $Y = \{m, n\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{m\}), (\{a_1\}, \{m\}), (\{a_2\}, \{m\}), (X, \{m\}), (X, Y)\}$. Then $(\phi, \phi), (\phi, \{n\}), (\phi, Y), (\{a\}, \{n\}), (\{a_1\}, Y), (\{a_2\}, \{n\}), (\{a_2\}, Y), (X, \{n\}), (X, Y)$ are $bg^{\star}$-closed sets. Then $bg^{\#}$-closed sets are $(\phi, \phi), (\phi, \{n\}), (\{a_1\}, \{n\}), (\{a_2\}, \{n\}), (X, \{n\}), (X, Y)$. Then the subset $(I_1, I_2) = (\{a_2\}, Y)$ is $bg^{\star}$-closed but not $bg^{\#}$-closed set in $\mathcal{B}^{\mathcal{M}}$.

4. Example 2.8(3) shows that the binary sg-closed sets are $(\phi, \phi), (\phi, \{n\}), (\{a_1\}, \phi), (\{a_1\}, \{n\}), (\{a_2\}, \phi), (\{a_2\}, \{n\}), (X, \phi), (X, \{n\}), (X, Y)$. Accordingly, the subset $(I_1, I_2) = (\{a_2\}, \phi)$ is bsg-closed, but not $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

5. $b\alpha g$-closed sets in Example 2.8(3) are $(\phi, \phi), (\phi, \{n\}), (\{a_3\}, \phi), (\{a_3\}, \{m\}), (\{a_3\}, \{n\}), (\{a_3\}, Y), (\{a_1, a_3\}, \phi), (\{a_1, a_3\}, \{m\}), (\{a_1, a_3\},$ $\{n\}), (\{a_1, a_3\}, Y), (\{a_2, a_3\}, \phi), (\{a_2, a_3\}, \{m\}), (\{a_2, a_3\}, \{n\}), (\{a_2, a_3\}, Y), (X, \phi), (X, \{m\}),$ $(X, \{n\}), (X, Y)$. The subset $(I_1, I_2) = (\phi, \{n\})$ is therefore $b\alpha g$-closed, but not $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

6. bgs-closed sets in Example 2.8(3) are $(\phi, \phi), (\phi, \{m\}), (\phi, \{n\}), (\phi, Y), (\{a_1\}, \phi), (\{a_1\}, \{n\}), (\{a_1\}, Y), (\{a_2\}, \{n\}), (\{a_3\}, \phi), (\{a_3\}, \{m\}),$ $(\{a_3\}, \{n\}), (\{a_3\}, Y), (\{a_1, a_3\}, \phi), (\{a_1, a_3\}, \{m\}), (\{a_1, a_3\}, \{n\}), (\{a_1\}, \{a_3\}, Y), (\{a_2, a_3\}, \phi),$ $(\{a_2, a_3\}, \{m\}), (\{a_2, a_3\}, \{n\}), (\{a_2, a_3\}, Y), (X, \phi), (X, \{m\}), (X, \{n\}), (X, Y)$. Consequently, the subset $(I_1, I_2) = (\{a_1\}, \{m\})$ has bgs-closed but not $bg^{\#}$-closed set in $\mathcal{B}^{\mathcal{M}}$.

7. Assume $X = \{a_1, a_2\}, Y = \{m, n\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{n\}), (\{a_1\}, \{m\}), (\{a_1\}, Y), (X, \{m\}), (X, Y)\}$. Next, the subsets

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$(\phi,\phi),(\phi,\{n\}),(\{a_2\},\phi),(\{a_2\},\{m\}),(\{a_2\},\{n\}),(\{a_2\},Y),(X,\phi),(X,\{m\}),(X,\{n\}),(X,Y)$. They are $bg^{\#}$-closed set and $(\phi,\phi),(\phi,\{m\}),(\phi,\{n\}),(\phi,Y),(\{a_1\},\phi),(\{a_1\},\{n\}),(\{a_2\},\phi),(\{a_2\},\{m\}),(\{a_2\},\{n\}),(\{a_2\},Y),$ $(X,\phi),(X,\{m\}),(X,\{n\}),(X,Y)$ are $bgsp$-closed set, as well. Accordingly, the subset $(I_1,I_2)=(\phi,\{m\})$ is $bgsp$-closed, but not $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

Remark 2.9 *When illustrated by the next examples, the $bg^{\#}$-closed set family is independent of the two bs-closed sets and $b\alpha$-closed sets.*

Example 2.10 *Using 2.8(3) as an example,* $(\phi,\phi),(\phi,\{n\}),(\{a_3\},\phi),(\{a_3\ \phi),(\{a_3\ \{n\}),(\{a_1,a_3\},\{m\}),(\{a_1,a_3\},Y),(\{a_2,a_3\},\{n\}),(X,Y)$ *are $b\alpha$-closed and* $(\phi,\phi),(\phi,\{n\}),(\{a_1\},\{m\}),(\{a_1\},Y),(\{a_2\},\{n\}),(\{a_3\},\phi),(\{a_3\},\{n\}),(\{a_1,a_3\},\{m\}),(\{a_1,a_3\},$ $\{m\}),(\{a_1,a_3\},Y),(X,Y)$ *are bs-closed. Subset $(I_1,I_2)=(\{a_1,a_3\},\{n\})$ is then $bg^{\#}$-closed, but it is neither $b\alpha$-closed nor bs-closed in $\mathcal{B}^{\mathcal{M}}$. Subset $(I_1,I_2)=(\phi,\{n\})$ is then $b\alpha$-closed and bs-closed, but not $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.*

Remark 2.11 *These results are shown in the diagrams.*

Diagram - I

$$\begin{array}{ccccccc}
b\text{-closed} & \longrightarrow & bg^{\#}\text{-closed} & \longrightarrow & bg^{\star}\text{-closed} & \longrightarrow & bg\text{-closed} \\
 & \searrow & \not\updownarrow & \searrow & \not\updownarrow & & \downarrow \\
 & & b\alpha\text{-closed} & & bg\alpha\text{-closed} & & b\alpha g\text{-closed} \\
 & & \downarrow & & & \searrow & \downarrow \\
 & & bs\text{-closed} & \longrightarrow & bsg\text{-closed} & \longrightarrow & bgs\text{-closed} \\
 & & \downarrow & & \downarrow & & \\
 & & b\beta\text{-closed} & \longrightarrow & bgsp\text{-closed} & & \\
 & & & \nwarrow \; bp\text{-closed} \; \nearrow & & &
\end{array}$$

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Diagram - II

$$\begin{tikzcd}
 & b\beta\text{-closed} & \\
bs\text{-closed} \arrow[r, phantom, "\nleftrightarrow"] & bg^{\#}\text{-closed} \arrow[u] \arrow[d] \arrow[r] & b\alpha g\text{-closed} \\
 & bsg\text{-closed} &
\end{tikzcd}$$

3 More properties of binary $g^{\#}$-closed sets

Definition 3.1 *The $b\alpha g$-kernel of (I_1, I_2) is the intersection of all $b\alpha g$-open subsets of $\mathcal{B}^{\mathcal{M}}$ that contain (I_1, I_2). It is represented by $b\alpha g$-$ker(I_1, I_2)$.*

Lemma 3.2 *$bg^{\#}$-closed is a binary subset (I_1, I_2) of a $\mathcal{B}^{\mathcal{M}}$. If and only if $cl_b(I_1, I_2) \subseteq b\alpha g$-$ker(I_1, I_2)$.*

Proof. Let (I_1, I_2) be $bg^{\#}$-closed. If $(I_1, I_2) \subseteq (E_1, E_2)$ and (E_1, E_2) are $b\alpha g$-open, then $cl_b(I_1, I_2) \subseteq (E_1, E_2)$. let $(i_1, i_2) \in cl_b(I_1, I_2)$. There is a $b\alpha g$-open set (E_1, E_2) that contains (I_1, I_2) such that $(i_1, i_2) \not\in (E_1, E_2)$ if $(i_1, i_2) \not\in b\alpha g$-$ker(I_1, I_2)$. We have $(i_1, i_2) \not\in cl_b(I_1, I_2)$, which is a contradiction since (E_1, E_2) is a $b\alpha g$-open set including (I_1, I_2).

Conversely, let $cl_b(I_1, I_2) \subseteq b\alpha g$-$ker(I_1, I_2)$. If (E_1, E_2) is any $b\alpha g$-open set containing (I_1, I_2), then $cl_b(I_1, I_2) \subseteq b\alpha g$-$ker(I_1, I_2) \subseteq (E_1, E_2)$. Therefore, (I_1, I_2) is $bg^{\#}$-closed.

Theorem 3.3 *Assuming that (I_1, I_2) is $bg^{\#}$-closed. There isn't a nonempty $b\alpha g$-closed set in (X, Y) if $cl_b(I_1, I_2) - (I_1, I_2)$.*

Proof. Suppose that (I_1, I_2) is $bg^{\#}$-closed. Let (H_1, H_2) be a $b\alpha g$-closed subset of $cl_b(I_1, I_2) - (I_1, I_2)$. Then $(I_1, I_2) \subseteq (H_1, H_2)^c$. But (I_1, I_2) is $bg^{\#}$-closed, therefore $cl_b(I_1, I_2) \subseteq (H_1, H_2)^c$. Consequently, $(H_1, H_2) \subseteq (cl_b(I_1, I_2))^c$. We already have $(H_1, H_2) \subseteq cl_b(I_1, I_2)$. Thus $(H_1, H_2) \subseteq cl_b(I_1, I_2) \cap (cl_b(I_1, I_2))^c$ and (H_1, H_2) is empty.

Theorem 3.4 *If (I_1, I_2) is $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$ and $(I_1, I_2) \subseteq (E_1, E_2) \subseteq cl_b(I_1, I_2)$, then (E_1, E_2) is $bg^{\#}$-closed.*

Proof. In $\mathcal{B}^{\mathcal{M}}$, let (H_1, H_2) be a $b\alpha g$-open set such that $(I_1, I_2) \subseteq (H_1, H_2)$. $cl_b(I_1, I_2) \subseteq (H_1, H_2)$ since (I_1, I_2) is $bg^{\#}$-closed. We have $cl_b(E_1, E_2) \subseteq (H_1, H_2)$ since $cl_b(E_1, E_2) \subseteq cl_b(I_1, I_2)$. (E_1, E_2) is therefore a $bg^{\#}$-closed set.

Theorem 3.5 *(I_1, I_2) is b-closed in $\mathcal{B}^{\mathcal{M}}$ if it is a $b\alpha g$-open and $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.*

Proof. Considering that (I_1, I_2) is $b\alpha g$-open and $bg^{\#}$-closed, $cl_b(I_1, I_2) \subseteq (I_1, I_2)$, so (I_1, I_2)

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is b-closed in $\mathcal{B}^{\mathcal{M}}$.

Theorem 3.6 *For every $(a_1, a_2) \in \mathcal{B}^{\mathcal{M}}$, either $\{a_1, a_2\}$ is $b\alpha g$-closed or $\{a_1, a_2\}^c$ is $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.*

Proof. Let $\{a_1, a_2\}$ not be $b\alpha g$-closed in $\mathcal{B}^{\mathcal{M}}$. The space $\mathcal{B}^{\mathcal{M}}$ itself is the only $b\alpha g$-open set that contains $\{a_1, a_2\}^c$, and $\{a_1, a_2\}^c$ is not $b\alpha g$-open. Hence, $cl_b(\{a_1, a_2\}^c) \subseteq \mathcal{B}^{\mathcal{M}}$, and $\{a_1, a_2\}^c$ is $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$ accordingly.

Proposition 3.7 *For a $\mathcal{B}^{\mathcal{M}}$, let (I_1, I_2) be a $bg^{\#}$-closed set. $pint_b(I_1, I_2)$ and $scl_b(I_1, I_2)$ are likewise $bg^{\#}$-closed sets if (I_1, I_2) is br-open.*

Proof. The fact that (I_1, I_2) is br-open in $\mathcal{B}^{\mathcal{M}}$. $Int_b(cl_b(I_1, I_2)) = (I_1, I_2)$. Consequently, $scl_b(I_1, I_2) = (I_1, I_2) \cup int_b(cl_b(I_1, I_2)) = (I_1, I_2)$ results. $scl_b(I_1, I_2)$ is hence $bg^{\#}$-closed within $\mathcal{B}^{\mathcal{M}}$. Since $pint_b(I_1, I_2) = (I_1, I_2) \cap int_b(cl_b(I_1, I_2)) = (I_1, I_2)$, There is a $bg^{\#}$-closed $pint_b(I_1, I_2)$.

Proposition 3.8 *A $bg^{\#}$-closed set of a $\mathcal{B}^{\mathcal{M}}$ is (I_1, I_2). It follows that $pcl_b(I_1, I_2)$ is $bg^{\#}$-closed if (I_1, I_2) is br-closed.*

Proof. Because (I_1, I_2) is br-closed in $\mathcal{B}^{\mathcal{M}}$, $(I_1, I_2) = cl_b(int_b(I_1, I_2))$. Then $pcl_b(I_1, I_2) = (I_1, I_2) \cup cl_b(int_b(I_1, I_2)) = (I_1, I_2)$. Consequently, $pcl_b(I_1, I_2)$ is $bg^{\#}$-closed.

Remark 3.9 *We may conclude given the subsequent example that both the converses of Propositions 3.7 and 3.8 are not true.*

Example 3.10 *The subsets $(\phi, \phi), (\phi, \{n\}), (X, \{m\})$, and $\mathcal{B}^{\mathcal{M}}$ are br-open in Example 2.8(7). Subset $(I_1, I_2) = (\{b\}, \{m\})$ is not br-open in that case. (I_1, I_2), however, is $bg^{\#}$-closed, and $scl_b(I_1, I_2) = (X, \{m\})$. $pint_b(I_1, I_2) = (\{b\}, \{m\})$ is likewise $bg^{\#}$-closed, and is $bg^{\#}$-closed. The subset $(E_1, E_2) = (\{b\}, \{n\})$ is therefore not br-closed. But in $\mathcal{B}^{\mathcal{M}}$, (E_1, E_2) is a $bg^{\#}$-closed, and $pcl_b(E_1, E_2) = (\{b\}, \{n\})$ is a $bg^{\#}$-closed.*

4 Conclusion

The main point of this paper is that we discussed the theory of $bg^{\#}$-closed and their related possession in $\mathcal{B}^{\mathcal{M}}$. Future research can focus on $bg^{\#}$-open maps and $bg^{\#}$-closed maps, as well as continuous functions (i.e., The binary topological space consists of the domain and the codomain.). Finding an inverted maps or function for $bg^{\#}$-closed to $bg^{\#}$-closed set is another way to expand. We may also discover a number of topological space regions with related characteristics. This exploration could lead to a deeper understanding of how these concepts interact with various types of continuity and compactness in binary topological spaces. Additionally, investigating the implications of $bg^{\#}$-closed sets on other fundamental topological constructs may reveal new connections and enhance our overall comprehension of the subject.

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