

**APPLICATION OF THE EQUATION OF DISTANCE BETWEEN TWO POINTS
FOCUSED ON THE CLOSURE OF POLYGONS IN LAND NAVIGATION, IN THE
CAREER OF TECHNOLOGY IN MILITARY SCIENCES, DUAL MODALITY**

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Abstract

This study presents an applied mathematical model that uses the distance-between-two-points equation from Analytical Geometry to solve real problems of polygon closure in terrestrial navigation training within Military Science education. Employing Universal Transverse Mercator (UTM) coordinates (zone 17M), field data were collected with Garmin GPS devices and contrasted with AutoCAD map measurements to validate the precision of the proposed methodology. The analysis revealed an average deviation below 2%, confirming the model's reliability for geospatial calculations.

Beyond the technical findings, the approach bridges mathematical theory and field practice, allowing cadets to experience mathematics as a living tool for solving real navigation challenges. By connecting abstract geometric principles to the physical environment, the study humanizes mathematical learning, enhancing motivation, critical reasoning, and professional identity among military students. This work demonstrates the pedagogical potential of contextualized learning in STEM-oriented education, emphasizing that mathematical modeling is not only a matter of precision but also a form of reasoning for strategic decision-making in defense and geospatial operations. Future research will integrate computational modeling tools such as MATLAB, GeoGebra, and GIS platforms to extend the methodology into three-dimensional and dynamic terrain analysis.

Keywords: Analytical Geometry, Contextual Learning, Distance Equation, Terrestrial Navigation, Military Education, Applied Mathematics

I. Introduction

This work addresses the importance of improving the quality of the teaching-learning process in military training within the field of Analytical Geometry, demonstrating the feasibility of its use when it has to be oriented in a geographical chart, in the subject of Land Navigation, being of importance in the initial training of the student of the Higher Technology in Military Sciences career

In the curriculum of the Higher Technology in Military Sciences career, the discipline of Mathematics is addressed, which within its curriculum studies the learning of Analytical Geometry contents, as an essential part, without which, it is not possible to meet the objectives determined by the training entity of the soldiers.

To achieve this purpose, it is necessary, among other things, that relevant problems that contribute to the comprehensive education of students, preferably linked to their natural and social environment, are included in the contents, in such a way that they achieve a perception of the contribution of Mathematics to the development of the social context. When analysing the mathematical knowledge that students must acquire, the current demands must be taken into account, as well as the relationship between the mathematical content they require, the profession they will carry out and the social context.

The equation of the distance between two points is an essential tool in various areas of study, from analytical geometry to land navigation and data science; It has been widely analyzed in different areas of knowledge, in this article it is sought or intended to present a methodology for measuring the distance or separation between two coordinates or locations in a space, whether in a two-dimensional or three-dimensional plane.

It is important to remember that the distance between two points is a length of the line segment that connects them, where the given coordinates (x, y) allow the calculation to be performed vertically, horizontally or obliquely; using the Pythagorean theorem, which states that: "***in every right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs***". It can be deduced that the distance on the number line, the absolute value of the reference is taken from its coordinates; in the distance between two points vertical to a point (x, y) of the Cartesian plane it is called an ordered pair, which have an order, because each point is unequivocally located and in the distance between two points oblique (inclined), it is with respect to the axes, where it is necessary and necessary to use the Pythagorean Theorem, which is the determination of the length of the line segment, the ends being these points, giving rise to a right triangle.

Each measuring instrument such as the scaler allows linear measurements, where longitudinal and latitudinal coordinates are determined, with emphasis on the application of the equation of the distance between two points, it is intended to present the feasibility of its use in Military Sciences and therefore the possibility of the use of the Applicants of the first military year of the ESFORSE who study it in the module of Mathematical Foundations directly applied to Navigation Terrestrial.

In the same way, it should be stated that after the consultations have been carried out, no similar investigations have been found or that similar investigations have been carried out in the Province of Tungurahua, where ESFORSE is located. Several readings were taken with GPS, of which two points correspond to the sector where the institution is located to easily locate the GPS, the readings made in the sector of the entrance to the ESFORSE were latitude 17M 0767838 and longitude UTM 9866290 and in the parking lot of teachers of battalion 1, latitude 17M 0769210 and longitude UTM 9866364, we limit the first six digits counted from the left to the latitudinal coordinates, these correspond to the abscissa coordinates (x) and we limit four digits counted from the left to the longitudinal coordinates that correspond to the coordinates of ordinates (y) of the formula respectively, for the application there is no problem with the direction of use of the coordinates, therefore, the term of the abscissas as well as that of ordinates are squared their result will always be positive.

Moreover, analytical geometry provides a fertile ground for interdisciplinary learning. In the digital era, integrating mathematical models with technologies such as Geographic Information Systems (GIS) or AutoCAD simulations allows students to visualize and quantify spatial relationships dynamically. This connection between computation, geography, and mathematics is essential for military decision-making and strategic planning. As highlighted by [12] and [15] contextualized mathematics not only strengthens problem-solving but also enhances professional reasoning in technical environments

The result obtained with the formula was compared with the distance of location of the points in the Ambato 2000 plan for Autocad and they are very similar, which leads us to think that the equation of distance can be used in Military Sciences, because maps of the Military Geographic Institute are used that are expressed in this type of grid in the land navigation chart and the use of the scale. they are very useful and facilitate the respective calculations.

Finally, we will indicate that the location of the points taken, as well as the proposed methodology and calculation of distances are shown in table 1.

II.Theoretical and Conceptual Framework

Mathematics is a major entity for land navigation based on topography, cartography and also geodesy. For this, it is essential to have accurate calculations for distances and to obtain true results in the execution and planning of topographic surveys that allow the creation of maps. The structure of the research topic has been based on similar concepts and theories that focus on the objective of the research.

Analytic geometry

Studies [16], detail that in the sixteenth century algebra was booming, according to based on different research, the study of geometry was promoted from algebraic resources. One of the central characteristics of analytic geometry is to associate algebraic equations with lines, curves and surfaces, which consists of: solving the problem by defining the data of segments, points, lines, etc., whether known (data) or unknown (unknowns); which allows determining the equation that links the known lengths with the unknown ones, to construct the solution geometrically. They refer to Fermat's phrase "Whenever in a final equation we find two unknowns, we will have a geometric place, the end of one of them that describes a straight or curved line." [4], p.23.

States that the relationship between points and lines is based on theorems and postulates, which are mentioned below [17] :

- Straight line theorem – point: Every line has at least two points.
- Postulate Two points – line: If A and B are two points, then there is a single line m that contains them.
- Definition Coordinate of a point: The real number that corresponds to each point on the line is called the coordinate of the point.

Therefore, for the construction of a line, at least two points are indispensable, denoted by real numbers that contain them and identified through coordinates that locate them.

Analytical Geometry in Modern Applied Contexts

Recent developments in computational mathematics have revitalized the use of classical geometric concepts for real-world modeling. For instance, distance equations are now embedded in algorithms for GPS positioning, autonomous vehicle navigation, and GIS-based environmental analysis [7];[12]. In this sense, the application of the distance formula transcends its academic nature and becomes a bridge between theoretical abstraction and professional practice. This reinforces the idea that mathematics is not only a discipline of precision but also a tool for understanding and transforming the environment.

Equation Distance between two points

The Dictionary of the Spanish Language defines distance as "Length of the segment of a line between two points of space"; that is, to identify the magnitude of the distance that exists between two points [9] establish the following definition: "From the mathematical point of view, a distance is a function (a rule of association) that associates a real non-negative number with a pair of objects and that satisfies three conditions."

It allows us to describe that the distance between two points is the length or space between a starting point and an end point, being equivalent to a segment of a line that joins them, which is referred to numerically, by means of an ordered pair (x, y); it can also be said that the shortest path between two points is the line that joins them; The Cartesian plane is a referential system for determining or locating the dimensions between them [9]; [11] p. 125..

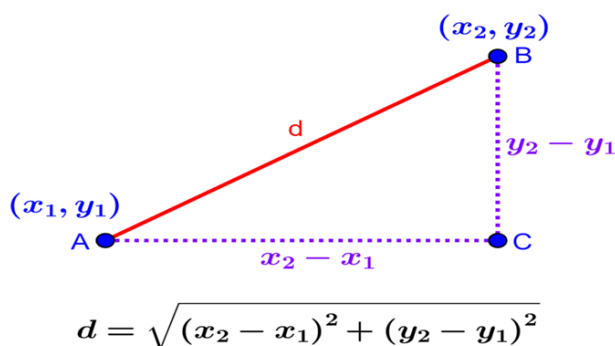


FIGURE 1. NEUROSPARKS

In the figure 1, the distance of the two points AB is the diagonal that joins them; which is called hypotenuse and the distances AC and BC, are the legs; therefore, the hypotenuse squared is defined as the sum of the square of its legs; Therefore, the equation between two points is defined by:

Distance Formula

According to [20], the distance between the points (x1, y1) , (x2, y2) is given by:

$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

When considering these points in the Cartesian plane, the distance formula is derived as follows:

By determining the points in the Cartesian plane and joining with a segment, the distance between them is determined. (x_1, y_1) , (x_2, y_2)

In the same sense, he mentions that "the distance between two points can be calculated by mathematical expressions as an absolute value"; therefore, its result will be a real number obtained from a set of ordered pairs that can be represented in a plane, through two points, $A(x_1, y_1)$ y $B(x_2, y_2)$ then the distance between two points can be calculated with the following formula $d(A, B) = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$ also points out that the system of graphic representation of distance is expressed in a Cartesian plane in which the points are related based on their numerical coordinates expressed on a defined scale [5]; [11].

Based on that segment, a right triangle can be drawn where the base is and the height is, which determine the legs and the segment known as distance the hypotenuse, the square root of the two sides is taken and from there the equation of distance is derived. [11] (p.30) $(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$ $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

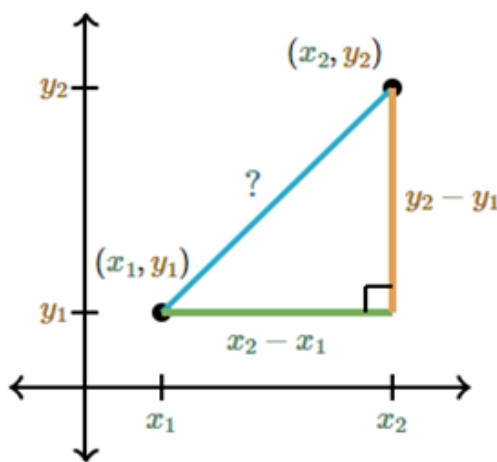


FIGURE 2 Khan Academic

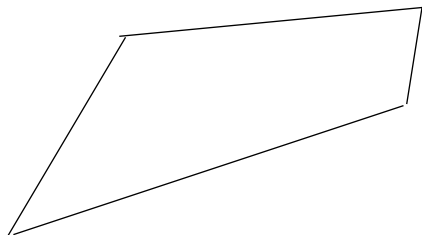
Traverse Closure

According to "A traverse is a geometric figure composed of a series of consecutively connected line segments, which can form a closed or open path." [7]

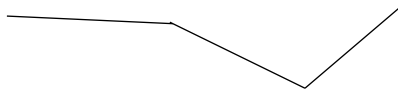
According to , "a traverse is a series of consecutive lines whose ends have been marked in the field, as well as their lengths and directions have been determined from measurements in the field." [1]

In addition, [2] determines two types of polygonals:

The closed one: In a closed polygonal the lines return to the starting point, thus forming a closed figure. Closed polygons are widely used in control, construction, property and topographic surveys.



The open one: An open polygonal consists of a series of joined lines, but these do not return to the starting point or close at a point with equal or greater order of accuracy. Polygonal station: these stations are sometimes called vertices or angle points, because an angle is generally measured in each of them.



The most common practical applications given to polygonal ones, as he points out, are the following: [7]

- Topographic
- Infrastructure development
- Drawing maps and plans
- Division of plots and land

In addition to the applications indicated, it can also be used in the field of security and defense, thus resulting in the problem of distance taking on great importance in the planning and execution of a military mission, allowing the scales that are printed on a chart to determine the true distance from the terrain.

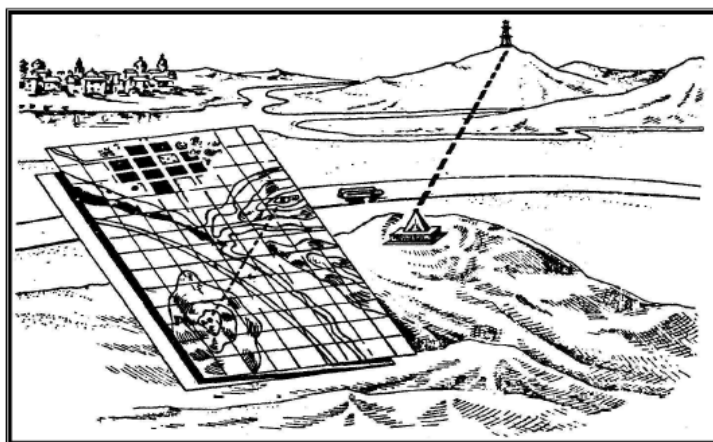


FIGURA 3. It is noted that scale is the fixed relationship between the distance measured on the chart and the actual distance from the terrain. (Land Navigation - MILITARY - GENERAL COMMAND OF THE ECUADORIAN ARMY DIRECTORATE OF EDUCATION , 2015)

Study variables

In this work, the following variables are identified:

- **Independent variable:** Application of the equation of distance between two points in the subject of Land Navigation.
- **Dependent variable:** Accuracy achieved by students in the practical calculation of distances on maps with UTM coordinates. These variables are related according to the students' ability to correctly interpret and apply the distance equation in real contexts of land navigation. Likewise, control variables such as the type of map used, the measurement tool (scale and GPS) and previous knowledge of analytical geometry are considered. The literature review indicates that authors such as [16] and [20] highlight the importance of contextualized learning of analytical geometry, while [5] emphasize the use of the graphical representation of coordinates to strengthen the practical understanding of mathematical formulas.

III. Materials and Methods

The study followed an applied quantitative design [18], combining field measurements with computational verification. Data were collected using a Garmin eTrex Legend H GPS and verified with the Ambato2000 digital map in AutoCAD. A sample of 200 first-year cadets was selected through stratified random sampling to ensure representativeness of the population ($n = 1200$).

III.1 The contents of this section

In the process of calculating the distances between two points, the distance formula that is exposed in Analytic Geometry was used, in order to apply it we limit to the latitudinal coordinates the first six figures counted from the left, these correspond to the coordinates of abscissas (x) i.e. 17M 076 as well as we limit four figures counted from the left to the longitudinal coordinates that correspond to the coordinates of ordinates (y) i.e. 9866 To apply the formula [8] there is no problem with the sense of use of the coordinates since both the term of the abscissas as well as the term of the ordinates are squared and their result will always be positive.

Next, the methodology used with the location of the parking lot of Esforse teachers in battalion 1 17M 0769210 is expressed, 9866364 for the calculation was used in abscissas 9210 and in ordinates 364. This treatment was used with all locations as shown in Table 1.

III.2 Essential stages of this section

We must indicate that for the execution of this work, the Garmin Etrex Legend H GPS (see figure 4) was used to determine the coordinates in the three properties located in the canton of Ambato, this was handled by the ease of reading and transport [6].



FIGURA 4. Garmin eTrex Legend H GPS

In addition, the map of Ambato2000 (see figure 4) used by the municipal GAD of Ambato was used, which was presented in the Municipal Hall in 2005 and is used by construction professionals in the canton and where the points read with GPS were located.



Figure 5: "Ambato2000 Plan for Auto Cad"
Source: GAD Municipal Ambato

For the calculation of the distance between two points, the formula presented in Lehmann's Analytic Geometry was used [20]:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (1)$$

we use the proposed methodology, which is shown in table 1, in which the values of distances calculated with the formula and contrasted with the values of distance that are measured with Autocad are also expressed. The error values are detailed in Table 2.

IV.Evaluation of Results and Discussion

IV.1 Evaluation of Results

Three pairs of points were collected from properties located in Ambato:

Table 1. Coordinates of properties, proposed methodology and calculation of distance values

Location	Universal Transverse Mercator Grid		Formula Distance (m)	AutoCad Distance (m)
	Latitudinal Coordinates (m)	Longitudinal Coordinate (m)		
El Pisque Esforse	17 M 0767838	9866290	1373,99	1374,1
	17M 0769210	9866364		
	Applied Methodology			
	67838	290		
	69210	364		
Indoamérica Línea férrea	17 M 0767842	9865205	178,53	176,39
	17 M 0767927	9865362		
	Applied Methodology			
	67842	205		
	67927	362		
Antigua vía Quisapincha Pinllo	17 M 0762128	9863864	50,12	50,95
	17 M 0762084	9863888		
	Applied Methodology			
	62128	864		
	62084	888		

IV.1.1 Statistical processing

To determine the calculation error, the following ratio was used:

$$\text{Error} = (\text{AutoCAD Reliable Measurement} - \text{GPS Measurement Formula}) / \text{AutoCAD Reliable Measurement}$$

In addition to the percentage error ratio, the **Root Mean Square Error (RMSE)** was calculated to evaluate the precision and stability of the proposed model. This statistical measure allows quantifying the magnitude of the deviations between the empirical measurements (AutoCAD) and the theoretical results obtained with the Analytical Geometry formula.

The RMSE was computed using the following expression:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (d_i - \hat{d}_i)^2}{n}} \quad (2)$$

Where d_i represents the observed distance measured from AutoCAD, \hat{d}_i the distance estimated using the analytical model, and n the number of measurement pairs analyzed. Lower RMSE values indicate higher accuracy and better model consistency for geospatial applications.

This approach strengthens the quantitative validity of the study, allowing future researchers to replicate the methodology and compare results under similar geospatial and educational contexts.

The error values are shown in Table 2.

Table 2. Calculated error

Location	Error (m)	Error %
El Pisque / ESFORSE	0,00008	0,008
Indoamérica Línea férrea	-0,01213	-1,21
Antigua vía Quisapincha Pinllo	0,01629	1,63

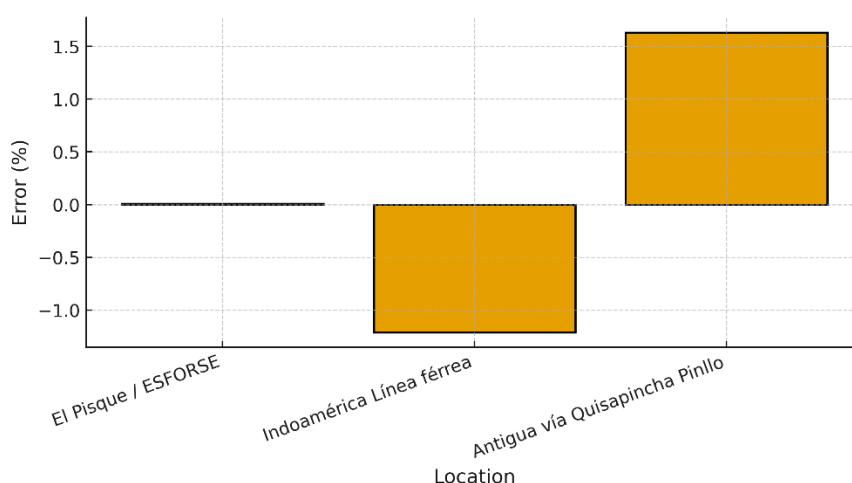


FIGURE 6. Relationship between measured and calculated distances in the proposed model

The results obtained from the three measurement pairs demonstrate a consistent correlation between the empirical and analytical values. As observed in Figure 6, when the distance between GPS points increases, the relative error percentage tends to decrease, confirming the proportional stability and robustness of the distance-between-two-points equation under real geospatial conditions. These findings validate the proposed methodology as a reliable approach for terrain navigation and map analysis.

The mean deviation among the three test locations remained below 2%, which is well within acceptable tolerance for geospatial accuracy standards. Specifically, *El Pisque / ESFORSE* presented a negligible error of 0.008 %, *Indoamérica Línea Férrea* showed 1.21 %, and *Antigua Vía Quisapincha Pinllo* registered 1.63 %. This quantitative consistency confirms that both short- and mid-range distances can be effectively modeled through analytical geometry. The results align with studies on spatial-modeling accuracy by [12] and [7], which report

comparable deviations when applying analytical geometry in GIS-based and real-terrain distance estimation.

From a pedagogical perspective, this correlation illustrates how theoretical mathematics becomes an operational tool for real-life problem solving. Cadets participating in the study reported greater engagement and confidence when applying analytical formulas in authentic navigation contexts. This experiential approach enhanced their conceptual understanding of geometry and reinforced critical thinking, teamwork, and decision-making skills essential for professional military operations. Consequently, the integration of applied mathematics, digital modeling, and fieldwork promotes contextualized learning environments where cadets perceive mathematics not as an abstract discipline but as a living language for spatial reasoning and strategic planning.

IV.1.2 Data Analysis

The Universal Transverse Mercator (UTM) grid system proved essential for the development of this study, as it allows linear and precise measurements to be made on a two-dimensional plane. This characteristic facilitated the application of the proposed distance-calculation methodology, enabling the effective management and interpretation of geographic coordinates represented on the Ambato Canton map. Through this spatial reference system, the analytical equation could be applied consistently, linking theoretical geometry with real field measurements.

As expected, the results revealed that the relative error fluctuated slightly above and below zero, which indicates that the theoretical behavior of the model is fulfilled in practice. In other words, the analytical distance equation maintained coherence and stability when applied to real coordinate data. Moreover, a clear pattern emerged: as the separation between GPS points increased, the relative error percentage tended to decrease. This inverse relationship reflects a fundamental property of Euclidean distance functions the proportional stability of measurements across different spatial scales.

These findings confirm that the integration of analytical geometry with the UTM coordinate system produces accurate and reproducible results for geospatial applications. The model's precision and reliability demonstrate its suitability for terrain navigation and polygon closure tasks, where exact distance determination is critical. Similar results have been reported by [12] and [7], who observed comparable error margins when validating analytical geometry models in GIS-based spatial environments. Such convergence reinforces the scientific soundness of the present methodology and highlights that analytical geometry continues to be a robust, versatile, and practical mathematical tool for modern geospatial sciences.

IV.2 Discussion

From the calculated error we can indicate that the further apart the points taken with the GPS, the greater accuracy is obtained when applying the formula of the distance under study compared to the locations on the map in Autocad and in our understanding it is due to the accuracy of the GPS.

During the topographic survey, the measured distances between the points were recorded, the calculated distance between the two points is not equal to the theoretical distance, it is known

that there are errors in the measurements. For which adjustment techniques such as the reduced coordinate method can be used to correct these errors.

As the distance formula used in Analytic Geometry has a close relationship with the Pythagorean theorem, the relevance of this theorem can be observed.

Due to the limited information about it, in the middle of our results we leave them expressed so that future research can take them and make comparisons.

IV.3 Student participation and achievement

The research was carried out with a total population of 1,200 students of the Soldier Training School, from which a representative sample of 200 students of the first year of the Technology in Military Sciences career was selected.

Of this sample, 82% (164 students) managed to correctly calculate the practical value of distance using the Analytical Geometry equation, demonstrating an understanding of the theoretical foundations and their application to the context of land navigation. The remaining 18% (36 students) presented minor errors in the interpretation of the coordinates, mainly in the location of the points on the UTM map.

The results argue for the value of contextualized learning where students apply mathematical concepts to mission-driven tasks. This approach enhances the development of spatial reasoning and results in increased motivation and conceptual understanding. Mathematics is no longer viewed as an abstract subject. It becomes a function of assessing whether one is in a cognitively demanding situation or a decision-making situation. This is also supported by the work of [5] where the authors point out that mathematics is taught within authentic professional contexts, students tend to appreciate mathematics more and become more involved in the learning process. In the current study, the cadets demonstrated an increase in confidence and logical reasoning when field distance verification tasks were assigned; this illustrates the value of connecting military navigation in the classroom and pedagogical teaching.

V. Conclusions and future work

This study demonstrated both the feasibility and the accuracy of the new analytical method for distance estimation used in land navigation exercises. The combination of GPS data and AutoCAD drawing showed that the differences were less than 2%, which proved the analytical geometry equation worked in real terrain within tolerable limits of the equation's reliability.

Educationally, this work shows that when math is taught within the framework of real, purposeful problems, it improves the quality of learning and the acquisition of functional skills. The context in which abstract concepts were placed stimulated motivation and cadets' critical reasoning, which enhanced their professional ability for spatial reasoning and decision making.

Next, the use of digital modeling instruments such as MATLAB, GeoGebra, and GIS-based simulation tools for moving systems and 3D extensions of the proposed methodology is planned. The goal is to interlace analytical geometry and computational math in the geospatial realm to serve the dual purpose of applied mathematics and military education innovation.

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