

**ON THE FORCING MONOPHONIC GD, GEODETIC GD AND STEINER GD
NUMBER OF A GRAPH**

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Abstract

The forcing geodetic, Steiner and monophonic global domination (GD) numbers of a graph are examined in this research. It is established that there is a connected graph G such that $f_{\bar{V}_m}(G) = f_{\bar{V}_g}(G) = f_{\bar{V}_s}(G) = a$, for every integer $a \geq 0$. Moreover, it is established that for every integer $a \geq 0$ there exists a connected graph G such that $f_{\bar{V}_m}(G) = a, f_{\bar{V}_g}(G) = f_{\bar{V}_s}(G) = 0$.

Keywords: Geodetic Number, Monophonic number. Geodetic GD, Steiner number, Steiner GD number, Monophonic GD number.

Introduction

Consider the graph $G = (V, E)$, where the vertex and the edge set is $V(G)$ and $E(G)$ (or V and E). Additionally, we proved that G has size $m = |E(G)|$ and order $n = |V(G)|$. We examine [2] for a list of fundamental graph theoretic terms. A vertex v is regarded as extreme if the subgraph formed by its neighbors is complete. Therefore, a vertex v is a universal vertex if $\deg_G(v) = n - 1$.

Let u and v be the vertices of a graph G that is connected. A $u - v$ geodesic is a substitute for the shortest $u - v$ path. The length of a $u - v$ geodesic in G is the (shortest path) distance and if the graph is obvious from the context, it is represented by the $d(u, v)$ or $d_G(u, v)$. Such that G 's diameter is the maximum separation between each pair of vertices is referred as $d(G)$ or d respectively. An edge connecting two non-adjacent vertices of a path P is referred to as a chord. A chord-less $u - v$ path is represented as a monophonic path. All vertices residing in a $u - v$ monophonic path, are included in the closed interval $J[u, v]$ for two vertices v and u . When v and u are next to each other, $J[u, v] = \{u, v\}$. Let $J[M] = \cup_{u,v \in M} J[u, v]$ for a set M of vertices. In such case, $M \subseteq J[M]$. A monophonic set of G is

described as a set $M \subseteq V(G)$ if $J[M] = V$. The lowest order of a monophonic set of G is its monophonic number $m(G)$. A m -set of G is any monophonic set of order $m(G)$ [1, 2].

Since any vertex in $V \setminus D$ is adjacent to atleast one vertex of D , then the subset $D \subseteq V(G)$ is represented as a dominant set. The lowest cardinality of such dominating sets of G is indicated by the domination number, $\gamma(G)$. Hence, a γ -set of G is a minimum dominant set. In [3], the concept of dominance was examined. If D is a dominating set in G and $\bar{G}$, then a subset $D \subseteq V$ is defined to as a GD set in G . The least cardinality of a minimum GD set in G is the GD number $\bar{\gamma}(G)$. In [3], the idea of GD in graphs was first presented. If M is both a monophonic set and a GD set of G , then $M \subseteq V$ is a monophonic GD set of G . The monophonic GD is referred as $\bar{\gamma}_m(G)$, is the minimal cardinality of a monophonic GD set of G . A $\bar{\gamma}_m$ -set of G is a monophonic GD set of cardinality $\bar{\gamma}_m(G)$. In [4], the idea of monophonic globally domination in graphs was first presented. The sequel makes use of the following theorems.

A geodesic is a $x - y$ path of length $d(x, y)$. A $u - v$ geodesic is a $u - v$ path of length $d(u, v)$. If v is an internal vertex of P , then v is said to reside on a geodesic P . For a non-empty set $S \subseteq V(G)$, $I[S] = \cup_{x,y \in S} I[x, y]$. The closed interval $I[x, y]$ is composed of x, y , and all vertices lying on some $x - y$ geodesic of G . A geodetic set of G has a geodetic number, denoted as $g(G)$, which is its lowest cardinality. If $I[S] = V(G)$ then a set $S \subseteq V(G)$ in a connected graph G is a geodetic set of G . In [1, 3, 4], the geodetic idea was examined. The term "geodetic GD set of G " refers to a geodetic set W of a linked graph G that is both a geodetic set and a GD set is represented by $\bar{\gamma}_g(G)$, is the lowest cardinality of a geodetic GD set.

The Steiner distance $d(W)$ of a connected graph's nonempty collection W of vertices When a connected subgraph of G contains W , its smallest size is G . A subgraph that is a tree by necessity is called a Steiner tree with respect to W , or Steiner W -tree. Such that when $W = \{u, v\}$ then $d(W) = d(u, v)$. $S(W)$ represents the set of all vertices of G that are located on a Steiner W -tree. If $S(W) = W$ then $G[W]$ is connected. Therefore W is referred to as a Steiner set for G if $S(W) = W$.

The Steiner set of minimum cardinality, or Steiner number $S(G)$ of G , is often referred to as an s -set of G or a minimum Steiner set. These concepts were studied in [1,2,4,5,6]. In G , the set $D \subseteq V$ of vertices is called a dominating set when each vertex $x \in V$ is either a member of D or near an element of D . A subset $D \subseteq V$ is called a GD set in G if it is a dominating set of both G and $\bar{G}$. For a GD set in G , the GD number $\bar{\gamma}(G)$ is its lowest cardinality. These concepts were studied in [3,8,9]. If W is both a Steiner set and a GD set of G , then a Steiner GD set of a linked graph G is defined as a Steiner set W of G . A Steiner GD set's lowest cardinality is represented by the Steiner GD number, which is written as $\bar{\gamma}_s(G)$. In [7], these ideas were examined. The sequel makes use of the following theorems.

Theorem 1.1. [7] Each Steiner GD set of G contains every extreme vertex of a connected graph G .

Theorem 1.2. [7] Assume that G is a connected graph and that W is the collection of all geodetic GD sets of G . Subsequently $f_{\bar{y}_g}(G) \leq \bar{y}_g(G) - |W|$.

Theorem 1.3. [7] Every geodetic global dominating set of a connected graph G contains every extreme vertex (universal vertex).

On The Forcing Monophonic Global Domination, Geodetic Global Domination and Steiner Global Domination Number of a Graph

Theorem 2.1. For any integer $a \geq 0$, there exists a connected graph G such that $f_{\bar{y}_m}(G) = f_{\bar{y}_g}(G) = f_{\bar{y}_s}(G) = a$.

Proof. Assume $P: x, y, z$ be a three-vertex path. On two vertices, consider $P_i: u_i, v_i$ ($1 \leq i \leq a$) be duplicates of the path. Let H be the graph is formed by adding the edges yu_i and zv_i ($1 \leq i \leq a$) to P and to each P_i ($1 \leq i \leq a$). Let G be the graph obtained from H by adding the vertices $z_1, z_2, \dots, z_{b-a-1}$ and presenting the novel edge, z_i ($1 \leq i \leq b - a - 1$) (Figure 1).

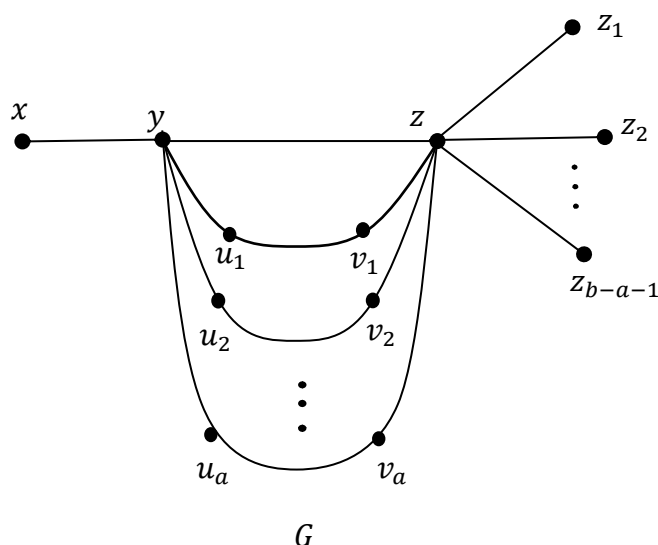


Figure 1. Forcing Monophonic Global Domination of G

Initially, it is demonstrated that $\bar{y}_m(G) = b$. Let $Z = \{x, z_1, z_2, \dots, z_{b-a-1}\}$ constitute the set of end vertices of G . For any monophonic GD set of G , Z is a subset, by Theorem 1.1, so $\bar{y}_m(G) \geq b - a - 1 + 1 = b - a$. However, $J[Z] \neq V(G)$, Z is not a monophonic global dominant set of G . Such that, $H_i = \{u_i, v_i\}$ ($1 \leq i \leq a$). Every, H_i ($1 \leq i \leq a$) has at least one vertex, making it an easily global monophonic dominant set. Therefore $\bar{y}_m(G) \geq b - a + a = b$, such that $M = Z \cup \{u_1, u_2, \dots, u_a\}$. Therefore, if M is a monophonic GD set of G , then $\bar{y}_m(G) = b$.

It is demonstrated that, $f_{\bar{\gamma}_m}(G) = a$. Then $f_{\bar{\gamma}_m}(G) \leq \bar{\gamma}_m(G) - |Z| = b - (b - a) = a$, according to Theorem 1.2, Given that $\bar{\gamma}_m(G) = b$ and that each $\bar{\gamma}_m$ -set of G contains Z , it is evident that each $\bar{\gamma}_m$ -set of G has the form $M = Z \cup \{c_1, c_2, \dots, c_a\}$, where $c_i \in H_i$ ($1 \leq i \leq a$). With $|T| < a$, let T be a proper subset of M . Next, a vertex, c_j ($1 \leq j \leq a$) such that $c_j \notin T$ exists. Consider b_j to be a vertex of H_j that is different from c_j . Therefore, $M_1 = \{b_j \cup (M - \{c_j\})\}$ is a $\bar{\gamma}_m$ -set that correctly contains T . Thus M is not the unique $\bar{\gamma}_m$ -set containing T . Hence, T is not a subset of M that is forced. This holds true for every minimum $\bar{\gamma}_m$ -set of G , so, $f_{\bar{\gamma}_m}(G) = a$. Analogously, $f_{\bar{\gamma}_g}(G) = f_{\bar{\gamma}_s}(G) = a$.

Theorem 2.2. For any integer $a \geq 0$, there exists a connected graph G such that $f_{\bar{\gamma}_m}(G) = 0, f_{\bar{\gamma}_g}(G) = f_{\bar{\gamma}_s}(G) = a$.

Proof. Consider a path on three vertices, $P: u, v, w$. On two vertices, let, $P_i: x_i, y_i$ ($1 \leq i \leq a$) be a duplicate of the path. Let H be the graph that is generated by combining the edges ux_i and wy_i ($1 \leq i \leq a$) to P and P_i ($1 \leq i \leq a$). Consider G to be the graph that is generated by taking H and adding the additional vertex $z_1, z_2, \dots, z_{b-a-1}$ and the edge wz_i ($1 \leq i \leq b - a - 1$) (Figure 2).

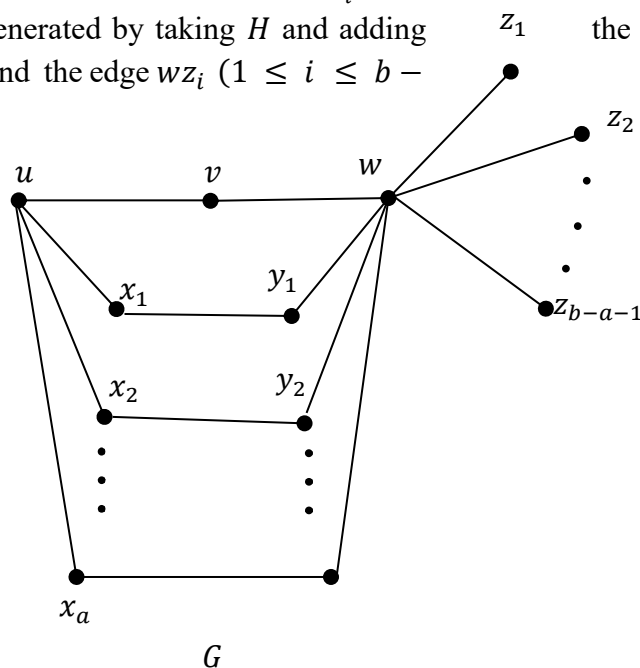


Figure 2. Geodetic Global Domination of G

Initially, it is demonstrated that $f_{\bar{\gamma}_m}(G) = 0$. Letting $Z = \{z_1, z_2, \dots, z_{b-a-1}\}$ represent the set of G 's end end vertices. Z is a subset of each monophonic GD set of G , by Theorem 1.1. Additionally, $\bar{\gamma}_m(G) \geq b - a - 1$ since $J[Z] \neq V(G)$, Z is not a monophonic GD set of G . Since $Z \cup \{x\}$, $x \notin Z$ is not a monophonic GD set of G , it can be readily confirmed that $\bar{\gamma}_m(G) \geq b - a - 1$. Let, $Z_1 = Z \cup \{u, w\}$. In such case $\bar{\gamma}_m(G) = 0$. Since Z_1 is the only $\bar{\gamma}_m$ -set of G and Z is a subset of all geodetic GD sets of G by Theorem 1.3, $\bar{\gamma}_g(G) \geq b - a - 1$.

Consider the following $Q_i = \{x_i, y_j\}$ ($1 \leq i \leq a$). Clearly, every GD set has at least one vertex from Q_i ($1 \leq i \leq a$) and the vertex, u . Consequently $\bar{\gamma}_g(G) \geq b - a - 1 + 1 + a = b$. Considering that $S = Z \cup \{u\} \cup \{x_1, x_2, \dots, x_a\}$. If S is a $\bar{\gamma}_g$ -set of G , then $\bar{\gamma}_g(G) = b$, $f_{\bar{\gamma}_g}(G) \leq b - |Z \cup \{u\}| = a$ for Theorem 1.3. Since $\bar{\gamma}_g(G) = b$ and every $\bar{\gamma}_g$ -set of G contains $Z \cup \{u\}$, it is evident that each $\bar{\gamma}_g$ -set of G is of the form $M = Z \cup \{u\} \cup \{c_1, c_2, \dots, c_a\}$, where, $c_i \in H_i$ ($1 \leq i \leq a$). With $|T| < a$, let T be a proper subset of M . Next, a vertex, c_j ($1 \leq j \leq a$) such that, $c_j \notin T$ exists. Consider, b_j to be a vertex of H_j , that is different from c_j . Therefore, $M_1 = (M - \{c_j\}) \cup \{b_j\}$ is a $\bar{\gamma}_g$ -set that correctly contains T . Hence, T is not a forcing subset of M . This holds true for every minimum $\bar{\gamma}_g$ -set of G , so $f_{\bar{\gamma}_g}(G) = a$. In a similar manner, it shows that $f_{\bar{\gamma}_s}(G) = a$.

Theorem 2.3. For every integers $a \geq 0$, there exists a connected graph G with respect to $f_{\bar{\gamma}_m}(G) = a, f_{\bar{\gamma}_g}(G) = f_{\bar{\gamma}_s}(G) = 0$.

Proof. Consider a path on three vertices, $P: x, y, z$. Let $(1 \leq i \leq a)$ be a duplicate of the path on three vertices, and let, $P_i: r_i, s_i, t_i$. Let H be the graph that is produced by adding the edges xr_i and yt_i ($1 \leq i \leq a$) to P and P_i ($1 \leq i \leq a$). Let G be the graph obtained from H by inserting the edge yz_i ($1 \leq i \leq b - a - 1$) and the new vertices $z_1, z_2, \dots, z_{b-a-1}$. Figure 3 shows the graph G .

Initially, it is established that, $f_{\bar{\gamma}_m}(G) = a$. Assuming $Z = \{x, z_1, z_2, \dots, z_{b-a-2}\}$ represent the set of G 's end vertices. Such that Z is a subset of each monophonic GD set of G , by Theorem 1.1 and so $\bar{\gamma}_m(G) = b - a - 1$. There is no monophonic global dominant set of G since $J[Z] \neq V(G)$. Suppose that, $H_i = \{s_i, t_i\}$ ($1 \leq i \leq a$). Clearly, any global monophonic dominant set has atleast one vertex from any H_i ($1 \leq i \leq a$) of the following such that $\bar{\gamma}_m(G) \geq b - a - 1 + 1 + a = b$. Consider $M = Z \cup \{z\} \cup \{s_1, s_2, \dots, s_a\}$ instead. Consequently, M is a global monophonic dominant set of G , hence $\bar{\gamma}_m(G) = b$.

It is then verified that, $f_{\bar{\gamma}_m}(G) = a$. Hence according to Theorem 1.2, $f_{\bar{\gamma}_m}(G) \leq \bar{\gamma}_m(G) - |Z| \cup \{x\} = b - (b - a + 1 - 1) = a$. Following that $\bar{\gamma}_m(G) = b$ and each $\bar{\gamma}_m$ -set of G contains $Z \cup \{x\}$, it is evident that every $\bar{\gamma}_m$ -set of G is of the type $M = Z \cup \{x\} \cup \{c_1, c_2, \dots, c_a\}$, where $c_i \in H_i$ ($1 \leq i \leq a$). With, $|T| < a$, consider T be a proper subset of M . Next, a vertex, c_j ($1 \leq j \leq a$) such that, $c_j \notin T$ exists. Consider, b_j to be a vertex of H_j that is different from, c_j . Therefore, $M_1 = (M - \{c_j\}) \cup \{b_j\}$ is a $\bar{\gamma}_m$ -set that correctly contains T . Hence, T is not a subset of M that is forced. This holds true for any minimum $\bar{\gamma}_m$ -set of G , so $f_{\bar{\gamma}_m}(G) = a$.

Following that, it is established that, $f_{\bar{\gamma}_g}(G) = 0$. A subset of each global geodetic set of G is Z , from Theorem 1.3. Therefore, $J[Z] \neq V(G)$, Z is not a geodetic set of G . There is no doubt that every global geodetic set of G only has the vertex S_i , from any H_i ($1 \leq i \leq a$). Consequently, $\bar{\gamma}_g(G) \geq -1 + 1 + a + b - a = b$. Suppose $S = Z \cup \{x\} \cup \{s_1, s_2, \dots, s_a\}$.

Thus, S is a unique $\bar{\gamma}_g$ -set of G since $\bar{\gamma}_g(G) = b$ and $f_{\bar{\gamma}_g}(G) = 0$. Similarly, it can be demonstrated that $f_{\bar{\gamma}_s}(G) = 0$.

Theorem 2.4. For each integer $a \geq 0$, there exists a connected graph G with respect to $f_{\bar{\gamma}_m}(G) = 0, f_{\bar{\gamma}_g}(G) = a, f_{\bar{\gamma}_s}(G) = 0$.

Proof. Consider a path on six vertices, $P: u, v, w, x, y, z$. Let $(1 \leq i \leq a)$ be a replica of the path on three vertices, and let $P_i: u_i, v_i, w_i$. Let H be the graph formed from P and P_i ($1 \leq i \leq a$) by adding the edges uu_i, yw_i, wu_i, wv_i and ww_i ($1 \leq i \leq a$). Let G be the graph that is generated by taking H and adding the new vertices $z_1, z_2, \dots, z_{b-a-2}$, and adding the edge yz_i ($1 \leq i \leq b-a-2$).

Initially, it is illustrated that $f_{\bar{\gamma}_m}(G) = 0$. Considering $Z = \{u, z_1, z_2, \dots, z_{b-a-2}\}$ represent the collection of G 's end vertices. Every monophonic GD set of G has a subset of Z , according to Theorem 1.1. Because $J[Z] \neq V(G)$, Z is not a monophonic GD set of G , $\bar{\gamma}_m(G) = b-a-2$. It is simple to confirm that x is a member of all global monophonic dominant sets. Assume that $Z_1 = Z \cup \{x\}$. The global monophonic dominating set of G is then Z_1 . Hence $\bar{\gamma}_m(G) = 0$, since Z_1 is a dominating set of G and $\bar{G}$. Thus Z_1 is a unique minimal GD set of G . It is then established that $\bar{\gamma}_g(G) = a$. The fact that Z_1 is a subset of each geodetic GD set of G is readily apparent. The geodetic GD set of G is not $I[Z_1] \neq V(G)$, Z_1 . Consider the following $H_i = \{u_i, v_i, w_i\}$ ($1 \leq i \leq a$). The presence of at least one vertex from each, H_i ($1 \leq i \leq a$) in every global geodetic dominating set can be readily confirmed, and thus $\bar{\gamma}_g(G) \geq b-a+a = b$. For example, let, $S_1 = Z_1 \cup \{u_1, u_2, \dots, u_a\}$. Therefore S_1 is a $\bar{\gamma}_g$ -set of G , hence $\bar{\gamma}_g(G) = b$.

Following that, $f_{\bar{\gamma}_g}(G) = a$ is proved. By Theorem 1.3, $f_{\bar{\gamma}_g}(G) \leq \bar{\gamma}_g(G) - |Z_1| = b - (b-a) = a$. Such that, $\bar{\gamma}_g(G) = b$ and any $\bar{\gamma}_g$ -set of G contains Z_1 , it is evident that each $\bar{\gamma}_g$ -set of G has the form $S = Z_1 \cup \{u_1, u_2, \dots, u_a\}$, where, $u_i \in H_i$ ($1 \leq i \leq a$). Since T is a valid subset of S with $|T| < a$. Then, a vertex like u_i ($1 \leq i \leq a$) there exists $u_i \notin T$. In contrast to, c_j , let, b_j be a vertex of H_j . This means that $\bar{\gamma}_g$ -set that correctly contains T is $S_1 = (S - \{u_i\}) \cup \{b_j\}$. Consequently, T is not a forcing subset of S . It follows that, $f_{\bar{\gamma}_g}(G) = a$, hence this is true for all minimum $\bar{\gamma}_g$ -set of G .

It is demonstrated that $\bar{\gamma}_s(G) = 0$. The fact that Z_1 is a subset of each Steiner GD set of G is readily apparent. There is no Steiner global dominant set of such that $S(Z_1) \neq V(G)$, Z_1 . It is readily apparent that the vertex v_i , from each H_i ($1 \leq i \leq a$). Then W is the only element present in every Steiner GD set of G . For instance, consider $W = Z_1 \cup \{v_1, v_2, \dots, v_a\}$. After that, W is the only Steiner GD set of G that is unique, therefore $\bar{\gamma}_s(G) = 0$.

Theorem 2.5. For every integer $a \geq 0$, there exists a connected graph G such that $f_{\bar{\gamma}_m}(G) = 0, f_{\bar{\gamma}_g}(G) = 0, f_{\bar{\gamma}_s}(G) = a$.

Proof. Consider a path on nine vertices, $P: r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9$. Let, $P_i: x_i, y_i, u_i, v_i, w_i$ ($1 \leq i \leq a$) as a duplicate of the path on five vertices. Consider H be the graph that is generated from P and P_i ($1 \leq i \leq a$) by adding the edges yy_i, yu_i, yv_i, r_2x_i and r_8w_i ($1 \leq i \leq a$). Let G be the graph that is generated by taking H and adding the new vertices x and z , as well as the edge yz, xr_5 . Figure 3 displays the graph G .

Initially, it is verified that, $f_{\bar{y}_m}(G) = 0$. Consider that the set of end vertices of G is $Z = \{x, z, r_1, r_9, r_5, y\}$. Every monophonic global dominant set of G has a subset of Z , according to Theorem 1.1. Considering that $J[Z] = V(G)$, Z is the only monophonic set of G , and therefore $f_{\bar{y}_m}(G) = 0$.

It is established that $f_{\bar{y}_g}(G) = 0$. Every geodetic GD set of G has a subset of Z , according to Theorem 1.3. Considering that $I[Z] = V(G)$, Z is the only geodetic set of G , $f_{\bar{y}_g}(G) = 0$.

Afterward, it is established that, $f_{\bar{y}_s}(G) = a$. Z is a subset of each Steiner GD set of G . There is no Steiner set of G such that $S(Z) \neq V(G)$, Z . For example, let, $H_i = \{y_i, u_i, v_i\}$ ($1 \leq i \leq a$). The fact that each Steiner GD set of G has exactly one vertex from each of the following is readily apparent: ($1 \leq i \leq a$) and so $\bar{y}_s(G) \geq a + b$. Consider $W = Z \cup \{y_1, y_2, \dots, y_a\}$. Therefore $S(Z) \neq V(G)$ hence $\bar{y}_s(G) = a$.

Following that, it is demonstrated that $f_{\bar{y}_s}(G) = a$. As per the Theorem 1.3, $f_{\bar{y}_s}(G) \leq b + a - |Z| = b + a - b = a$. Since, $\bar{y}_g(G) = a + b$ and for any $\bar{y}_s$ -set of G contains, Z_1 , it is evident that every, $\bar{y}_s$ -set of G has the form $W = Z \cup \{y_1, y_2, \dots, y_a\}$, where, $y_i \in H_i$ ($1 \leq i \leq a$). Assume that T is a valid subset of W when $|T| < a$. Afterwards, a vertex y_i ($1 \leq i \leq a$) exists, $y_i \notin T$. Let b_j be a vertex of H_j distinct from c_j . This means that a $\bar{y}_s$ -set that correctly contains T is $W_1 = (W - \{y_i\}) \cup \{b_j\}$. Consequently, T is not a forcing subset of W . It follows that, $f_{\bar{y}_s}(G) = a$. Therefore it is minimum $\bar{y}_s$ -set of G .

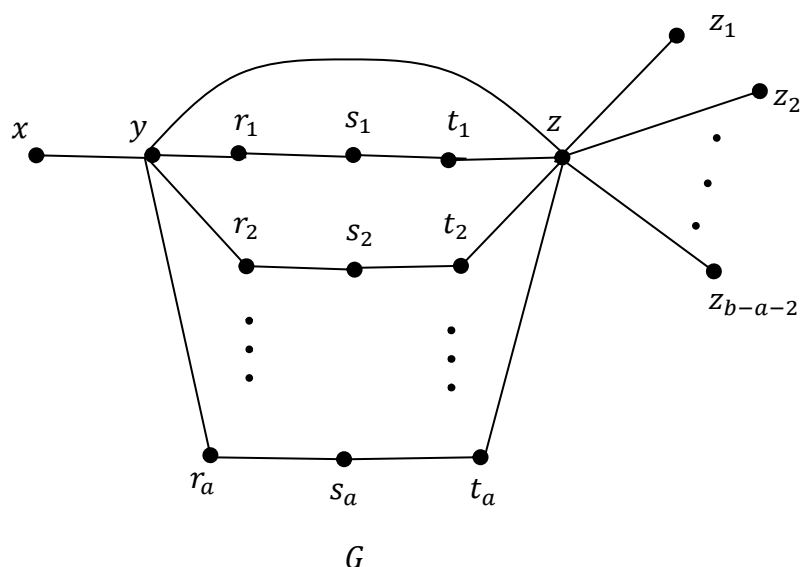


Figure 3. Steiner Global Domination Number of G

References

- [1] G. Chartrand and P. Zhang, The Steiner number of a graph, *Discrete Math.* 242, (2002) 41–54.
- [2] G. Chartrand and P. Zhang, The forcing geodetic number of a graph, *Discuss. Math. Graph Theory*, 19 (1999), 45-58.
- [3] M. B. Frondoza and S. R. Canoy Jr., The edge Steiner number of a graph, *Bull. Malays. Math. Sci. Soc.* 35 (2012) 53–69.
- [4] Haynes, TW, Hedetniemi, ST & Slater, PJ 1998, ‘Fundamentals of Domination in Graphs’, Marcel Dekker, New York, 1998.
- [5] J John, G Edwin, P Arul Paul Sudhahar, The Steiner domination number of a graph, *International journal of mathematics and computer applications research*, 3(3), (2013), 37-42.
- [6] A.P. Santhakumaran and J. John, The edge Steiner number of a graph, *Discuss. Math. Graph Theory*. 31, (2011), 611 – 624.
- [7] J. Suja and V.Sujin Flower, The edge Steiner Global Domination number of a Graph, *Nanotechnology Perceptions*, 20 No.6 (2024), 3871- 3876.
- [8] J. Suja and V.Sujin Flower, The Steiner Global Domination number of a graph, *Library Progress International*, 44 No.3(2024), 27998-28003.
- [9] S.K. Vaidya & R. M. Pandit, Some results on GD sets, *Proyecciones Journal of Mathematics*, 32(3), (2013), 235-244.