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TRANSIENT ANALYSIS OF MULTISERVER WITH ENVIRONMENT AND RETENTION OF IMPATIENT CUSTOMERS

Natasha and Dr. Deepak Gupta*

Research Scholar, Department of Mathematics and Humanities, MMEC, Maharishi Markandeshwar (Deemed to be University), Mullana- 133203

*Professor, Department of Mathematics and Humanities, Maharishi Markandeshwar (Deemed to be University), Mullana- 133203

Abstract

The transient study of a limited capacity queuing system with many servers, environment effects, and impatient customer retention is done in this paper. Here, the impact of environmental change and retention is covered in detail using mathematical expressions.

Keyword: Transient Analysis; Retention; Environment; Reneging

1. Introduction

The M/M/N queue has been the topic of thorough and methodical research since its inception. In recent years, emphasis has been drawn to extension, which includes the impact of retention. In this regard, the article by N.K. Jain et al. has been specifically cited. The authors of the report [6] acknowledged the impact of the disaster on a number of scientific and technological fields, but this paper also discusses customer retention. Customers that exhibit impatient behavior, such as balking or reneging at various times, must be persuaded to do so.

Since its introduction, the M/M/N queue has been the subject of extensive and systematic investigation. Extension, which encompasses the effect of retention, has received more attention in recent years. The paper by N.K. Jain et al. has been expressly referenced in this context. Although this document also addresses customer retention, the writers of the research [1] highlighted the influence of the accident on several scientific and technological domains. It is necessary to persuade customers who display impatient behavior, such as balking or reneging at different times.

The concept of retaining loyal clients was first forth by Lee [5] in a discrete-time line. Kumar and Sharma [3] investigated the retention of reneged clients and the ephemeral behavior of the M/M/C queuing system with balking. Kumar and Sharma [6] looked into an M/M/c/N queueing model and the retention of reneged clients.

This paper is organized as follows: Section 2 describes the model and its assumptions; Section 3 gives a mathematical description of the model; Section 4 presents an analysis of the model in a transitory state; and Section 5 displays the article's conclusion.

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2. Model's Description and Assumption

The model's underlying assumptions are as follows:
1. The queuing system has two states: R and S. With an α rate to move from state S to state R and a β rate to go from R to S, the system operates as a Poisson process.

2. The inter-arrival and service times have a negative exponential distribution. In state R, the queuing system has a rate of arrival of λ_1 and a rate of service of μ_1, whereas in state S, the rate of arrival is 0 (i.e., $\lambda = 0$) and the rate of service is μ_2. At any one time, only one of states R and S is in operation.

3. Each customer entering the line must wait for his service to begin for a certain period of time T, say, when the system is in state R. If it doesn't begin by then, he has the option to either renounce with probability p or remain in line for his duty with complementing probability. The exponential distribution of time variable T in unfavorable

with parameter ξ has been hypothesized. The system is in state S when there is no reneging present.

4. The queue discipline is FCFS, and the system has a finite function for N. "The first person to arrive will receive service first."

3. Mathematical Description of the Model

Let in state R, system have probability denoted by Pn(t) and at time t there are n number of clients.Let in state S, system have probability denoted by Qn(t) and at time t number of clients are same as that in state R.

Initially when the system is empty and is in environmental state R then we have

$$P_n(0) = \begin{cases} 1; & n = 0 \\ 0; & \text{otherwise} \end{cases} \tag{1}$$

$$Q_n(0) = 0; \text{ for all n} \tag{2}$$

Equations Governing the System

$$\frac{dP_0(t)}{dt} = -(\lambda_1 + \beta)P_0(t) + \mu_1 P_1(t) + \alpha Q_0(t) + \sum_{n=0}^{N} P_n(t) \tag{3}$$

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$$\frac{dP_n(t)}{dt} = -(\lambda_1 + \mu_1 + \beta + (n-1)\xi p)P_n(t) + \lambda_1 P_{n-1}(t) + (\mu_1 + n\beta p)P_{n+1}(t) + \alpha Q_n(t) \quad (4)$$

$$\frac{dP_N(t)}{dt} = -(\mu_1 + \beta + (N-1)\xi p)P_N(T) + \lambda_1 P_{N-1}(t) + (\mu_1 + \beta + (N-1)\xi p)P_N(t) + \alpha Q_N(t) \quad (5)$$

$$\frac{dQ_0(t)}{dt} = -\alpha Q_0(t) + \mu_2 Q(t)_1 + \beta P_0(t) + \sum_{n=0}^{N} Q_n(t) \quad (6)$$

$$\frac{dQ_n(t)}{dt} = -(\mu_2 + \alpha)Q_n(t) + \mu_2 Q_{n+1}(t) + \beta P_n(t) \quad (7)$$

$$\frac{dQ_N(t)}{dt} = -(\mu_2 + \alpha)Q_N(t) + \beta P_n(t) \quad (8)$$

4. Transient Analysis of the Model

Laplace Transform of f(t) be

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \quad (9)$$

Taking Laplace Transform of the equations (3)-(8) and using the initial conditions, we have

$$(s + \lambda_1 + \beta)\bar{P}_0(s) - 1 = \mu_1 \bar{P}_1(s) + \alpha \bar{Q}_0(s) + \sum_{n=0}^{N} \bar{P}_n(s) \quad (10)$$

$$(s + \lambda_1 + \mu_1 + \beta + (n-1)\xi p)\bar{P}_n(s) = \lambda_1 \bar{P}_{n-1}(s) + (\mu_1 + n\xi p)\bar{P}_{n+1}(s) + \alpha \bar{Q}_n(s) \quad (11)$$

$$(s + \mu_1 + \beta + (N-1)\xi p)\bar{P}_N(s) = \lambda_1 \bar{P}_{N-1}(s) + \alpha \bar{Q}_N(s) \quad (12)$$

$$(s + \alpha)\bar{Q}_0(s) = \mu_2 \bar{Q}_1(s)\beta \bar{P}_0(s) + \sum_{n=0}^{N} \bar{Q}_n(s) \quad (13)$$

$$-(s + \mu_2 + \alpha)\bar{Q}_n(s) = \mu_2 \bar{Q}_{n+1}(s) + \beta \bar{P}_n(s) \quad (14)$$

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$-(s+\mu_2+\alpha)\bar{Q}_N(s) =$
$\beta\ \bar{P}_N(s)$ (15)

Defining the probability generating functions by

$P(z,s) = \sum_{n=0}^{N} \bar{P}_n(s) z^n$ (16)

$Q(z,s) = \sum_{n=0}^{N} \bar{Q}_n(s) z^n$ (17)

$U(z,s) = \sum_{n=0}^{N} \bar{U}_n(s) z^n$ (18)

Where

$U(z,s) = P(z,s) + Q(z,s)$ (19)

$\bar{U}(z,s) = \bar{P}(z,s) + \bar{Q}(z,s)$ (20)

$[z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p-\lambda_1 z^2-\mu_1]P(z,s)-\alpha z Q(z,s)+\mu_1(1-z)\bar{P}_0-\lambda_1 z^{m+1}(1-z)\bar{P}_N - z - z\sum_{n=0}^{N}\bar{P}_n(s)-(n-1)\xi p\bar{P}_0 = 0$
(21)

$\boldsymbol{\beta} z P(z,s)+Q(z,s)[\mu_2-z(s+\mu_2+\alpha)]-\mu_2(1-z)\bar{Q}_0(s)+z\sum_{n=0}^{N}\bar{Q}_n(s)=0$
(22)

On solving above equations

$P(z,s)=\{\alpha\mu_2 z(1-z)\bar{Q}_0-\alpha z^2\sum_{n=0}^{N}\bar{Q}_n(s)-[\mu_1(1-z)-(n-1)\xi p][\mu_2-z(s+\mu_2+\alpha]\bar{P}_0+\lambda_1 z^{m+1}(1-z)([\mu_2-z(s+\mu_2+\alpha)]\bar{P}_N+z(\mu_2-z(s+\mu_2+\alpha))+z\sum_{n=0}^{N}\bar{P}_n(s)(\mu_2-z(s+\mu_2+\alpha))\}/\{\alpha\boldsymbol{\beta} z^2+(\mu_2-z(s+\mu_2+\alpha))(z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)-\lambda_1 z^2-\mu_1)\}$ (23)

And

$Q(z,s)=\{\boldsymbol{\beta} z(\mu_1(1-z)-(n-1)\xi p)\bar{P}_0+\mu_2(1-z)\ (z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)-\lambda_1 z^2-\mu_1)\bar{Q}_0+[\lambda_1 z^2+\mu_1-z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p]z\sum_{n=0}^{N}\bar{Q}_n(s)-\boldsymbol{\beta}\lambda_1 z^{N+2}(1-z)\bar{P}_N-\boldsymbol{\beta} z^2-\ \boldsymbol{\beta} z^2\sum_{n=0}^{N}\bar{P}_n(s)\}/\{\alpha\boldsymbol{\beta} z^2+(\mu_2-z(s+\mu_2+\alpha))(z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)-\lambda_1 z^2-\mu_1)\}$ (24)

$R(z,s) = \{(1-z)[\ \alpha\mu_2+\mu_2(z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)-\lambda_1 z^2-\mu_1)\}\bar{Q}_0+\{\lambda_1 z^2-(z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)+\mu_1)\}-\alpha z\}\ \ z\sum_{n=0}^{N}\bar{Q}_n(s)+[\mu_1(1-z)-(n-1)\xi p]\{\boldsymbol{\beta} z+[z(s+\mu_2+\alpha)-\mu_2]\}\ +[\lambda_1 z^{m+1}([\mu_2-z(s+\mu_2+\alpha)]-\beta\lambda_1 z^{N+2}](1-z)\ \bar{P}_N+z(\mu_2-z(s+\mu_2+\alpha))-\boldsymbol{\beta} z^2+z[(\mu_2-z(s+\mu_2+\alpha))-\beta z]\ \sum_{n=0}^{N}\bar{P}_n(s)\}/\{\alpha\boldsymbol{\beta} z^2+(\mu_2-z(s+\mu_2+\alpha))(z(s+\lambda_1+\mu_1+\beta+(n-1)\xi p)-\lambda_1 z^2-\mu_1)\}$ (25)

Furthermore, relation (25) exists for all values of z, including the three zeros of the denominator, and is a polynomial in z. Therefore, by setting the numerator to zero and replacing the three zeros m_1, m_2, m_3 of the denominator, $\bar{P}_0(s), \bar{Q}_0(s)$ and $\bar{U}_M(s)$ are produced.

5. **Conclusion**

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This study explicitly finds the temporary solution of a multiple server queuing system with reneging customer retention in two environment situations. Using the Laplace transform, a transient numerical analysis is also conducted to examine the system's behavior.

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