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HEAT AND MASS TRANSFER ON A COUETTE FLOW THROUGH POROUS MEDIUM WITH FINE DUST

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Abstract:

This study investigates the influence of fine dust particles on heat and mass transfer of a Couette flow between two porous plates, employing the perturbation technique. The flow is generated between two infinite parallel plates, with the upper plate moving at a uniform velocity while the lower plate remains stationary. The presence of dust particles modifies the fluid's momentum, energy, and concentration profiles. The governing partial differential equations, which describe the conservation of mass, momentum, energy, and concentration, are formulated for a two-phase flow model—comprising a viscous fluid phase and a uniformly distributed dust phase. The perturbation method is applied to linearize the nonlinear coupled equations under small parameter assumptions. Analytical expressions for velocity, temperature, and concentration fields are derived and discussed for various physical parameters such as the dust particle relaxation time, Prandtl number, Schmidt number, and mass concentration parameter. The results show that the presence of fine dust significantly reduces the fluid velocity and enhances heat and mass transfer rates depending on the particle loading.

Keywords: Couette flow, Fine dust particles, Two-phase flow, Heat and mass transfer, Perturbation technique, Prandtl number, Schmidt number, Dust–fluid interaction.

Introduction

The problem of three dimensional fluid flows between porous plates has been of great interest due to its importance in geophysics, metallurgy, aerodynamics and extrusion of plastic sheets and other engineering processes such as petroleum engineering, chemical engineering, composite or ceramic engineering and heat exchangers.

Gersten and Gross (1974) studied the effect of sinusoidal transverse suction velocity distribution on the flow and heat transfer over a plane wall. This suction velocity distribution leads to a cross flow and hence to a three-dimensional flow over the surface. Singh et al (1978) analyzed theoretically of fluid flow along a porous plate placed normal to the flow, where the system is exposed to a crosswise sinusoidal velocity profile. This forcing gives rise to a three-dimensional nature in the flow field. For the asymptotic limit case, the wall shear stress in the direction of primary flow is calculated for numerous standards of the buoyancy parameter G.

The effect of mixed convection and mass transfer of the three dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in the presence of transverse sinusoidal suction velocity oscillating with time and constant free stream velocity was studied by Ahmed and Liu (2010).

A study on the effect of periodic suction velocity and viscous dissipative heat on three dimensional flow of an incompressible viscous fluid through an infinite vertical porous plate when the free stream velocity oscillates in time about a non-zero constant mean was carried out by Sahin Ahmed (2010).

Bhupendra Kumar Sharma, Tara Chand and Chaudhary (2011) analyzed the mixed convection three dimensional steady laminar flow of a viscous incompressible fluid past an infinite vertical plate. Guria and Jana (2006) analyzed the three dimensional fluctuating Couette flows through the porous plates with heat transfer considering unsteady Couette flow of a viscous incompressible fluid between two horizontal porous flat plates. The stationary plate was subjected to a periodic suction and plate in uniform motion was subjected to uniform injection.

Dolat Khan et al. (2024) analyzed the two-step MHD fluctuating flow of Brinkman-type dusty fluid between parallel non-conducting plates, taking into account the effects of heat and mass transfer. Employing the Poincaré–Lighthill perturbation method, they studied the effects of parameters such as the Grashof number, magnetic field, and dust concentration on the fluid velocity.

Aziz Ullah Awan et al. (2025) studied the momentum, heat transfer, and entropy generation in an MHD Maxwell nanofluid with conducting dust particles along a sloping surface. By employing similarity transformations and the shooting method, they examined mixed convection, magnetic field, and dusty-fluid interaction effects on velocity, temperature, and Bejan number with increased entropy production due to the elastic nature of the fluid.

In this work, the absence of concentration in Gayathri *et al* (2018) motivated us to take up the present work wherein we study, heat and mass transfer over a porous medium in the presence of concentration. In order to examine the accuracy of our result, the effect of various parameters on velocity, temperature and concentration are calculated and plotted.

Flow Description and Governing Equations

Consider the Couette flow of a viscous incompressible dusty fluid between two infinite parallel flat porous plates. The coordinate system is introduced with its origin on the lower stationary plate lying horizontally on the $xy-$ plane and the upper plate is placed at a distance d which is subjected to a uniform motion U. The y – axis is taken perpendicular to the plates. The lower and the upper plates are assumed to be at constant temperatures T_0 and T_1 respectively, with T_1>T_0. The upper plates is subjected to a constant suction V_0 and the lower to a transverse exponential injection velocity distribution of the form$v = 1 + \epsilon e^{\alpha z}$.

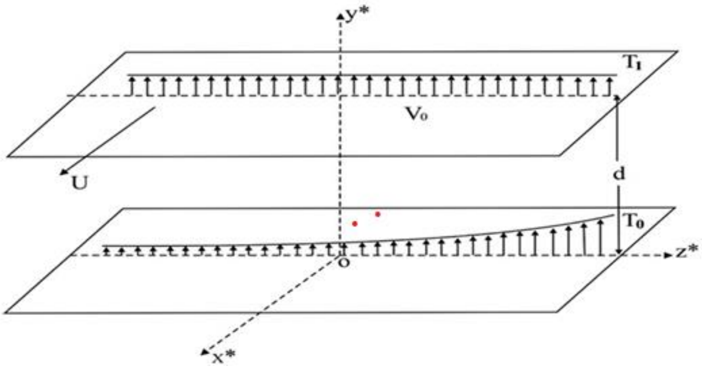

Fig 1. Schematic Diagram of the flow configuration

The basic equations that govern the steady three dimensional flow of viscous fluid are given as:

For Fluid phase

$\nabla \cdot \vec{q} = 0$ (1)

$\rho[(\vec{q}^*, \nabla)\vec{q}^*] = -\nabla^* p^* + \mu \nabla^{*^2} q^- + kN\left(\vec{q}_p^v - \vec{q}\right)$ (2)

$\rho c_p (\vec{q}^* \cdot \nabla^*) T = K \nabla^{*^2} T + k_p N\left(T_p^* - T^*\right)$ (3)

$(q^* \cdot \nabla^*) c = D_m \nabla^2 c$ (4)

For particle phase

$\nabla^* \cdot \vec{q}_p^* = 0$ (5)

$m_p N\left(\vec{q}_p^* \cdot \nabla^*\right)\vec{q}_p^* = kN\left(\vec{q}^* - \vec{q}_p^*\right)$ (6)

$\rho_p C_{pp}\left(\vec{q}_p^* \cdot \nabla^*\right) T_p^* = h_p N\left(T^* - T_p^*\right)$ (7)

With the corresponding boundary conditions

$u = 0; \quad v = v_0(1 + \epsilon e^{\alpha Z}); \quad w = 0; \quad T = T_0; \quad C = C_0$

$u_p = 0; \quad v_p = v_0(1 + \epsilon e^{\alpha Z}); \quad w_p = 0; T_p = T_0$ at $y = 0$

$u = u_0; \quad v = v_0; \quad w = 0; T = T_1; \quad C = C_1$

$u_p = u_0; \quad v_p = v_0; \quad w_p = 0; T_p = T_1$ at $y = 1$ (8)

By introducing the following dimensionless parameters,

$(x^*, y^*, z^*) = d(x, y, z); \; p^* = \rho_0 v_0^2 p \; ; \; (u^*, v^*, w^*) = v_0(u, v, w)$

$(u_d^*, v_d^*, w_d^*) = v_0(u_d, v_d, w_d); \; \dfrac{\mathrm{T} - T_0}{T_1 - T_0} = \theta; \quad \dfrac{C - C_0}{C_1 - C_0} = \phi; \quad \dfrac{T_p - T_0}{T_1 - T_0} = \theta_p$

Here, p is the non-dimensional pressure,

ρ is the density of the fluid,

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N is the number density of the dust particle,

k is the stokes drag constant,

m is the mass of the dust particle, respectively.

The boundary conditions for the problem are

Hence the non-dimensional equations are

For Fluid phase

$$\nabla \cdot \vec{q} = 0 \tag{9}$$

$$\langle \vec{q} \cdot \nabla \rangle \vec{q} = -\nabla \rho + \frac{1}{R_e} \nabla^2 \vec{q} + \frac{f}{\tau} (\vec{q}_p - \vec{q}) \tag{10}$$

$$(\vec{q} \cdot \nabla)\theta = \frac{1}{p_r} \frac{1}{R_e} \nabla^2 \theta + \delta(\theta_p - \theta) \tag{11}$$

$$(\vec{q} \cdot \nabla)\phi = \frac{1}{S_c} \frac{1}{R_e} \nabla^2 \phi \tag{12}$$

For Particle phase

$$\nabla \cdot \vec{q}_p = 0 \tag{13}$$

$$(\vec{q}_p \cdot \nabla)\vec{q}_p = \frac{1}{\tau} (\vec{q} - \vec{q}_p) \tag{14}$$

$$(\vec{q}_p \cdot \nabla)\theta_p = \delta\lambda\beta(\theta - \theta_p) \tag{15}$$

Boundary Conditions are

$$u = 0; \quad v = 1 + \epsilon e^{\alpha z}; \quad w = 0; \quad \theta = 0; \quad \phi = 0$$

$$u_p = 0; \quad v_p = 1 + \epsilon e^{\alpha z}; \quad w_p = 0; \quad \theta_p = 0 \qquad \text{at } y = 0$$

$$u = 1; \quad v = 1; \quad w = 0; \quad \theta = 1; \quad \phi = 1$$

$$u_p = 1; \quad v_p = 1; w_p = 0; \quad \theta_p = 1 \qquad \text{at } y = 1 \tag{16}$$

Writing the equations (9) - (15) in component form, we get

For Fluid phase

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{17}$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{R_e} \left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] + \frac{f}{\tau} (u_p - u) \tag{18}$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R_e} \left[\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] + \frac{f}{\tau} (v_p - v). =$$

$$-\frac{\partial p}{\partial z} + \frac{1}{R_e} \left[\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] + \frac{f}{\tau} (w_p - w) \tag{19}$$

$$v \frac{\partial \theta}{\partial y} + \omega \frac{\partial \theta}{\partial z} = \frac{1}{P_r R_e} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \delta(\theta_p - \theta) \tag{20}$$

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$$v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}=\frac{1}{S_c\mathrm{Re}}\left[\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right] \tag{21}$$

For Particle phase

$$\frac{\partial V_p}{\partial y}+\frac{\partial\omega_p}{\partial z}=0 \tag{22}$$

$$\left(v_p\frac{\partial u_p}{\partial y}+w_p\frac{\partial u_p}{\partial z}\right)=\frac{1}{\tau}\left(u-u_p\right) \tag{23}$$

$$\left(v_p\frac{\partial v_p}{\partial y}+w_p\frac{\partial v_p}{\partial z}\right)=\frac{1}{\tau}\left(v-v_p\right) \tag{24}$$

$$\left(v_p\frac{\partial w_p}{\partial y}+w_p\frac{\partial w_p}{\partial z}\right)=\frac{1}{\tau}\left(\mathrm{w}-w_p\right) \tag{25}$$

$$\left(v_\rho\frac{\partial\theta_\rho}{\partial y}+\omega_\rho\frac{\partial\theta_\rho}{\partial z}\right)=\delta\lambda\beta\left(\theta-\theta_\rho\right) \tag{26}$$

3. Solution of the problem

In order to solve the differential equation (17) - (28), we assume the solution in the following form

$$u(y,z,t)\ =u_0(y)+\epsilon u_1(y,z,t)+\epsilon^2u_2(y,z,t)+\cdots$$
$$v(y,z,t)\ =v_0(y)+\epsilon v_1(y,z,t)+\epsilon^2v_2(y,z,t)+\cdots$$
$$w(y,z,t)=w_0(y)+\epsilon w_1(y,z,t)+\epsilon^2w_2(y,z,t)+\cdots$$
$$u_p(y,z,t)=u_{p_0}(y)+\epsilon u_{p1}(y,z,t)+\epsilon^2u_{p2}(y,z,t)+\cdots$$
$$v_p(y,z,t)\ =v_{p_0}(y)+\epsilon v_{p_1}(y,z,t)+\epsilon^2v_{p_2}(y,z,t)+\cdots$$
$$w_p(y,z,t)=w_{p_0}(y)+\epsilon w_{p_1}(y,z,t)+\epsilon^2w_{p_2}(y,z,t)+\cdots$$
$$p(y,z,t)=p_0(y)+\epsilon p_1(y,z,t)+\epsilon^2p_2(y,z,t)+ \tag{27}$$

when $\epsilon=0$, the problem reduces to

$$\frac{\partial v_0}{\partial y}\ =0\ ,v_0=1 \tag{28}$$

$$\frac{\partial^2u_0}{\partial y^2}-Re\frac{\partial u_0}{\partial y}-\frac{f}{\tau}Reu_0(y)=-\frac{f}{\tau}Re\ \ u_{p_0}(y) \tag{29}$$

$$\frac{\partial P_0}{\partial y}=0\Rightarrow\ p_0\ =k=1 \tag{30}$$

$$\frac{\partial^2\theta_0}{\partial y^2}-PrRe\frac{\partial\theta_0}{\partial y}-PrRe\ \delta\theta_0(y)=-\ PrRe\ \delta\ \ \theta_{p_0} \tag{31}$$

$$\frac{\partial\phi_0}{\partial y}=\frac{1}{Sc\ Re}\frac{\partial^2\phi_0}{\partial y^2} \tag{32}$$

$$\frac{\partial v_{p0}}{\partial y}=0;v_{P_0}=1 \tag{33}$$

$$\frac{\partial u_{p0}}{\partial y}=\frac{1}{\tau}\left[u_0(y)-u_{p_0}(y)\right] \tag{34}$$

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$$\frac{\partial \theta_{p_0}}{\partial y} = \delta\lambda\beta\theta_0(y) - \delta\lambda\beta\theta_{p_0} \tag{35}$$

Hence the solution is given by

$$w_0 = w_{p_0} = 0 \tag{36}$$

$$u_0(y) = A_3 + A_4 e^{m_1 y} + A_5 e^{m_2 y} \tag{37}$$

$$u_{p_0} = A_6 e^{\frac{-y}{\tau}} + A_3 + A_4 \frac{e^{m_1 y}}{(m_1\tau+1)} + A_5 \frac{e^{m_2 y}}{(m_2\tau+1)} \tag{38}$$

$$\theta_0(y) = A_7 + A_8 e^{m_3 y} + A_9 e^{m_4 y} \tag{39}$$

$$\theta_{p_0} = A_{10} e^{-\lambda_\beta \delta y} + A_7 + A_8 B_{17} e^{m_3 y} + A_9 B_{18} e^{m_4 y} \tag{40}$$

$$\phi_0(y) = A_1 + A_2 e^{Sc\,Re\,y} \tag{41}$$

When $\epsilon \neq 0$, substituting (27) in equations (9) –(15), comparing the coefficients of ϵ and neglecting the higher order terms, we get the following equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{42}$$

$$\frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \frac{1}{\text{Re}}\left\{\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right\} + \frac{f}{\tau}\left(u_{p_1} - u_1\right) \tag{43}$$

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\text{Re}}\left[\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right] + \frac{f}{\tau}\left(v_{p_1} - v_1\right) \tag{44}$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\text{Re}}\left[\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right] + \frac{f}{\tau}\left(w_{p_1} - w_1\right) \tag{45}$$

$$\frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} = \frac{1}{\text{PrRe}}\left\{\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right\} + \delta\left(\theta_{p_1} - \theta_1\right) \tag{46}$$

$$\frac{\partial \phi_1}{\partial y} + v_1 \frac{\partial \phi_0}{\partial y} = \frac{1}{S_c\text{Re}}\left\{\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right\} \tag{47}$$

$$\frac{\partial v_{p_1}}{\partial y} + \frac{\partial w_{p_1}}{\partial z} = 0 \tag{48}$$

$$\frac{\partial u_{p_1}}{\partial y} + v_{p_1} \frac{\partial u_{p_0}}{\partial y} = \frac{1}{\tau}\left(u_1 - u_{p_1}\right) \tag{49}$$

$$\frac{\partial v_{p_1}}{\partial y} = \frac{1}{\tau}\left(v_1 - v_{p_1}\right) \tag{50}$$

$$\frac{\partial w_{p_1}}{\partial y} = \frac{1}{\tau}\left(w_1 - w_{p_1}\right) \tag{51}$$

$$\frac{\partial \theta_{p_1}}{\partial y} + v_1 \frac{\partial \theta_{p_0}}{\partial y} = \lambda\beta\delta\left(\theta_1 - \theta_{p_1}\right) \tag{52}$$

To solve the equations (42) to (52) we assume the following complex form

$$f_1(y,z) = e^{\alpha z} \cdot f_{11}(y) \tag{53}$$

$$\omega_{p_{11}} = \left(-\frac{1}{\alpha}\right)\frac{\partial v_{p_{11}}}{\partial y} \tag{54}$$

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$$\omega_{11} = \left(-\frac{1}{\alpha}\right)\frac{\partial v_{11}}{\partial y} \tag{55}$$

where $f_1(y,z) = \{u_1; v_1; w_1; u_{p1}; v_{p1}; w_{p1}; \theta_1; \theta_{p_1};\ \phi_1\}$ and

$$f_{11}(y) = \{u_{11}; v_{11}; w_{11}; u_{p11}; v_{p11}; w_{p11}; \theta_{11}; \theta_{p_{11}};\ \phi_{11}\}$$

Corresponding boundary conditions are

$$u_{11} = 0\,; \quad v_{11} = 1; \quad w_{11} = 0; \quad \theta_{11} = 0\,; \quad \phi_{11} = 0$$

$$u_{p_{11}} = 0\,; \ v_{p_{11}} = 1; \ w_{p_{11}} = 0; \quad \theta_{p_{11}} = 0 \qquad at\ \mathrm{y} = 0$$

$$u_{11} = 0\,; \quad v_{11} = 0; \quad w_{11} = 0; \quad \theta_{11} = 0\,; \quad \phi_{11} = 0$$

$$u_{p_{11}} = 0\,; \ v_{p_{11}} = 0; \ w_{p_{11}} = 0; \quad \theta_{p_{11}} = 0 \qquad at\ \mathrm{y} = 1 \tag{56}$$

using the boundary conditions equations (56) and also using equations (53), (54) and (55) following set of solutions are obtained

$$p_{11} = A_{11}e^{i\alpha y} + A_{12}e^{-i\alpha y} \tag{57}$$

$$v_{11} = A_{13}e^{m_5 y} + A_{14}e^{m_6 y} + A_{15}e^{m_7 y} + B_{31}e^{i\alpha y} \tag{58}$$

$$v_{P11} = A_{13}\left(\frac{1}{\tau m_5 + 1}\right)e^{m_5 y} + A_{14}e^{m_6 y} + A_{16}e^{-\frac{y}{\tau}}$$
$$+A_{15}\frac{1}{(\tau m_7+1)}e^{m_7 y} + B_{31}\frac{1}{(i\tau\alpha+1)}e^{i\alpha y} \tag{59}$$

$$u_{11} = A_{17}e^{m_5 y} + A_{18}e^{m_6 y} + A_{19}e^{m_7 y} + B_{56}e^{(m_1+m_5)y} + B_{57}e^{(m_1+m_6)y}$$
$$+B_{58}e^{(m_1+m_7)y} + B_{59}e^{(m_2+m_5)y} + B_{60}e^{(m_2+m_6)y} + B_{61}e^{(m_2+m_7)y}$$
$$+B_{62}e^{(i\alpha+m_1)y} + B_{63}e^{(i\alpha+m_2)y} + B_{64}e^{\left(m_1-\frac{1}{\tau}\right)y} + B_{65}e^{\left(m_2-\frac{1}{\tau}\right)y.} \tag{60}$$

$$u_{p11} = A_{20}e^{-\frac{y}{\tau}} + A_{17}B_{81}e^{m_5 y} + A_{18}B_{82}e^{m_6 y} + A_{19}B_{83}e^{m7y} + B_{84}e^{(m_1+m_5)y}$$
$$+B_{85}e^{(m_1+m_6)y} + B_{86}e^{(m_1+m_7)y} + B_{87}e^{(m_2+m_5)y} + B_{88}e^{(m_2+m_6)y}$$
$$+B_{89}e^{(m_2+m_7)y} + B_{90}e^{(i\alpha+m_1)y} + B_{91}e^{(i\alpha+m_2)y} + B_{92}e^{\left(m_1-\frac{1}{\tau}\right)y}$$
$$+B_{93}e^{\left(m_2-\frac{1}{\tau}\right)y} + B_{94}e^{\left(m_5-\frac{1}{\tau}\right)y} + B_{95}e^{\left(m_6-\frac{1}{\tau}\right)y} + B_{96}e^{\left(m_7-\frac{1}{\tau}\right)y}$$
$$+B_{97}e^{-\frac{2}{\tau}y} + B_{98}e^{\left(i\alpha-\frac{1}{\tau}\right)y} \tag{61}$$

$$\theta_{11} = A_{21}e^{m_8 y} + A_{22}e^{m_9 y} + A_{23}e^{m_{10} y}B_{131}e^{(m_5+m_3)y} + B_{132}e^{(m_3+m_6)y} + B_{133}e^{(m_3+m_7)y}$$
$$+ B_{134}e^{(m_3+i\alpha)y} + B_{135}e^{(m_4+m_5)y} + B_{136}e^{(m_4+m_6)y} + B_{137}e^{(m_4+m_7)y}$$
$$+ B_{138}e^{(m_4+i\alpha)y} + B_{139}e^{-\left(\lambda\delta\beta+\frac{1}{\tau}\right)y} + B_{140}e^{\left((m_3-\frac{1}{\tau}\right)y} + B_{141}e^{\left((m_4-\frac{1}{\tau}\right)y}$$
$$+ B_{142}e^{((m_5-\lambda\delta\beta)y} + B_{143}e^{((m_6-\lambda\delta\beta)y}$$
$$+B_{144}e^{((m_7-\lambda\delta\beta)y} + B_{145}e^{(i\alpha-\lambda\delta\beta)y} \tag{62}$$

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$$\theta_{p_{11}} = A_{21}B_{146}e^{m_8 y} + A_{22}B_{147}e^{m_9 y} + A_{23}e^{m_{10}y}B_{148} + A_{23}e^{-\lambda\delta\beta y}$$
$$+B_{149}e^{(m_3+m_5)y} + B_{150}e^{(m_3+m_6)y} + B_{151}e^{(m_3+m_7)y} + B_{152}e^{(m_3+i\alpha)y}$$
$$+B_{153}e^{(m_4+m_5)y} + B_{154}e^{(m_4+m_6)y} + B_{155}e^{(m_4+m_7)y} + B_{156}e^{(m_4+i\alpha)y}$$
$$+B_{157}e^{-\left(\lambda\delta\beta+\frac{1}{\tau}\right)y} + B_{158}e^{\left((m_3-\frac{1}{\tau}\right)y} + B_{159}e^{\left((m_4-\frac{1}{\tau}\right)y} + B_{160}e^{((m_5-\lambda\delta\beta)y}$$
$$+B_{161}e^{((m_6-\lambda\delta\beta)y} + B_{162}e^{((m_7-\lambda\delta\beta)y} + B_{163}e^{(i\alpha-\lambda\delta\beta)y} \quad (63)$$

$$\phi_1(y) = A_{25}e^{m_{11}y} + A_{26}e^{m_{12}y} + B_{182}e^{(m_5+Sc\,Re)y} + B_{183}e^{(m_6+Sc\,Re)y}$$
$$+B_{184}e^{(m_7+Sc\,Re)y} + B_{185}e^{i\alpha+Sc\,Re)y} \quad (64)$$

$$q_W = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Skin Friction

The skin friction at the wall due to main flow is given by

$$\tau_x = -\left(\frac{du}{dy}\right)_{y=0} = -\left(\frac{du_0}{dy}\right)_{y=0} + \in\left(\frac{du_1}{dy}\right)_{y=0} + O(\in^2)$$

The skin friction at the wall due to cross flow is given by

$$\tau_z = -\left(\frac{du}{dy}\right)_{y=1} = -\left(\frac{du_0}{dy}\right)_{y=1} + \in\left(\frac{du_1}{dy}\right)_{y=1} + O(\in^2)$$

Nusselt number

The rate of heat transfer is

$$q_W = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

The non dimensional form of Nusselt number at $y = 0$ is

$$Nu_x = -\left(\frac{d\theta}{dy}\right)_{y=0} = -\left(\frac{d\theta_0}{dy}\right)_{y=0} + \in\left(\frac{d\theta_1}{dy}\right)_{y=0} + O(\in^2)$$

The non dimensional form of Nusselt number at $y = 1$ is

$$Nu_x = -\left(\frac{d\theta}{dy}\right)_{y=0} = -\left(\frac{d\theta_0}{dy}\right)_{y=0} + \in\left(\frac{d\theta_1}{dy}\right)_{y=0} + O(\in^2)$$

Sherwood number

The rate mass transfer is

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$$q_c = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

The non dimensional form of Sherwood number at $y = 0$ is

$$Sh_x = -\left(\frac{d\phi}{dy}\right)_{y=0} = -\left(\frac{d\phi_0}{dy}\right)_{y=0} + \epsilon\left(\frac{d\phi_1}{dy}\right)_{y=0} + O(\epsilon^2)$$

The non dimensional form of Sherwood number at $y = 1$ is

$$Sh_x = -\left(\frac{d\phi}{dy}\right)_{y=0} = -\left(\frac{d\phi_0}{dy}\right)_{y=0} + \epsilon\left(\frac{d\phi_1}{dy}\right)_{y=0} + O(\epsilon^2)$$

Results and Discussion

The velocity, temperature and concentration profiles are plotted numerically. The effect of various pertinent parameters on the profiles is discussed through these plots. Further, skin friction, Nusselt number and Sherwood number are also analyzed for various dimensionless parameters.

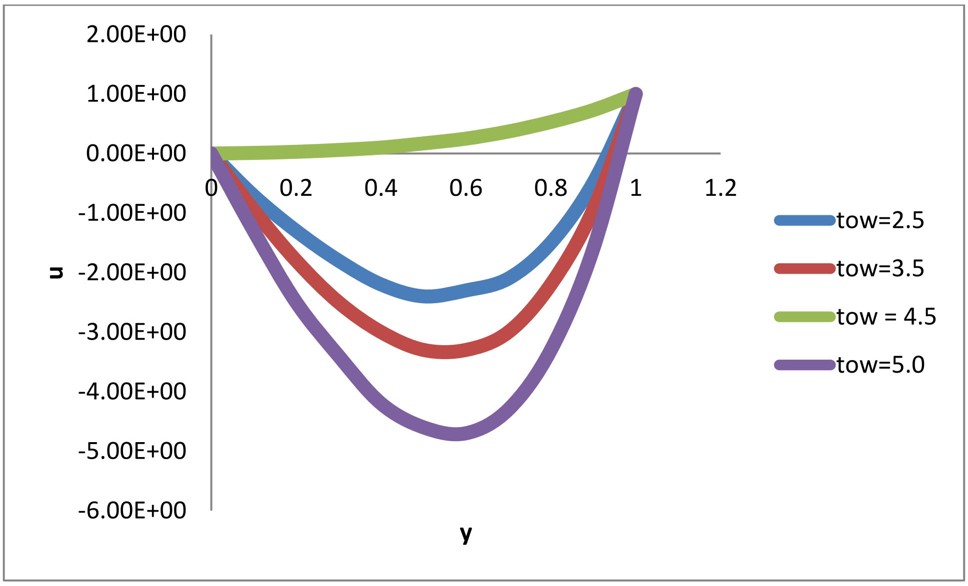

Fig 2. Main flow velocity as a function of relaxation time parameter (τ)

We know that relaxation time measures the speed with which the dust particle blends with the fluid. We can see from this Fig 2. that the relaxation time parameter is having an enhancing effect on the magnitude of the main flow velocity of fluid. In this graph Prandtl number is fixed as 0.71.

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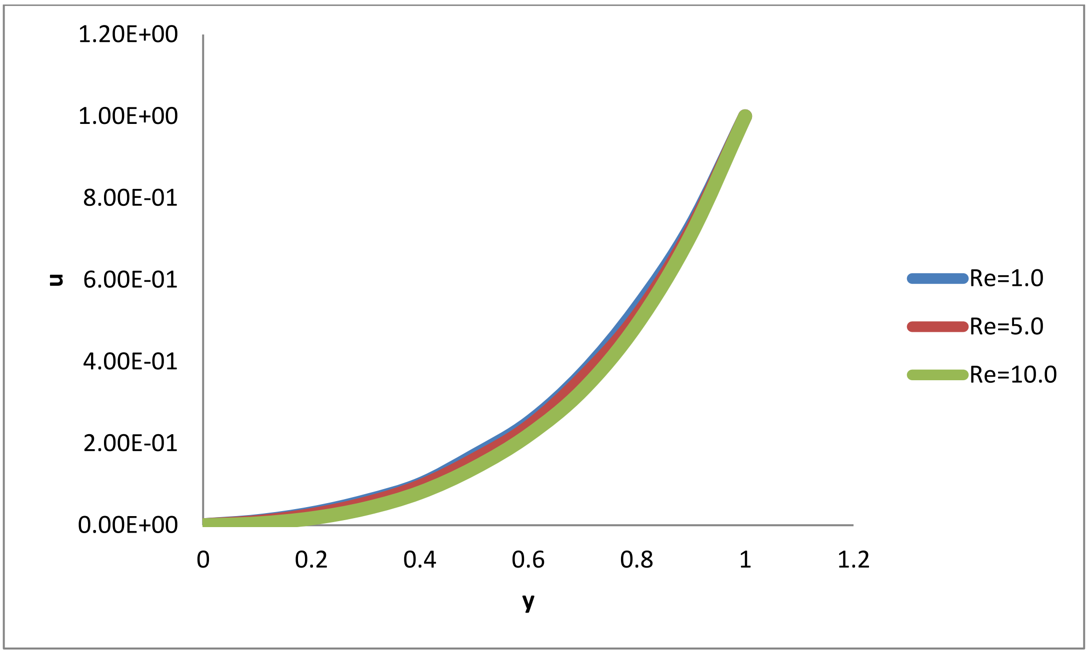

Fig 3. Main flow velocity as a function of Reynolds number (*Re*)

Reynolds number which measures the impact of inertial force to viscous force is shown in this figure. It is evident from the above Fig. 3 that as Reynolds number increases the main flow velocity decreases. This can be considered as an important impact of the presence of the dust particles and the viscosity. Both the presence of the dust particles and the viscous force are found to retard the flow velocity.

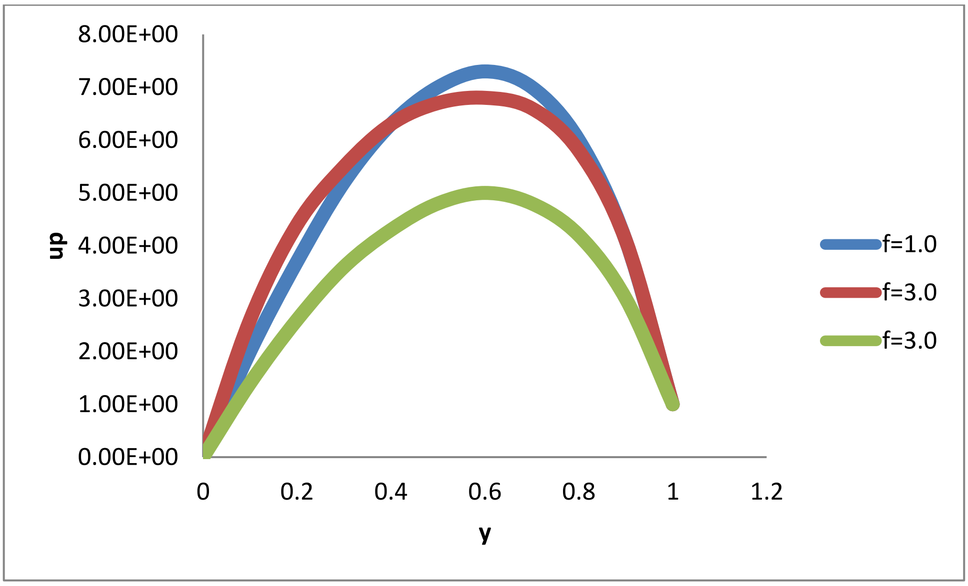

Fig. 4 Main flow velocity for various f

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The behavior of the fluid as a function of the mass concentration f of the dust particles is shown through Fig. 4. It is observed that, When the mass concentration parameter is increasing the fluid velocity of the dust particles slows down.

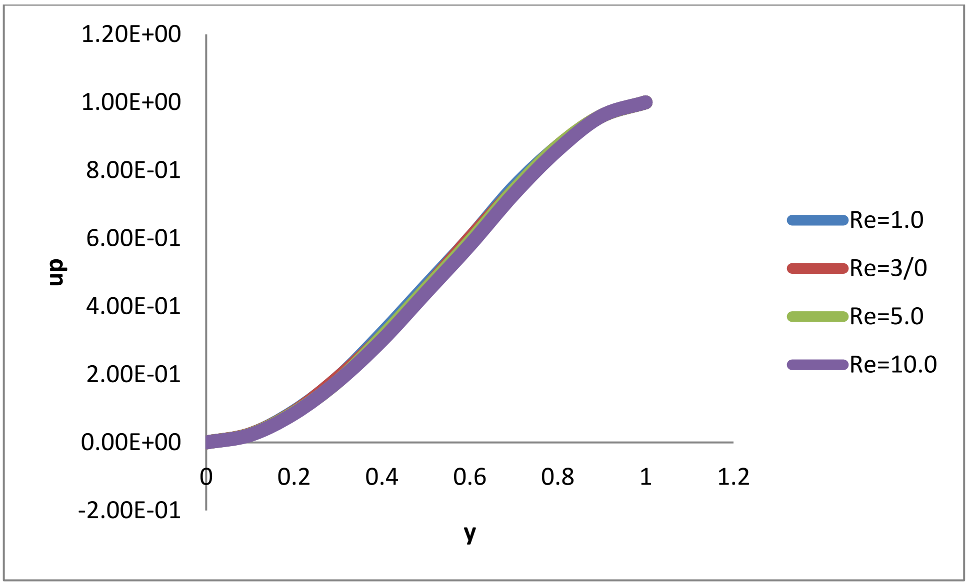

Fig. 5 Main flow velocity of dust particle phase for various Re

In Fig. 5 the influence of the Reynolds number on the main flow velocity of the dust particles is seen. It is found that, though the velocity of the dust particles is found to be decreasing due to increase in the Reynolds number it is not very significant.

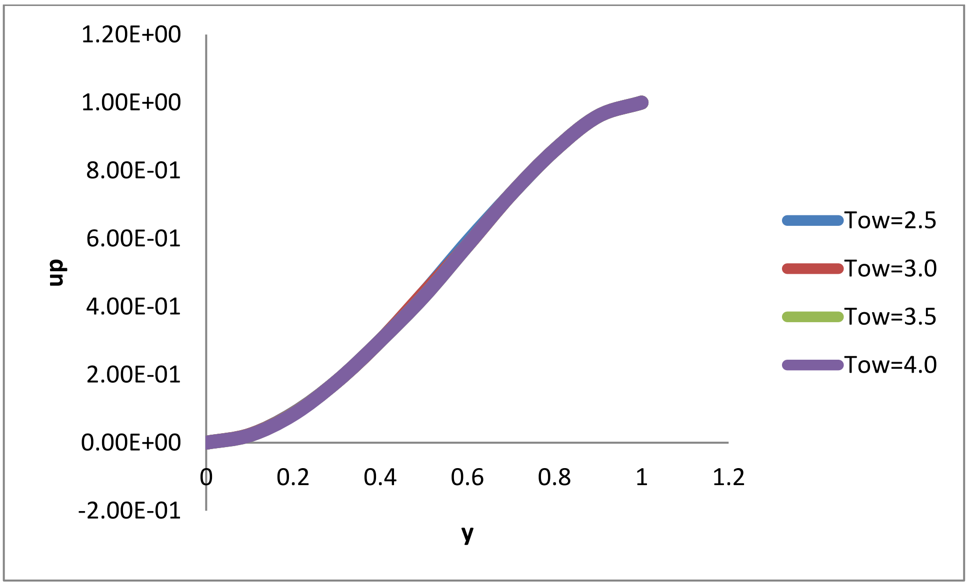

Fig. 6 Main flow velocity of dust particle phase for various τ

Fig. 6 discusses the main flow velocity of dust particles for various τ. From Fig 6 we can conclude that, the parameter τ emphasizes the quickness with which dust particles blend with

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the main flow. In this figure we can observe that the velocities of the dust particles are not modified greatly with the increase in the relaxation time parameter τ.

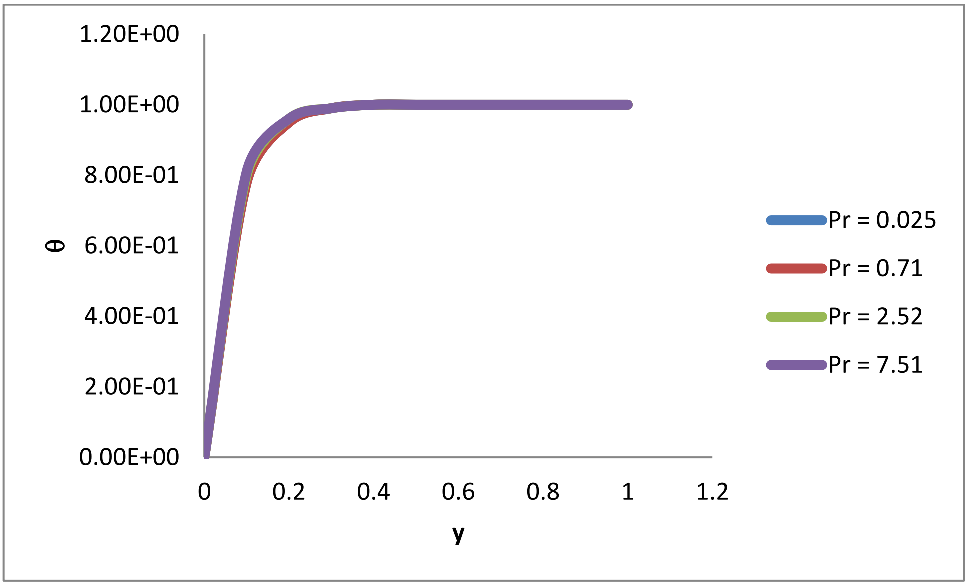

Fig 7 Temperature profile for various Pr (fluid Phase)

The temperature profile for various Prandtl number is shown through Fig. 7. From Fig. 7, we can conclude that, with the increase in Pr, the temperature profilr for the fluid phase increases.

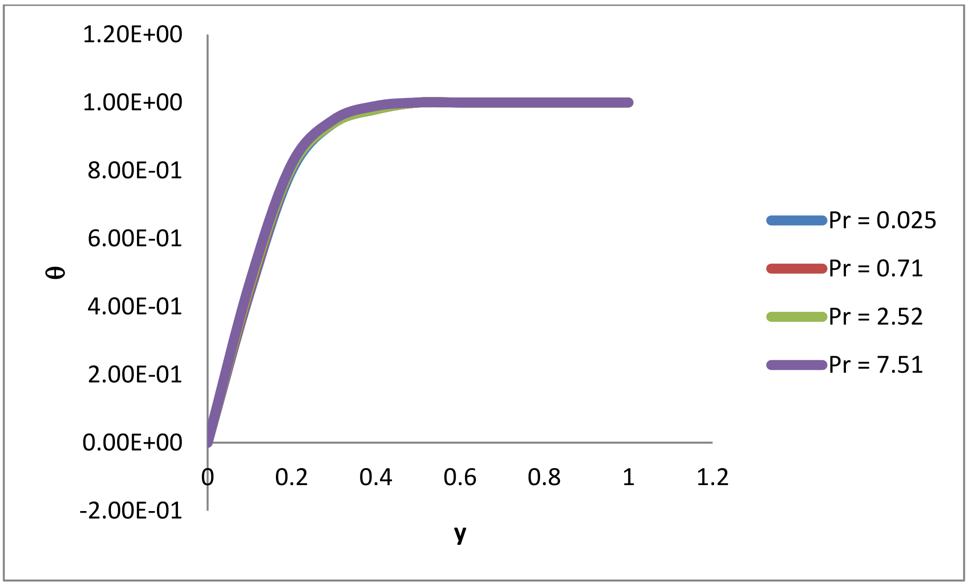

Fig. 8 Temperature profile for various Pr (Particle Phase)

Fig 8 presents the temperature profile for various Pr. It is observed that, with the enhancement of Pr in particle phase, the temperature profile increases but not much variation is seen.

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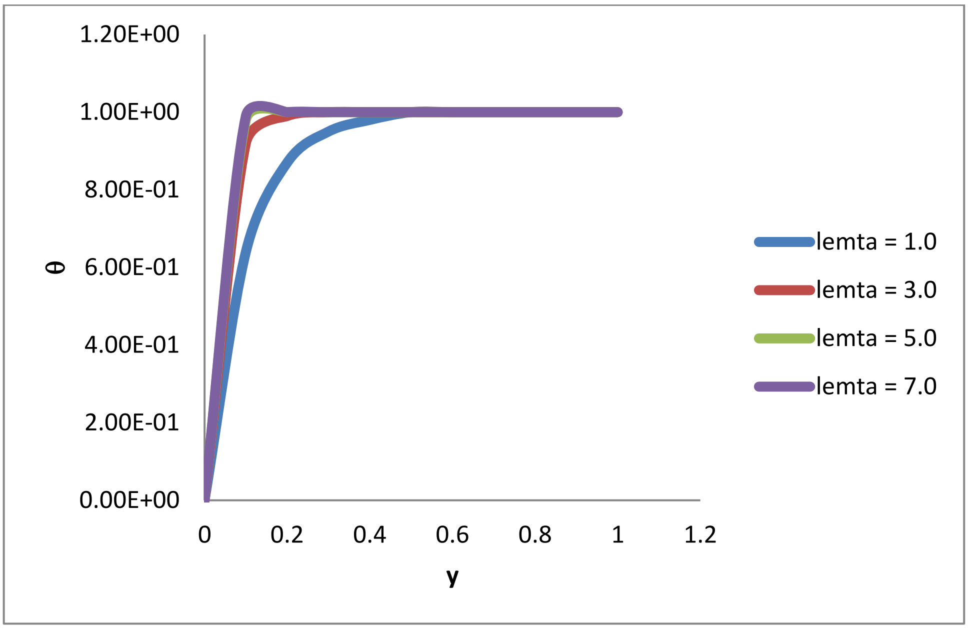

Fig. 9 Temperature profile for various λ (Fluid phase)

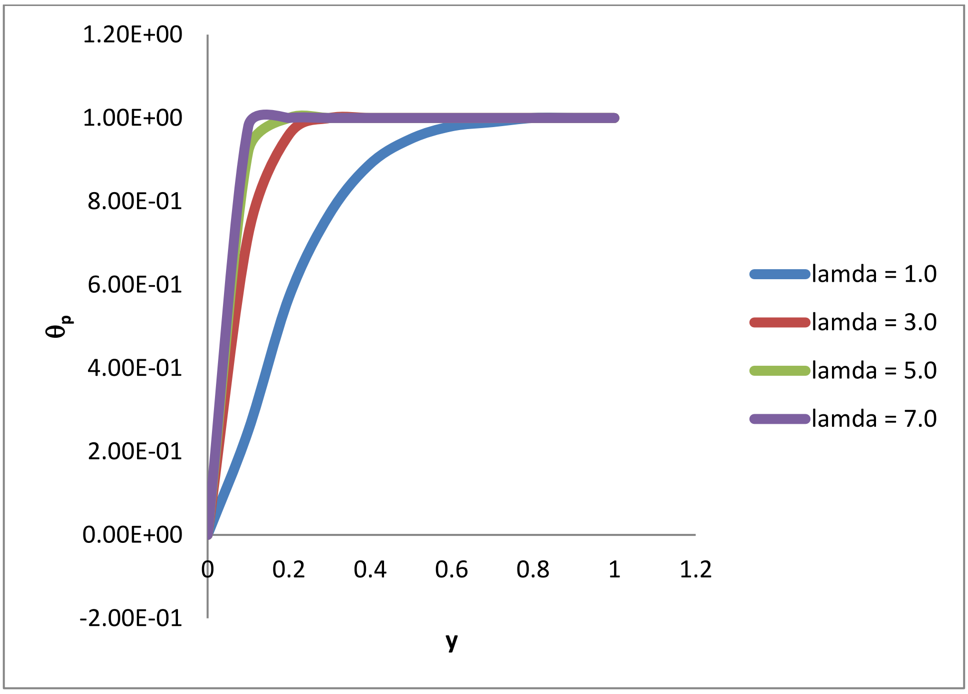

Fig. 10 Temperature profile for various λ (Particle phase)

The temperature profile for various λ for both fluid and particle phase are shown through Figs. 9 and 10. This can be readily understood from these figures that, temperature profile enhances with the enhancement of λ.

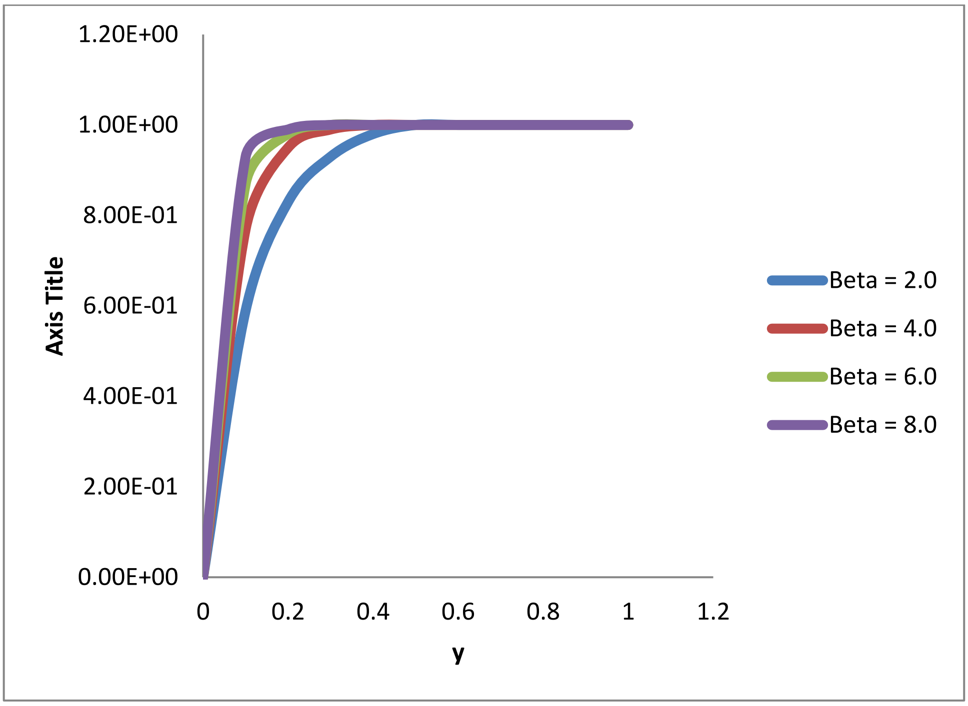

Fig. 11 Temperature profile for various β (Fluid Phase)

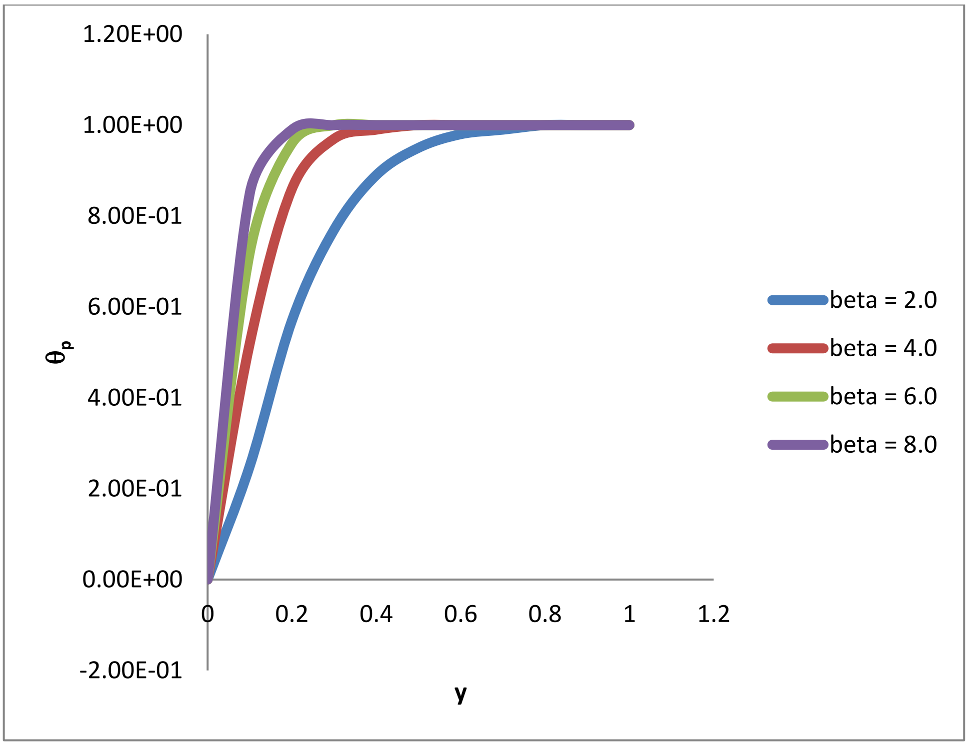

Fig. 12 Temperature profile for various β (Particle Phase)

The effect of temperature on fluid and particle phase for various β is depicted in Fig. 11 and Fig. 12. It can be seen from these figures that, the temperature field increases with the raise of β.

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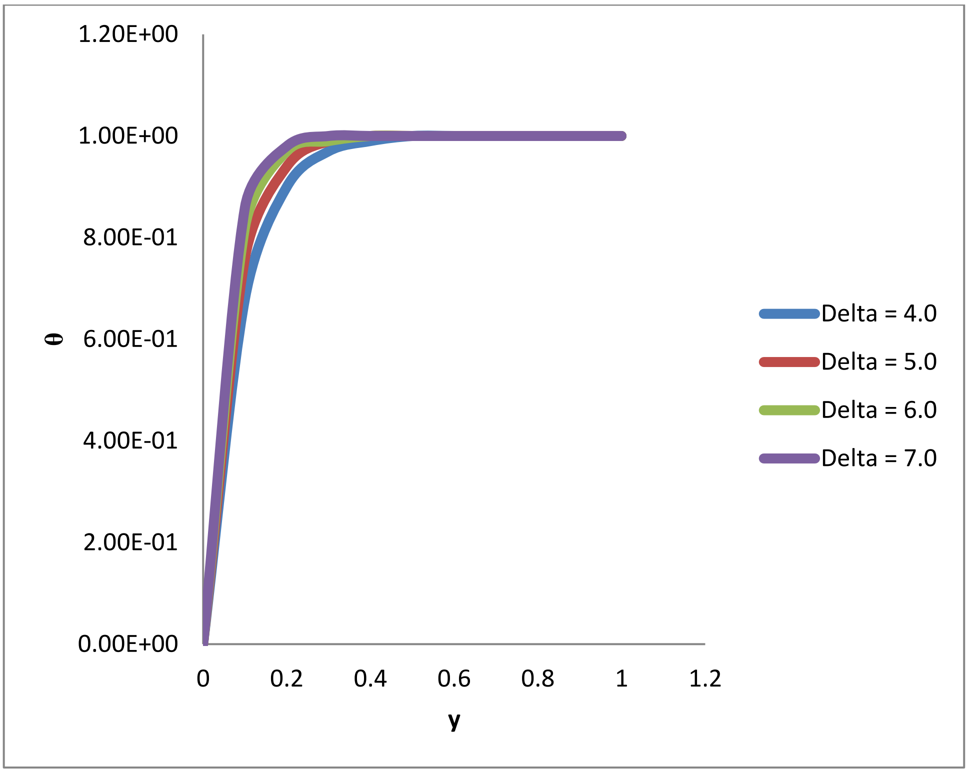

Fig. 13 Temperature profile for various δ(Fluid Phase)

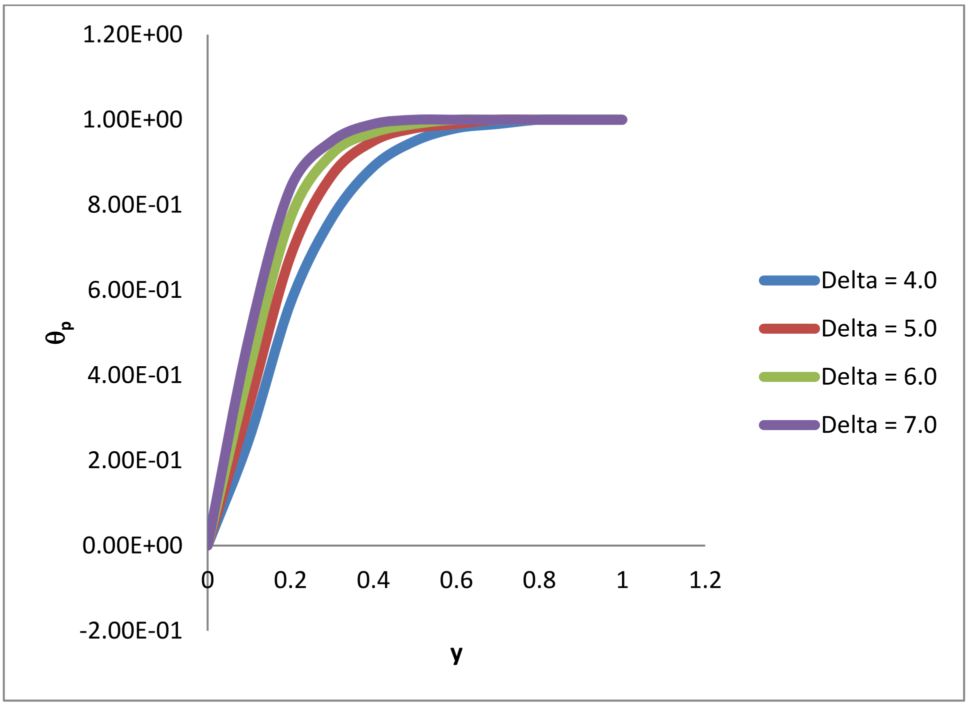

Fig. 14 Temperature profile for various δ(Particle Phase)

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The effect of δ on temperature filed for both fluid and particle phase are shown through Figs. 13 and 14. The temperature increases significantly with the increase in δ.

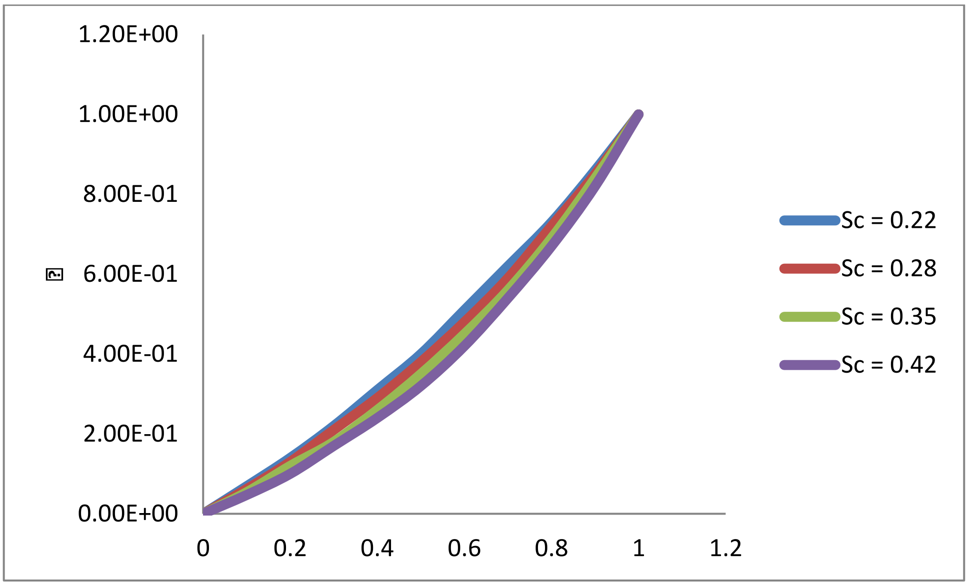

Fig. 15 Concentration profile for various Sc

The effect of Schmidt number on concentration field is observed in Fig. 15. An increase in δ, decreases the concentration profile.

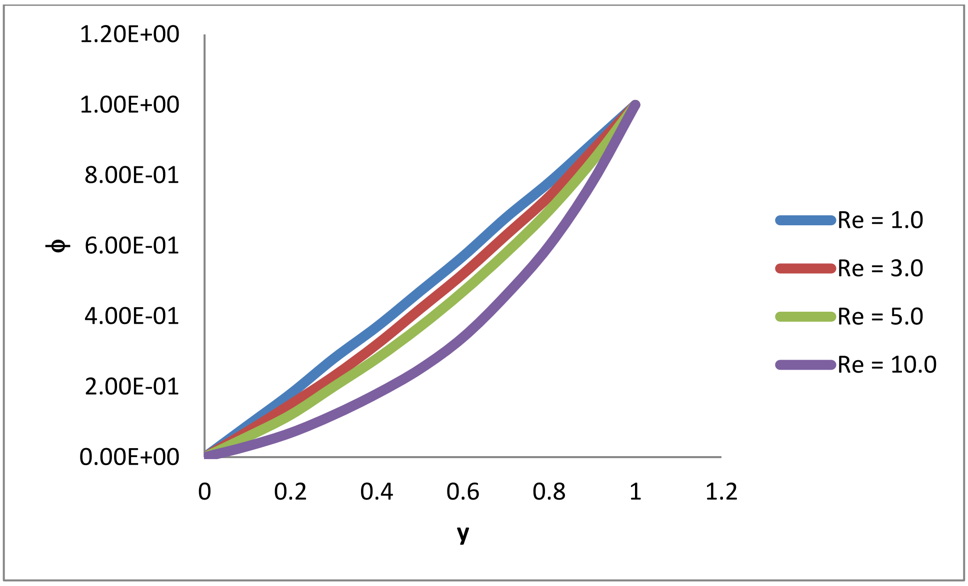

Fig. 16 Concentration profile for various Re

The impact of Reynolds number on Concentration filed is shown through Fig. 16. We can conclude that, with the increase in Re, the concentration filed decreases.

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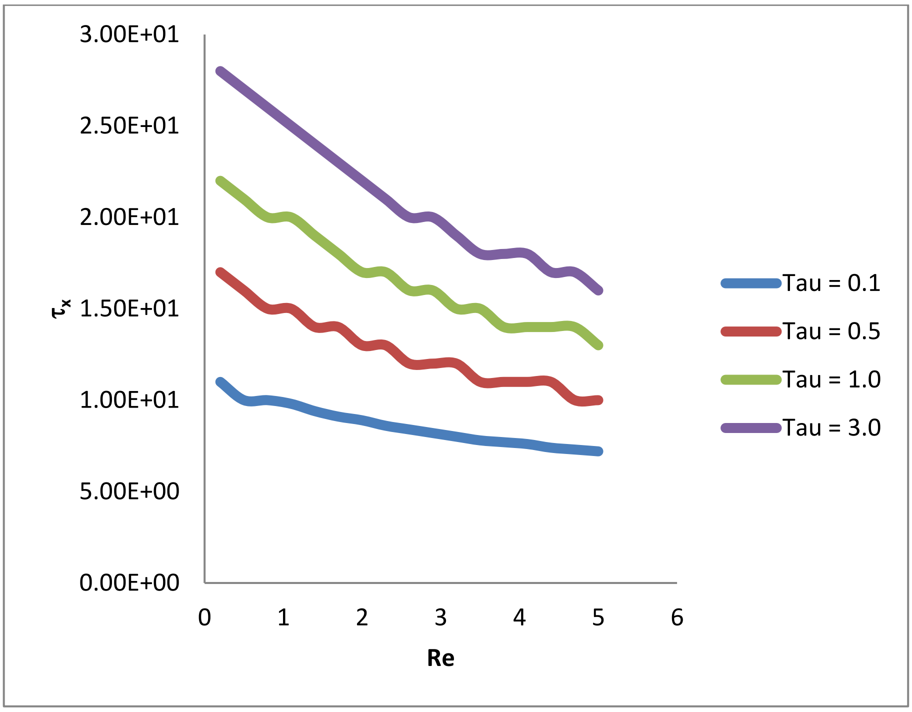

Fig. 17 Skin friction for various τ at y = 0

The impact of skin friction on τ is depicted in Fig. 17. It can be shown from the figure that, skin friction at y = 0 increases with the increase in τ

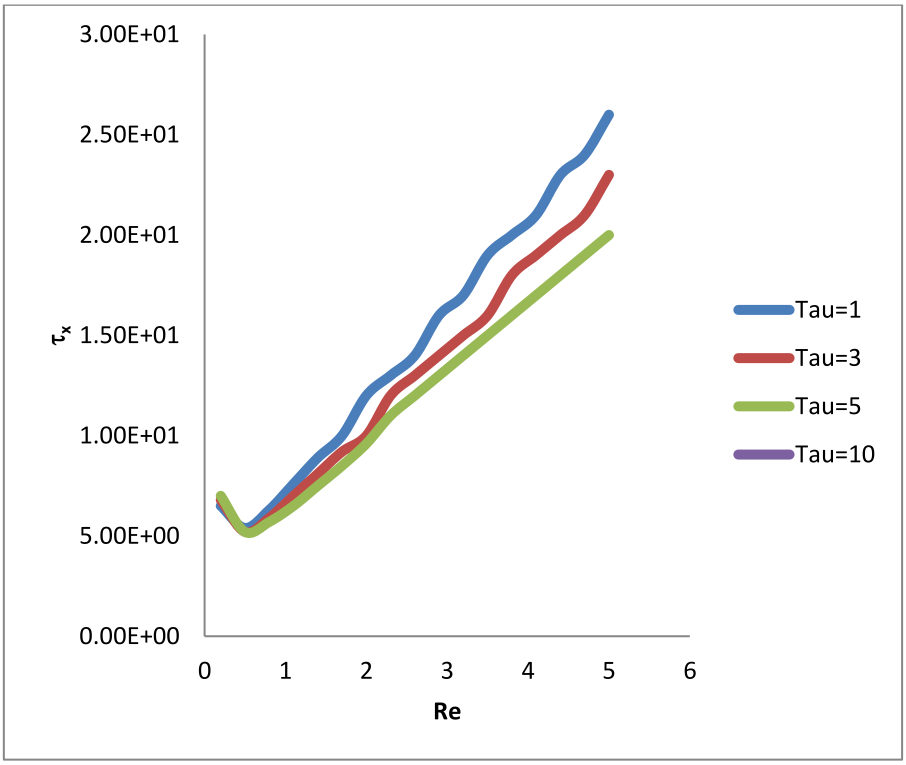

Fig. 18 Skin friction for various τ at y = 1

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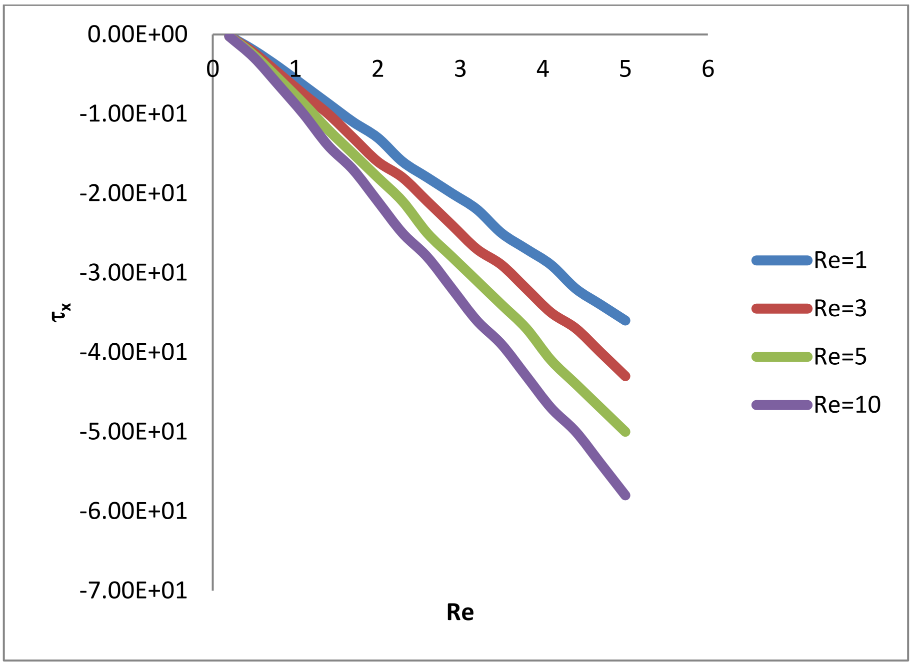

Fig. 19 Skin friction for various Re at y = 1

The effect of skin friction for various τ and Re is shown through Fig. 18 and 19. As expected, skin friction at the upper plate decreases with the enhancement of τ and Re.

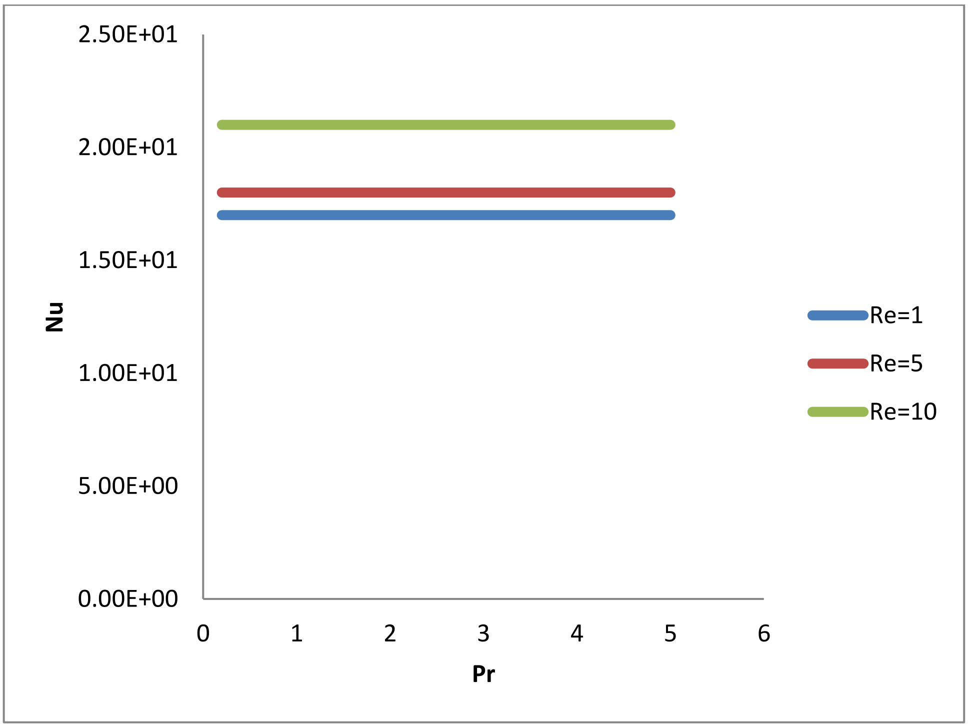

Fig. 20 Nusselt number for various Re at y = 0

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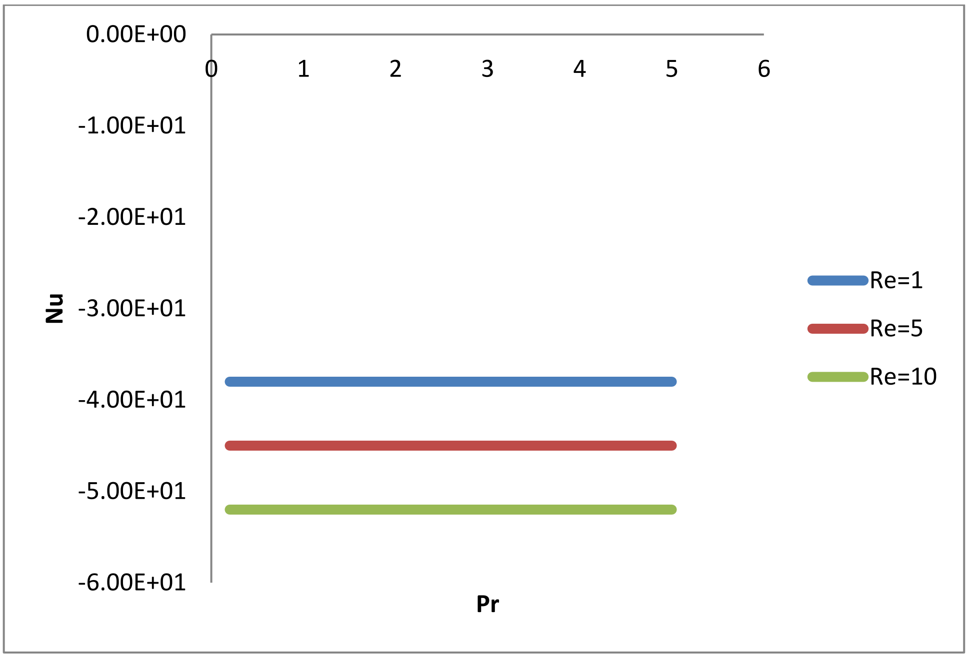

Fig. 21 Nusselt number for various at y = 1

Fig. 20 and 21 presents the impact of Nusselt number for various Reynolds number at lower and upper plates. This is readily understood from figures that, Nusselt number enhances with the enhancement of Re at the lower plate, depreciates with the enhancement of Re at the upper plate.

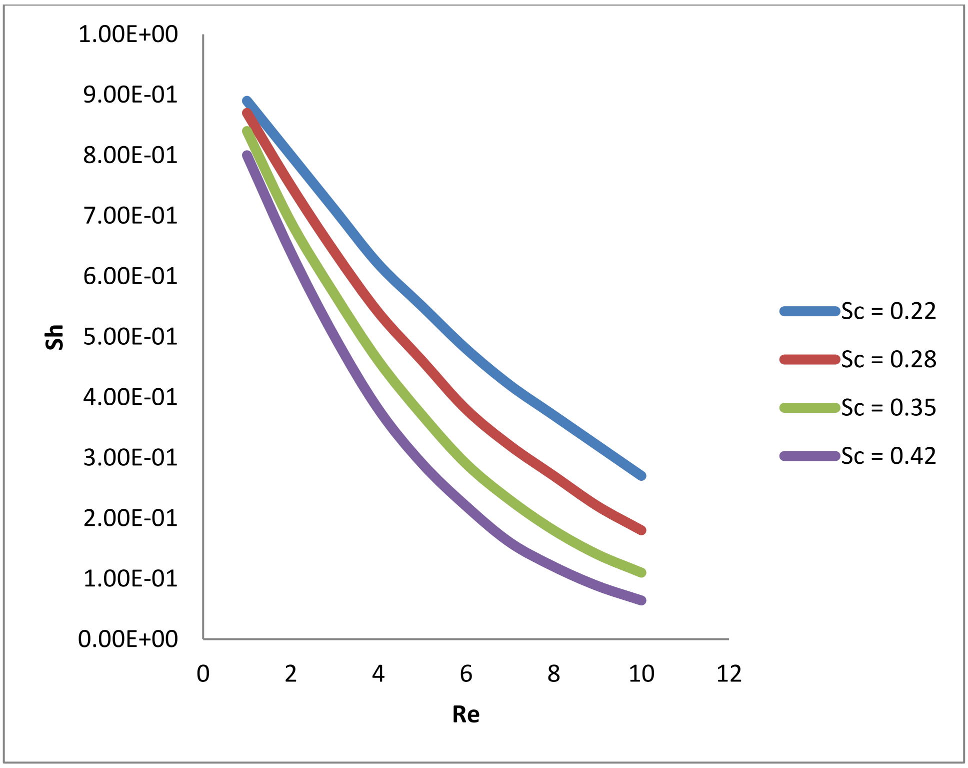

Fig. 22 Sherwood number for various Sc at y = 0

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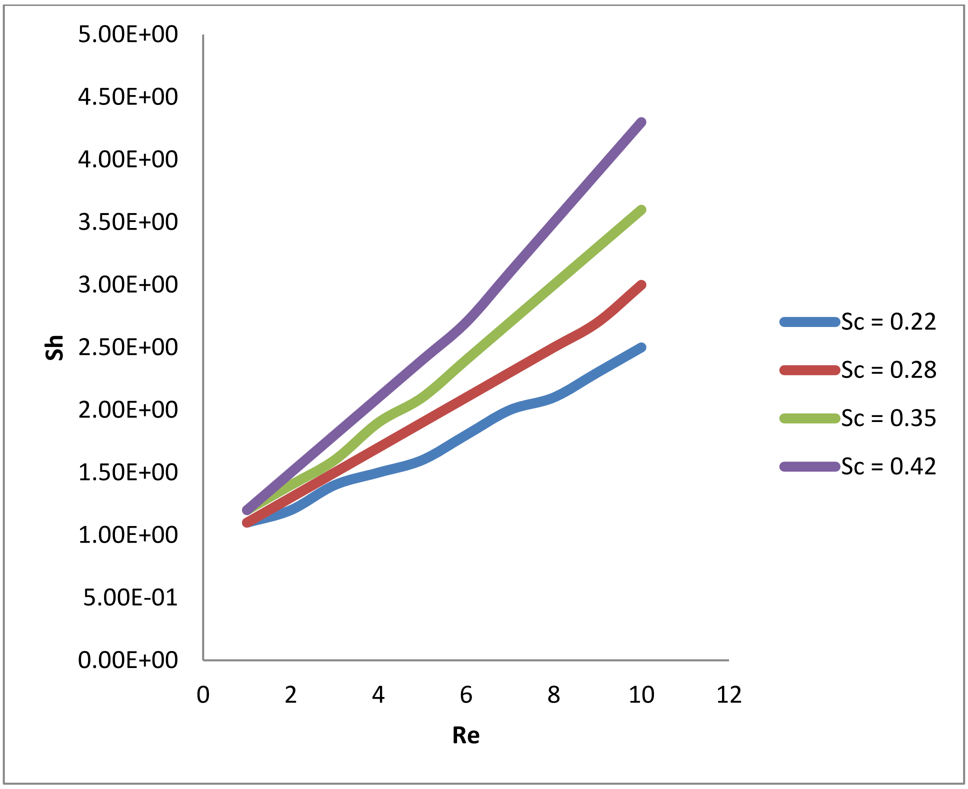

Fig. 23 Sherwood number for various Sc at y = 1

Fig 22 and 23 represent the impact of Schmidt number on the mass transfer at boundaries . It can be seen that, Sherwood number decreases with the increase in Sc at the lower plate, increases with the rise of Re and Sc at the upper plate.

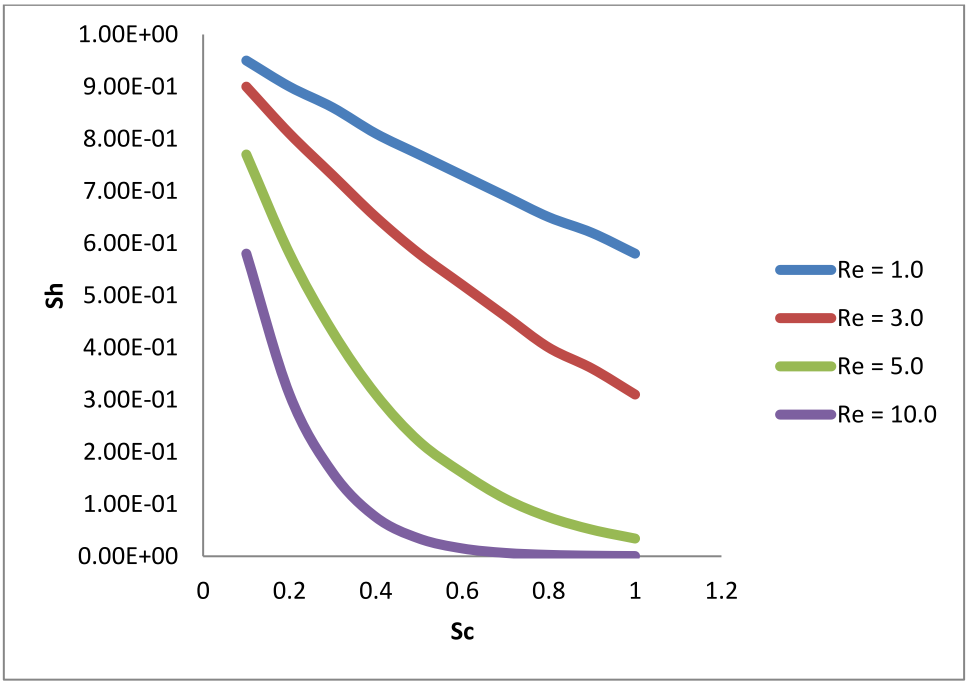

Fig. 24 Sherwood number for various Re at y = 0

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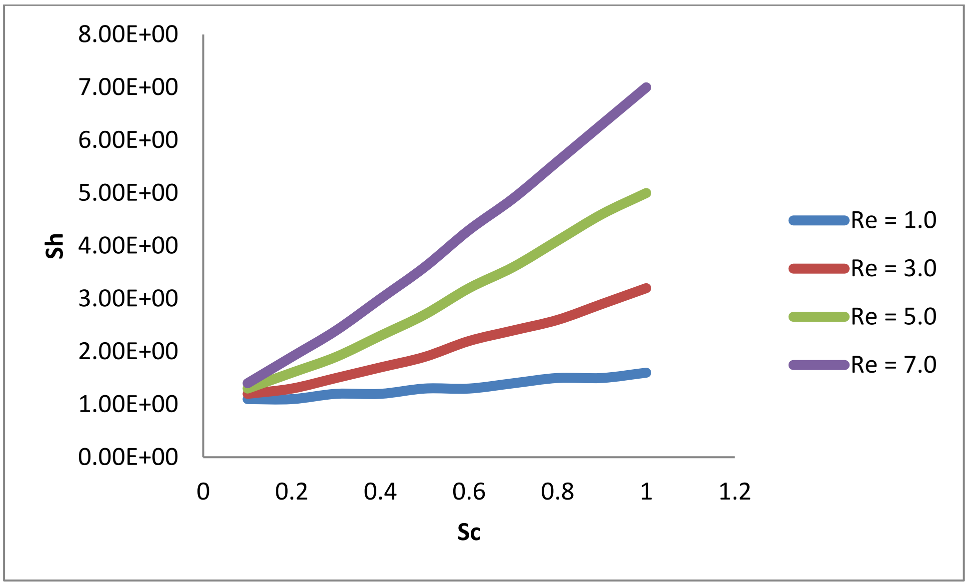

Fig. 25 Sherwood number for various Re at y = 1

Fig 24 and 25 represents the impact of Sherwood number on Re. It can be understood that, Sherwood number decreases as Sc rises with the increase in Sc at the lower plate, increases with the rise of Re and Sc at the upper plate.

Conclusion

The effect of dust particles on three dimensional Couette flow between two porous plates of a viscous incompressible fluid is analyzed in this research paper. The lower plate is stationary while the upper plate is undergoing uniform motion in its plane. These plates are, respectively subjected to transverse exponential injection and its corresponding removal by constant suction.

The key findings are

✓ The main flow velocity enhances with the increase in relaxation time parameter, decreases with the enhancement of Reynolds number, mass concentration parameter.

✓ The main flow velocity of dust particles decreases with the increase in Reynolds number, increases with the rise of Prandtl number.

✓ The temperature profile enhances with the raise of λ, β and δ for both fluid and particle phase.

✓ The concentration field decreases with the increase in δ and Re.

✓ The skin friction increases at the lower plate, decreases at the upper plate with the increase in τ and Re.

✓ Nusselt number enhances with the enhancement of Re at the lower plate, depreciates with the enhancement of Re at the upper plate.

✓ Sherwood number decreases with the increase in Sc at the lower plate, increases with the rise of Re and Sc at the upper plate.

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Appendix

$V_0 = 1$

$W_0 = 0$

$V_{P_0} = 1$

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$W_{P_0} = 0$

$B_1 = e^{S_c R_e} - 1$

$B_2 = -1$

$B_3 = 1$

$A_1 = B_2/B_1$

$A_2 = B_3/B_1$

$\phi_0(y) == A_1 + A_2 e^{S_c R_e y}$

$$m_1 = \frac{-\left(\frac{1}{\tau} - R_e\right) + \sqrt{\left(\frac{1}{\tau} - R_e\right)^2 + 4\left(\frac{R_e}{\tau} - \frac{f_e}{\tau}\right)}}{2}$$

$$m_2 = \frac{-\left(\frac{1}{\tau} - R_e\right) - \sqrt{\left(\frac{1}{\tau} - R_e\right)^2 + 4\left(\frac{R_e}{\tau} - \frac{f_e}{\tau}\right)}}{2}$$

$$B_4 = \frac{1}{m_1\tau + 1} - 1$$

$$B_5 = \frac{1}{m_2\tau + 1} - 1$$

$B_6 = e^{m_1} - 1$

$B_7 = e^{m_2} - 1$

$$B_8 = \frac{e^{m_1}}{m_1\tau + 1} - 1$$

$$B_9 = \frac{e^{m_2}}{m_2\tau + 1} - 1$$

$B_{10} = e^{-1/\tau}$

$B_{11} = B_6B_9 - B_6B_5B_{10} - B_7B_8 + B_7B_4B_{10}$

$B_{12} = B_9 - B_5B_{10} - B_7$

$B_{13} = B_6 - B_8 + B_4B_{10}$

$B_{14} = -B_6B_5 + B_4B_7 + B_5B_8 - B_4B_9$

$B_{15} = \lambda\beta\delta - P_r R_e$

$B_{16} = -P_r R_e \lambda\beta\delta - P_r R_e \delta$

$A_4 = B_{12}/B_{11}$

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$A_5 = B_{13}/B_{11}$

$A_6 = B_{14}/B_{11}$

$A_3 = -A_4 - A_5$

$u_0 = A_3 + A_4 e^{m_1 y} + A_5 e^{m_2 y}$

$u_{p_0} = A_3 + A_4 \frac{e^{m_1 y}}{m_1 \tau + 1} + A_5 \frac{e^{m_2 y}}{m_2 \tau + 1} + A_6 e^{-y/\tau}$

$m_3 = \frac{-B_{15} + \sqrt{B_{15}{}^2 - 4B_{16}}}{2}$

$m_4 = \frac{-B_{15} - \sqrt{B_{15}{}^2 - 4B_{16}}}{2}$

$B_{17} = \frac{\lambda\beta\delta}{m_3 + \lambda_\beta \delta}$

$B_{18} = \frac{\lambda\beta\delta}{m_3 + \lambda\beta\delta}$

$B_{19} = B_{17} - 1$

$B_{20} = B_{18} - 1$

$B_{21} = e^{m_3} - 1$

$B_{22} = e^{m_4} - 1$

$B_{23} = B_{17} e^{m_3} - 1$

$B_{24} = B_{18} e^{m_4} - 1$

$B_{25} = e^{-\lambda\beta\delta}$

$B_{26} = B_{21}B_{24} - B_{21}B_{20}B_{25} - B_{22}B_{23} + B_{22}B_{19}B_{25}$

$B_{27} = B_{24} - B_{20}B_{25} - B_{22}$

$B_{28} = B_{21} - B_{23} + B_{25}B_{19}$

$B_{29} = -B_{20}B_{21} + B_{22}B_{19} + B_{23}B_{20} - B_{19}B_{24}$

$A_8 = B_{27}/B_{26}$

$A_9 = B_{28}/B_{26}$

$A_{10} = \frac{B_{29}}{B_{26}}$

$A_7 = -A_8 - A_9$

$\theta_0(y) = A_7 + A_8 e^{m_3 y} + A_9 e^{m_4 y}$

$\theta_{P_0}(y) = A_{10}e^{-\lambda\beta\delta y} + A_7 + A_8B_{17}e^{m_3y} + A_9B_{18}e^{m_4y}$

$P_{11}(y) = A_{11}e^{i\alpha y} + A_{12}e^{-i\alpha y}$

$B_{30} = R_e\left(-\alpha^2 + \frac{i\alpha}{\tau}\right)$

$g_2 = 1$

$g_2 = \left(\frac{1}{\tau} - R_e\right)$

$g_3 = \alpha^2 - \frac{R_e}{\tau} - \frac{f\ Re}{\tau}$

$g_4 = \frac{\alpha^2}{\tau}$

$g_5 = -27g_1^2g_4 + 9g_1g_2g_3 - 2g_2^3$

$g_6 = (3g_1g_3 - g_2^2)^3$

$g_7 = \left((g_5^2 + 4g_6)^{1/2} - g_5\right)^{1/3}$

$m_5 = \frac{g_7}{3(2)^{1/3}g_1} - \frac{(2)^{1/3}g_6}{3g_1g_7} - \frac{g_2}{3g_1}$

$m_6 = -\frac{(1 - i\sqrt{3})g_7}{6(2)^{1/3}g_1} + \frac{(1 + i\sqrt{3})g_6}{3(4)^{1/3}g_1g_7} - \frac{g_2}{3g_1}$

$m_7 = -\frac{(1 + i\sqrt{3})g_7}{6(2)^{1/3}g_1} + \frac{(1 - i\sqrt{3})g_6}{3(4)^{1/3}g_1g_7} - \frac{g_2}{3g_1}$

$B_{31} = \frac{B_{30}}{(i\alpha)^3 + g_2(i\alpha)^2 + g_3(i\alpha) + g_4}$

$B_{32} = \frac{1}{\tau m_6 + 1} - \frac{1}{\tau m_5 + 1}$

$B_{33} = \frac{1}{\tau m_7 + 1} - \frac{1}{\tau m_5 + 1}$

$B_{34} = 1 - \left[B_3\left(\frac{1}{i\tau\alpha + 1}\right) + (1 - B_{31})\left(\frac{1}{\tau m_5 + 1}\right)\right]$

$B_{35} = e^{m_6} - e^{m_5}$

$B_{36} = e^{m_7} - e^{m_5}$

$B_{37} = -\left(B_{31}e^{i\alpha} + (1 - B_{31})e^{m_5}\right)$

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$$B_{38} = \frac{e^{m_6}}{\tau m_6 + 1} - \frac{e^{m_5}}{\tau m_5 + 1}$$

$$B_{39} = \frac{e^{m_7}}{\tau m_7 + 1} - \frac{e^{m_5}}{\tau m_5 + 1}$$

$$B_{40} = e^{-1/\tau}$$

$$B_{41} = -\left[B_{31}\frac{e^{i\alpha}}{i\tau\alpha + 1} + (1 - B_{31})\frac{e^{m_5}}{\tau m_5 + 1}\right]$$

$$B_{42} = -B_{38}B_{31} + B_{39}B_{35} + B_{40}(B_{32}B_{36} - B_{33}B_{35})$$

$$B_{43} = -B_{36}B_{41} + B_{39}B_{37} + B_{40}(B_{34}B_{36} - B_{33}B_{37})$$

$$B_{44} = -B_{38}B_{37} + B_{41}B_{35} + B_{40}(B_{32}B_{37} - B_{34}B_{35})$$

$$B_{45} = B_{38}(B_{33}B_{37} - B_{37}B_{36}) - B_{39}(B_{32}B_{37} - B_{34}B_{35}) + B_{41}(B_{32}B_{36} - B_{33}B_{35})$$

$$A_{14} = \frac{B_{43}}{B_{42}}$$

$$A_{15} = \frac{B_{44}}{B_{42}}$$

$$A_{16} = \frac{B_{45}}{B_{42}}$$

$$V_{11} = A_{13}e^{m_5 y} + A_{14}e^{m_6 y} + A_{15}e^{m_7 y} + B_{31}e^{i\alpha y}$$

$$v_{p11} = A_{13}\left(\frac{1}{\tau m_5 + 1}\right)e^{m_5 y} + A_{14}\frac{e^{\tau m_6}}{(\tau m_6 + 1)} + \frac{A_{15}}{\tau m_7 + 1}e^{m_7 y} + A_{16}e^{-y/\tau} + B_{31}\frac{1}{i\tau\alpha + 1}e^{i\alpha y}$$

$$B_{46} = A_4A_{13}m_1m_5(\mathrm{Re}) + \mathrm{Re}m_1^2A_4A_{13} + \frac{\mathrm{Re}}{\tau}A_{13}A_4m_1 + \left(\frac{fRe}{\tau}\right)A_4A_{13}\frac{m_1}{m_1\tau + 1}\frac{1}{(\tau m_5 + 1)}$$

$$B_{47} = A_4A_{14}m_1m_6(Re) + \mathrm{Re}m_1^2A_4A_{14} + \frac{Re}{\tau}A_{14}A_4m_1 + \left(\frac{fRe}{\tau}\right)A_4A_{14}\frac{m_1}{m_1\tau + 1}\frac{1}{(\tau m_6 + 1)}$$

$$B_{48} = A_4A_{15}m_1m_7(Re) + ReA_4A_{15}m_1^2 + \frac{Re}{\tau}A_{15}A_4m_1 + \left(\frac{fRe}{\tau}\right)A_4A_{15}\frac{1}{m_7\tau + 1}\frac{m_1}{m_1\tau + 1}$$

$$B_{49} = A_5A_{13}m_2m_5(Re) + ReA_5A_{13}m_2^2 + \frac{Re}{\tau}A_{13}A_5m_2 + \left(\frac{fRe}{\tau}\right)A_5A_{13}\frac{1}{m_5\tau + 1}\frac{m_2}{m_2\tau + 1}$$

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$$B_{50} = \mathrm{Re}A_5A_{14}m_2m_6 + A_5A_{14}m_2^2\mathrm{Re} + \frac{Re}{\tau}A_{14}A_5m_2 + \left(\frac{fRe}{\tau}\right)A_5A_{14}\frac{m_2}{m_2\tau+1}\frac{1}{\tau m_6+1}$$

$$B_{51} = \mathrm{Re}\ A_5A_{15}m_2m_7 + A_5A_{15}m_2^2\mathrm{Re} + \frac{Re}{\tau}A_{15}A_5m_2 + A_5A_{15}\frac{m_2}{m_2\tau+1}\frac{1}{\tau m_7+1}\frac{fRe}{\tau}$$

$$B_{52} = \mathrm{Re}B_{31}A_4m_1^2 + \frac{\mathrm{Re}}{\tau}B_{31}A_4m_1 + B_{31}\frac{1}{i\tau\alpha+1}\left(\frac{fR_e}{\tau}\right)A_4\frac{m_1}{m_1\tau+1}$$

$$B_{53} = R_eB_{31}A_5m_2^2 + \frac{Re}{\tau}B_{31}A_5m_2 + B_{31}\frac{1}{i\tau\alpha+1}\left(\frac{fR_e}{\tau}\right)A_5\frac{m_2}{m_2\tau+1}$$

$$B_{54} = f\frac{\mathrm{Re}}{\tau}A_4\frac{m_1}{m_1\tau+1}A_{16}$$

$$B_{55} = \frac{f\mathrm{Re}}{\tau}A_5\frac{m_2}{m_2\tau+1}A_{16}.$$

$$B_{56} = \frac{B_{46}}{(m_1+m_5)^3 + \left(-Re+\frac{1}{\tau}\right)(m_1+m_5)^2 + \left(-\frac{Re}{\tau}+\alpha^2-\frac{Re}{\tau}\right)(m_1+m_5) + \frac{\alpha^2}{\tau}}$$

$$B_{57} = \frac{B_{47}}{(m_1+m_6)^3 + \left(-R_e+\frac{1}{\tau}\right)(m_1+m_6)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(m_1+m_6) + \frac{\alpha^2}{\tau}}$$

$$B_{58} = \frac{B_{48}}{(m_1+m_7)^3 + \left(-R_e+\frac{1}{\tau}\right)(m_1+m_7)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(m_1+m_7) + \frac{\alpha^2}{\tau}}$$

$$B_{59} = \frac{B_{49}}{(m_2+m_5)^3 + \left(-R_e+\frac{1}{\tau}\right)(m_2+m_5)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(m_2+m_5) + \frac{\alpha^2}{\tau}}$$

$$B_{60} = \frac{B_{50}}{(m_2+m_6)^3 + \left(-R_e+\frac{1}{\tau}\right)(m_2+m_6)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(m_2+m_6) + \frac{\alpha^2}{\tau}}$$

$$B_{61} = \frac{B_{51}}{(m_2+m_7)^2 + \left(-R_e+\frac{1}{\tau}\right)(m_2+m_7)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(m_2+m_7) + \frac{\alpha^2}{\tau}}$$

$$B_{62} = \frac{B_{52}}{(i\alpha+m_1)^2 + \left(-R_e+\frac{1}{\tau}\right)(i\alpha+m_1)^2 + \left(-\frac{R_e}{\tau}+\alpha^2-\frac{R_e}{\tau}\right)(i\alpha+m_1) + \frac{\alpha^2}{\tau}}$$

$$B_{63} = \frac{B_{53}}{(i\alpha+m_2)^2 + \left(-Re+\frac{1}{\tau}\right)(i\alpha+m_2)^2 + \left(\frac{-Re}{\tau}+\alpha^2-\frac{Re}{\tau}\right)(i\alpha+m_2) + \frac{\alpha^2}{\tau}}$$

$$B_{64} = \frac{B_{54}}{\left(m_1-\frac{1}{\tau}\right)^3 + \left(-Re+\frac{1}{\tau}\right)\left(m_1-\frac{1}{\tau}\right)^2 + \left(\frac{-Re}{\tau}+\alpha^2-\frac{Re}{\tau}\right)\left(m_1-\frac{1}{\tau}\right) + \frac{\alpha^2}{\tau}}$$

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$$B_{65} = \frac{B_{55}}{\left(m_2 - \frac{1}{\tau}\right)^3 + \left(-Re + \frac{1}{\tau}\right)\left(m_2 - \frac{1}{\tau}\right)^2 + \left(-\frac{Re}{\tau} + \alpha^2 - \frac{Re}{\tau}\right)\left(m_2 - \frac{1}{\tau}\right) + \frac{\alpha^2}{\tau}}$$

$$B_{66} = \frac{B_{56}}{\tau} - A_4 A_{13} \frac{m_1}{m_1\tau + 1} \frac{1}{\tau m_5 + 1}$$

$$B_{67} = \frac{B_{57}}{\tau} - A_4 A_{14} \frac{m_1}{m_1\tau + 1} \frac{1}{\tau m_6 + 1}$$

$$B_{68} = \frac{B_{58}}{\tau} - A_4 A_5 \frac{m_1}{m_1\tau + 1} \frac{1}{\tau m_7 + 1}$$

$$B_{69} = \frac{B_{59}}{\tau} - A_5 A_{13} \frac{m_2}{m_2\tau + 1} \frac{1}{\tau m_5 + 1}$$

$$B_{70} = \frac{B_{60}}{\tau} - A_5 A_{14} \frac{m_2}{m_2\tau + 1} \frac{1}{m_6\tau + 1}$$

$$B_{71} = \frac{B_{61}}{\tau} - A_5 A_{15} \frac{m_2}{m_2\tau + 1} \frac{1}{m_7\tau + 1}$$

$$B_{72} = \frac{B_{62}}{\tau} - A_4 B_{36} \frac{m_1}{m_1\tau + 1} \frac{1}{i\tau\alpha + 1}$$

$$B_{73} = \frac{B_{63}}{\tau} - A_4 B_{31} \frac{m_2}{m_2\tau + 1} \frac{1}{i\tau\alpha + 1}$$

$$B_{74} = \frac{B_{64}}{\tau} - A_4 \frac{m_1}{m_1\tau + 1} A_6$$

$$B_{75} = \frac{1}{\tau} B_{65} - A_5 \frac{m_2}{m_2\tau + 1} A_6$$

$$B_{76} = \frac{1}{\tau} A_6 A_{13} \frac{1}{\tau m_5 + 1}$$

$$B_{77} = \frac{1}{\tau} A_6 A_{14} \frac{1}{\tau m_6 + 1}$$

$$B_{78} = \frac{A_6}{\tau} A_5 \frac{1}{\tau m_7 + 1}$$

$$B_{79} = \frac{A_6^2}{\tau}$$

$$B_{80} = \frac{A_6}{\tau} B_{31} \left(\frac{1}{i\tau\alpha + 1}\right)$$

$$B_{81} = \frac{1}{\tau} \frac{1}{\tau m_5 + 1}$$

$$B_{82} = \frac{1}{\tau m_6 + 1}$$

$$B_{83} = \frac{1}{\tau m_7 + 1}$$

$$B_{84} = \frac{\tau}{\tau(m_1 + m_5) + 1} B_{66}$$

$$B_{85} = \frac{\tau}{\tau(m_1 + m_6) + 1} B_{67}$$

$$B_{86} = \frac{\tau}{\tau(m_1 + m_7) + 1} B_{68}$$

$$B_{87} = \frac{\tau}{\tau(m_2 + m_5)\tau + 1)} B_{69}$$

$$B_{88} = \frac{\tau}{[(m_2 + m_6)\tau + 1]} B_{70}$$

$$B_{89} = \frac{\tau}{[(m_2 + m_7)\tau + 1]} B_{71}$$

$$B_{90} = \frac{\tau}{[(\mathrm{i}\alpha + m_1)\tau + 1]} B_{72}$$

$$B_{91} = \frac{\tau}{[(i\alpha + m_2)\tau + 1]} B_{73}$$

$$B_{92} = \frac{\tau}{\left[\left(m_1 - \frac{1}{\tau}\right)\tau + 1\right]} B_{74}$$

$$B_{93} = \frac{\tau}{\left[\tau\left(m_2 - \frac{1}{\tau}\right) + 1\right]} B_{75}$$

$$B_{94} = \frac{\tau}{\left[\tau\left(m_5 - \frac{1}{\tau}\right) + 1\right]} B_{76}$$

$$B_{95} = \frac{\tau}{\left[\tau\left(m_6 - \frac{1}{\tau}\right) + 1\right]} B_{77}$$

$$B_{96} = \frac{\tau}{\left[\tau\left(m_7 - \frac{1}{\tau}\right) + 1\right]} B_{78}$$

$$B_{97} \ = \frac{-\tau}{\tau\left(-\frac{2}{\tau}\right) + 1} B_{79}$$

$$B_{98} = \frac{-\tau}{\tau(i\alpha - \frac{1}{\tau})} B_{80}$$

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$$B_{99} = -\{B_{56} + B_{57} + B_{58} + B_{59} + B_{60} + B_{61} + B_{62} + B_{63} + B_{64} + B_{65}\}$$

$$B_{100} = -\Big\{B_{56}e^{(m_1+m_5)} + B_{57}e^{m_1+m_6} + B_{58}e^{m_1+m_7} + B_{59}e^{m_2+m_5} + B_{60}e^{m_2+m_6} + B_{61}e^{m_2+m_7} + B_{62}e^{i\alpha+m_1} + B_{63}e^{i\alpha+m_2} + B_{64}e^{m_1-\frac{1}{\tau}} + B_{65}e^{m_2-\frac{1}{\tau}}\Big\}$$

$B_{101}=-[B_{84}+B_{85}+B_{86}+B_{87}+B_{88}+B_{89}+B_{90}+B_{91}+B_{92}+B_{93}+B_{94}+B_{95}+B_{96}+B_{97}+B_{98}]$

$B_{102}=-[B_{84}e^{m1+m5}+B_{85}e^{m1+m6}+B_{86}e^{m1+m7}+B_{87}e^{m2+m5}+B_{88}e^{m2+m6}+B_{89}e^{m2+m7}+B_{90}e^{m1+i\alpha}+B_{91}e^{m2+i\alpha}+B_{92}e^{m1-1/\tau}+B_{93}e^{m2-1/\tau}+B_{94}e^{m5-1/\tau}+B_{95}e^{m6-1/\tau}+B_{96}e^{m7-1/\tau}+B_{97}e^{-2/\tau}+B_{98}e^{(i\alpha-1/\tau)}$

$B_{103}=e^{m6}-e^{m5}$

$B_{104}= e^{m7}-e^{m5}$

$B_{105}=B_{100}-B_{99}$

$B_{106}=B_{82}-B_{81}$

$B_{107}=B_{83}-B_{81}$

$B_{108}=B_{101}-B_{99}$

$B_{109}=B_{82}e^{m6}-B_{81}e^{m5}$

$B_{110}=B_{83}e^{m7}-B_{81}e^{m5}$

$B_{111}=B_{102}-B_{99}$

$B_{112}=B_{103}(B_{107}e^{-1/\tau}-B_{110})-B_{104}(B_{106}e^{-1/\tau}-B_{109})$

$B113=B105(B_{107}e^{-1/\tau}-B110)- B_{104}(B_{108}e^{-1/\tau}-B_{111})$

$B_{114}=B_{103}(B_{108}e^{-1/\tau}-B_{111})-B_{105}(B_{106}e^{-1/\tau}-B_{109})$

$B_{115}=B_{103}(B_{107}B_{111}-B_{108}B_{110})-B_{104}(B_{106}B_{110}-B_{108}B_{109})+B_{105}(B_{106}B_{110}-B_{107}B_{109})$

$A_{18}=B_{113}/B_{112}$

$A_{19}=B_{114}/B_{112}$

$A_{20}=B_{115}/B_{112}$

$A_{17}=B_{99}-A_{18}-A_{19}$

$B_{116}= [PrRe(A_{13}A_8m_3m_5+A_{13}A_8m_3^2+\beta\delta\lambda A_{13}A_8m_3 +\delta A_{13}(1/\tau m_6+1)A_8B_{17}m_3)]$

$B_{117}= [PrRe(A_8 A_{14} m_3 m_6 +A_{14}A_8m_3^2+\beta\delta\lambda A_{14}A_8m_3$

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$+\delta A_{14}(1/\tau m_5+1)A_8B_{17}m_3)]$

$B_{118}=[PrRe(A_8\ A_{15}\ m_3\ m_7+A_{15}A_8m_3^2+\beta\delta\lambda A_{14}A_8m_3$

$+\delta A_{14}(1/\tau m_5+1)A_8B_{17}m_3)]$

$B_{119}=[PrRe(A_8\ B_{31}\ m_3\ i\alpha+B_{31}A_8m_3^2+\beta\delta\lambda B_{13}A_8m_3$

$+\delta B_{31}(1/\tau i\alpha+1)A_8B_{17}m_3)]$

$B_{120}=[PrRe(A_9\ A_{13}\ m_5\ m_4+A_{13}A_9m_4^2+\beta\delta\lambda A_{13}A_9m_4$

$+\delta A_{13}(1/\tau m_5+1)A_9B_{18}m_4)]$

$B_{121}=[PrRe(A_9\ A_{14}\ m_6\ m_4+A_{14}A_9m_4^2+\beta\delta\lambda A_{14}A_9m_4$

$+\delta A_{14}(1/\tau m_6+1)A_9B_{18}m_4)]$

$B_{122}=[PrRe(A_9\ A_{15}\ m_7\ m_4+A_{15}A_9m_4^2+\beta\delta\lambda A_{15}A_9m_4$

$+\delta A_{15}(1/\tau m_7+1)A_9B_{18}m_4)]$

$B_{123}=[PrRe(A_9\ B_{31}\ m_4\ i\alpha+B_{13}A_9m_4^2+\beta\delta\lambda B_{13}A_9m_4^2$

$+\delta B_{31}(1/\tau i\alpha+1)A_9B_{18}m_4)]$

$B_{124}=Pr\ Re\ \delta\ A_{16}\ A_{10}(-\lambda\beta\delta)e^{-(\lambda\delta\beta+1/\tau)y}$

$B_{125}=Pr\ Re\ \delta\ A_{16}\ A_8\ B_{17}\ m_3e^{(m3-1/\tau)y}$

$B_{126}=Pr\ Re\ \delta\ A_{16}\ A_9\ B_{18}\ m_4\ e^{(m4-1/\tau)y}$

$B_{127}=Pr\ Re\ \delta\ A_{10}\ (-\lambda\beta\delta).A_{13}(1/\tau m_5+1)$

$B_{128}=Pr\ Re\ \delta\ A_{10}\ (-\lambda\beta\delta).A_{14}(1/\tau m_6+1)$

$B_{129}=Pr\ Re\ \delta\ A_{10}\ (-\lambda\beta\delta).A_{15}(1/\tau m_7+1)$

$B_{130}=B_{31}\ (1/i\alpha\tau+1)\ A_{10}\ (-\lambda\beta\delta)$

$g_{21}=1$

$g_{22}=(\delta\lambda\beta-Pr\ Re)$

$g_{23}=(\alpha^2-Pr\ Re\ \delta^2\lambda\beta)$

$g_{24}=\alpha^2$

$g_{25}=-27g_{21}^2g_{24}+9\ g_{21}\ g_{22}\ g_{23}-2g_{22}^3$

$g_{26}=(3\ g_{21}\ g_{23}-g_{22}^2)^3$

$g_{27}=((g_{25}^2+4\ g_{26})^{1/2}-g_{25})^{1/3}$

$m_8=g_{27}/3(2)^{1/3}g_{21}-(2)^{1/3}g_{26}/3\ g_{21}g_{27}-g_{22}/3\ g_{21}$

$m_9=-(1-i\sqrt{3})g_{27}/6(2)^{1/3}g_{21}+(1+i\sqrt{3})g_{26}/3(4)^{1/3}\ g_{21}g_{27}-g_{22}/3\ g_{21}$

$m_{10}=-(1+i\sqrt{3})g_{27}/6(2)^{1/3}g_{21}+(1-i\sqrt{3})g_{26}/3(4)^{1/3}\ g_{21}g_{27}-g_{22}/3\ g_{21}$

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$B_{131}=B_{116}/(m_5+m_3)^3+g_{22}\ (m_5+m_3)^2+g23\ (m_5+m_3)+ g_{24}$

$B_{132}=B_{117}/(m_3+m_6)^3+g_{22}\ (m_6+m_3)^2+g23\ (m_6+m_3)+ g_{24}$

$B_{133}=B_{118}/(m_7+m_3)^3+g_{22}\ (m_7+m_3)^2+g23\ (m_7+m_3)+ g_{24}$

$B_{134}=B_{119}/(i\alpha+m_3)^3+g_{22}\ (i\alpha+m_3)^2+g23\ (i\alpha+m_3)+ g_{24}$

$B_{135}=B_{120}/(m_4+m_5)^3+g_{22}\ (m_4+m_5)^2+g23\ (m_4+m_5)+ g_{24}$

$B_{136}=B_{121}/(m_4+m_6)^3+g_{22}\ (m_4+m_6)^2+g23\ (m_4+m_6)+ g_{24}$

$B_{137}=B_{122}/(m_4+m_7)^3+g_{22}\ (m_4+m_7)^2+g23\ (m_4+m_7)+ g_{24}$

$B_{138}=B_{123}/(m_4+i\alpha)^3+g_{22}\ (m_4+i\alpha)^2+g23\ (m_4+i\alpha)+ g_{24}$

$B_{139}=B_{124}/-(\lambda\beta\delta+1/\tau)^3+g_{22}\ -(\lambda\beta\delta+1/\tau)^2+g23\ -(\lambda\beta\delta+1/\tau)+ g_{24}$

$B_{140}=B_{125}/(m_3-1/\tau)^3+g_{22}\ (m_3-1/\tau)^2+g23\ (m_3-1/\tau)+ g_{24}$

$B_{141}=B_{126}/(m_4-1/\tau)^3+g_{22}\ (m_4-1/\tau)^2+g23\ (m_4-1/\tau)+ g_{24}$

$B_{142}=B_{127}/(m_5-\lambda\beta\delta)^3+g_{22}\ (m_5-\lambda\beta\delta)^2+g23\ (m_5-\lambda\beta\delta)+ g_{24}$

$B_{143}=B_{128}/(m_6-\lambda\beta\delta)^3+g_{22}\ (m_6-\lambda\beta\delta)^2+g23\ (m_6-\lambda\beta\delta)+ g_{24}$

$B_{144}=B_{129}/(m_7-\lambda\beta\delta)^3+g_{22}\ (m_7-\lambda\beta\delta)^2+g23\ (m_7-\lambda\beta\delta)+ g_{24}$

$B_{145}=B_{130}/(i\alpha-\lambda\beta\delta)^3+g_{22}\ (i\alpha-\lambda\beta\delta)^2+g23\ (i\alpha-\lambda\beta\delta)+ g_{24}$

$B_{146}= \lambda\beta\delta/m_8+ \lambda\beta\delta$

$B_{147}= \lambda\beta\delta/m_9+ \lambda\beta\delta$

$B_{148}= \lambda\beta\delta/m_{10}+ \lambda\beta\delta$

$B_{149}= (\lambda\beta\delta\ B_{131}-(A_{13}/\tau\ m_5+1)\ A_8B_{17}m_3)\ 1/(m_5+m_3+ \lambda\beta\delta)$

$B_{150}= (\lambda\beta\delta\ B_{132}-(A_{14}/\tau\ m_6+1)\ A_8B_{17}m_4)\ 1/(m_6+m_3+ \lambda\beta\delta)$

$B_{151}= (\lambda\beta\delta\ B_{133}-(A_{15}/\tau\ m_7+1)\ A_8B_{17}m_3)\ 1/(m_7+m_3+ \lambda\beta\delta)$

$B_{152}= (\lambda\beta\delta\ B_{134}-(B_{31}/\tau\ i\alpha +1)\ A_8B_{17}m_3)\ 1/(i\alpha +m_3+ \lambda\beta\delta)$

$B_{153}= (\lambda\beta\delta\ B_{135}-(A_{13}/\tau\ m_5+1)\ A_9B_{18}m_4)\ 1/(m_4 +m_5+ \lambda\beta\delta)$

$B_{154}= (\lambda\beta\delta\ B_{136}-(A_{14}/\tau\ m_6+1)\ A_9B_{18}m_4)\ 1/(m_4 +m_6+ \lambda\beta\delta)$

$B_{155}= (\lambda\beta\delta\ B_{137}-(A_{15}/\tau\ m_7+1)\ A_9B_{18}m_4)\ 1/(m_4 +m_7+ \lambda\beta\delta)$

$B_{156}= (\lambda\beta\delta\ B_{138}-(A_{31}/\tau\ i\alpha +1)\ A_9B_{18}m_4)\ 1/(m_4 + i\alpha + \lambda\beta\delta)$

$B_{157}= (\lambda\beta\delta\ B_{139}-A_{10}(- \lambda\beta\delta)A_{16})$

$B_{158}= (\lambda\beta\delta\ B_{140}-A_8\ B_{17}m_3A_{16}(1/(m_3-1/\tau) +\lambda\beta\delta)$

$B_{159}= (\lambda\beta\delta\ B_{141}-A_9\ B_{18}m_4A_{16}(1/(m_4-1/\tau) +\lambda\beta\delta)$

$B_{160}= (\lambda\beta\delta\ B_{142}-(A_{13}/ \tau\ m_5+1)\ A_{10}(\lambda\beta\delta).1/m_5$

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$B_{161}= (\lambda\beta\delta\ B_{143}-(A_{14}/\ \tau\ m_6+1)\ A_{10}(\lambda\beta\delta).1/m_6$

$B_{162}= (\lambda\beta\delta\ B_{144}-(A_{15}/\ \tau\ m_7+1)\ A_{10}(\lambda\beta\delta).1/m_7$

$B_{163}= (\lambda\beta\delta\ B_{145}-(B_{31}/\ \tau\ i\alpha+1)\ A_{10}(\lambda\beta\delta).1/\ i\alpha$

$B_{164}= -[B_{131}+ B_{132}+ B_{133}+ B_{134}+ B_{135}+ B_{136}+ B_{137}+ B_{138}+ B_{139}+ B_{140}+ B_{141}+ B_{142}+ B_{143}+ B_{144}+ B_{145}]$

$B_{165}= -[B_{131}e^{(m5+m3)}+ B_{132}\ e^{(m6+m3)}+ B_{133}\ e^{(m7+m3)}+ B_{134}\ e^{(i\alpha+m3)}+ B_{135}\ e^{(m5+m4)}+ B_{136}\ e^{(m4+m6)}+ B_{137}\ e^{(m4+m7)}+ B_{138}\ e^{(m4+ i\alpha)}+ B_{139}\ e^{-(\delta\beta\lambda+1/\tau)}+ B_{140}\ e^{(m3-1/\tau)}+ B_{141}\ e^{(m4-1/\tau)}+ B_{142}\ e^{(m5-\delta\lambda\beta)}+ B_{143}\ e^{(m6-\lambda\delta\beta)}+ B_{144}\ e^{(m7-\lambda\beta\delta)}+ B_{145}\ e^{(i\alpha-\lambda\beta\delta)}]$

$B_{166}= -[B_{149}+ B_{150}+ B_{151}+ B_{152}+ B_{153}+ B_{154}+ B_{155}+ B_{156}+ B_{157}+ B_{158}+ B_{159}+ B_{160}+ B_{161}+ B_{162}+ B_{163}]$

$B_{167}= -[B_{149}\ e^{(m5+m3)}+ B_{150}\ e^{(m6+m3)}+ B_{151}\ e^{(m7+m3)}+ B_{152}\ e^{(i\alpha+m3)}+ B_{153}\ e^{(m5+m4)}+ B_{154}\ e^{(m4+m6)}+ B_{155}\ e^{(m4+m7)}+ B_{156}\ e^{(m4+ i\alpha)}+ B_{157}\ e^{-(\delta\beta\lambda+1/\tau)}+ B_{158}\ e^{(m3-1/\tau)}+ B_{159}\ e^{(m4-1/\tau)}+ B_{160}\ e^{(m5-\delta\lambda\beta)}+ B_{161}\ e^{(m6-\lambda\delta\beta)}+ B_{162}\ e^{(m7-\lambda\beta\delta)}+ B_{163}\ e^{(i\alpha-\lambda\beta\delta)}]$

$B_{168}=e^{m9}-e^{m8}$

$B_{169}=e^{m10}-e^{m8}$

$B_{170}= B_{165}- B_{164}$

$B_{171}= B_{147}- B_{146}$

$B_{172}= B_{148}- B_{146}$

$B_{173}= B_{166}- B_{164}$

$B_{174}= B_{147}e^{m9}- B_{146}e^{m8}$

$B_{175}= B_{148}e^{m10}- B_{146}e^{m8}$

$B_{176}= e^{-\lambda\beta\delta}$

$B_{177}= B_{167}- B_{164}B_{146}e^{m8}$

$B_{178}= B_{168}(B_{172}\ B_{176}- B_{175})- B_{169}(B_{171}\ B_{176}- B_{174})$

$B_{179}= B_{170}(B_{172}\ B_{176}- B_{175})- B_{169}(B_{173}\ B_{176}- B_{177})$

$B_{180}= B_{168}(B_{173}\ B_{176}- B_{177})- B_{170}(B_{171}\ B_{176}- B_{174})$

$B_{181}= B_{168}(B_{172}\ B_{177}- B_{173}\ B_{175})- B_{169}(B_{171}\ B_{177}- B_{173}\ B_{174})+ B_{170}(B_{171}\ B_{175}- B_{172}\ B_{174})$

$A_{22}= B_{179}/\ B_{178}$

$A_{23}= B_{180}/\ B_{178}$

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$A_{24}= B_{181}/ B_{178}$

$A_{21}= B_{164}-A_{22}-A_{23}$

$B_{182}=A_2\, Sc^2\, Re^2\, A_{13}/(m_5+Sc\, Re)^2-Sc\, Re\, (m_5+Sc\, Re)+\alpha^2$

$B_{183}=A_2\, Sc^2\, Re^2\, A_{14}/(m_6+Sc\, Re)^2-Sc\, Re\, (m_6+Sc\, Re)+\alpha^2$

$B_{184}=A_2\, Sc^2\, Re^2\, A_{15}/(m_7+Sc\, Re)^2-Sc\, Re\, (m_7+Sc\, Re)+\alpha^2$

$B_{185}=A_2\, Sc^2\, Re^2\, B_{31}/(i\alpha+Sc\, Re)^2-Sc\, Re\, (i\alpha+Sc\, Re)+\alpha^2$

$B_{186}=-[B_{182}+ B_{183}+ B_{184}+ B_{185}]$

$B_{187}=-[B_{182}e^{m5+Sc\, Re}+ B_{183}\, e^{m6+Sc\, Re} + B_{184}\, e^{m7+Sc\, Re} + B_{185}\, e^{i\alpha+Sc\, Re}]$

$A_{25}=(B_{186}e^{m12}-B_{187})/(e^{m12}-e^{m11})$

$A_{26}=(B_{187}-B_{186}e^{m11})/(e^{m12}-e^{m11})$