

AN ITERATIVE APPROACH FOR SOLVING FRACTIONAL DIFFERENTIAL EQUATIONS USING YJ ADOMIAN DECOMPOSITION METHOD

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Abstract

This paper investigates the use of the Adomian decomposition method (ADM) with Yasser-Jassim transform to solve fractional-order differential equations (FDEs). The ADM is applied to both linear and nonlinear FDEs, with a focus on demonstrating its efficiency and accuracy in solving complex systems that involve fractional derivatives. The method is shown to decompose nonlinear equations into solvable series, providing an effective approach for obtaining analytical solutions without requiring simplifying assumptions. Numerical examples are presented to compare YJADM with traditional methods, such as numerical integration and perturbation techniques, showcasing its advantages in terms of convergence and solution precision. The results confirm the method's potential to solve practical and theoretical problems in fractional calculus, contributing to the advancement of this field.

Keywords: Yasser Jassim transform, Linear and Nonlinear Systems, Adomian Analysis, Analytical Solutions, Fractional Derivatives, Fractional equations.

Introduction

Fractional calculus (FC) has many applications in mathematics and has attracted many researchers. The publicity of FC is due to the variety of the fractional derivatives operators, such as the Caputo fractional derivative, the Riemann-Liouville fractional derivative (RLFD), the Atangana-Baleanu fractional derivative (ABFD), the Caputo-Fabrizio fractional derivative (CFFD), and many others. The ABFD and CFFD were recently used in modeling physical phenomena. The ABFD and CFFD are great compromises in modeling real world phenomena [1-5]. In recent years, a many of approximate and analytical methods have been utilized to solve the PDEs with LFDOs such as fractional Adomian decomposition method, variational iteration method, homotopy perturbation method and other methods [6-79].

The Adomian decomposition method (ADM) is a well-known systematic method for practical solution of linear or nonlinear and deterministic or stochastic operator equations and fractional equations, including ordinary differential equations (ODEs), partial differential equations (PDEs), integral equations, integro-differential equations, etc. The ADM is a powerful technique, which provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering. The ADM has been successfully applied in various studies for solving FDEs in diverse fields, including physics, control systems, and material science [77].

This research paper aims to apply the Adomian decomposition method to solve fractional differential equations of arbitrary order using Yasser Jassim transforms, focusing on linear and nonlinear cases. The study explores the advantages of using the Adomian decomposition method for solving nonlinear systems, focusing on its computational efficiency and accuracy. The paper also compares the Adomian decomposition method with other traditional methods, such as numerical and perturbation techniques, highlighting its superiority in terms of solution accuracy and speed of convergence.

Yasser Jassim Adomian Decomposition Method (YJADM)

Integral transforms have played a fundamental role in mathematics, serving as powerful tools for simplifying and solving differential, integral, and fractional equations. By converting differential equations into algebraic ones, these transforms significantly facilitate the solution process. This typically involves applying the transform, solving the resulting algebraic equation, and then employing the inverse transform to retrieve the original function [78].

In this section, we used the Yasser Jassim transformation [79] with fractional Adomian decomposition method to solve fractional differential equations. We derived the general form of the Yasser Jassim Adomian equation, then took the linear form and the non-linear form and analyzed them as an example. We also took the state of the system and then solved it using the Yasser Jassim Adomian method.

Consider the following fractional nonlinear PDEs:

$${}^c D_t^\gamma w(x, t) + R[w(x, t)] + N[w(x, t)] = g(x, t), t > 0, n - 1 < \gamma \leq n \quad (1)$$

where ${}^c D_t^\gamma w(x, t)$ is the derivative of $w(x, t)$ in Caputo sense, R, N differential operators, including linear and nonlinear and $g(x, t)$ is the energy term.

Now by taking HT for both sides of Eq. (1), we obtain

$$H\{{}^c D_t^\gamma w(x, t) + R[w(x, t)] + N[w(x, t)]\} = H\{g(x, t)\}, \quad (2)$$

$$\frac{H\{w(x, t)\}}{\sqrt{a}^\gamma} - \sum_{k=0}^{n-1} \frac{aw^{(k)}(0)}{\sqrt{a}^{\gamma-k-1}} = H\{g(x, t)\} - H\{R[w(x, t)] + N[w(x, t)]\}, \quad (3)$$

or

$$H\{w(x, t)\} = \sum_{k=0}^{n-1} \sqrt{a}^{1+k} w^{(k)}(x, 0) + \sqrt{a}^{\gamma} H\{g(x, t)\} - \sqrt{a}^{\gamma} H\{R[w(x, t)] + N[w(x, t)]\}. \quad (4)$$

Applying inverse Yasser Jassim transform on both sides of Eq. (4), we find

$$w(x, t) = \sum_{k=0}^{n-1} \frac{t^k}{k!} w^{(k)}(x, 0) + H^{-1} \left(\sqrt{a}^{\gamma} H\{g(x, t)\} - H^{-1} \left(\sqrt{a}^{\gamma} H\{R[w(x, t)] + N[w(x, t)]\} \right) \right). \quad (5)$$

To broaden the solution, the adomian parameter p^n is employed, let

$$w = \sum_{n=0}^{\infty} w_n, \quad (6)$$

and the nonlinear term is decomposed as

$$N[w(x, t)] = \sum_{n=0}^{\infty} A_n. \quad (7)$$

where

$$A_n = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left(\sum_{i=0}^n p^i u_i \right)_{p=0}.$$

Substituting (6) and (7) in (5), we get

$$\sum_{n=0}^{\infty} w_n = \sum_{k=0}^{n-1} \frac{t^k}{k!} w^{(k)}(x, 0) + H^{-1} \left(\sqrt{a}^{\gamma} H\{g(x, t)\} - \left[H^{-1} \left(\sqrt{a}^{\gamma} H \left\{ R[w(x, t)] + \sum_{n=0}^{\infty} A_n \right\} \right) \right] \right). \quad (8)$$

Now we take some examples to find approximate solutions

Applications of YJ-ADM

Example 1. Let us consider the fractional integro-differential equation

$${}^c D_t^{\frac{1}{2}} w(t) = 1 + I_0^t(w(t)), \quad (9)$$

with initial condition

$$w(0) = 1.$$

Taking HT for both sides of Eq. (9), we obtain

$$\frac{H(w(t))}{(\sqrt{a})^{\frac{1}{2}}} - \frac{a}{(\sqrt{a})^{-\frac{1}{2}}} = a\sqrt{a} + (\sqrt{a})^{\frac{1}{2}} H(w(t))$$

$$H(w(t)) - \frac{a(\sqrt{a})^{\frac{1}{2}}}{(\sqrt{a})^{-\frac{1}{2}}} = a(\sqrt{a})^{\frac{1}{2}+1} + \sqrt{a} H(w(t))$$

$$H[w(t)](1 - \sqrt{a}) = a(\sqrt{a})^{\frac{1}{2}+1} + a\sqrt{a}$$

Applying the inverse of YJ, we get

$$H^{-1}[H[w(t)]] = H^{-1}\left[\frac{a(\sqrt{a})^{\frac{1}{2}+1}}{1-\sqrt{a}} + \frac{a\sqrt{a}}{1-\sqrt{a}}\right]$$

$$w(t) = tE_{2,2}(t^2) + E_{1,1}(t).$$

Example 2. Consider the fractional Burger equation

$${}^cD_t^\gamma w(x, t) + ww_x = w_{xx} ; \quad 0 < \gamma \leq 1; \quad (10)$$

subject to initial condition

$$w(x, 0) = x.$$

Taking HT for both sides of Eq. (10), we obtain

$$\frac{H(w(x, t))}{\sqrt{a}^\gamma} - \frac{aw(0)}{\sqrt{a}^{\gamma-1}} + H[ww_x] = H[w_{xx}]$$

$$H[w(x, t)] = a\sqrt{a}x - \sqrt{a}^\gamma H[ww_x] + \sqrt{a}^\gamma H[w_{xx}]$$

$$\begin{aligned} w(x, t) &= H^{-1}[a\sqrt{a}x] - H^{-1}\left[\sqrt{a}^\gamma H[ww_x]\right] + H^{-1}\left[\sqrt{a}^\gamma H[w_{xx}]\right] \\ &= x - H^{-1}\left[\sqrt{a}^\gamma H[ww_x]\right] + H^{-1}\left[\sqrt{a}^\gamma H[w_{xx}]\right] \end{aligned}$$

Suppose that

$$ww_x = \sum_{n=0}^{\infty} A_n, \quad w = \sum_{n=0}^{\infty} w_n$$

Then, we have

$$\begin{aligned} w_0(x, t) &= x \\ w_1(x, t) &= H^{-1} \left[\sqrt{a}^\gamma H[w_{0_{xx}}] \right] - H^{-1} \left[\sqrt{a}^\gamma H[A_0] \right] \\ &= -\frac{t^\gamma}{\Gamma(\gamma+1)} x \\ w_2(x, t) &= H^{-1} \left[\sqrt{a}^\gamma H[w_{1_{xx}}] \right] - H^{-1} \left[\sqrt{a}^\gamma H[A_1] \right] \\ &= H^{-1} \left[\sqrt{a}^\gamma a \sqrt{a}^{\gamma+1} 2x \right] \\ &= \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} 2x \\ &\vdots \end{aligned}$$

Therefore, the approximate solution is

$$\begin{aligned} w(x, t) &= \sum_{n=0}^{\infty} w_n(x, t) \\ &= x - \frac{t^\gamma}{\Gamma(\gamma+1)} x + \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} 2x - \dots \end{aligned}$$

If $\gamma = 1$, we have

$$w(x, t) = \frac{x}{1+t}.$$

Example 3. Consider the nonlinear system of fractional differential equations

$${}^c D_t^\gamma w(x, t) - w_{xx} - 2ww_x + (wv)_x = 0, \quad (11)$$

$${}^c D_t^\gamma v(x, t) - v_{xx} - 2v v_x + (wv)_x = 0,$$

Subject to initial conditions

$$w(x, 0) = \sin x,$$

$$v(x, 0) = \sin x.$$

Applying YJ transform to (11), we obtain

$$\frac{H(w(x, t))}{\sqrt{a}^\theta} - \frac{a}{\sqrt{a}^{\theta-1}} \sin x = H(w_{xx} + 2ww_x - (wv)_x)$$

$$\frac{H(v(x, t))}{\sqrt{a}^\gamma} - \frac{a}{\sqrt{a}^{\gamma-1}} \sin x = H(v_{xx} + 2vv_x - (wv)_x),$$

or

$$w(x, t) = \sin x + H^{-1} \left[\sqrt{a}^\theta H(w_{xx} + 2ww_x - (wv)_x) \right]$$

$$v(x, t) = \sin x + H^{-1} \left[\sqrt{a}^\gamma H(v_{xx} + 2vv_x - (wv)_x) \right].$$

Now, let us consider

$$w = \sum_{n=0}^{\infty} w_n, \quad 2ww_x = \sum_{n=0}^{\infty} A_n,$$

$$v = \sum_{n=0}^{\infty} v_n, \quad 2vv_x = \sum_{n=0}^{\infty} M_n.$$

Therefore, we have

$$\sum_{n=0}^{\infty} w_n(x, t) = \sin x + H^{-1} \left[\sqrt{a}^\theta H \left(\left(\sum_{n=0}^{\infty} w_n \right)_{xx} + \sum_{n=0}^{\infty} A_n - \left(\sum_{n=0}^{\infty} w_n \sum_{n=0}^{\infty} v_n \right)_x \right) \right],$$

$$\sum_{n=0}^{\infty} v_n(x, t) = \sin x + H^{-1} \left[\sqrt{a}^\gamma H \left(\left(\sum_{n=0}^{\infty} v_n \right)_{xx} + \sum_{n=0}^{\infty} M_n - \left(\sum_{n=0}^{\infty} w_n \sum_{n=0}^{\infty} v_n \right)_x \right) \right].$$

Hence, we get

$$w_0(x, t) = \sin x,$$

$$v_0(x, t) = \sin x,$$

$$w_1 = H^{-1} \left[\sqrt{a}^\theta H[-\sin x + 2\sin x \cos x - 2\sin x \cos x] \right]$$

$$= H^{-1} \left[\sqrt{a}^\theta H[-\sin x] \right]$$

$$= -\sin x H^{-1} \left[a\sqrt{a}^{\theta+1} \right]$$

$$= -\sin x \frac{t^\theta}{\Gamma(\theta + 1)}.$$

$$v_1 = H^{-1} \left[\sqrt{a}^\gamma H[-\sin x + 2\sin x \cos x - 2\sin x \cos x] \right]$$

$$\begin{aligned}
 &= H^{-1} \left[\sqrt{a}^\gamma H[-\sin x] \right] \\
 &= -\sin x H^{-1} \left[a \sqrt{a}^{\gamma+1} \right] \\
 &= -\sin x \frac{t^\gamma}{\Gamma(\gamma+1)}.
 \end{aligned}$$

$$\begin{aligned}
 w_2(x, t) &= \sin x \frac{t^{2\theta}}{\Gamma(2\theta+1)} - 2 \sin x \cos x \frac{t^{2\theta}}{\Gamma(2\theta+1)} + 2 \sin x \cos x \frac{t^{\theta+\gamma}}{\Gamma(\theta+\gamma+1)} \\
 v_2(x, t) &= \sin x \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} - 2 \sin x \cos x \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + 2 \sin x \cos x \frac{t^{\theta+\gamma}}{\Gamma(\theta+\gamma+1)}
 \end{aligned}$$

⋮

and so on.

Therefore, the solution of (11) is given by

$$\begin{aligned}
 w(x, t) &= \sin x \left(1 - \frac{t^\theta}{\Gamma(\theta+1)} + \frac{t^{2\theta}}{\Gamma(2\theta+1)} - \dots \right) \\
 &\quad + 2 \sin x \cos x \left(-\frac{t^{2\theta}}{\Gamma(2\theta+1)} + \frac{t^{\theta+\gamma}}{\Gamma(\theta+\gamma+1)} \dots \right) \\
 v(x, t) &= \sin x \left(1 - \frac{t^\gamma}{\Gamma(\gamma+1)} + \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} - \dots \right) \\
 &\quad + 2 \sin x \cos x \left(-\frac{t^{2\gamma}}{\Gamma(2\gamma+1)} + \frac{t^{\theta+\gamma}}{\Gamma(\theta+\gamma+1)} \dots \right)
 \end{aligned}$$

Setting $\theta = \gamma$, we obtain

$$\begin{aligned}
 w(x, t) &= \sin x \left(1 - \frac{t^\theta}{\Gamma(\theta+1)} + \frac{t^{2\theta}}{\Gamma(2\theta+1)} - \dots \right) = E_\theta(-t^\theta) \sin x \\
 v(x, t) &= \sin x \left(1 - \frac{t^\gamma}{\Gamma(\gamma+1)} + \frac{t^{2\gamma}}{\Gamma(2\gamma+1)} - \dots \right) = E_\gamma(-t^\gamma) \sin x
 \end{aligned}$$

If $\theta = \gamma = 1$, we get

$$\begin{aligned}
 w(x, t) &= e^{-t} \sin x. \\
 v(x, t) &= e^{-t} \sin x.
 \end{aligned}$$

Conclusion

In this study, we addressed the fractional Burger equation, one of the fundamental partial differential equations with wide applications in physics and engineering. A novel semi analytical strategy was proposed by combining the Adomian decomposition method with the

newly introduced Yasser–Jassim integral transform. The convergence of the method was carefully analyzed, and sufficient conditions were established to guarantee accurate and efficient solutions. Moreover, the incorporation of the Caputo fractional derivative provided a more precise description of the solutions and offered greater flexibility in modeling practical phenomena.

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