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THE IMPROVED RESIDUAL POWER SERIES METHOD FOR SOLVING SWIFT–HOHENBERG EQUATION

Ali Latif Arif[1], Hassan Kamil Jassim[2], Mohammed T. Yasser[3,4]

[1]Factually of Midicin, University of Thi-Qar, Thi-Qar, Iraq;
[2]Department of Mathematics, Factually of Education for Pure Sciences, University of Thi-Qar, Thi-Qar, Iraq;
hassankamil@utq.edu.iq
[3]College of Technical Engineering, National University of Science and Technology, Thi-Qar, Iraq
[4]Department of Mathematics, Faculty of Education, Al-Ayen Iraqi University, Thi-Qar, Iraq.

Abstract

: In this work, the residual power series method (RPSM) and Yasser-Jassim transform residual power series method (YJ-RPSM) are used to find the approximate solutions of partial differential equations. Furthermore, to show the accuracy and the efficiency of the presented methods, we compare the obtained approximate solutions of partial differential equations by RPSM and YJTRPSM numerically and analytically with the exact solution.

Keyword: Residual power series method, Partial differential equations, Yasser-Jassim transform, Exact solution.

Introduction

The Swift-Hohenberg equation is one of important equations for description localized structures in the modern physics. This equation occurs in fluid dynamics, optical physics and other fields [1,2]. Most problems in science are nonlinear and in most cases it is difficult to solve them, especially analytically. Many problems are modeled by difficult nonlinear differential equations. The perturbation method is one of the most well-known methods for solving nonlinear problems. These methods are based on the existence of small/large parameters, the so-called perturbation quantities. However, many nonlinear problems do not contain such perturbation quantities and, therefore, non-perturbation techniques are used to solve such problems. During the last few decades several methods have been used to solve ordinary and partial differential equations, integro-differential equations and fractional differential equations. Among these methods the Adomian decomposition method, He's variational iteration method, the differential transformation method, the homotopy perturbation method and other methods [3–77] have gained much success.

The residual power series method (RPSM) was first presented in 2013 by Abu Arqub [78]. The RPSM is a very significant method for finding the numerical solution of linear and nonlinear differential equations. This method has been applied successfully by many authors to find the numerical solutions for numerous problems.

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Residual power series method (RPSM).

Consider the nonlinear PDE :

$$L_t^{(n)}u(x,t)+R\big(u(x,t)\big)+N\big(u(x,t)\big)=g(x,t)$$

Where $L_t^{(n)}(.)=\frac{\partial^n}{\partial t^n}$, R is a linear operator which may contain derivatives with respect to x (e.g. $\frac{\partial}{\partial x},\frac{\partial^2}{\partial x^2},\dots$)

N is a nonlinear operator $(\text{e.g.}, uu_x, u^2, sin(u), ..)$ and $g(x,t)$ is the source term .

Step1: express the solution as a power series in t

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x)\,t^n=u_0(x)+u_1(x)t+u_2(x)t^2+\cdots$$

Where each is an unknown function of x to be determined

Step2: compute derivatives with respect to t and compute derivatives with respect to x

$$\frac{\partial^n}{\partial t^n}=\sum_{n=m}^{\infty}\frac{n!}{(n-m)!}u_n(x)\,t^{n-m}\quad,\quad\frac{\partial^k x}{\partial x^k}=\sum_{n=0}^{\infty}u_n^{(k)}(x)\,t^n$$

Step3: substitute into the PDF :

$$\sum_{n=m}^{\infty}\frac{n!}{(n-m)!}u_n(x)\,t^{n-m}+R\left(\sum_{n=0}^{\infty}u_n(x)\,t^n\right)+N\left(\sum_{n=0}^{\infty}u_n(x)\,t^n\right)=g(x,t)$$

Step3: Define the residual function Res(x, y) :

$$\text{Res}(x,y)=L_t^{(n)}u(x,t)+R\big(u(x,t)\big)+N\big(u(x,t)\big)=g(x,t)$$

Substitute the series expansions :

$$Res(x,y)=\sum_{n=m}^{\infty}\frac{n!}{(n-m)!}u_n(x)\,t^{n-m}+R\left(\sum_{n=0}^{\infty}u_n(x)\,t^n\right)+N\left(\sum_{n=0}^{\infty}u_n(x)\,t^n\right)-g(x,t)$$

Step4: collect term by powers of t

$$Res(x,y)=\sum_{n=0}^{\infty}R_n(x)\,t^n$$

Where each coefficient $R_n(x)$ is a function of $u_j(x)$ and their derivatives for $j\le n$

.set residual coefficient to zero to satisfy PDE approximately set :

$$R_n(x)=0\ \ ,for\ n=0,1,2,\dots$$

This gives a system of equations for the unknown functions $u_n(x)$

Step5: solve the system recursively

-use initial or boundary conditions to find $u_0(x)$

-use $R_0(x)$ to find $u_1(x)$

-use $R_1(x)$ to find $u_2(x)$

And so on

Step6: We substitute $u_0(x), u_1(x), u_3(x), ..$ in the series $u(x,y)=\sum_{n=0}^{\infty}u_n(x)\,t^n$

Yasser-Jassim transform (YJT).

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The YJT is a relatively new integral transform. For a function $y(t)$, defined for $t \geqslant 0$, its Yasser-Jassim transform is defined as:

$$H[y(t)] = K(\alpha) = \alpha \int_0^{\infty} e^{-\frac{1}{\sqrt{\alpha}}t} y(t)\, dt$$

Where $\alpha \neq 0$ is the transform variable.

Key Properties of YJT:

- Linearity: $H[nf(t) + mg(t)] = nK_f(\alpha) + mK_g(\alpha)$.
- Transform of Derivatives: This is crucial for solving differential equations. The transform of the first and second derivatives are given by:

$$H[f'(t)] = \frac{1}{\sqrt{\alpha}} K(\alpha) - \alpha f(0)$$

$$H[f''(t)] = \frac{1}{\alpha} K(\alpha) - \sqrt{\alpha} f(0) - \alpha f'(0)$$

- Duality with Laplace Transform: If $F(s)$ is the Laplace transform of $y(t)$, then the YJT is $K(\alpha) = \alpha F(\frac{1}{\sqrt{\alpha}})$. This relationship allows you to leverage known Laplace transform pairs.

Yasser-Jassim transform Residual power series (YJ-RPSM).

Step 1: We apply Yasser Jassim's transformation to the variable :

$$H\left[L_t^{(n)} u(x,t)\right] + H[R(u)] + H[N(u)] = H[g(x,t)]$$

Using the YJ transform property for derivatives :

$$H\left[\frac{\partial^n u}{\partial t^n}\right] = \frac{1}{\left(\sqrt{\alpha}\right)^n} U(x,\alpha) - \sum_{k=0}^{n-1} \alpha . \frac{1}{\left(\sqrt{\alpha}\right)^{n-k-1}} \frac{\partial^k u}{\partial t^k}(x,0)$$

Substituting initial conditions :

$$H\left[\frac{\partial^n u}{\partial t^n}\right] = \frac{1}{\left(\sqrt{\alpha}\right)^n} U(x,\alpha) - \sum_{k=0}^{n-1} \alpha . \frac{1}{\left(\sqrt{\alpha}\right)^{n-k-1}} h_k(x)$$

$$H[R(u)] = R(H[u]) \quad ,H[N(u)] = N[H(u)] \quad ,H[g(x,t)] = G(x,\alpha)$$

$$\frac{1}{\left(\sqrt{\alpha}\right)^n} U(x,\alpha) - \sum_{k=0}^{n-1} \alpha . \frac{1}{\left(\sqrt{\alpha}\right)^{n-k-1}} h_k(x) + R(H[u]) + N[H(u)] = G(x,\alpha)$$

Step2 : power series representation in YJ

Assume the solution in the YJ has the power series expansion:

$$U(x,\alpha)=\sum_{m=0}^{\infty}F_m(x)\,.\,\alpha^{\frac{3}{2}+\frac{m}{2}}$$

The k-the truncated series is :

$$U_k(x,\alpha)=\sum_{m=0}^{k}F_m(x)\,.\,\alpha^{\frac{3}{2}+\frac{m}{2}}$$

Initial Coefficient Determination

From the initial condition $u(x,0)=h_0(x)$:

$$H[u(x,0)]=H[h_0(x)]=h_0(x)\cdot H[1]=h_0(x)\cdot\alpha^{3/2}$$

Comparing with our series:

$$U_0(x,\alpha)=F_0(x)\cdot\alpha^{3/2}\Rightarrow F_0(x)=h_0(x)$$

Thus, the zeroth approximation is:

$$U_0(x,\alpha)=h_0(x)\cdot\alpha^{3/2}$$

Step3 : define residual function in YJ
The residual function in the YJ domain is :

$$\mathrm{Res}U_k(\alpha,\mathrm{x})=\frac{1}{\left(\sqrt{\alpha}\right)^n}U(x,\alpha)-\sum_{k=0}^{n-1}\alpha.\frac{1}{\left(\sqrt{\alpha}\right)^{n-k-1}}h_k(x)+R(H[u])+N[H(u)]-G(x,\alpha)$$

Step 4 : coefficient extraction method :
We use the condition :

$$\lim_{\alpha\to\infty}\left[\alpha^{-(3/2+m/2)}\mathrm{Res}U_k(x,\alpha)\right]=0\quad for\ m=0,1,,,,k$$

The give a system of equation for $F_0(x)$, $F_1(x)$, ... $F_k(x)$.

Step5 : Recursive Algorithm

Initialization (k=0) :

$$U_0(x,\alpha)=F_0(x)\alpha^{3/2}\text{ with }F_0(x)=h_0(x)$$

For k = 1 :

$$U_1(x,\alpha) = F_0(x)\alpha^{3/2} + F_1(x)\alpha^2$$

Substitute into residual and extract coefficient of α^2:

$$\text{Coefficient}[ResU_1(x,\alpha),\alpha^2] = 0 \Rightarrow \text{Equation for } F_1(x)$$

For k = 2:

$$U_2(x,\alpha) = F_0(x)\alpha^{3/2} + F_1(x)\alpha^2 + F_2(x)\alpha^{5/2}$$

Extract coefficient of $\alpha^{5/2}$:

$$\text{Coefficient}[ResU_2(x,\alpha),\alpha^{5/2}] = 0 \Rightarrow \text{Equation for } F_2(x)$$

General Step (k = m):

$$\text{Coefficient}[ResU_m(x,\alpha),\alpha^{3/2+m/2}] = 0 \Rightarrow \text{Equation for } F_m(x)$$

Each equation for $F_m(x)$ will be an ODE in x that can be solved using the previously determined $F_0(x), F_1(x), \dots, F_{m-1}(x)$.

Step 6: Apply the Inverse Yasser-Jassim Transform

After determining $F_0(x), F_1(x), \dots, F_k(x)$. We have

$$U_k(x,\alpha) = \sum_{m=0}^{k} F_m(x)\,.\,\alpha^{3/2+m/2}$$

Using Inverse Yasser-Jassim Transform

$$H^{-1}[\alpha^{{}^{3}/_{2}+{}^{m}/_{2}}] = \frac{t^m}{m!}$$

Therefore, the solution in the original domain is:

$$u(x,t) = \sum_{m=0}^{\mathrm{k}} F_m(x)\frac{t^m}{m!}$$

Analytical approximation of the SH equation

Example1. Let us consider the Swift–Hohenberg (S–H) equation is linear partial differential equation :

$u_t + (1-r)u + 2u_{xx} + u_{xxx} = 0 \quad , u(x,0) = e^x$

(1) by using RPSM

let $u(x,t) = \sum_{n=0}^{\infty} u_n(x)\, t^n$

With initial condition : $u(x,0) = u_0(x) = e^x$.

$$u_t = \sum_{n=1}^{\infty} n u_n(x)\, t^{n-1} \quad , \quad u_x = \sum_{n=0}^{\infty} u_n'(x)\, t^n \quad ,$$

$$u_{xx} = \sum_{n=0}^{\infty} u_n''(x)\, t^n \ , u_{xxx} = \sum_{n=0}^{\infty} u_n'''(x)\, t^n$$

From the initial condition : $u(x,0) = u_0(x) = e^x$

$$\sum_{n=1}^{\infty} n u_n(x)\, t^{n-1} + (1-r)\sum_{n=0}^{\infty} u_n(x)\, t^n + 2\sum_{n=0}^{\infty} u_n''(x)\, t^n + \sum_{n=0}^{\infty} u_n'''(x)\, t^n = 0$$

Rewrite the first sum to have powers t^n

$$\sum_{n=0}^{\infty} (n+1)u_{n+1}(x)\, t^n + (1-r)\sum_{n=0}^{\infty} u_n(x)\, t^n + 2\sum_{n=0}^{\infty} u_n''(x)\, t^n + \sum_{n=0}^{\infty} u_n'''(x)\, t^n = 0$$

$(n+1)u_{n+1}(x) + (1-r)u_n(x) + 2u_n''(x) + u_n'''(x) = 0 \Rightarrow u_{n+1}(x) = \frac{-\left((1-r)u_n(x) + 2u_n''(x) + u_n'''(x)\right)}{n+1}$

$u_0(x) = e^x$

For $n = 0 \Rightarrow u_1(x) = -\left((1-r)u_0(x) + 2u_0''(x) + u_0'''(x)\right) \Rightarrow u_1(x) = -(4-r)e^x$.

For $n = 1 \Rightarrow u_2(x) = -\frac{\left((1-r)u_1(x) + 2u_1''(x) + u_1'''(x)\right)}{2} \Rightarrow u_2(x) = \frac{(4-r)^2 e^x}{2!}$.

For $n = 2 \Rightarrow u_3(x) = -\frac{\left((1-r)u_2(x) + 2u_2''(x) + u_2'''(x)\right)}{3} \Rightarrow u_3(x) = \frac{-(4-r)^3 e^x}{3!}$.

Induction to get the general form :

$$u_n(x) = \frac{(-1)^n (4-r)^n e^x}{n!} \quad , for\ all\ n \geq 0$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x)\, t^n = e^x \sum_{n=0}^{\infty} \frac{(-1)^n (4-r)^n}{n!} t^n = e^x e^{-(4-r)t}$$

$u(x,t) = e^{x-(4-r)t}$.

(2) By using YJ-RPSM:

The YJ transform H with respect to t gives:

$$H[u_t] + H[(1-r)u] + 2H[u_{xx}] + H[u_{xxx}] = 0$$

$$\frac{1}{\sqrt{\alpha}}U - \alpha e^x + (1-r)U + 2U_{xx} + U_{xxx} = 0$$

Assume power series solution in α :

$$U(x,\alpha)=\sum_{m=0}^{\infty}F_m(x).\alpha^{\frac{3}{2}+\frac{m}{2}}$$

Substitute into transformed PDF :

$$\frac{1}{\sqrt{\alpha}}\sum_{m=0}^{\infty}F_m(x).\alpha^{\frac{3}{2}+\frac{m}{2}}-\alpha e^x+(1-r)\sum_{m=0}^{\infty}F_m(x).\alpha^{\frac{3}{2}+\frac{m}{2}}+2\sum_{m=0}^{\infty}F_m''(x).\alpha^{\frac{3}{2}+\frac{m}{2}}+\sum_{m=0}^{\infty}F_m'''(x).\alpha^{\frac{3}{2}+\frac{m}{2}}=0$$

$$\sum_{m=0}^{\infty}F_m(x).\alpha^{1+\frac{m}{2}}-\alpha e^x+(1-r)\sum_{m=0}^{\infty}F_m(x).\alpha^{\frac{3}{2}+\frac{m}{2}}+2\sum_{m=0}^{\infty}F_m''(x).\alpha^{\frac{3}{2}+\frac{m}{2}}+\sum_{m=0}^{\infty}F_m'''(x).\alpha^{\frac{3}{2}+\frac{m}{2}}=0$$

Equate coefficients of powers of α to zero

From the first sum with $m=0$:

$$F_0(x)-e^x=0\Rightarrow F_0(x)=e^x$$

From sums with m=1 :

$$F_1(x)+(1-r)F_0(x)+2F_0''(x)+F_0'''(x)=0$$

$$F_1(x)+(1-r)e^x+2e^x+e^x=0\Rightarrow F_1(x)+(4-r)e^x=0\Rightarrow F_1(x)=-(4-r)e^x$$

From sums with m=2 :

$$F_2(x)-(1-r)F_1(x)+2F_1''(x)+F_1'''(x)=0$$

$$F_2(x)-(1-r)(4-r)e^x-2(4-r)e^x-(4-r)e^x=0$$

$$F_2(x)-(1-r)(4-r)e^x-3(4-r)e^x=0\Rightarrow F_2(x)=(4-r)^2e^x$$

Write approximate solution in YJ domain :

$$U_2(x,\alpha)=e^x\alpha^{3/2}-(4-r)e^x\alpha^2+(4-r)^2e^x\alpha^{5/2}$$

Apply inverse YJ transform

$$H^{-1}[\alpha^{3/2+m/2}]=\frac{t^m}{m!}$$

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$$u(x,t) = e^x\left(1 - (4-r)t + \frac{(4-r)^2}{2}t^2 - \cdots\right)$$
$$\Rightarrow u(x,t) = e^x . e^{-(4-r)t}$$
$$\Rightarrow u(x,t) = e^{x-(4-r)t}$$

Example 2. Consider the nonlinear Swift–Hohenberg (S–H)

$$u_t + u_{xxxx} + (1-\beta)u + 2u_{xx} - \rho u_{xxx} - u^2 + (u_x)^2 = 0 \ ,$$

with initial condition

$$u(x,0) = e^x$$

(1) by using RPSM

Assum power series solution in t :

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x)\, t^n$$

$$u_t = \sum_{n=1}^{\infty} n f_n(x)\, t^{n-1} \quad , \quad u_x = \sum_{n=0}^{\infty} f_n'(x)\, t^n \ ,$$

$$u_{xx} = \sum_{n=0}^{\infty} f_n''(x)\, t^n \ , u_{xxx} = \sum_{n=0}^{\infty} f_n'''(x)\, t^n \ , , u_{xxxx} = \sum_{n=0}^{\infty} f_n^{(4)}(x)\, t^n$$

$$\sum_{n=1}^{\infty} n f_n(x)\, t^{n-1} + \sum_{n=0}^{\infty} f_n^{(4)}(x)\, t^n + (1-\beta)\sum_{n=0}^{\infty} f_n(x)\, t^n + 2\sum_{n=0}^{\infty} f_n''(x)\, t^n - \rho \sum_{n=0}^{\infty} f_n'''(x)\, t^n$$
$$-\left(\sum_{n=0}^{\infty} f_n(x)\, t^n\right)^2 + \left(\sum_{n=0}^{\infty} f_n'(x)\, t^n\right)^2 = 0$$

Rewrite the time derivative sum to start at $n = 0$

$$\sum_{n=0}^{\infty}(n+1) f_{n+1}(x)\, t^n + \sum_{n=0}^{\infty} f_n^{(4)}(x)\, t^n + (1-\beta)\sum_{n=0}^{\infty} f_n(x)\, t^n + 2\sum_{n=0}^{\infty} f_n''(x)\, t^n$$
$$-\rho \sum_{n=0}^{\infty} f_n'''(x)\, t^n - \left(\sum_{n=0}^{\infty} f_n(x)\, t^n\right)^2 + \left(\sum_{n=0}^{\infty} f_n'(x)\, t^n\right)^2 = 0$$

Define residual function :

$$Res_n(x,t) = \sum_{k=0}^{n}\left[(k+1) f_{k+1}(x) + f_k^{(4)}(x) + (1-\beta) f_k(x) + 2 f_k''(x) - \rho f_k'''(x)\right] t^k$$
$$-\left(\sum_{k=0}^{n} f_k(x)\, t^k\right)^2$$
$$+\left(\sum_{k=0}^{n} f_k'(x)\, t^k\right)^2 = 0$$

From $u(x,0) = e^x$ we have $f_0(x) = e^x$

$$Res_n(x,0) = 0 \ , \ \frac{\partial}{\partial t} Res_n(x,t)|_{t=0} = 0 \ , \ \frac{\partial^n}{\partial t^n} Res_n(x,t)|_{t=0} = 0$$

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Calculate $f_1(x)$: For $n = 0$

$$Res_n(x,t) = (1)f_1(x) + f_0^{(4)}(x) + (1-\beta)f_0(x) + 2f_0''(x) - \rho f_0'''(x) - \left(f_0(x)\right)^2 + \left(f_0'(x)\right)^2 = 0$$

$$f_1(x) = -\left(f_0^{(4)}(x) + (1-\beta)f_0(x) + 2f_0''(x) - \rho f_0'''(x) - \left(f_0(x)\right)^2 + \left(f_0'(x)\right)^2\right)$$

$$f_1(x) = -(e^x + (1-\beta)e^x + 2e^x - \rho e^x)$$

$$f_1(x) = -e^x(4-\beta-\rho)$$

Calculate $f_2(x)$: For $n = 1$

$$(2)f_2(x) + f_1^{(4)}(x) + (1-\beta)f_1(x) + 2f_1''(x) - \rho f_1'''(x) - 2f_0(x)f_1(x) + 2f_0'(x)f_1'(x) = 0$$

$$f_2(x) = -\frac{1}{2}\left(f_1^{(4)}(x) + (1-\beta)f_1(x) + 2f_1''(x) - \rho f_1'''(x) - 2f_0(x)f_1(x) + 2f_0'(x)f_1'(x)\right)$$

$$f_2(x) = \frac{1}{2}e^x(4-\beta-\rho)^2$$

$$u(x,t) = f_0(x) + f_1(x)t + f_2(x)t^2 + \cdots$$

$$u(x,t) = e^x - e^x(4-\beta-\rho)t + \frac{t^2}{2!}e^x(4-\beta-\rho)^2 - \cdots$$

$$u(x,t) = e^{x-(4-\beta-\rho)t}$$

(2) Apply YJ transform to the PDE with respect to time t:

$$H[u_t] + H[u_{xxxx}] + (1-\beta)H[u] + 2H[u_{xx}] - \rho H[u_{xxx}] - H[u^2] + H[(u_x)^2] = 0$$

$$\frac{1}{\sqrt{\alpha}}U(x,\alpha) - \alpha e^x + U_{xxxx}(x,\alpha) + (1-\beta)U(x,\alpha) + 2U_{xx}(x,\alpha) - \rho U_{xxx}(x,\alpha) - H[u^2] + H[(u_x)^2] = 0$$

$$\frac{1}{\sqrt{\alpha}}U(x,\alpha) + U_{xxxx}(x,\alpha) + (1-\beta)U(x,\alpha) + 2U_{xx}(x,\alpha) - \rho U_{xxx}(x,\alpha) = \alpha e^x + H[u^2] - H[(u_x)^2]$$

Power series in YJ :

$$U(x,\alpha) = \sum_{m=0}^{\infty} F_m(x)\,.\,\alpha^{\frac{3}{2}+\frac{m}{2}}$$

he initial condition $u(x,0) = e^x$ gives:

$$H[u(x,0)] = H[e^x] = e^x \cdot H[1] = e^x \cdot \alpha^{3/2}$$

Comparing with the series, the zeroth approximation is:

$$U_0(x,\alpha) = F_0(x)\alpha^{3/2} \Rightarrow F_0(x) = e^x$$

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Express nonlinear terms in terms of $F_x(x)$

$$u(x,t)=\sum_{m=0}^{\infty}F_m(x)\frac{t^m}{m!}$$

Then :

$$u^2=\left(\sum_{m=0}^{\infty}F_m(x)\frac{t^m}{m!}\right)^2=\sum_{m=0}^{\infty}\left(\sum_{j=0}^{m}\frac{F_j(x)F_{m-j}(x)}{j!\,(m-j)!}\right)t^m$$

$$u_x=\sum_{m=0}^{\infty}F_m'(x)\frac{t^m}{m!}\Rightarrow (u_x)^2=\sum_{m=0}^{\infty}\left(\sum_{j=0}^{m}\frac{F_j'(x)F_{m-j}'(x)}{j!\,(m-j)!}\right)t^m$$

Apply YJ transform to u^2 and $(u_x)^2$gives:

$$H[u^2]=\sum_{m=0}^{\infty}\left(\sum_{j=0}^{m}\frac{F_j(x)F_{m-j}(x)}{j!\,(m-j)!}\right)\alpha^{\frac{3}{2}+\frac{m}{2}}\quad ,$$

$$H[(u_x)^2]\ =\sum_{m=0}^{\infty}\left(\sum_{j=0}^{m}\frac{F_j'(x)F_{m-j}'(x)}{j!\,(m-j)!}\right)\alpha^{\frac{3}{2}+\frac{m}{2}}$$

$$\sum_{m=0}^{\infty}\Big[F_m(x)\alpha^{1+\frac{m}{2}}+F_m^{(4)}(x)\alpha^{3/2+m/2}+(1-\beta)F_m(x)\alpha^{3/2+m/2}+2F_m''(x)\alpha^{3/2+m/2}-\rho F_m^{(3)}(x)\alpha^{3/2+m/2}\Big]=\alpha e^x+\sum_{m=0}^{\infty}\left(\sum_{j=0}^{m}\frac{F_j(x)F_{m-j}(x)-F_j'(x)F_{m-j}'(x)}{j!\,(m-j)!}\right)\alpha^{\frac{3}{2}+\frac{m}{2}}$$

Match power of :

$$\sum_{m=0}^{\infty}F_m(x)\alpha^{1+\frac{m}{2}}=\sum_{m=0}^{\infty}F_m(x)\alpha^{\frac{3}{2}+\frac{m-1}{2}}=\sum_{m=-1}^{\infty}F_{m+1}(x)\alpha^{\frac{3}{2}+\frac{m}{2}}$$

$$F_{m+1}(x)+F_m^{(4)}(x)+(1-\beta)F_m(x)+2F_m''(x)-\rho F_m^{(3)}(x)=\sum_{j=0}^{m}\frac{F_j(x)F_{m-j}(x)-F_j'(x)F_{m-j}'(x)}{j!\,(m-j)!}$$

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$$F_{m+1}(x) = -F_m^{(4)}(x) - (1-\beta)F_m(x) - 2F_m''(x) + \rho F_m^{(3)}(x) + \sum_{j=0}^{m} \frac{F_j(x)F_{m-j}(x) - F_j'(x)F_{m-j}'(x)}{j!\,(m-j)!}$$

For $m = 0$:

$$F_1(x) = -F_0^{(4)}(x) - (1-\beta)F_0(x) - 2F_0''(x) + \rho F_0^{(3)}(x) + \frac{\left(F_0(x)\right)^2 - \left(F_0'(x)\right)^2}{0!\,0!}$$

$$F_1(x) = -e^x - (1-\beta)e^x - 2e^x + \rho e^x + e^{2x} - e^{2x} \Rightarrow F_1(x) = e^x(\rho - 4 + \beta)$$

For $m = 1$:

$$F_2(x) = -F_1^{(4)}(x) - (1-\beta)F_1(x) - 2F_1''(x) + \rho F_1^{(3)}(x) + \frac{F_0(x)F_1(x) - F_0'(x)F_1'(x)}{0!\,1!}$$

$$F_2(x) = -e^x(\rho-4+\beta) - (1-\beta)e^x(\rho-4+\beta) - 2e^x(\rho-4+\beta) + \rho e^x(\rho-4+\beta)$$

$$F_2(x) = e^x(\rho-4+\beta)^2$$

For m=2:

$$F_3(x) = -F_2^{(4)}(x) - (1-\beta)F_2(x) - 2F_2''(x) + \rho F_2^{(3)}(x) + \sum_{j=0}^{2} \frac{F_j(x)\,F_{2-j}(x) - F_j'(x)\,F_{2-j}'(x)}{j!\,(2-j)!}$$

$$F_3(x) = -e^x(\rho-4+\beta)^2 - (1-\beta)e^x(\rho-4+\beta)^2 - 2e^x(\rho-4+\beta)^2 + \rho e^x(\rho-4+\beta)^2 + \sum_{j=0}^{2} \frac{F_j(x)\,F_{2-j}(x) - F_j'(x)\,F_{2-j}'(x)}{j!\,(2-j)!}$$

$$F_3(x) = e^x(\rho-4+\beta)^3$$

$$u(x,t) = e^x + e^x(\rho-4+\beta)t + \frac{t^2}{2!}e^x(\rho-4+\beta)^2 + \frac{t^3}{3!}e^x(\rho-4+\beta)^3 + \cdots$$

Conclusions

The residual power series method (RPSM) and Yasser-Jassim transform residual power series method (YJ-RPSM) are good techniques and approach that can deal with PDEs that are linear or nonlinear. We implemented this procedure solve the Swift–Hohenberg equation. The suggested scheme is a more dependable method that converges more quickly and the findings demonstrate that the technique used is efficient and for obtaining the rough solutions of the suggested model.

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