International Journal of Applied Mathematics
Volume38 No.3s,2025
ISSN:1311-1728(printed version);ISSN:1314-8060(on-line version)

STAR CHROMATIC INDEX OF LINE GRAPHS OF THE FAMILY OF LADDER GRAPHS

HEMALATHA P[1], SHANMUGA VADIVU D[2]

[1]Vellalar College for Women, Erode-12, India

e-mail: dr.hemalatha@gmail.com

[2]Velalar College of Engineering and Technology, Erode-12, India

e-mail: dsvvcet@gmail.com

Abstract

A star edge coloring of a graph G is a proper edge coloring without bichromatic paths or cycles of length 4.The smallest integer k such that G admits a star edge coloring with k colors is the star chromatic index of G and is denoted by $\chi^{'}(G)$. The Line Graph $L(G)$ of a graph G is the one in which $V(L(G)$ corresponds to the edges of G and two vertices of $V(L(G))$ are adjacent if and only if their corresponding edges in G meet at a common vertex in G. In this paper, we obtain the star chromatic index of the line graph of the ladder graphs, triangular ladder graphs and diagonal ladder graphs.

Keywords: Star Edge Coloring, Star Chromatic Index, Family of Ladder Graphs, Line Graphs.

2020 AMS Subject Classification: 05C15, 05C76

1 Introduction

All graphs considered in this paper are finite, simple and undirected. Let P_n and C_n respectively denote a path and a cycle of length n. An edge coloring of a graph $G = (V, E)$ is a function $C: E \to N$, in which any two adjacent edges $e, f \in E$ are assigned different colors. The function C is known as the edge coloring function. Agraph G for which there exists an edge coloring which requires k colors is called k-edge colorable, while such a coloring is called a k-edge coloring. The smallest number k for which there exists a k-edge coloring of G is called the chromatic index of a graph $G$and is denoted by $\chi'(G)$. A star edge coloring of a graph G is a proper edge coloring where atleast three distinct colors are used on the edges of every induced paths and cycles of length four in G. i.e., there is neither an induced bichromatic path nor an induced bichromatic cycle of length four in$G$. The minimum number of colors forwhich G admits a star edge coloring is called the star chromatic index of G and is denoted by $\chi_{st}^{'}(G)$.

The Cartesian product of two graphs $G$and H, denoted by $G\square H$, is the graph with vertex set $V(G) \times V(H)$ and two vertices $u = (u_1, v_1), v = (u_2, v_2)$ are adjacent in $G\square H$ if either $u_1 = u_2$ and v_1 is adjacent to v_2 or $v_1 = v_2$ and u_1 is adjacent to u_2. For a graph G and a subgraph $H$of G, $G\backslash H$ denotes the graph G'inwhich $V(G') = V(G)$and$E(G') = E(G) - E(H)$.

Received : August05,2025

Additional graph theoretic terminologies used in this paper can be found in [1]. The Star edge coloring was first introduced by Liu and Deng in 2008 [8]. Dvorak, Mohar, and Samal [4] explored the extremal values for the star chromatic index of complete graphs. Ludmila Bezegova et al. [2] examined this parameter for trees and outerplanar graphs, relating it to their maximum degree Δ. Omoomi and Dastjerbi[11] analyzed the star chromatic index for several Cartesian product graphs, including path-cycle products, d-dimensional grids, hypercubes, and toroidal grids, and proposed corresponding upper bounds. Star edge coloring has applications in various fields including scheduling, network design and storage optimization [5]. In this paper, the star chromatic index of the Line Graph of Ladder Graph, Triangular Ladder Graph and Diagonal Ladder Graph have been discussed.

2 Preliminaries

A *Ladder graph* L_{2n} [9] is a special case of cartesian product of paths P_m and P_n with $m = 2$. i.e, $L_{2n} = P_2 \square P_n$. A *Triangular Ladder Graph* $TL_n, n \geq 2$, is a graph obtained from L_n by adding the edges $E_i = \{(u_{i+1}, v_i) : 0 \leq i \leq n-2\}$ to $E(L_n)$. A *Diagonal Ladder Graph* DL_n , $n \geq 2$,is a graph obtained from L_n by adding the edges, $E_i = \{(u_{i,} v_{i+1}) : 1 \leq i \leq n-1\} \cup E_i = \{(u_{i+1,} v_i) : 1 \leq i \leq n-1 \; to \; E(L_n)$. The *Line Graph* $\mathcal{L}(G)$[3] of a graph G is defined as follows: i) Each edge of g represents avertex of $\mathcal{L}(G)$ii) Two vertices of $\mathcal{L}(G)$ are adjacent if and only if their corresponding edges meet at a vertex of G. The Example of ladder graph $L_n$and line graph $\mathcal{L}(L_n)$of ladder graph are shown in Figures 1 and 2. The vertex set of line graph of ladder graph is given by $V(\mathcal{L}(L_n)) = \{f_{i,}, g_i, h_i, 1 \leq i \leq n-1\} \cup \{h_n\}$ where the two parallel paths which serves as the sides of the ladder graph are represented as $f_i \; and \; g_i, 1 \leq i \leq n-1$ and the rungs of the ladder graph are represented by $h_i, 1 \leq i \leq n$.

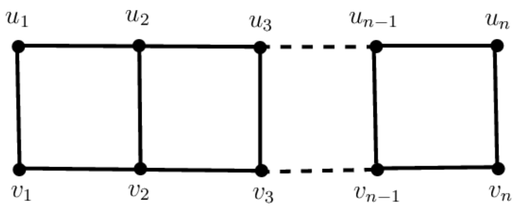

Figure 1: Ladder Graph L_n

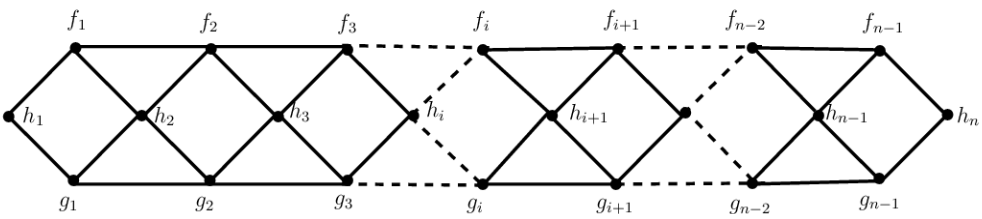

Figure 2: Line Graph of Ladder Graph $\mathcal{L}(L_n)$

Received : August05,2025

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printedversion);ISSN:1314-8060(on-line version)

The following theorem gives the star chromatic index of the cartesian product of paths P_m and P_n and in particular for ladder graph, $P_2 \square P_n$, $n \geq 2$.

Theorem 2.1. [11] For given natural numbers m, n with $2 \leq m \leq n$, we have

$$\chi'_{st}(P_m \square P_n) = \begin{cases} 3, if\ m = 2, n = 2 \\ 4, if\ m = 2, n \geq 2 \\ 5, if\ m = 3, n \in \{3,4\} \\ 6, otherwise \end{cases}$$

2 Star Chromatic Index of Line Graph of Ladder Graphs

In this section, the star chromatic index of the family of line graph of ladder graphs such as $\mathcal{L}(L_n)$, $\mathcal{L}(TL_n)$, $\mathcal{L}(DL_n)$ have been obtained

Theorem 3.1. The star chromatic index of the Line Graph of the Ladder Graph is given

by $\chi'_{st}(\mathcal{L}(L_n)) = \begin{cases} 3, if\ n = 2 \\ 5, if\ n = 3 \\ 7, if\ n = 4,5 \\ 8, if\ n \geq 6 \end{cases}$

Proof

Let $V(\mathcal{L}(L_n)) = \{f_i, g_i, h_i, 1 \leq i \leq n-1\} \cup \{h_n\}$

$$E(\mathcal{L}(L_n)) = \{(f_i, f_{i+1}), 1 \leq i \leq n-2\} \cup \{(g_i, g_{i+1}), 1 \leq i \leq n-2\} \cup \{(h_i, g_i), 1 \leq i \leq n-1\}$$
$$\cup\ \{(g_i, h_{i+1}), 1 \leq i \leq n-1\} \cup \{(h_i, f_i), 1 \leq i \leq n-1\} \cup \{(f_i, h_{i+1}), 1 \leq i \leq n-1\}$$

Let F be a function defined by $F: E \to C$, where C ⊆ N. Now we have the star edge coloring of $G = \mathcal{L}(L_n)$for the given positive integers n as given below:

Case 1: n = 2

In this case, $G = \mathcal{L}(L_2)$is isomorphic to the ladder graph L_2 and by Theorem 2.1, $\chi'_{st}(\mathcal{L}(L_2)) = 3$.

Case 2: n = 3

In this case, $G = \mathcal{L}(L_3)$, and the maximum degree of the Graph G is 4. First we color the edges that are incident with the vertex of maximum degree 4 using four colors say {0, 1, 2, 3} see Figure 3. There exists two edge disjoint paths P_4 including this maximum degree vertex . In each path, two edges are already colored and the two edges which are not already colored can be colored using the remaining two colors from the list {0, 1, 2, 3}. Now the edges that connect the internal vertices of these paths cannot be colored using {0, 1, 2, 3}, as these two edges are adjacent with all the edges already colored using {0, 1, 2, 3}. Thus, a fifth color must be introduced to color these two edges and hence $\chi'_{st}(\mathcal{L}(L_3)) = 5$.

Volume38No.3s,2025
ISSN:1311-1728(printed version);ISSN:1314-8060(on-line version)

Case 3: n = 4, 5

Consider $G = \mathcal{L}(L_4)$, which contains $G_1 = \mathcal{L}(L_3)$as a subgraph. As $\mathcal{L}(L_3)$ is 5-star edge colorable, it is clear that $\chi'_{st}(G) \geq 5$. From the structure of G, we see that it contains four vertices of maximum degree 4. We can start the coloring of edges at any one of these 4 vertices say at v using the colors from {0, 1, 2, 3, 4}. Now there exists two distinct paths of length 6, through the vertices of degree 4 in the middle layer and we want to color those edges using the five colors. In this way, we are left out with 4 edges that connect the internal vertices of the distinct paths, but cannot be colored using the permutations of five colors .Hence we use sixth and seventh colors for these four edges and hence $\chi'_{st}(\mathcal{L}(L_4)) = 7$. Similarly we can prove $\chi'_{st}(\mathcal{L}(L_5)) = 7$.

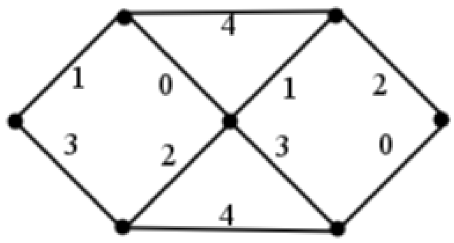

Figure 3: 5-Star edge coloring of $\mathcal{L}(L_3)$

Case 4: n ≥ 6

Consider the case, $G = \mathcal{L}(L_6)$ which contains $G_2 = \mathcal{L}(L_5)$ as a subgraph. It is clear that $\chi'_{st}(G_2) = 7$ and it is not possible to color the edges of G with the same set of colors already used in the coloring of G_2. So we introduce eighth color for the edges of G / G_2. Therefore, $\chi'_{st}(\mathcal{L}(L_6)) = 8$. By using the similar argument, we can prove that $\chi'_{st}(\mathcal{L}(L_n)) = 8$, for all n ≥ 6 and the star edge coloring pattern of $G = \mathcal{L}(L_n)$ for n ≥ 6 is as follows:

If $1 \leq i \leq n-2$

$$\mathcal{F}((f_i, f_{i+1}) = \begin{cases} 6, & if\ i \equiv 0 (mod\ 3) \\ 7, & if\ i \equiv 1\ (mod\ 3) \\ 0, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

If $1 \leq i \leq n-2$

$$\mathcal{F}((g_i, g_{i+1}) = \begin{cases} 7, & if\ i \equiv 0 (mod\ 3) \\ 2, & if\ i \equiv 1\ (mod\ 3) \\ 6, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

$$\mathcal{F}\left(\left(h_{i,} g_i\right) = 7, if\ i = 1\right.$$

If $2 \leq i \leq n-1$

$$\mathcal{F}((h_i, g_i) = \begin{cases} 2, & if\ i \equiv 0 (mod\ 3) \\ 6, & if\ i \equiv 1\ (mod\ 3) \\ 3, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

Received : August05,2025

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printed version);ISSN:1314-8060(on-line version)

If $1 \le i \le n-2$

$$\mathcal{F}((g_i, h_{i+1}) = \begin{cases} 4, & if\ i \equiv 0(mod\ 3) \\ 1, & if\ i \equiv 1\ (mod\ 3) \\ 0, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

$$\mathcal{F}((g_{n-1}, h_n) = \begin{cases} 0, & if\ i \equiv 0(mod\ 3) \\ 7, & if\ i \equiv 1\ (mod\ 3) \\ 1, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

$$\mathcal{F}((h_{i,} f_i) = 6, if\ i = 1$$

If $2 \le i \le n-1$

$$\mathcal{F}((h_i, f_i) = \begin{cases} 1, & if\ i \equiv 0(mod\ 3) \\ 5, & if\ i \equiv 1\ (mod\ 3) \\ 2, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

If $1 \le i \le n-1$

$$\mathcal{F}((f_i, h_{i+1}) = \begin{cases} 3, & if\ i \equiv 0(mod\ 3) \\ 0, & if\ i \equiv 1\ (mod\ 3) \\ 5, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

Due to the structure of $\mathcal{L}(L_n)$, $\mathcal{F}$ has a repeating pattern for n ≥ 6 and hence it sufficies to show that $\mathcal{L}(L_7)$ has a 8-star edge coloring. The star edge coloring of $\mathcal{L}(L_7)$ is shown in Figure 4. Thus, $\chi'_{st}(\mathcal{L}(L_n)) = 8, for\ n \ge 6.$

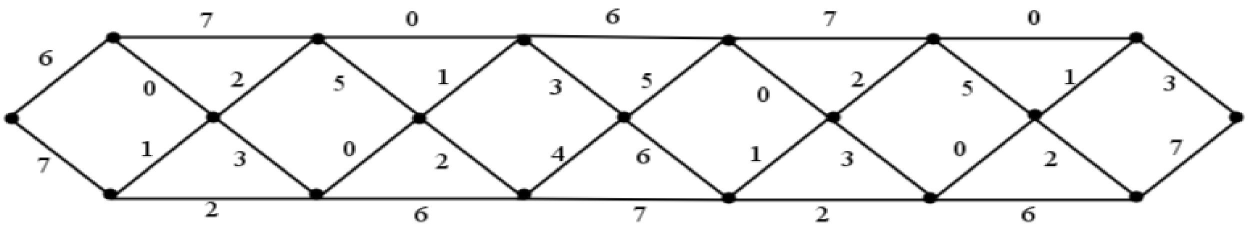

Figure 4: 8-Star edge coloring of $\mathcal{L}(L_7)$

Theorem 3.1. The star chromatic index of the Line Graph of the Triangular Ladder Graph is given by

$$\chi'_{st}(\mathcal{L}(TL_n)) = \begin{cases} 7, if\ n = 2 \\ 9, if\ n = 3 \\ 10, if\ n = 4 \\ 11, if\ n = 5,6 \\ 12, if\ n \ge 7 \end{cases}$$

Proof

Let $V(\mathcal{L}(TL_n)) = \{f_{i,} g_i, h_i, k_i, 1 \le i \le n-1\} \cup \{h_n\}$

$$E(\mathcal{L}(TL_n)) = \{(f_{i,}f_{i+1}), 1 \le i \le n-2\} \cup \{(g_{i,}g_{i+1}), 1 \le i \le n-2\} \cup \{(h_{i,}g_i), 1 \le i \le n-1\}$$
$$\cup \{(g_{i,}h_{i+1}), 1 \le i \le n-1\} \cup \{(h_{i,}f_i), 1 \le i \le n-1\} \cup \{(f_{i,}h_{i+1}), 1 \le i \le n-1\}$$
$$\cup \{(k_{i,}h_i), 1 \le i \le n-2\} \cup \{(k_{i,}g_i), 1 \le i \le n-2\} \cup \{(k_{i,}f_i), 1 \le i \le n-2\}$$
$$\cup \{(k_{i,}h_{i+1}), 1 \le i \le n-2\} \cup \{(k_{i,}g_{i+1}), 1 \le i \le n-3\} \cup \{(k_{i,}f_{i-1}), 2 \le i \le n-2\}$$

Let F be a function defined by $F : E \to C$, where C ⊆ N. Now we have the star edge coloring of $G = \mathcal{L}(TL_n)$ for the given positive integers n as given below:

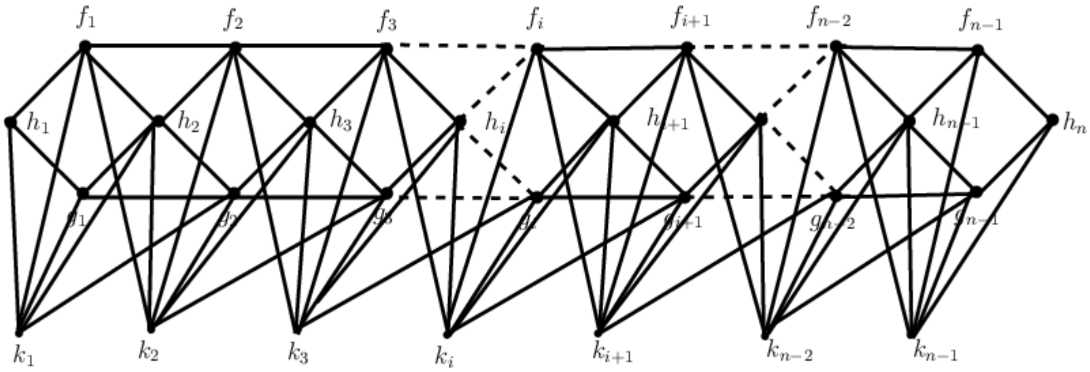

Figure5: Line graph of the Triangular Ladder graph TL_n

Case 1: n = 2

In this case, $G = \mathcal{L}(TL_2)$ and $|V(G)| = 4$, the maximum degree in G is four say d(v) = 4, other vertices have degree 3 and are adjacent to v. Hence, if we color thefour edges incident at v with 4 colors from the list {0,1,2,3}, then it is not possibleto use all the four colors but one to the remaining edges which for a cycle of length 4 in G. Thus we introduce three new colors namely {4,5,6} to color 3 edges of C_4 and one color from the list {0,1,2,3}. Therefore, $\chi'_{st}(\mathcal{L}(TL_2)) = 7$.

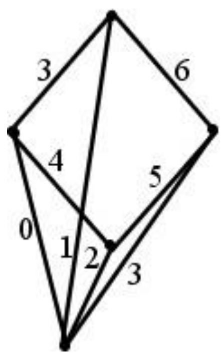

Figure 6: 7-Star edge coloring of $\mathcal{L}(TL_2)$

Received : August05,2025

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printed version);ISSN:1314-8060(on-line version)

Case 2: n = 3

In this case, $G = \mathcal{L}(TL_3)$and $|V\ (G)|\ =\ 9$, the maximum degree in G is six say $d(v)\ =\ 6$, and it is adjacent to other vertices of degree 4 or 5 . Hence, we color thesix edges incident at v with 6 colors from the list {0,1,2,3,4,5}, and also $G_2 = \mathcal{L}(TL_2)$ is a subgraph of G and $\chi'_{st}(\mathcal{L}(TL_2))\ =\ 7$ then it is not possible to use the same sixcolors to the remaining edges which forms a cycle or path of length 4 in G. Thuswe introduce three new colors namely {6,7,8} to color the edges of G . Therefore, $\chi'_{st}(\mathcal{L}(TL_3))\ =\ 9$.

Case 3: n = 4

In this case, $G = \mathcal{L}(TL_4)$ contains $G_3 = \mathcal{L}(TL_3)$ as a subgraph and it is clear that $\chi'_{st}(\mathcal{L}(TL_3))\ =\ 9$ and in the graph G, it is not possible to color the edges with the sameset of colors already used in the coloring of G_3, so we introduce the tenth color forthe edges of $G\backslash G_3$. Therefore, $\chi'_{st}(\mathcal{L}(TL_4))\ =\ 10$.

Case 4: n = 5, 6 and n ≥ 7

The proof ofother cases are discussed in the following table:

n	Graph G	Maximal induced subgraph	Lower bound for $\chi'_{st}(G)$	$\chi'_{st}(G)$
5	$\chi'_{st}(\mathcal{L}(TL_5))$	$\chi'_{st}(\mathcal{L}(TL_4))$	10	11
6	$\chi'_{st}(\mathcal{L}(TL_6))$	$\chi'_{st}(\mathcal{L}(TL_5))$	10	11
7	$\chi'_{st}(\mathcal{L}(TL_7))$	$\chi'_{st}(\mathcal{L}(TL_6))$	11	12

Table 1: Star Edge Coloring of $\boldsymbol{\mathcal{L}(TL_n)}$

By using the similar argument, we can prove that $\chi'_{st}(\mathcal{L}(TL_n))\ =\ 12$, for all n ≥ 7.

Note that $E(G)\ =\ E(G')\ \cup\ E(G\backslash G')$, where $G'\ =\ L(L_n)$. As we mentioned previouslythe edges of the subgraphG'can be colored using the same coloring pattern as givenin Theorem 5.1, Hence the star edge coloring pattern for the edges of $G\backslash G'$ is asfollows:

If $1\ \le\ i\ \le\ 5$,

$$\mathcal{F}(k_i, h_i) = \begin{cases} 0, if\ i = 1 \\ 5, if\ i = 2 \\ 4, if\ i = 3 \\ 8, if\ i = 4 \\ 8, if\ i = 5 \end{cases}$$

$$\mathcal{F}(k_i, g_i) = \begin{cases} 4, if\ i = 1 \\ 7, if\ i = 2 \\ 1, if\ i = 3 \\ 0, if\ i = 4 \\ 5, if\ i = 5 \end{cases}$$

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printedversion);ISSN:1314-8060(on-line version)

$$\mathcal{F}(k_i, f_i) = \begin{cases} 5, if\ i = 1 \\ 8, if\ i = 2 \\ 8, if\ i = 3 \\ 4, if\ i = 4 \\ 10, if\ i = 5 \end{cases}$$

$$\mathcal{F}(k_i, h_{i+1}) = \begin{cases} 8, if\ i = 1 \\ 6, if\ i = 2 \\ 9, if\ i = 3 \\ 5, if\ i = 4 \\ 7, if\ i = 5 \end{cases}$$

$$\mathcal{F}(k_i, g_{i+1}) = \begin{cases} 9, if\ i = 1 \\ 3, if\ i = 2 \\ 10, if\ i = 3 \\ 9, if\ i = 4 \\ 1, if\ i = 5 \end{cases}$$

$$\mathcal{F}(k_i, f_{i-1}) = \begin{cases} 1, if\ i = 2 \\ 3, if\ i = 3 \\ 10, if\ i = 4 \\ 11, if\ i = 5 \end{cases}$$

If $6 \le i \le n-1$,

$$\mathcal{F}((k_i, h_i) = \begin{cases} 4, & if\ i \equiv 0 (mod\ 3) \\ 0, & if\ i \equiv 1\ mod\ 3 \\ 5, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

$$\mathcal{F}((k_i, g_i) = \begin{cases} 11, & if\ i \equiv 0 (mod\ 3) \\ 1, & if\ i \equiv 1\ mod\ 3 \\ 4, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

$$\mathcal{F}((k_i, f_i) = \begin{cases} 8, & if\ i \equiv 0 (mod\ 3) \\ 3, & if\ i \equiv 1\ mod\ 3 \\ 6, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

$$\mathcal{F}((k_i, h_{i+1}) = \begin{cases} 10, & if\ i \equiv 0 (mod\ 3) \\ 9, & if\ i \equiv 1\ mod\ 3 \\ 1, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

$$\mathcal{F}((k_i, f_{i-1}) = \begin{cases} 9, & if\ i \equiv 0 (mod\ 3) \\ 7, & if\ i \equiv 1\ mod\ 3 \\ 2, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

If $6 \le i \le n-2$,

$$\mathcal{F}((k_i, g_{i+1}) = \begin{cases} 0, & if\ i \equiv 0 (mod\ 3) \\ 8, & if\ i \equiv 1\ mod\ 3 \\ 9, & if\ i \equiv 2\ mod\ 3 \end{cases}$$

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printedversion);ISSN:1314-8060(on-line version)

Due to the structure of $\mathcal{L}(TL_n)$, $\mathcal{F}$ has a repeating pattern for $n > 6$ and hence it sufficies to show that $\mathcal{L}(TL_{10})$ has a 12-star edge coloring. The star edge coloring of $\mathcal{L}(TL_{10})$ is shown in Figure 7. Thus, $\chi'_{st}(\mathcal{L}(TL_n)) = 12, for\ n \geq 7$.

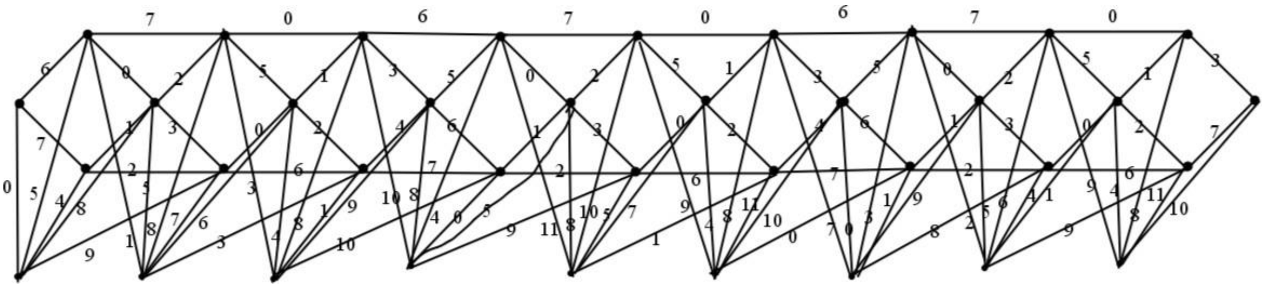

Figure 7: 12-Star edge coloring of $\mathcal{L}(TL_{10})$

Theorem 3.2. The star chromatic index of the Line Graph of the Diagonal Ladder Graph is given by

$$\chi'_{st}(\mathcal{L}(DL_n)) = \begin{cases} 8, if\ n = 2 \\ 11, if\ n = 3 \\ 12, if\ n = 4 \\ 13, if\ n = 5,6 \\ 14, if\ n \geq 7 \end{cases}$$

Proof

Let $V\big(\mathcal{L}(DL_n)\big) = \{f_{i,}g_i, h_i, k_i, 1 \leq i \leq n-1\} \cup \{h_n\}$

$$E\big(\mathcal{L}(DL_n)\big) = \{(f_{i,}f_{i+1}), 1 \leq i \leq n-2\} \cup \{(g_{i,}g_{i+1}), 1 \leq i \leq n-2\} \cup \{(h_{i,}g_i), 1 \leq i \leq n-1\}$$
$$\cup\ \{(g_{i,}h_{i+1}), 1 \leq i \leq n-1\} \cup \{(h_{i,}f_i), 1 \leq i \leq n-1\} \cup \{(f_{i,}h_{i+1}), 1 \leq i \leq n-1\}$$
$$\cup\ \{(k_{i,}h_i), 1 \leq i \leq n-2\} \cup \{(k_{i,}g_i), 1 \leq i \leq n-2\} \cup \{(k_{i,}f_i), 1 \leq i \leq n-2\}$$
$$\cup\ \{(k_{i,}h_{i+1}), 1 \leq i \leq n-2\} \cup \{(k_{i,}g_{i+1}), 1 \leq i \leq n-3\} \cup \{(k_{i,}f_{i-1}), 2 \leq i \leq n-2\}$$
$$\cup \{(m_{i,}h_i), 1 \leq i \leq n-1\} \cup \{(m_{i,}f_i), 1 \leq i \leq n-1\} \cup \{(m_{i,}h_{i+1}), 1 \leq i \leq n-1\}$$
$$\cup \{(m_{i,}g_i), 1 \leq i \leq n-1\} \cup \{(m_{i,}f_{i+1}), 1 \leq i \leq n-1\} \cup \{(m_{i,}k_{i+1}), 1 \leq i \leq n-2\}$$
$$\cup\ \{(m_{i,}k_{i-1}), 2 \leq i \leq n-1\} \cup \{(m_{i,}g_{i-1}), 2 \leq i \leq n-1\}$$

Note that $V\ (\mathcal{L}(TL_n)) \subseteq V\ (\mathcal{L}(DL_n))\ and\ E(\mathcal{L}(TL_n)) \subseteq E(\mathcal{L}(DL_n))$ and hence we consider this in finding the star chromatic index of $\mathcal{L}(DL_n)$.

Received : August05,2025

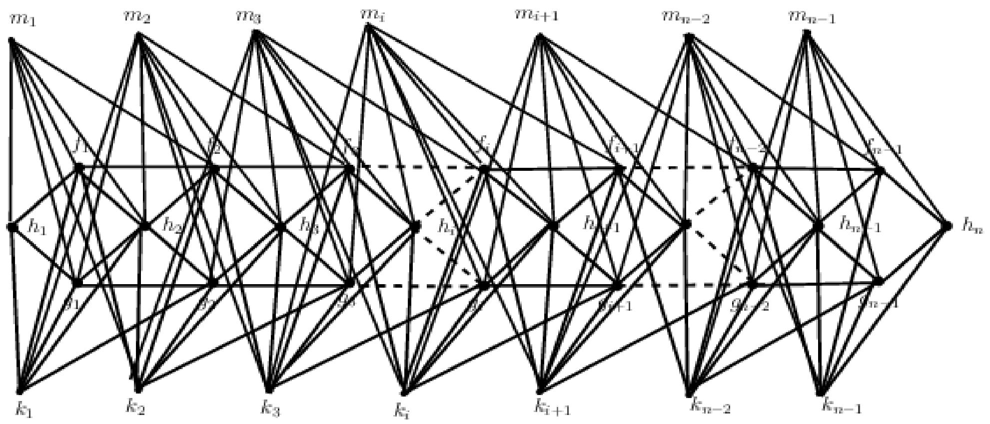

Figure 8: Line Graph of the Diagonal Ladder Graph $\mathcal{L}(DL_n)$

Let F be a function defined by $F: E \to C$, where $C \subseteq N$. Now we have the star edge coloring of $G = \mathcal{L}(DL_n)$for the given positive integers n as given below:

Case 1: n = 2

In this case, $G = \mathcal{L}(DL_2)$ contains $G_1 = \mathcal{L}(TL_2)$ as a subgraph and it is clear that $\chi'_{st}(\mathcal{L}(TL_2)) = 7$ and in the graph G, it is not possible to color the edges with the sameset of colors already used in the coloring of G_1, so we introduce the eighth color forthe edges of $G \backslash G_1$. Therefore, $\chi'_{st}(\mathcal{L}(DL_2)) = 8$.

Case 4: n = 3,4,5, 6 and n ≥ 7

The proof ofother cases are discussed in the following table:

n	Graph G	Maximal induced subgraph	Lower bound for $\chi'_{st}(G)$	$\chi'_{st}(G)$
3	$\chi'_{st}(\mathcal{L}(DL_3))$	$\chi'_{st}(\mathcal{L}(TL_3))$	10	11
4	$\chi'_{st}(\mathcal{L}(DL_4))$	$\chi'_{st}(\mathcal{L}(TL_4))$	10	12
5	$\chi'_{st}(\mathcal{L}(DL_5))$	$\chi'_{st}(\mathcal{L}(TL_5))$	11	13
6	$\chi'_{st}(\mathcal{L}(DL_6))$	$\chi'_{st}(\mathcal{L}(TL_6))$	11	13
7	$\chi'_{st}(\mathcal{L}(DL_7))$	$\chi'_{st}(\mathcal{L}(TL_7))$	12	14

Table 2: Star Edge Coloring of $\mathcal{L}(DL_n)$

By using the similar argument, we can prove that $\chi'_{st}(\mathcal{L}(DL_n)) = 14$, for all n ≥ 7.

Note that $E(G) = E(G') \cup E(G \backslash G')$, where $G' = L(TL_n)$. As we mentioned previouslythe edges of the subgraphG'can be colored using the same coloring pattern as givenin Theorem 3.2, Hence the star edge coloring pattern for the edges of $G \backslash G'$ is asfollows:

If $1 \leq i \leq 5$,

$$\mathcal{F}(m_i, h_i) = \begin{cases} 2, if\ i = 1 \\ 9, if\ i = 2 \\ 9, if\ i = 3 \\ 0, if\ i = 4 \\ 4, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, f_i) = \begin{cases} 4, if\ i = 1 \\ 6, if\ i = 2 \\ 11, if\ i = 3 \\ 2, if\ i = 4 \\ 12, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, h_{i+1}) = \begin{cases} 6, if\ i = 1 \\ 11, if\ i = 2 \\ 2, if\ i = 3 \\ 9, if\ i = 4 \\ 11, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, g_i) = \begin{cases} 3, if\ i = 1 \\ 10, if\ i = 2 \\ 10, if\ i = 3 \\ 11, if\ i = 4 \\ 5, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, f_{i+1}) = \begin{cases} 9, if\ i = 1 \\ 8, if\ i = 2 \\ 4, if\ i = 3 \\ 5, if\ i = 4 \\ 9, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, k_{i-1}) = \begin{cases} 1, if\ i = 2 \\ 12, if\ i = 3 \\ 7, if\ i = 4 \\ 2, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, k_{i+1}) = \begin{cases} 10, if\ i = 1 \\ 12, if\ i = 2 \\ 3, if\ i = 3 \\ 3, if\ i = 4 \\ 1, if\ i = 5 \end{cases}$$

$$\mathcal{F}(m_i, g_{i-1}) = \begin{cases} 5, if\ i = 2 \\ 5, if\ i = 3 \\ 1, if\ i = 4 \\ 8, if\ i = 5 \end{cases}$$

If $6 \leq i \leq n-1$,

$$\mathcal{F}((m_i, h_i) = \begin{cases} 6, & if\ i \equiv 0 (mod\ 3) \\ 9, & if\ i \equiv 1\ (mod\ 3) \\ 11, & if\ i \equiv 2 (mod\ 3) \end{cases}$$

International Journal of Applied Mathematics
Volume38No.3s,2025
ISSN:1311-1728(printedversion);ISSN:1314-8060(on-line version)

$$\mathcal{F}((m_i, f_i) = \begin{cases} 10, & if\ i \equiv 0(mod\ 3) \\ 10, & if\ i \equiv 1\ (mod\ 3) \\ 9, & if\ i \equiv 2\ (mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, h_{i+1}) = \begin{cases} 0, & if\ i \equiv 0(mod\ 3) \\ 6, & if\ i \equiv 1\ (mod\ 3) \\ 10, & if\ i \equiv 2(mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, g_i) = \begin{cases} 5, & if\ i \equiv 0(mod\ 3) \\ 0, & if\ i \equiv 1\ (mod\ 3) \\ 12, & if\ i \equiv 2(mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, f_{i+1}) = \begin{cases} 2, & if\ i \equiv 0(mod\ 3) \\ 8, & if\ i \equiv 1\ (mod\ 3) \\ 13, & if\ i \equiv 2(mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, k_{i-1}) = \begin{cases} 3, & if\ i \equiv 0(mod\ 3) \\ 12, & if\ i \equiv 1\ (mod\ 3) \\ 6, & if\ i \equiv 2(mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, k_{i+1}) = \begin{cases} 4, & if\ i \equiv 0(mod\ 3) \\ 7, & if\ i \equiv 1\ (mod\ 3) \\ 4, & if\ i \equiv 2(mod\ 3) \end{cases}$$

$$\mathcal{F}((m_i, g_{i-1}) = \begin{cases} 12, & if\ i \equiv 0(mod\ 3) \\ 1, & if\ i \equiv 1\ (mod\ 3) \\ 7, & if\ i \equiv 2(mod\ 3) \end{cases}$$

Due to the structure of$\mathcal{L}(DL_n)$, $\mathcal{F}$ has a repeating pattern for n ≥ 7and hence it sufficies to show that $\mathcal{L}(DL_{10}$)has a 14-star edge coloring. The star edge coloring of $\mathcal{L}(DL_{10})$ is shown in Figure 9. Thus, $\chi'_{st}(\mathcal{L}(DL_n)) = 14, for\ n \geq 7.$

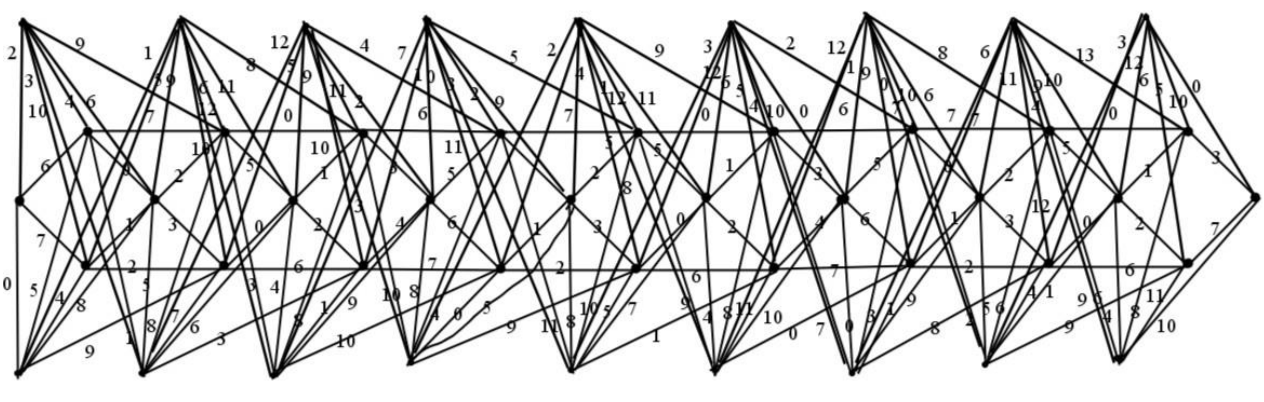

Figure 9: 14-Star edge coloring of $\mathcal{L}(DL_n)$

Received : August05,2025

ISSN:1311-1728(printedversion);ISSN:1314-8060(on-line version)

Conclusion

In this paper, the star chromatic index of the line graph of ladder graph$\mathcal{L}(L_n)$ and the line graph of the family of ladder graphs such as Triangular ladder graph and the Diagonal ladder graph , $\mathcal{L}(TL_n)$ and $\mathcal{L}(DL_n)$ have also been obtained.

References

[1] Bondy J A, Murty U S R, Graph Theory with Applications, London, Macmillan,(1976).

[2] Benzegova L, et al., Star Edge Coloring of Some Classes of Graphs, J.GraphTheory, 81(1) (2016), 73 -82

[3] Chenxu Yang, Deng Chinxao, On the Local Metric Dimension of Line Graphs,Journal of Interconnection Networks, 2350026 (2023), 1 - 15.

[4] Dvorak Z , Mohar B and Samal R, Star Chromatic Index, J.Graph Theory, 72,(2013), 313 - 326.

[5] Evangeline Lydia W, Vijaya Xavier Parthipan J, Restrained Star Edge Coloringof Graphs and its Application in Optimal and Safe Storage Practices, RatioMathematica - Journal of Mathematics, Statistics, and Applications 48, (2023),564 - 575.

[6] Hui Lei, Yongtangshi, A Survey On Star Edge Coloring of Graphs, arXiv:2009.08017V1 [math.CO], 17 Sep 2020.

[7] Jethruth Emelda Mary L, et al., A Study on Star Chromatic Number of SomeSpecial Classes of Graphs, GlobalJournal of Pure and Applied Mathematics,13(9), (2017), 4569 – 4593.

[8] Liu X S, Deng K, An Upper Bound on the Star Chromatic Index of Graphs with$\delta \geq 7$, J.Lanzhou Univ(nat-sci), 44, (2008), 94 - 95.

[9] Ramesh G ,Sathiya A, Some New Results on Ladder Graphs , InternationalJournal of Research and Analytical Reviews , 5, (2018), 296 - 302.

[10] Sateesh Kumar T V, Meenakshi S, Family of Ladder Graphs are Properly Lucky,Journal of Pharmaceutical Negative Results, 13, Special Issue 3, (2022), 1579 -1583.

[11] OmooniB and Dastjerbi M V, Star Edge Coloring of Cartesian Product ofGraphs, arXiv: 1802.01300, 2018.

Received : August05,2025