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MODELING THE SPREAD OF TUBERCULOSIS IN DISASTER SITUATIONS USING SEIR MODEL

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Abstract

The spread of tuberculosis (TB) and disaster are two events that both have a negative impact, but when they occur together, there is often

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a neglect of the spread of TB. The number of individuals piling up in refugee camps with different disease histories could be a place for TB to spread. This study aims to analyze parameters that are important factors in modeling the spread of TB in disaster situations with the SEIR model. This study involved 75 respondents from TB program managers in cities and sub-districts. Respondents involved in focus group discussions (FGD) are those who have handled TB cases in disaster situations such as floods, earthquakes, and social disasters. The results obtained are that there are two groups of obstacles in the management of TB cases in disaster situations, namely environmental factors and habitual factors. These two factors influence the process of case discovery and healing. Based on the results obtained, the effort that needs to be made in disaster situations is to increase the healing process to more than three times under normal conditions. This needs to be done because a buildup of infected populations will occur and be visible after a few months after the disaster.

Math. Subject Classification: 92D25, 92D30
Key Words and Phrases: TB models, health disaster management, environmental factor, habitual factor, TB treatment.

1 Introduction

TB can be prevented, treated, and cured, but the disease is a serious public health problem worldwide, affecting the most vulnerable groups of society. In addition, although the cost of diagnosis is low, lack of examination, delay in investigation, and misdiagnosis are common causes of delayed treatment and unreported cases, making it difficult to take action to break the chain of disease transmission. Therefore, scalable, low-cost, and rapid-response solutions are essential to prevent populations from advanced and critical tuberculosis. Middle-aged people, who are between the ages of 18 and 50 at their most productive age, are most affected by TB. Based on records, the estimated global incidence rate peaked in 2002 at 141 cases per 100,000 population, and gradually in 2010 the number of cases fell to 128 per 100,000 population [1]. In Indonesia, TB cases have risen and placed Indonesia to rank 2nd TB in the world.

Various efforts have been made in various countries, for example the use of artificial intelligence to detect TB in Chest radiographs (CXR) which has made significant progress over the last three decades [2], [3]. Despite widespread publicity and awareness regarding the implementation of various control measures, such as Bacillus Calmette Guerin (BCG) vaccination, direct observation ther-

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apy strategies (DOTS) of the TB cessation section at WHO that focus on case finding and short-term chemotherapy, the global burden of TB has increased in recent decades [1]. In countries where TB infection is more rampant, infants are often given the BCG vaccine because it can prevent severe TB in children [4]. For example, the BCG vaccine is not recommended for use by the general public in the United States due to the fact that it only has the best effect on children. Until now, a number of new TB vaccines are still in various stages of development and testing. Pampaloni et al. [5] states that immigrants and refugees arriving in Italy are sent directly to the Port. The screening results showed that at least 25% of refugees had a productive cough and detected some infected with TB. Research [5] also states that screening programs reduce dissemination based on mathematical model simulations performed.

Omede et al. [6] highlights the challenges posed by drug resistance in the treatment of tuberculosis and highlights the importance of understanding disease dynamics through mathematical modeling to develop effective strategies in its control. Many mathematical models have been established to describe the conditions under which TB spreads. In general, there are succeptible (S), exposed (E), infected (I), and recovery (R) populations [7]. In population S, researchers have used several ways to form accurate S, such as dividing it into age levels [8], [9], levels related to doses at a certain age [10], and infant populations given the vaccine [11]. In population E, it relies heavily on several genes in TB coreceptors (e.g. HLA and non-HLA) that are associated with susceptibility and resistance to tuberculosis and the rate of progression to active TB. The speed of population E to I led some researchers to divide it into latent with long progress, exposed with normal time, and exposed which quickly became infected [12]. Whereas in population I, there are often studies that divide it into treatment populations [13], infection without knowledge and with knowledge of TB [14], and quarantine. While the results of infection treatment will be distributed into several population compartments such as perfect and inperfect recovery [15].

TB is also often associated with some vulnerable groups, including groups coinfected with HIV [16], groups with lung problems such as smokers [17] and Covid19 [18], [19], children under the age of 15 [20]–[23] and groups with comorbid disorders such as diabetes [24]. Ideal modeling often ignores that these groups are separate from the general public. In fact, in certain circumstances this group can be in the same environment. In this study, the model built is based on the assumption that all vulnerable groups are in the same environment as the general group as in a disaster situation. Irregularities caused by disasters can undermine the order of programs for TB elimination. Therefore, this study

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aims to model the spread of TB in disaster situations by considering certain parameters that arise in disaster situations.

2 Method

The study was conducted in Aceh Province, Indonesia with the consideration that this area has a representative disaster history for the model. Disasters recorded in the history of data collection include natural disasters (earthquakes, landslides, and floods) and social disasters (conflicts and refugees from other countries). The parameters considered in this study are environmental and habitual parameters obtained from the results of focus group discussions in 5 cities in Aceh Province. There were 75 respondents from TB officers at the city and sub-district levels who gave opinions.

The initial state of the individual can be said to be susceptible (S) which can then turn into a latent / exposed individual (E), but this individual has not been fully infected with the disease (I). If individual E becomes infected then it can be said that the individual turns into I. Once infected and then becomes cured, this individual becomes a cured individual (R). In general, these changes can be seen in Figure 1 below:

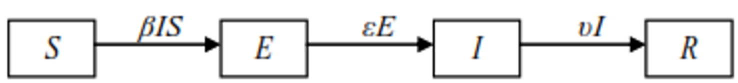

Figure 1: SEIR type epidemic model

Changes in vulnerable groups caused by the interaction of vulnerable individuals with sick individuals with transmission rate parameters of β which will then be included in the latent group, resulting in an increase in the number of individuals in the latent group. Individuals in the latent group will decrease due to the active bacteria in the patient's body, so that they will be included in the sick group with a change rate parameter of ϵ, this results in an increase in individuals in the sick group. The number of individuals in the sick group will decrease if the individual has recovered from the disease and will be included in the cured population with a cure rate parameter of ν, this results in an increase in individuals in the cured group. The number of individuals who have recovered is assumed not to be infected with TB disease again, so a system of differential equations can be made for the main model of SEIR type epidemics as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta SI \\
\frac{dE}{dt} &= \beta SI - \varepsilon E \\
\frac{dI}{dt} &= \varepsilon E - \nu I \\
\frac{dR}{dt} &= \nu I \\
N &= S + E + I + R
\end{aligned}
\tag{1}
$$

Information:
N : Total population (inhabitants)
S : groups of susceptible individuals (inhabitants)
E : Latent groups of individuals (inhabitants)
I : groups of sick individuals (inhabitants)
R : groups of recovery individuals (inhabitants)
β : contagion speed parameters (per years)
ϵ : Parameters of the rate of change from latent individual to pain (per years)
ν : Rate parameters of recovery (per years).

The model is a bilinear-form model, where the number of interactions between individuals per unit time increases by the increase in total population (N). While the standard form is used when the number of interactions of sick individuals is limited or grows lower than the total population (N).

Systematically, the system of differential equations (1) can be written as:

$$\dot{x}(t) = f(t, x), \tag{2}$$

with:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

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Then,

$$\dot{x}(t) = \frac{dx}{dt} = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \vdots \\ \frac{dx_n}{dt} \end{pmatrix}, f(t,x) = \begin{pmatrix} f_1(t, x_1, x_2, \ldots, x_n) \\ f_2(t, x_1, x_2, \ldots, x_n) \\ \vdots \\ f_n(t, x_1, x_2, \ldots, x_n) \end{pmatrix} \tag{3}$$

$x_1, x_2, \ldots, x_n$ are dependent variable and t is independent. If the right segment of equation (3), the variable t is not expressed explicitly, then the system is said to be an autonomous system and mathematically can be written $x(t) = f(x)$.

If each function $f_1, f_2, \ldots, f_n$ in equation (3) is a linear function of the variable t and the variable $x_1, x_2, \ldots, x_n$, then the system is called a system of linear differential equations. If $x(t)$ is a variable bound to a variable t, $\dot{x}(t)$ is a derivative of $x(t)$ towards t and $a(t)$ is the coefficient $x(t)$, then a system of n number of n-order linear differential equations can be written in the form:

$$\begin{aligned} a_{11}x + a_{12}x' + \cdots + a_{1(n-1)}x^{(n-1)} + a_{1n}x^{(n)} &= f_1(t) \\ a_{21}x + a_{22}x' + \cdots + a_{2(n-1)}x^{(n-1)} + a_{2n}x^{(n)} &= f_2(t) \\ &\vdots \\ a_{n1}x + a_{n2}x' + \cdots + a_{n(n-1)}x^{(n-1)} + a_{nn}x^{(n)} &= f_n(t) \end{aligned} \tag{4}$$

The system of n first-order linear differential equations are:

$$\begin{aligned} \dot{x}_1(t) &= a_{11}x_1(t) + a_{12}x_2(t) + \cdots + a_{1n}x_n(t) + f_1(t) \\ \dot{x}_2(t) &= a_{21}x_1(t) + a_{22}x_2(t) + \cdots + a_{2n}x_n(t) + f_2(t) \\ &\vdots \\ \dot{x}_n(t) &= a_{n1}x_1(t) + a_{n2}x_2(t) + \cdots + a_{nn}x_n(t) + f_n(t) \end{aligned} \tag{5}$$

System (5) can be written in matrix form:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} + \begin{pmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{pmatrix} \tag{6}$$

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which can be written in the form of:

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + f(x) \tag{7}$$

If the function $f(x) = 0$, then system (7) is said to be homogeneous which can written $\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t)$. If the system coefficient is a constant, then system (7) can be written in the form:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \tag{8}$$

Briefly, equation (8) can be written $\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t)$. If each function $f_1, f_2, \ldots, f_n$, Equation (3) is not a linear function of the independent variable t and the dependent variable $x_1, x_2, \ldots, x_n$, then the system is called a system of nonlinear differential equations. Autonomous systems containing first-order nonlinear differential equations can be written in the form of:

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, x_2, \ldots, x_n) \\ \frac{dx_2}{dt} &= f_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(x_1, x_2, \ldots, x_n). \end{aligned} \tag{9}$$

Solving equation (1) using assumptions in equation (9) by the ode45 function in the software used in this study, namely MATLAB R2013a which functions as a tool to simulate and display a graph.

3 Results and Discussion

Findings in the FGD

Based on the results of FGD in primary data collection at five locations in Aceh province, namely Lhokseumawe, North Aceh, Bener Meriah, Central Aceh and Banda Aceh City, the development of disease outbreaks that occurred in the community showed that there was a decrease in the exposure value (E), which reflected the number of individuals exposed or exposed to the disease. This

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increase can be due to lack of adherence to the preventive measures implemented, such as infection control measures, isolation, or the use of masks. An increase in exposure value indicates that preventive measures taken have not been successful in reducing the level of exposure to the disease.

The description of the value of infected (I) is not detected due to several factors, including when individuals exposed to mycobacterium tuberculosis bacteria do not show symptoms so that the individual does not want to check with officers / health services. This happens because there is still a strong social stigma in some levels of society. Individuals infected with TB may worry about disclosing their TB status to others and face discrimination/exclusion. Then, many cases of infection (I) become Expose cases (E) due to low individual compliance in taking medication for several reasons, namely individuals feel that they have recovered the duration of treatment is too long. After only a few days of taking the medication, the individual feels the symptoms diminish or even disappear completely. This makes them think that they are cured and no longer need to take medicine. On the other hand, the recovery value (R) decreases, which describes the number of individuals who recover from the disease and become immune to infection. A decrease in rehabilitation scores indicates a decrease in overall recovery in the population. In addition, there is an increase in the susceptible value (S), which describes the number of individuals susceptible to infection. An increase in susceptible values can occur if there is a decrease in individual immunity in the population or if there are population groups that have not been exposed or previously infected. Increased susceptible values indicate that there is still a large portion of the population that is susceptible to infection and needs to be protected or vaccinated to prevent wider spread of the disease.

There are at least 30 parameters disclosed in FGD, but this study is grouped based on respondents' recommendations so that the remodeling process is carried out with the addition of a new parameter, namely relapse (φ) or the process of population R returning to population S. The increase in the number of S population is also affected by birth without disease (Λ). Parameters of natural death (μ) and death from disease (μ_i) for the relevant compartments. The unknown cure parameter is very small, but it needs to be considered as the rate of change in the number of populations from E to I given the symbol ψ. The notification parameter brings S to I with the rate of change α. The strategy for using the ode45 function is that all state variables become in percent form to N

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i.e. denoted by regular letters without capitals.

$$
\begin{aligned}
\frac{ds}{dt} &= \Lambda + \phi r - (\alpha + \beta) si - \mu s \\
\frac{de}{dt} &= \beta si - (\varepsilon + \psi + \mu) e \\
\frac{di}{dt} &= \alpha si + \varepsilon e - (\nu + \mu_i) i \\
\frac{dr}{dt} &= \psi e + \nu i - (\phi + \mu) r
\end{aligned}
\tag{10}
$$

Simulation

The simulation results are based on TB distribution and notification target data in Aceh Province as initial values for formula solutions (9) and (10). The total population is $N = 5,407,855$, where the number of infections that are the burden of TB is $I = 19,160$. The exposure value of E obtained is 103464 with recovery according to the assumption $R = 0.8I$. While for population S is the remainder of the reduction of N with all other populations. The values of the parameters are obtained from the distribution of middle values of the recording data of four quarters of TB progress in Aceh and are presented in Table 1.

Table 1: Parameter values

No	Parameters	Value (per years)
1	Birthrate Λ	0.001964
2	Mortality rate μ	0.001964
3	Infected Mortality rate μ_i	0.039
4	Relapse ϕ	$0 - 0.95$
5	Infected rate α	0.0468
6	Infected rate β	0.0468
7	Recovery rate ε	0.1472
8	Recovery rate ψ	0.1472

Source: Primary and secondary data from the recording of the TB notification application of the Aceh Provincial Health Office, 2023

Using the parameters obtained in Table 1 which are substituted in equation 10, the following simulation is obtained:

The relapse parameter is based on complaints from TB officers wh o found that cases that have been marked as cured will have a susceptibility to contracting TB with higher cases. The model is only able to put this cured population

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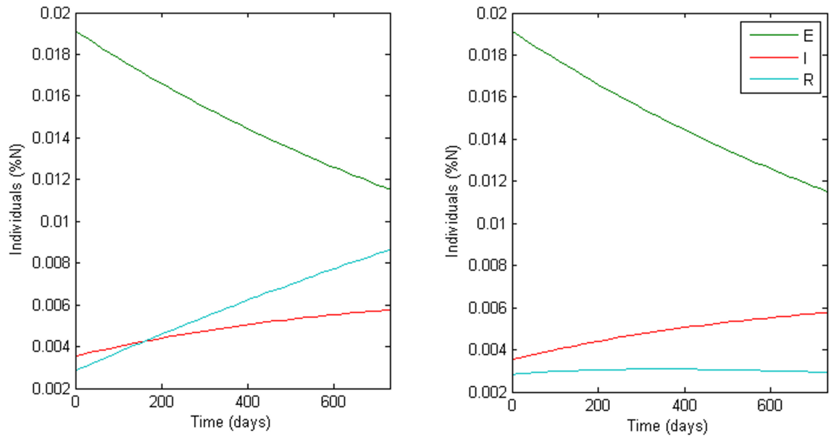

(a) No Relapse (b) Relapse

Figure 2: Comparison of the effect of relapse parameters on models

into vulnerable again and cannot accommodate moving to the exposed or infected population compartment to higher. This is clearly due to the fact that when granting cured status, officers procedural ensure that there are no more physical signs of TB. Such conditions will also return to pain but not in the near future. This review of relapse was also stated in another study with a range of 1.4 years [25],[26]. Figure 2(a) shows that there is a balance point between infected and recovery, when in fact it will not happen because it is clear that it will be very difficult to find a relatively fast recovery in the second quarter of each year. Assumptions are built in accordance with conditions in the field, meaning that there is a need to consider the parameters of repeated infections in the TB spread model with this data. Figure 2(b) explains that there is still a long way to go to achieve recovery and infected equilibrium.

The model using the parameters in Figure 2(b) will look for combinations that can show equilibrium, but beforehand it is necessary to learn in advance about the working area of each parameter against changes in state variables. Figure 3 shows the sensitivity of state variables if the screening, contact search and notification parameters from S to I or E to I are raised. The exposed, infected, and recovery variables only affect the second year, which means it is very difficult to perform this search. Each parameter is assigned a multiplier up to two times but the result will still be visible in the second year and the value is also insignificant. In FGD, this is explained by respondents as a barrier factor. The

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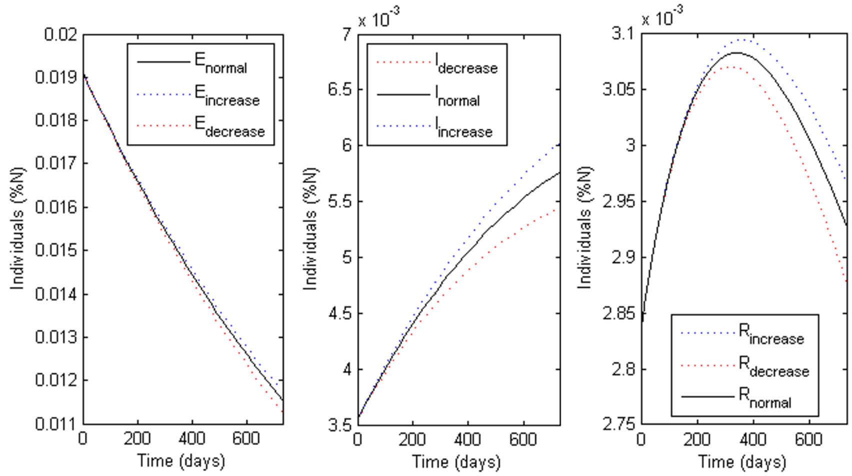

(a) Exposed (b) Infected (c) Recovery

Figure 3: Comparison of the effect of screening, contact search and notification parameters on the model

large population demands sufficient personnel for screening and contact tracing services. That is, not only human resources, but also facilities also need to be well prepared. Operational funding support and the availability of sputum bottles are clearly derivative problems of this control effort. Furthermore, contact tracing is promising in several studies in India and the Philippines [27], [28]. Other study [29] given that this study is a study at certain times such as disasters that must gather large populations in communities that do not sort and give special space to TB sufferers, optimistically, healing parameters can be studied with efficiency several times the normal situation.

Here the study attempts to find the equilibrium of the model by multiplying the healing parameter by the constant c. This constant aims to analyze the extent to which the model is able to illustrate that TB infection cases will fall below the cure line. In Figure 4(a), it occurs at the end of the second year which means that the third year is the decisive year whether there will be a new wave or has been successfully weakened. The same would happen in Figure 4(b). Whereas in Figures 4(c) and 4(d), it can be seen that the recovery ability attempted from $c = 3.2$ to 3.3 greater than the current one will result in a line of infection that continues to fall. The simulations showed that by the end of the first year the number of infections was already below the initial number of infections included in the model. This means that the model has found that the

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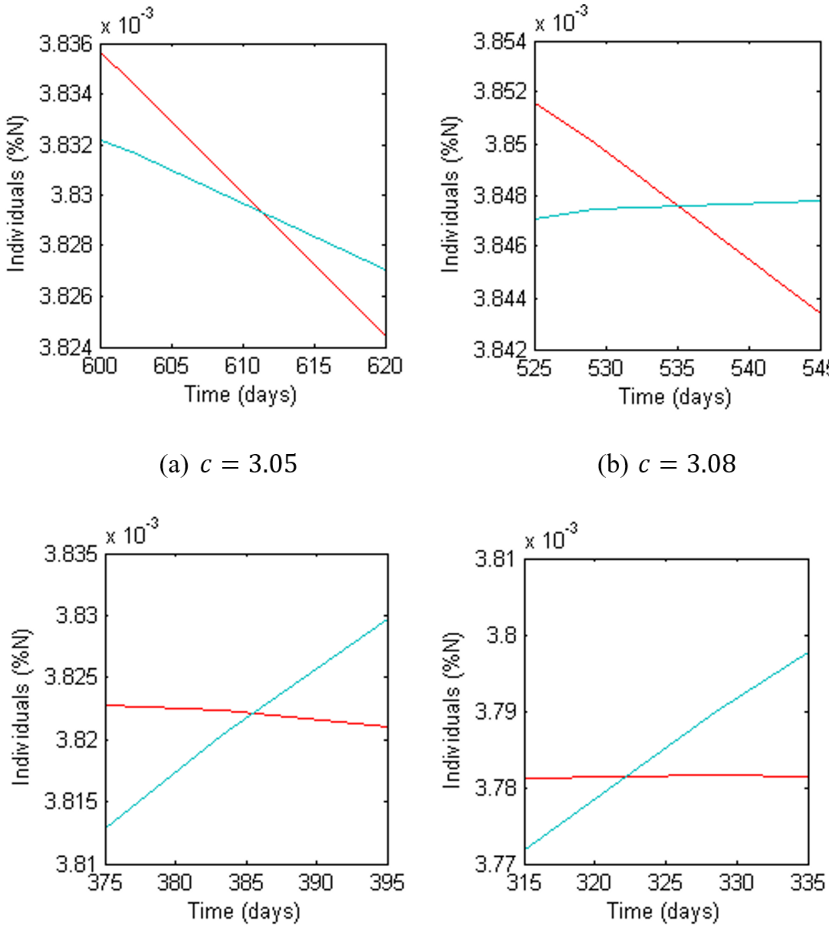

(a) $c = 3.05$ (b) $c = 3.08$

(c) $c = 3.2$ (d) $c = 3.3$

Figure 4: Comparison of the influence of treatment success parameters

effort that must be given to make the patient recover needs to be increased to 3.2 times the normal state.

4 Conclusion

Based on the results of the study, disaster situations that occur in just a short period can have a major impact on the spread for the next year. With normal treatment, the iceberg phenomenon will appear in the second year with the

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number of infections greater than the initial infection. An effort that can be done is to run the program under normal circumstances in the process of screening, contact tracing, and notifications. Optimistically, massive healing can also be done, which is up to more than times the healing effort in normal conditions. This can be done because the influence of environmental and habitual parameters is different at the time of the disaster, i.e. the infection population is in an area that can be fully monitored by TB officers. This model upgrade is only limited to projected impact in one to two years. In further research, it is recommended to use optimization control methods with Hamiltonians to obtain results that are more quickly balanced.

Ethical Statement

This study was performed with approval from the Ethics committee of the Health Polytechnic of the Ministry of Health Aceh (LB.02.03/38/2023).

References

[1] K. M. Owolabi, E. Pindza, "A nonlinear epidemic model for tuberculosis with Caputo operator and fixed point theory," *Healthc. Anal.*, **2** (2022), 100111.

[2] A. Ueslei da Fonseca et al., "A novel tuberculosis diagnosis approach using feed-forward neural networks and binary pattern of phase congruency," *Intell. Syst. with Appl.*, **21** (2024), 200317.

[3] S. Vats et al., "Incremental learning-based cascaded model for detection and localization of tuberculosis from chest x-ray images," *Expert Syst. Appl.*, **238** (2024), 122129.

[4] P. Roy, J. Vekemans, A. Clark, C. Sanderson, R. C. Harris, R. G. White, "Potential effect of age of BCG vaccination on global paediatric tuberculosis mortality: a modelling study," *Lancet Glob. Heal.*, **7** (2019), e1655–e1663.

[5] A. Pampaloni et al., "'Diagnosis on the Dock' project: A proactive screening program for diagnosing pulmonary tuberculosis in disembarking refugees and new SEI model," *Int. J. Infect. Dis.*, **106** (2021), 98–104.

International Journal of Applied Mathematics
Volume 38 No. 7s 2025,
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

[6] B. I. Omede, O. J. Peter, W. Atokolo, B. Bolaji, T. A. Ayoola, "A mathematical analysis of the two-strain tuberculosis model dynamics with exogenous re-infection," *Healthc. Anal.*, **4** (2023), 100266.

[7] K. Das, B. S. N. Murthy, S. A. Samad, M. H. A. Biswas, "Mathematical transmission analysis of SEIR tuberculosis disease model," *Sensors Int.*, **2** (2021), 100120.

[8] H. Khan, K. Alam, H. Gulzar, S. Etemad, S. Rezapour, "A case study of fractal-fractional tuberculosis model in China: Existence and stability theories along with numerical simulations," *Math. Comput. Simul.*, **198** (2022), 455–473.

[9] Fatmawati, U. Dyah Purwati, F. Riyudha, H. Tasman, "Optimal control of a discrete age-structured model for tuberculosis transmission," *Heliyon*, **6** (2020), e03030.

[10] K. K. Radtke et al., "Alternative dosing guidelines to improve outcomes in childhood tuberculosis: a mathematical modelling study," *Lancet Child Adolesc. Heal.*, **3** (2019), 636–645.

[11] S. Rashid, M. K. Iqbal, A. M. Alshehri, R. Ashraf, F. Jarad, "A comprehensive analysis of the stochastic fractal–fractional tuberculosis model via Mittag-Leffler kernel and white noise," *Results Phys.*, **39** (2022), 105764.

[12] D. Okuonghae, "A mathematical model of tuberculosis transmission with heterogeneity in disease susceptibility and progression under a treatment regime for infectious cases," *Appl. Math. Model.*, **37** (2013), 6786–6808.

[13] W. Shatanawi et al., "A fractional dynamics of tuberculosis (TB) model in the frame of generalized Atangana–Baleanu derivative," *Results Phys.*, **29** (2021), 104739.

[14] S. Olaniyi, S. F. Abimbade, F. M. Chuma, O. A. Adepoju, O. D. Falowo, "A fractional-order tuberculosis model with efficient and cost-effective optimal control interventions," *Decis. Anal. J.*, **8** (2023), 100324.

[15] D. Aldila et al., "A tuberculosis epidemic model as a proxy for the assessment of the novel M72/AS01E vaccine," *Commun. Nonlinear Sci. Numer. Simul.*, **120** (2023), 107162.

Received: August 08 ,2025

[16] O. M. Otunuga, "Analysis of the impact of treatments on HIV/AIDS and Tuberculosis co-infected population under random perturbations," *Infect. Dis. Model.*, **9** (2024), 27–55.

[17] T. Faniran, A. Ali, M. O. Adewole, B. Adebo, O. O. Akanni, "Asymptotic behavior of Tuberculosis between smokers and non-smokers," *Partial Differ. Equations Appl. Math.*, **5** (2022), 100244.

[18] Y. Marimuthu, B. Nagappa, N. Sharma, S. Basu, K. K. Chopra, "COVID-19 and tuberculosis: A mathematical model based forecasting in Delhi, India," *Indian J. Tuberc.*, **67** (2020), 177–181.

[19] K. G. Mekonen, L. L. Obsu, "Mathematical modeling and analysis for the co-infection of COVID-19 and tuberculosis," *Heliyon*, **8** (2022), e11195.

[20] P. J. Dodd, E. Gardiner, R. Coghlan, J. A. Seddon, "Burden of childhood tuberculosis in 22 high-burden countries: a mathematical modelling study," *Lancet Glob. Heal.*, **2** (2014), e453–e459.

[21] S. Charalambous et al., "Scaling up evidence-based approaches to tuberculosis screening in prisons," *Lancet Public Heal.*, **8** (2023), e305–e310.

[22] P. K. Yadav, P. Goel, "Treatment seeking dilemma for tuberculosis as timed strategic prisoner's dilemma game," *Phys. A Stat. Mech. its Appl.*, **632** (2023), 129297.

[23] M. Knipper et al., "The need for protecting and enhancing TB health policies and services for forcibly displaced and migrant populations during the ongoing COVID-19 pandemic," *Int. J. Infect. Dis.*, **113** (2021), S22–S27.

[24] S. Selvaraju et al., "Prevalence and factors associated with tuberculosis infection in India," *J. Infect. Public Health*, **16** (2023), 2058–2065.

[25] Y. Li, X. Liu, Y. Yuan, J. Li, L. Wang, "Global analysis of tuberculosis dynamical model and optimal control strategies based on case data in the United States," *Appl. Math. Comput.*, **422** (2022), 126983.

[26] I. Ullah, S. Ahmad, M. Zahri, "Investigation of the effect of awareness and treatment on Tuberculosis infection via a novel epidemic model," *Alexandria Eng. J.*, **68** (2023), 127–139.

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[27] A. S. Bhadauria, H. N. Dhungana, V. Verma, S. Woodcock, T. Rai, "Studying the efficacy of isolation as a control strategy and elimination of tuberculosis in India: A mathematical model," *Infect. Dis. Model.*, **8** (2023), 458–470.

[28] J. S. Calderon, K. E. Perry, S. S. Thi, L. L. Stevens, "Innovating tuberculosis prevention to achieve universal health coverage in the Philippines," *Lancet Reg. Heal. - West. Pacific*, **29** (2022), 100609.

[29] R. M. G. J. Houben et al., "Feasibility of achieving the 2025 WHO global tuberculosis targets in South Africa, China, and India: a combined analysis of 11 mathematical models," *Lancet Glob. Heal.*, **4** (2016), e806–e815.