

A Modified Algorithm for Solving Transportation Problem by using Least Cost Method

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Abstract

In this paper, we present a modified tool of Two-step exact algorithm for transportation problem. We use some basic ideas of Least Cost Cell method and Modified Distribution method. The modified algorithm is equally effective for balanced transportation problem as well as unbalanced transportation problem. The effectiveness of algorithm is discussed by considering several problems with experimental setup followed by result analysis because Optimal route selection for delivering product is the most key concern for companies related to supply chain. Route selection is an important aspect for such companies as it greatly affects in the financial section.

Keywords: supply chain management, balanced and unbalanced transportation problems, Modified tool for distribution methods(SDG-12)

1 Introduction

The Transportation Problem (Taha, 2004; Winston et al., 2004; Greenberg, 2012; Woodruff, 2013) features a specialized data structure in its solution, represented

as a transportation graph that forms a significant area within integer linear programming. The Transportation Problem was first formalized by Monge in 1781. Significant advancements were later made during World War II by Soviet mathematician Leonid Kantorovich, leading to the problem also being referred to as the Monge-Kantorovich Transportation Problem (Kantorovich, 1960). It has been extensively studied in the field of mathematical programming and engineering, where it is sometimes discussed under related topics such as facility location and allocation problems.

The very first significant contribution to the Transportation Problem was made by Hitchcock (1941) in his study titled 'The Distribution of a Product from several Sources to Numerous Localities.' The development of transportation methods involving multiple sources and destinations is largely credited to both Hitchcock and Koopmans (Koopmans, 1947). The problem earned its name due to its widespread applications in determining optimal transportation routes. George B. Dantzig (1951) further advanced the field by applying Linear Programming techniques to solve transportation models. He later introduced the primal simplex transportation method (Dantzig, 1963) for solving such problems. In 1958, Reinfeld and Vogel proposed the Vogel's Approximation Method, a widely used heuristic algorithm based on penalty cost calculations (Reinfeld et al., 1958).

An algorithm based on the Greedy approach, known as the Least Cost Method, was developed by Dantzig (Dantzig, 1963b). Williams and colleagues (Abadie et al., 1963) applied the decomposition principle of Dantzig and Wolfe (Dantzig et al., 1960) to the solution of Hitchcock's Transportation Problem. Efrayimson and Ray (1966) explored the use of a branch and bound algorithm for solving the plant location problem—a specialized form of discrete programming. Sharp et al. (1970) developed an optimal solution method for the production transportation problem. Roy and Gelders (1981) addressed the transportation of liquid bottled products using a 0–1 integer programming model and obtained the optimal solution via the branch and bound approach. Marcotte and Soland (1986) later introduced a simplified branch and bound algorithm. Lobel (1997) proposed an optimal vehicle routing and scheduling approach for public transportation systems. Equi et al. (1997) presented a combined model integrating transportation and scheduling problems. Ting et al. (2003) applied a dynamic programming model for route planning in shipping operations. Veena and Kowalski (2006) investigated transportation problems involving mixed constraints, while Caputo et al. (2006) provided an optimal solution framework for road transport activities.

This paper proposes an exact algorithm based on the least cost selection strategy of the Least Cost Method, with necessary modifications to derive an improved basic feasible solution. The paper is divided into nine sections; Section-1 is concerned with literature review and introduction of transportation problem. Section 2 discusses the model and basic equation for transportation problems. In the Sec-

tion 3 we discusses the type of transportation problems s.t. balanced and unbalanced problems. Section 4 focuses on solution of types for the same. Section 5 some important basic terminologies related to initial solution. Section 6 discusses related work and section 7 proposed the some tools or algorithm. we discusses algorithms for related and proposed work followed by examples in Section 8 and conclusion and result discussion of work in Section 9.

2 Model and Basic Equations

We assume the transportation problem consist of n destinations and m origins, where

X_{ij} = the amount of products/goods transported from the i^{th} origin to the j^{th} destination.

α_{ij} = the cost required in transporting per unit product from the i^{th} origin to the j^{th} destination.

η_i = the number of units required at the i^{th} origin.

μ_j = the number of units available at the j^{th} destination.

Let we consider the basic linear transportation problem as:

$$Z = \sum_{i=1}^m \sum_{j=1}^n \alpha_{ij} X_{ij} \quad (1)$$

where subject to constraints are

$$Z = \sum_{i=1}^m \sum_{j=1}^n \alpha_{ij} X_{ij} \quad (2)$$

$$\eta_i \geq \sum_{j=1}^n X_{ij}; \forall i \in I = (1, 2, 3, \dots, m)$$

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$\mu_j \leq \sum_{i=1}^m X_{ij}; \forall j \in J = (1, 2, 3, \dots, n)$ (4) and $X_{ij} \geq 0, \forall (i, j) \in I \times J$, for not balanced transportation problem, $\eta_{ij} \geq \mu_{ij}$ or $\eta_{ij} \leq \mu_{ij}$

Destination→ Source↓	1	2	...	N	Supply
1	α_{11}	α_{12}	...	α_{1n}	η_1
2	α_{21}	α_{22}	...	α_{2n}	η_2
...	...		...	...	
m	α_{m1}	α_{m2}	...	α_{mn}	η_m
Demand	μ_1	μ_2	...	μ_n	$\sum_{i=1}^m \eta_i \sum_{j=1}^n \mu_j$

3 Two types of problem

The above minimization objective function is valid when

$$\text{Total demand (Demand)} = \text{Total supply (Supply)}.$$

But common scenario in real life problems

$$\text{Total demand (Demand)} \neq \text{Total supply (Supply)}.$$

Hence we requires balancing of problems . The transportation problem is a type of optimization problem in linear programming where the objective is to transport goods from a set of suppliers to a set of consumers at the minimum cost. Such two type of transportation problem occur:- (a) Balanced & Unbalanced.

3.1 Balanced Transportation Problem

The total supply equals the total demand. This is the ideal situation where Supply is exactly equal to Demand occurred. Such situation can occur for small sized problems, but it is usually uncommon for large sized problems such that delivering product in entire city.

$$\sum \text{Supply} = \sum \text{Demand}$$

3.2 Unbalanced Transportation Problem

In the most of real life problems, s.t. transportation of products within whole entire city, Demand and Supply values are not always equal, hence such type of the problem becomes unbalanced.

$$\sum \text{Supply} \neq \sum \text{Demand}$$

If total supply is greater than total demand, then a dummy demand is added. If total demand is greater than total supply, then a dummy supply is added. The cost for dummy allocations is assumed to be zero. Some authors discussed unbalanced transportation problem in their work “Modified Vogel’s approximation method for the unbalanced transportation problem” Balakrishnan (1990) and Anubhav kumar prasad (2020). The initial basic feasible solution was being calculated without balancing which may produce different result for LCM and in this case it will always select dummy row/column, as it contains always zero.

4 Two types of Solution

4.1 Degenerate Solution

The term degenerate solution was coined for Simplex method (Zoutendijk (1960), Klee et al. (1970)) when one of the basic variables become zero. and may cause cycling, i.e. initial table is again achieved. However, pivoting rules (Gal (1993)) such as Bland's rule (Pan(1990)) can avoid degeneracy. In case of transportation problem, degeneracy occurs when number of assignments made is less than $m + n - 1$, where m and n are number of rows and columns respectively. This demands need for one extra assignment, $\in$ to least cost unassigned cell with non-loop condition to proceed for optimality check. Optimality check is performed using Stepping stone (Chranes et al. (1954)) or Modified distribution method (Dantzig (1963)). This extra assignment helps in forming closed path (loop) in each and every vacant cell for Stepping stone method and in calculation of u and v values for Modified distribution method. Shafaat et al. (1988) showed that degeneracy does not means that initial solution obtained is non optimal and proposed algorithm for making this extra assignment.

4.2 Non-Degenerate Solution

Non-degeneracy is the ideal condition when total assignments equals $m + n - 1$ with m rows and n columns. This condition is required for calculation of u and v values for Modified distribution method as assignments less than $m + n - 1$ will halt the process for calculation of $u - v$ values needed.

5 Some Basic Terminology for Solution

5.1 Basic Solution

The Initial solution should satisfy constraints described in previous Section, i.e. total assignments should sum up to total of Demand or Supply values.

5.2 Feasible Solution

In obtaining the initial solution for a transportation problem, it is essential that all basic variables are non-negative. This requirement ensures that only valid allocations are made, meaning each assignment to a particular cell must be equal to the minimum of the corresponding supply and demand values. This constraint prevents illegal or infeasible assignments and maintains the integrity of the solution.

5.3 Basic Feasible Solution

A Basic Feasible Solution (BFS) in a transportation problem must satisfy both feasibility and basic solution conditions. Specifically, it requires that all allocations are non-negative and that the supply and demand constraints are fully met. In other words, only valid (feasible) assignments should be made, with the allocated quantities summing exactly to the corresponding supply or demand values.

6 Some Related work

6.1 Least Cost Method (LCM)

The Least Cost Method (LCM) is a widely used heuristic algorithm for solving transportation problems, originally developed by Dantzig (1963). It employs a greedy approach for making allocations, similar in concept to Kruskal's algorithm for finding a minimum spanning tree (Ahuja, 2017), where the edge with the smallest weight is selected iteratively.

In LCM, the cell with the lowest transportation cost is identified across the entire cost matrix, and the maximum feasible allocation—defined as the minimum of the corresponding supply and demand—is made to that cell. In cases where multiple cells share the same minimum cost, the selection among them is made arbitrarily.

Although LCM is one of the simplest heuristic methods, it can be time-consuming due to the repeated search for the least cost cell throughout the matrix. The detailed algorithm for LCM is provided in next Section.

6.2 Modified Distribution Method (MODI)

The Modified Distribution Method (MODI), also known as the U-V Method (Dantzig, 1963), is used to test the optimality of a Basic Feasible Solution (BFS) obtained through any initial allocation method, such as the Least Cost Method (LCM). This technique is applicable only to non-degenerate solutions; therefore, if the initial solution is degenerate, it must first be converted into a non-degenerate form.

If the solution is found to be non-optimal, the MODI method improves it by reallocating resources to unassigned cells in a manner that reduces the overall transportation cost. This reallocation is performed using a loop formation strategy, which systematically adjusts the allocations while maintaining feasibility. The process involves the use of specific parameters, which are detailed in the subsequent discussion.

By using following formula find the values of variables u_i and v_j are

$$\alpha_{ij} = u_i + v_j$$

where α_{ij} represents allocated cells;

The process starts with assigning the value either u_i or v_j to 0 depending on whether maximum assignment is in i_{th} row or j_{th} column.

Loop is created if one of the opportunity cost, C_{ij} is negative value. If so, then loop is created for cell with most negative C_{ij} value. Calculation of C_{ij} is made using following formula:

$$C_{ij} = \alpha_{ij} - u_i - v_j$$

. In case if any of C_{ij} value becomes 0, its find alternate path do exist.

A minimum of four cells is required to form a closed loop, and the total number of cells in the loop must be even.

Movement within the loop is restricted to horizontal and vertical directions, and only cells with existing allocations are used to construct the loop.

Adjustments to allocations are made only along the boundary cells of the loop. Specifically, allocations in alternate cells are modified: starting from the newly considered (unallocated) cell, allocations in even-numbered positions are decreased, while those in odd-numbered positions are increased.

The amount of adjustment is equal to the smallest allocation found among the even-numbered cells in the loop, ensuring that no negative allocations result from the update.

7 Some Algorithms

7.1 Algorithm for Least Cost Method (LCM)

Step 1: Check whether the given transportation problem is balanced, i.e., the total supply equals the total demand (also known as the rim condition). If it is balanced, proceed to Step 3; otherwise, go to Step 2.

Step 2: Balance the transportation problem by introducing a dummy row or column, depending on whether there is excess supply or demand. Assign zero transportation costs to all cells in the dummy row or column.

Step 3: Identify the cell with the minimum transportation cost from the cost matrix and allocate as much as possible to that cell. The allocation should be equal to the lesser of the available supply or demand for that cell. In case of a tie (i.e., multiple cells with the same minimum cost), break the tie arbitrarily.

Step 4: Adjust the supply and demand values based on the allocation made. If the supply or demand for a row or column becomes zero, that row or column is excluded from further consideration in subsequent allocations.

Step 5: Repeat Steps 3 and 4 until all supply and demand values are satisfied (i.e., reduced to zero).

Step 6: Calculate the total transportation cost by summing the products of allocated quantities and their corresponding cell costs.

This algorithm provides an initial Basic Feasible Solution (BFS) by allocating resources in a cost-efficient manner. Though simple, the method can be computationally intensive due to the need to repeatedly search for the least cost cell in the entire matrix.

7.2 Algorithm for Modified Distribution Method (MODI)

Step 1: Check the solution for non-degeneracy. If the number of allocations is less than $m + n - 1$, the solution is degenerate. In such a case, introduce an infinitesimally small value (ϵ) to the least-cost unoccupied cell(s) to ensure the correct number of basic variables. This step is essential for the accurate computation of the dual variables u_i & v_j

Step 2: Calculate the dual variables u_i & v_j for all rows ($i = 1, 2, \dots, m$) and columns ($j = 1, 2, \dots, n$) such that for each occupied cell, the cost satisfies the condition:

$$\alpha_{ij} = u_i + v_j$$

Begin by assigning any one of the u_i values arbitrarily (typically $u_1 = 0$ and solve for the remaining variables.

Step 3: calculate the opportunity cost C_{ij} for each unoccupied cell using the formula:

$$C_{ij} = \alpha_{ij} - (u_i + v_j)$$

Step 4: Evaluate the optimality of the current solution based on the calculated C_{ij} values-

If all $C_{ij} > 0$: the current solution is optimal and unique.

If at least one $C_{ij} = 0$ and the rest are positive: the solution is optimal but has alternate solutions.

If any $C_{ij} < 0$: the solution is not optimal. then we Proceed to Step 6.

Step 5: If Step 4 indicates optimality, the algorithm terminates. Otherwise, proceed to improve the solution.

Step 6 (Loop Formation and Reallocation): Construct a closed loop starting from the cell with the most negative C_{ij} . Follow these rules:
 The loop must include an even number of cells (minimum of four), alternating between occupied and unoccupied cells.

Movement is restricted to horizontal and vertical paths, and the loop must pass only through allocated (occupied) cells.

Assign $+\vartheta$ & $-\vartheta$ alternately along the loop, beginning with at the unoccupied cell with the most negative C_{ij} .

The value of ϑ is the minimum allocated value among the cells marked with $-\vartheta$.

Update the allocations by adding ϑ to the $+\vartheta$ cells and subtracting ϑ from the $-\vartheta$ cells.

Step 7: Repeat Steps 2 through 6 until all $C_{ij} \geq 0$, indicating that an optimal solution has been reached.

Summary:

Step 1 handles degeneracy to ensure a valid BFS.

Steps 2 and 3 calculate dual variables and opportunity costs.

Step 4 checks for optimality.

Step 6 is the core of the MODI method, where loop formation is used to reallocate and reduce total cost.

The process iterates until no further improvement is possible, ensuring the solution is optimal.

7.3 Proposed Algorithm

Step 1 First we check for the problem type

(a) If balanced then go to the step 3 (b) If unbalanced then go to the step 2

Step 2 Balance the table

- (a) if Supply > Demand then introducing dummy column
- (b) if Supply < Demand then introducing dummy row.

Step 3(a) Select two least cell (least cost cell greater than 0) from each row with First Come First Serve Basis in case of non-uniqueness satisfying step3b. For dummy row, there will be no selection.

(b). select two additional cell only if an assignment already exists in that column for the specified row.

Step 4: Adjust the supply and demand values based on the allocation made. If the supply or demand for a row or column becomes zero, that row or column is excluded from further consideration in subsequent allocations.

Step 5: Repeat Steps 3 and 4 until all supply and demand values are satisfied (i.e., reduced to zero).

Step 6: Calculate the total transportation cost by summing the products of allocated quantities and their corresponding cell costs.

This algorithm provides an initial Basic Feasible Solution (BFS) by allocating resources in a cost-efficient manner.

8 Examples

(1)

	W1	W2	W3	W4	Supply
F1	5	1	3	3	34
F2	3	3	5	4	15
F3	6	4	4	3	12
F4	4	1	4	2	20
Demand	21	26	17	17	81/81

Since the data is balanced (Das et al., 2014), the unique cell selection step can be performed.

	W1	W2	W3	W4	Supply
F1	5	[1] ₁₃	[3] ₁₇	[3] ₄	34
F2	[3] ₁₅	3	5	4	15
F3	[6] ₅	4	4	[3] ₇	12
F4	[4] ₁	[1] ₁₃	4	2	20
Demand	21	26	17	17	81/81

Assignments with total cost=13+51+12+45+30+21+4+13+12=201.

2.

	W1	W2	W3	W4	W4	Supply
F1	5	4	8	6	5	600
F2	4	5	4	3	2	400
F3	3	6	5	8	4	1000
Demand	450	400	200	250	300	1600/2000

	W1	W2	W3	W4	W4	W5	Supply
F1	5	4	8	6	5	0	600
F2	4	5	4	3	2	0	400
F3	3	6	5	8	4	0	1000
Demand	450	400	200	250	300	400	2000/2000

Since the given problem is unbalanced, with supply exceeding demand (Kumaraguru et al., 2014), it is necessary to introduce a dummy column, C1, to balance the problem.

	W1	W2	W3	W4	W4	W5	Supply
F1	5	[4] ₄₀₀	8	[6] ₂₀₀	5	0	600
F2	4	5	4	3	[2] ₂₀₀	[0] ₂₀₀	400
F3	[3] ₄₅₀	6	[5] ₂₀₀	[8] ₅₀	[4] ₁₀₀	[0] ₂₀₀	1000
Demand	450	400	200	250	300	400	2000/2000

Assignments with total cost= 1600+1200+400+1350+1000+400+400=6350

3.

	W1	W2	W3	W4	W5	Supply
Factory A	6	4	1	5	2	30
Factory B	3	2	5	4	6	20
Factory C	4	6	3	2	7	25
Factory D	5	3	6	4	5	15
Demand	20	25	30	15	25	115/90

Since the given problem is unbalanced with supply less than demand (Girmay et al., 2013), a dummy row, R1, must be introduced to balance the problem.

	W1	W2	W3	W4	W5	Supply
Factory A	6	4	1	5	2	30
Factory B	3	2	5	4	6	20
Factory C	4	6	3	2	7	25
Factory D	5	3	6	4	5	15
Dummy	0	0	0	0	0	25
Demand	20	25	30	15	25	115/115

	W1	W2	W3	W4	W5	Supply
Factory A	6	4	[1] ₁₅	5	[2] ₁₅	30
Factory B	3	[2] ₂₀	5	4	6	20
Factory C	4	6	[3] ₁₀	[2] ₁₅	7	25
Factory D	[5] ₁₀	[3] ₅	6	4	5	15
Dummy	[0] ₁₀	0	[0] ₅	0	0	25
Demand	20	25	30	15	25	115/115

Assignments with total cost= 15+30+40+ 30+30+50+15=210

9 Conclusions and Result Discussion

Heuristic algorithms are employed because they are designed to provide near-optimal solutions within a reasonable computation time. The success of the Least Cost Method (LCM) lies in its simple algorithm, which follows a greedy approach to assign allocations to the least cost cells. However, this simplicity comes at the cost of increased computation time due to the effort required in identifying the minimum cost cells during each iteration. Furthermore, since LCM does not guarantee an optimal solution in a single iteration, additional time is often needed for optimality checks using methods such as MODI or the Stepping Stone method.

This paper proposes a Two-Step Exact Algorithm for solving the problem. The proposed algorithm is straightforward to implement and includes a built-in optimality checker to verify the initial feasibility of the solution. Unique cell selection can be selected efficiently using Key-Value Pair approach (Prasad et al.(2018a)). The first step of the algorithm adopts the fundamental concept of cell selection for assignment based on the Least Cost Method. In the proposed algorithm, two least cells (i.e., least cost cells greater than zero) are selected from each row. In cases of non-uniqueness, selection is made on a First-Come, First-Serve basis. If a column receives multiple assignments, an additional cell is selected from the corresponding row to ensure feasibility. The Least Cost Method is then applied to assign allocations from the selected two least cells. If further assignments are required, the remaining cells are also utilized using the same Least Cost Method approach. The proposed algorithm is easy to implement and incorporates a built-in optimality checker to evaluate solution optimality. Moreover, two least cell selection can be performed efficiently using a Key-Value Pair approach. The algorithm was evaluated on 100 problem sets with matrix sizes ranging from 3×3 to 100×100, encompassing both balanced and unbalanced scenarios. Representative sample problems are included to illustrate the functionality and effectiveness of the proposed approach.

References

- [1] Ahuja, R. K. (2017). Network flows: theory, algorithms, & applications. Pearson Education.
- [2] Ahuja, R. K., Magnanti, T. L., Orlin, J. B. & Reddy, M. R. (1995). Applications of network optimization. Handbooks in Operations Research and Management Science, 7, 1-83.
- [3] Caputo, A. C., Fracocchi, L. & Pelagagge, P. M. (2006). A genetic approach for freight transportation planning. Industrial Management & Data Systems, 106 (5), 719-738.
- [4] Das, U. K., Babu, M. A., Khan, A. R. & Uddin, M. S. (2014). Advanced Vogel's Approximation Method (AVAM): a new approach to determine penalty cost for better feasible solution of transportation problem. International Journal of Engineering Research & Technology (IJERT), 3(1), 182-187.
- [5] Girmay, N. & Sharma, T. (2013). Balance an Unbalanced Transportation Problem by a Heuristic approach. International Journal of Mathematics & Its Application, 1(1), 16.
- [6] Greenberg, H. J. (2012). A Computer-Assisted Analysis System for Mathematical Programming Models & Solutions: A User's Guide for ANALYZE. Springer Science & Business Media Hentenryck, P. V. & Michel, L. (2009). Constraint-based local search. The MIT press.
- [7] Kumaraguru, S., Kumar, B.S. & Revathy, M. (2014). Comparative study of various methods for solving transportation problem, 3(9), 246.
- [8] Prasad et al., Proceedings on Engineering Sciences, Vol. 02, No. 3 (2020) 269-280, doi: 10.24874/PES02.03.006
- [9] Prasad, A. K. & Pankaj (2018). A Study of KVP Approach on Some Transportation Models. In 2018 International Conference on Current Trends towards Converging Technologies (ICCTCT). IEEE, 1-5.
- [10] Prasad, A. K. & Pankaj (2018). Loop formation in Transportation Problem: Semi Cycle Formation in Restricted Graph. Journal of Advanced Research in Dynamical & Control Systems, 10(13), 1625-1632.
- [11] Taha, H. A. (2004). Operations research: An introduction (for VTU). Pearson Education India.

- [12] Ting, S. C. & Tzeng, G. H. (2003). Ship scheduling & cost analysis for route planning in liner shipping. *Maritime Economics & Logistics*, 5(4), 378-392.
- [13] Sheema Sadia, Ahteshamul Haq* Qazi Mazhar Ali (2023). "Solid Transportation Problem with Multiple Objectives under Integer and Non-Integer Solution" *International Journal of Agricultural and Statistical Sciences*, 19(2), 843-850.