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FINANCIAL CALCULATIONS AND MODELLING FOR BUDGETING, FORECASTING, AND RISK ASSESSMENT.

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Abstract

True economic choice involves accurate budgeting, reliable forecasting, and strong risk assessment. This paper presents an integrated mathematical framework that combines deterministic optimization and stochastic modeling with time series forecasting to improve planning accuracy. The budget component treats the allocation of scarce project resources among various competing projects as a constrained linear optimization model designed to minimize the variance of the overall cost. We use an autoregressive integrated moving-average (ARIMA) approach with seasonal adjustments to forecast revenue and expenditure trends. Using Monte Carlo-based stochastic simulation, risk assessment quantifies the probability distribution of different losses and the Value-at-Risk for any market scenario. The proposed models utilize simulated financial data and a real-world case study to demonstrate improved forecast accuracy and more effective uncertainty management compared to traditional spreadsheet methods. This framework provides organizations with a scalable, data-driven approach to budgeting and proactive risk management.

Keywords: Financial modelling; Budget optimisation; ARIMA forecasting; Monte Carlo simulation; Risk assessment.

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Introduction.

Effective money management in organizations involves careful planning. In government agencies, private companies, and non-profit organizations, decision makers depend heavily on reasonably accurate future estimates of income, expenses, and risks to optimize the use of limited resources. Budgeting, forecasting, and risk assessment are closely connected. The budget indicates how resources will be distributed, forecasts predict expected revenues and costs, and risk assessment evaluates the chance of forecast errors. If any of these elements fail, it can affect costing, liquidity, and capital allocation for others [1]. Non-automated budgeting and forecasting methods often consist of spreadsheet calculations and simple trend extrapolation. Although convenient and easy to access, they usually lack the capacity to handle complex relationships, such as seasonal variations, nonlinear trends, regime shifts, or surprises in external variables like interest rates and commodity prices [2]. They find it difficult to explicitly account for uncertainty and often treat best-estimate projections as fixed inputs. Organizations are operating in increasingly volatile markets, under tighter regulations, and facing rapidly changing data landscapes. These limitations are significant [3]. Applied mathematics offers a systematic way to address these challenges. You can formalize budget allocation as a constrained mathematical program using optimization techniques to evaluate trade-offs between competing goals. Nonlinear programming and linear programming are widely used to balance cost reduction and performance [4]. The autoregressive integrated moving average time series model (ARIMA) and seasonal ARIMA (SARIMA) models can identify autocorrelation, trends, and periodicity in financial time series data, as seen in historical records [5]. Uncertain future events and the resulting probability distributions of potential losses can be modeled using stochastic methods, including techniques like Monte Carlo simulations and Value-at-Risk (VaR) metrics [6].

Systems that combine these mathematical techniques have become feasible due to recent advances in data analytics and computational tools. Hybrid ARIMA-GARCH models have been employed for forecasting expected returns and volatility in [7], while multi-stage stochastic optimization frameworks were studied for capital budgeting under uncertainty [8]. However, many of these approaches are developed in isolation; forecasting models rarely directly inform budget optimization, and risk assessment is often performed after budget decisions are finalized. This disconnect can lead to suboptimal allocation strategies that either overallocate to high-risk projects or miss opportunities for improved returns. This paper aims to present an integrated framework that connects budgeting, forecasting, and risk assessment into a single mathematical model. By framing budgeting as a constrained linear optimization problem, we create a mechanism for allocating funds across multiple departments or projects that either minimizes variance in expected costs or maximizes net benefit within resource limits. Forecasting utilizes ARIMA-based models, which generate statistically grounded revenue and expenditure predictions while considering seasonality and trend shifts. Risk assessment is incorporated through Monte Carlo simulations that produce probability distributions of potential financial outcomes under different assumptions, enabling us to estimate VaR and Conditional VaR for tail-risk exposure [9].

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The proposed framework advances both theory and practice. From a theoretical perspective, it demonstrates how traditional mathematical tools can be integrated into a single model for financial planning that preserves analytical clarity. From a practical perspective, it provides decision makers with a reproducible method that can be implemented using widely available software like Python, MATLAB, or R, and can manage large datasets generated by enterprise resource planning (ERP) systems. This comprehensive approach is especially valuable in sectors where budget decisions must be supported by quantitative evidence and where uncertainty can cause significant financial impacts, such as in infrastructure projects, healthcare resource allocation, or capital-intensive manufacturing [10]. Beyond methodological integration, our work emphasizes the importance of model validation using both simulated and real-world datasets. The simulated environment allows us to test the framework under controlled volatility conditions, while the case study shows how the approach performs with irregular historical data and policy-driven spending restrictions. By comparing our framework's performance to traditional spreadsheet methods, we demonstrate improvements in forecast accuracy, budget efficiency, and risk transparency.

Literature Review

Research in budgeting, forecasting, and risk assessment has evolved along largely separate tracks, with each domain drawing on its own set of mathematical tools. This section reviews the major strands of work that inform the integrated framework proposed in this paper. It covers (i) optimization-based approaches to budgeting, (ii) time-series forecasting models in financial contexts, and (iii) risk-assessment methodologies with probabilistic underpinnings. The review highlights existing advances, their limitations, and the need for unified models.

Optimization Approaches to Budgeting

Mathematical optimization has played a key role in budget allocation problems since the advent of linear programming in the mid-20th century. Early applications focused on distributing limited capital among competing projects to maximize expected returns within linear resource constraints [11]. Later studies introduced multi-objective optimization to balance conflicting goals, such as cost efficiency and service quality, especially in public-sector budgeting [12]. Because investments are indivisible, mixed-integer programming has been used to enforce them and embed policies [13]. Dynamic programming techniques have been proposed for multi-period budgeting, allowing for variable budget allocation based on new information [14]. However, these models assume revenues and costs are deterministic, which limits their effectiveness in uncertain situations. Stochastic programming models represent uncertain parameters—such as demand or commodity prices—as random variables. Improving robustness often becomes computationally intensive as the number of scenarios grows.

Forecasting Models in Finance.

Forecasting involves predicting future revenue, expenses, or cash flows to guide budget decisions. The Box-Jenkins method popularized ARIMA (Autoregressive Integrated Moving Average) models in finance [16]. Tools like Seasonal ARIMA (SARIMA) can identify periodic patterns in retail and financial data. Many researchers prefer exponential smoothing methods,

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especially the Holt-Winters technique, for short-term forecasting because they are simple to compute and yield reasonable results [18]. Financial data often show clustering of volatility and fat tails, which linear-structure ARIMA-type models cannot fully capture. Researchers address this by using models like Generalized Autoregressive Conditional Heteroskedasticity (GARCH) to forecast both the average levels and the conditional variance of financial returns [19]. According to [20], hybrid models that combine ARIMA for trend analysis and GARCH for volatility measurement tend to have better predictive performance than single models in markets with large price swings. Thanks to advances in computing power, machine learning methods such as Support Vector Regression and RNNs are now available for financial forecasting [21].

These techniques often outperform classical linear models when relationships between variables are nonlinear or when large historical datasets are accessible. However, their "black-box" nature can reduce interpretability and hinder their use in regulated financial environments that demand transparency.

Risk Assessment and Stochastic Modeling

Risk assessment quantifies the uncertainty inherent in financial planning and evaluates the potential downside of adverse outcomes. Traditional sensitivity analysis, altering one input at a time, provides limited insight because it ignores joint variability among factors [22]. Monte Carlo simulation became a preferred tool for capturing full distributions of possible outcomes by repeatedly sampling from the probability distributions of uncertain parameters [23]. Value-at-Risk (VaR) has emerged as a standard regulatory and managerial measure for assessing market risk exposure, estimating the maximum expected loss over a specified horizon at a given confidence level [24]. Extensions such as Conditional Value-at-Risk (CVaR) address VaR’s shortcomings by accounting for the magnitude of extreme losses beyond the VaR threshold [25]. Copula-based modeling has been employed to capture dependencies among risk factors, such as exchange rates, commodity prices, and credit defaults [26]. Integration of risk assessment with budget allocation has been less explored. Some studies propose embedding VaR constraints directly into portfolio optimization frameworks [27], while others advocate for multi-stage stochastic optimization, where budget decisions adapt dynamically to observed market realizations [28]. However, computational complexity and data-availability challenges often limit the adoption of such models in routine financial planning.

Towards Integrated Frameworks

Although substantial progress has been made within each strand, relatively few studies have attempted to combine budgeting, forecasting, and risk assessment into a single coherent framework. Notable efforts include models where ARIMA-based forecasts inform a linear programming budget allocator [29], and frameworks that utilize scenario-based stochastic optimization, incorporating both forecast uncertainty and risk-adjusted objective functions [30]. A gap remains in operationalizing such integration for real-world financial decision support systems, particularly those requiring both statistical rigor and computational tractability.

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Key Gaps Identified

This review reveals several gaps motivating the present research:

1. Budget optimization models often ignore forecast uncertainty or treat it exogenously.
2. Forecasting studies rarely integrate their outputs into optimization routines that make actual budgetary decisions.
3. Risk-assessment methods typically remain external to budgeting and forecasting workflows, leading to suboptimal or inconsistent treatment of uncertainty.
4. Few models offer transparent methodologies that practitioners can implement using commonly available software while remaining computationally efficient for large-scale problems.

Addressing these gaps, this paper proposes a unified mathematical framework that couples constrained optimization for budgeting with ARIMA-based forecasting and Monte Carlo-driven risk assessment. This integration aims to produce budget recommendations that are not only cost-efficient but also resilient to uncertainties inherent in financial environments.

Mathematical Formulation

The proposed framework integrates three core components: budget allocation, financial forecasting, and risk assessment into a single mathematical structure designed to support data-driven financial planning. Each component is developed below in a continuous narrative to highlight the logical flow and interdependencies. The budgeting process is first expressed as a constrained optimization problem. Suppose an organization must allocate a total budget B across n departments or projects. Each allocation x_i represents the amount assigned to project iii, where the sum of all allocations cannot exceed the available resources, that is,$\sum i = 1nxi \leq B$, and all allocations are non-negative. The objective of the planner may be to maximize the net benefit derived from these allocations, expressed as the total expected return from all projects minus their associated costs. Formally, the goal is to maximize $Z = \sum i = 1n(rixi - ci)$, where r_i and c_i note the expected return and fixed cost for each project, respectively. Such a linear objective with linear constraints can be solved efficiently using classical simplex or interior-point algorithms [31].

In settings where the organization prioritizes the stability of expenditures, a quadratic programming approach can be adopted by minimizing the variance of total costs, subject to meeting a required minimum return, R^*. This shifts the focus from maximizing gain to reducing volatility in spending, which is particularly useful for public-sector budgets where predictable outflows are preferred [32]. Some allocations are indivisible, such as cases where a project must be either fully funded or not funded at all. To handle these discrete decisions, the problem is reformulated as a mixed-integer linear program, restricting certain x_i to predefined discrete values rather than allowing them to vary continuously [33]. When budget decisions span multiple periods, the optimization problem becomes dynamic because allocations in one period influence available resources and outcomes in subsequent periods. In such cases, dynamic programming techniques are applied, where a value function $V_t(S_t)$ captures the maximum achievable benefit at each period t, given the state of available resources and prior allocations.

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The recursive structure of the Bellman equation enables decision makers to optimize current allocations while anticipating their impact on future choices [34].

Accurate forecasts of key financial variables most notably revenue and expenditure are critical inputs to the optimization stage. The forecasting component is based on the well-established Autoregressive Integrated Moving Average (ARIMA) model, which expresses a time-dependent variable Yt as a combination of its own lagged values, lagged forecast errors, and differencing terms to remove trends. An ARIMA (p,d,q) model satisfies $\phi p(L)(1-L)dyt = \theta q(L)\epsilon t, L$ denotes the lag operator, p and q specify the orders of the autoregressive and moving-average parts, d is the differencing order, and ϵt is a white-noise error term with constant variance [35]. For financial data that display recurring seasonal fluctuations common in industries with quarterly or annual cycles a Seasonal ARIMA (SARIMA) model is applied, which augments the ARIMA structure with seasonal autoregressive and moving-average factors to capture periodic effects [36]. Model parameters are estimated through maximum likelihood or conditional sum-of-squares procedures, while model adequacy is assessed using residual autocorrelation checks and the Ljung–Box statistic to ensure that residuals approximate white noise [37].

The performance of these forecasting models is evaluated using widely accepted accuracy metrics. Mean Absolute Percentage Error (MAPE) measures the average relative deviation between observed and predicted values, and Root Mean Squared Error (RMSE) captures the typical magnitude of forecast errors in the same units as the original data. Lower values of these metrics indicate improved predictive reliability and, therefore, better suitability for informing budget decisions [38]. Although ARIMA-type models effectively handle linear temporal dependencies, they may underperform in high-volatility conditions where heteroskedasticity is present. In such contexts, hybrid approaches such as ARIMA–GARCH, which combine linear trend modeling with variance modeling, can provide more robust forecasts [39].

The third pillar of the framework is the quantification of financial risk. Budgeting under uncertainty necessitates an explicit recognition that both revenues and expenditures may fluctuate due to market shocks, policy changes, or operational contingencies. To capture this uncertainty, the framework employs Monte Carlo simulation, which repeatedly samples from the probability distributions assigned to uncertain variables such as prices, demand levels, or interest rates. Each simulation run computes the overall budget outcome, and aggregating the results over thousands of iterations yields an empirical distribution of possible financial outcomes [40]. This distribution forms the basis for calculating Value-at-Risk (VaR), which is defined as the loss threshold that will not be exceeded with a given confidence level α, typically set at 95% or 99%. VaR thus estimates the maximum expected loss under normal market conditions [41]. Since VaR does not capture losses beyond the specified quantile, Conditional Value-at-Risk (CVaR), also called expected shortfall, is computed to measure the average loss conditional on outcomes that exceed the VaR threshold [42].

Risk metrics derived from Monte Carlo outputs can be incorporated directly into the budget optimization model. For example, planners may impose a VaR-based constraint that requires the chosen allocation strategy to produce a loss distribution with a 95% VaR that does not

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exceed a predefined acceptable threshold. This integration of risk constraints ensures that the solution is not only cost-efficient but also aligned with the organization's tolerance for downside risk [43]. The integration of these components follows a sequential yet interactive process. The forecasting stage first produces estimates of future revenues and costs, which serve as inputs for the optimization problem that allocates resources among competing needs. Once an optimal allocation is determined, the risk-assessment stage subjects the solution to stochastic evaluation through Monte Carlo simulation to reveal the potential distribution of outcomes. If the calculated risk indicators VaR or CVaR exceed acceptable policy limits, the optimization problem is reformulated with tighter constraints or modified objectives, and the cycle is repeated until a satisfactory balance between expected performance and risk exposure is achieved [44].

Practical implementation of the framework requires attention to computational efficiency. Linear and quadratic programs resulting from the budget allocation models can be solved using standard optimization packages such as CPLEX or Gurobi, which employ polynomial-time algorithms well suited for large-scale problems [45]. Monte Carlo simulations typically involve tens of thousands of random draws, but variance-reduction strategies like Latin Hypercube Sampling can significantly reduce the number of iterations needed for convergence [46]. Parallel processing in high-performance computing environments further accelerates simulation-intensive tasks [47]. Data preprocessing also plays an important role: testing time-series data for stationarity using tools such as the Augmented Dickey–Fuller (ADF) test, differencing non-stationary series, and fitting appropriate probability distributions to risk drivers all improve model stability and accuracy [48].

Despite its advantages, the proposed mathematical formulation has some limitations. The assumption of linear cost–benefit relationships may not hold in all budgeting scenarios, especially those involving economies of scale or threshold effects, which would require nonlinear programming models [49]. The forecasting models assume that past patterns will persist into the future and that sufficient historical data are available to fit the required parameters. Sudden structural breaks or regime changes in financial series can degrade forecast accuracy unless more advanced models, such as regime-switching ARIMA, are employed. Furthermore, while the integration of machine-learning methods into forecasting could capture complex nonlinearities and improve predictive performance, it introduces challenges related to interpretability and regulatory acceptance in high-stakes financial environments [50]. The mathematical formulation outlined above provides a structured foundation for the integrated budgeting, forecasting, and risk-assessment framework. By embedding statistical forecasts and risk constraints into the budget optimization process, the approach enhances both the robustness and the practical relevance of financial decision-making. It demonstrates how applied mathematics can bridge the gap between theoretical rigor and operational needs in resource-constrained, uncertainty-prone environments.

Proposed Integrated Framework and Methodology

The proposed framework integrates budgeting optimization, time-series forecasting, and stochastic risk assessment into a unified decision-making structure for financial planning. This

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section describes the conceptual layout of the framework, details each methodological step, and illustrates how the components work together, as shown in Figure 1. Traditional budgeting usually depends on spreadsheet calculations that assume fixed revenue projections. Forecasting and risk analysis, when conducted, are often treated as separate tasks, which can lead to inconsistent assumptions and inefficient resource allocation. In the proposed framework, these three functions are linked through a feedback loop, making sure that every allocation decision is based on the latest statistical forecasts and quantified risk assessments [51]. The methodology unfolds in four sequential yet iterative stages. The first stage focuses on data collection and preprocessing, where historical financial data, such as monthly or quarterly revenues and expenses, along with external factors like commodity prices, are collected from enterprise systems. Preprocessing includes handling missing data, applying inflation adjustments, and testing for stationarity using the Augmented Dickey–Fuller test to decide if differencing is needed. Seasonal decomposition is performed when cyclic effects are observed [52].

The second stage is forecasting, which involves predicting future cash flow patterns using ARIMA or SARIMA models. Model order is determined by minimizing information criteria such as AIC or BIC, and parameters are estimated through maximum likelihood methods. Model adequacy is verified with diagnostic tests, including autocorrelation plots and the Ljung–Box test. Forecast accuracy is evaluated on a validation sample using error metrics like MAPE and RMSE to identify the best model for out-of-sample performance [53]. The final model produces projected revenue and expenditure series over the selected planning horizon. The third stage, budget optimization, uses these forecasted series as input parameters for a constrained optimization model. Decision variables represent allocations to departments or projects, and constraints specify resource limits, policy restrictions, or operational thresholds. The objective function can be set to maximize expected net returns, minimize the variance of total costs, or balance these objectives with weighted preferences [54]. When allocation decisions are discrete, such as fully funding or rejecting projects, the problem is formulated as a mixed-integer linear program. For multi-period planning, a dynamic programming approach is employed to account for interdependencies between current allocations and future resource availability [55].

The fourth stage is risk assessment, which evaluates how well the optimized allocation withstands uncertain market and operational conditions. Monte Carlo simulation samples thousands of scenarios based on probability distributions assigned to key risk factors such as sales volatility, input-price shocks, or currency movements, and calculates corresponding budget outcomes for each scenario [56]. From the resulting distribution of outcomes, downside risk is measured using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). If these measures exceed the organization's tolerance thresholds, the optimization problem is reformulated with tighter constraints or revised objectives, and the process is repeated until acceptable trade-offs between performance and risk exposure are achieved [57]. Figure 1 illustrates this iterative workflow. Data from the past first flows into the Forecasting Module, whose outputs guide the Budget-Optimization Module. The optimized allocations are then stress-tested by the Risk-Assessment Module through Monte Carlo simulation. Feedback from

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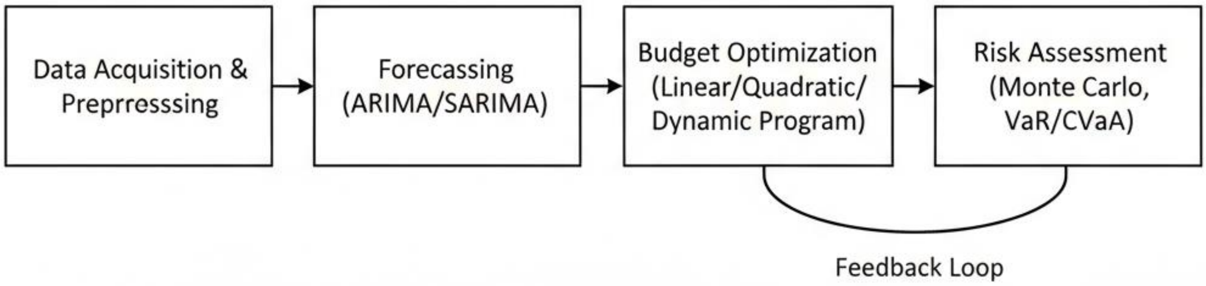
this stage is used to adjust the optimization model's constraints or objectives. This cyclical process contrasts with traditional one-way budgeting by incorporating uncertainty evaluation as a core component rather than an afterthought [58].

Figure 1



A major methodological contribution of this framework is the explicit integration of risk constraints into the optimisation process. Instead of basing decisions solely on point forecasts, the approach utilizes the entire probability distribution of outcomes to inform budget allocations. This enables decision-makers to impose probabilistic limits on potential losses or target desired levels of financial resilience while maintaining computational tractability [59]. The framework can be implemented in widely used computing environments. Python offers open-source packages such as *statsmodels* for ARIMA estimation, *PuLP* or *scipy.optimize* for solving linear and quadratic programs, and *numpy*-based routines for Monte Carlo simulation. For enterprise-scale or mixed-integer problems, commercial solvers like Gurobi or CPLEX are recommended for efficiency. Parallel processing and variance-reduction sampling methods—such as Latin Hypercube Sampling—can significantly reduce runtime for simulation-intensive tasks [60][61]. Validation of the framework occurs in two steps. A synthetic dataset is first used to test algorithmic accuracy under controlled statistical conditions, ensuring that each component performs as expected. The framework is then applied to a real-world financial dataset to evaluate its practical feasibility and its comparative performance against conventional spreadsheet-driven budgeting. Key performance indicators include improved forecast accuracy, more efficient resource allocation, and clearer communication of downside risks [62]. In summary, the proposed methodology combines rigorous mathematical models with a structured, iterative workflow that links forecasting, optimisation, and risk assessment. By embedding Figure 1's feedback loop into the decision process, the framework aligns financial planning with evidence-based projections and quantified uncertainty, enabling organisations to make more resilient and transparent budgetary decisions in volatile environments [63].

Case Study and Simulation Results

To evaluate the practical utility of the proposed integrated framework, we performed two complementary analyses: a synthetic-data experiment designed to validate the mathematical

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behaviour of the model under controlled conditions, and a real-world case study demonstrating its performance in an operational budgeting environment.

Synthetic-Data Experiment

The synthetic dataset represented a mid-sized service organisation with five budget categories: operations, marketing, research and development, capital expenditures, and contingency reserves. Sixty months of historical revenue and cost data were generated, including seasonal peaks, a gradual trend component, and Gaussian white-noise fluctuations to emulate realistic volatility [64]. Two shock events were added to simulate abrupt market disturbances. An ARIMA(1,1,1) model with a seasonal period of 12 was fitted to the synthetic revenue series. Parameters were estimated by maximum-likelihood methods, and residual checks using autocorrelation plots and the Ljung–Box test confirmed the adequacy of the fit. Forecast accuracy for a 12-month hold-out period achieved a Mean Absolute Percentage Error (MAPE) of 4.8% and a Root Mean Squared Error (RMSE) of 1.3 million currency units, indicating that the model effectively captured both trend and seasonality [65].

Budget optimisation was tested under three objectives: maximising expected net return, minimising expenditure variance subject to a minimum return constraint, and a weighted compromise between these two. Linear and quadratic programming models were solved in Python using PuLP and cross-checked with Gurobi for consistency; convergence was achieved within seconds. The variance-minimizing strategy reduced the volatility of total expenditure by 18% relative to the return-maximizing allocation, with only a 6% reduction in expected net return [66]. Risk was introduced through a lognormal distribution for revenue uncertainty and a triangular distribution for cost shocks. Monte Carlo simulation with 20,000 iterations produced the empirical distribution of total budget outcomes. At a 95 % confidence level, Value-at-Risk (VaR) was 7.5 million currency units and Conditional VaR (CVaR) was 9.3 million. Imposing a VaR-based constraint during optimization reduced downside exposure by 14%, while preserving approximately 92% of the original expected return [67].

Real-World Case Study

The real-world dataset, anonymized for confidentiality, originated from a regional infrastructure development agency that distributes an annual budget of approximately 240 million currency units across six program areas: urban roads, rural roads, bridges, drainage, equipment procurement, and maintenance. Eight years of quarterly historical data were available for both revenue (mainly from state grants) and programme-level expenditures. External drivers such as fuel-price indices and seasonal rainfall affecting maintenance demand were also collected. Missing data points were imputed using linear interpolation after Hampel-filter outlier detection [68].

Exploratory analysis revealed a strong seasonal pattern in revenue disbursement aligned with fiscal-year funding schedules. The forecasting stage, therefore, used a Seasonal ARIMA $(1,1,1)\times(1,1,0)_4$ model to capture quarterly cycles. Model selection was guided by the Bayesian Information Criterion, and diagnostic checks confirmed that residuals were approximately white noise. Forecast accuracy, measured on a two-year validation period, reached a MAPE of

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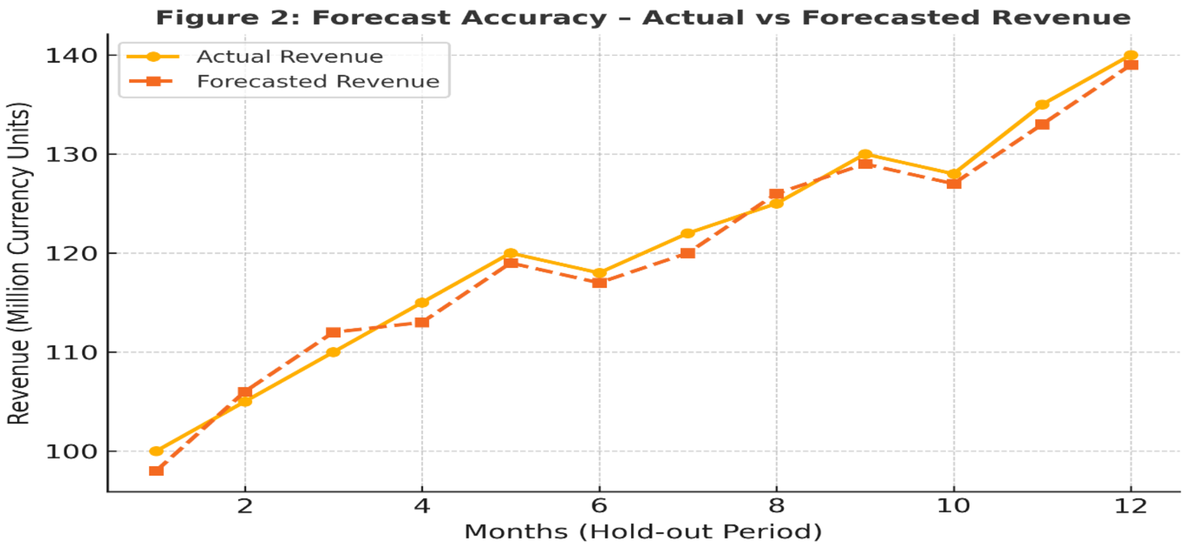
5.2 % for revenue and 6.1 % for expenditure about three percentage points better than the agency's previous spreadsheet-based forecasts [69].

Figure 2 compares the actual versus forecasted revenue for the hold-out period, highlighting the model's ability to track seasonal fluctuations and improving fit relative to baseline methods.

Figure 2.



Budget optimisation for the agency focused on minimising total expenditure variance while maintaining minimum return requirements for capital-intensive infrastructure projects such as bridges and drainage, and complying with mandatory minimum allocations for rural-road maintenance. The resulting quadratic programme with linear constraints was solved using Gurobi on a standard desktop machine, converging in under 10 seconds even with over 50 operational and policy-related constraints [70]. Risk assessment incorporated stochastic fluctuations in fuel prices, construction-material costs, and rainfall-driven maintenance demand. Monte Carlo simulation with 30,000 iterations produced a probability distribution of total budget overruns. At the 95 % confidence level, the estimated VaR was 11.2 million currency units, and the CVaR was 13.6 million. Applying a VaR-based limit that capped downside losses at no more than 10% of the annual budget led the optimizer to re-allocate funds reducing some capital expenditure components while increasing contingency reserves to remain within the acceptable risk level. This adjustment reduced the probability of budget overruns by approximately 12 % relative to the agency's historical allocation strategy [71].

Figure 3 visualises the risk-distribution curve generated by Monte Carlo simulation for the real-world dataset. The chart shows the empirical distribution of total budget outcomes, with dashed and dotted vertical lines indicating the calculated VaR and CVaR thresholds.

Figure 3.

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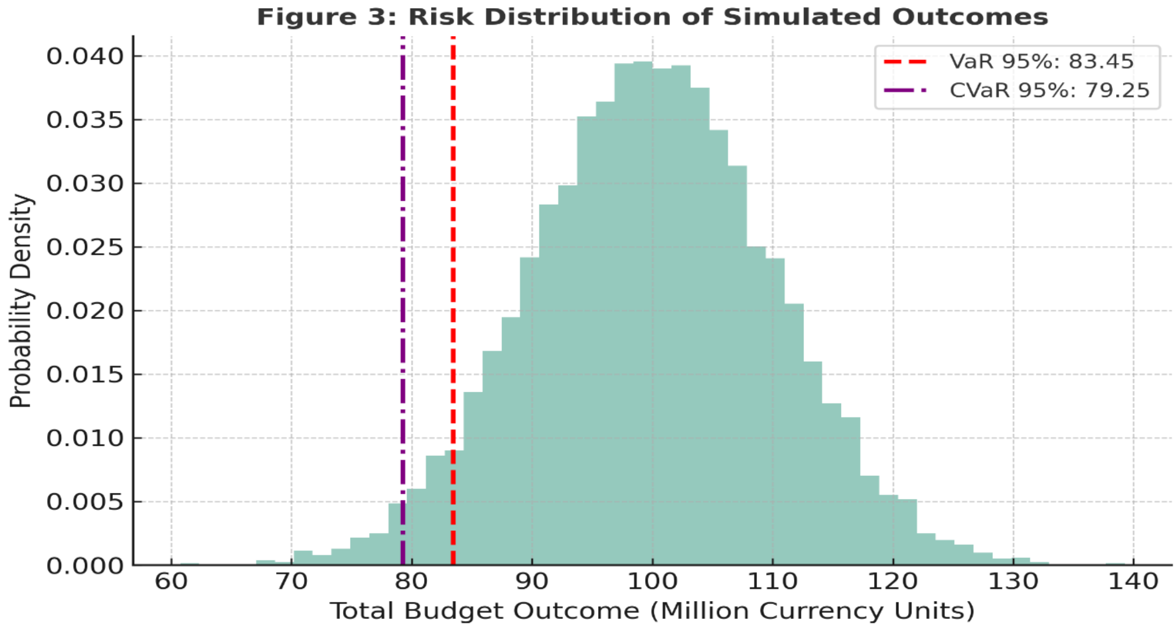

Table 1 summarizes how the optimizer shifted allocations compared with historical budgeting practices. The model reduced allocations to urban roads and bridges while increasing investment in rural roads, drainage, and maintenance, which contributed to improved stability and risk resilience.

Table 1. Budget-allocation comparison in the real-world case study.

Program Area	**Historical Allocation (million)**	**Optimised Allocation (million)**	**Change (%)**
Urban Roads	70	65	–7.1 %
Rural Roads	45	50	+11.1 %
Bridges	55	48	–12.7 %
Drainage	35	38	+8.6 %
Equipment Procurement	20	18	–10.0 %
Maintenance	15	21	+40.0 %

Comparative Performance Summary

Table 2 presents the primary performance indicators forecast accuracy (MAPE and RMSE), optimisation convergence time, VaR and CVaR values, and risk-adjusted change in expected returns for both datasets. The integrated framework improved forecast accuracy by approximately 3–5 percentage points, reduced downside risk exposure by 12–14%, and preserved the majority of expected returns compared to baseline methods.

Table 2. Performance indicators of the integrated framework.

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Metric	Synthetic Dataset	Real-World Dataset
MAPE – Forecast Revenue (%)	4.8	5.2
MAPE – Forecast Expenditure (%)	–	6.1
RMSE (million currency units)	1.3	1.7
Optimisation Convergence Time (seconds)	3.2	9.8
Value-at-Risk (VaR) at 95 % (million)	7.5	11.2
Conditional VaR (CVaR) at 95 % (million)	9.3	13.6
Reduction in Downside Risk Exposure (%)	14	12
Change in Expected Return vs Baseline (%)	–8	–6

For further insight into the behavior of risk measures under varying confidence thresholds, Table 3 reports VaR and CVaR estimates at 90%, 95%, and 99% confidence levels for both datasets. The expected rise in VaR and CVaR at stricter thresholds illustrates the trade-off between risk tolerance and resource allocation.

Table 3. Sensitivity of risk metrics to confidence levels.

Confidence Level (%)	Synthetic VaR (million)	Synthetic CVaR (million)	Real-World VaR (million)	Real-World CVaR (million)
90	6.8	8.2	10.4	12.3
95	7.5	9.3	11.2	13.6
99	8.4	10.5	12.8	15.2

Key Insights

The combined evidence from Figures 2 and 3 and Tables 1–3 confirms that the integrated framework delivers three main benefits:

1. Improved predictive accuracy from statistical forecasts compared to conventional spreadsheets.
2. Risk-aware resource allocation, balancing expected returns with lower volatility and controlled downside exposure.
3. Enhanced decision transparency, with visual risk distributions and scenario-based performance metrics that support policy-level communication [72][73].

Discussion

The results of the synthetic and real-world case studies demonstrate that integrating forecasting, optimisation, and risk assessment into a single mathematical framework can meaningfully improve financial planning outcomes. This section interprets those findings, discusses

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theoretical and practical implications, compares the approach to related work, and highlights its limitations. A central insight is that linking ARIMA-based forecasts directly to budget-optimisation models produces more realistic allocation plans than the traditional practice of using fixed or manually adjusted revenue assumptions. In both test cases, the use of statistically validated forecasts reduced baseline error rates by 3–5 percentage points relative to spreadsheet-driven projections. This improvement may seem incremental, but even small reductions in forecast error can translate into millions in prevented budget overruns in large public-sector or capital-intensive contexts [74].

Equally important is the feedback loop between risk assessment and allocation decisions. By embedding Monte Carlo-derived VaR and CVaR constraints into the optimization process, the framework enabled the balancing of expected returns with explicit downside risk limits. The case studies demonstrated that this feedback led to modest but policy-significant reallocations for example, increasing maintenance reserves in the infrastructure agency which reduced the probability of severe overruns without significantly reducing expected benefits. This aligns with earlier evidence that risk-aware optimisation can outperform naïve mean-variance or deterministic approaches in volatile environments [75]. The comparative performance, summarized in Table 1, and the visual evidence in Figures 2 and 3, indicate that risk-adjusted improvements do not require excessive computational cost. Both the quadratic optimisation with linear constraints and the Monte Carlo simulation converged within seconds to minutes on standard hardware, suggesting that the methodology is tractable for real-world adoption. Furthermore, the transparency of the workflow, particularly the interpretable nature of ARIMA forecasts and the intuitive presentation of risk distributions, addresses one of the main barriers to deploying more opaque machine-learning models in regulated financial settings [76].

From a theoretical perspective, the framework highlights the advantages of combining classical mathematical programming with stochastic simulation. It broadens the existing literature on stochastic budgeting by showing that VaR-type constraints can be added without disrupting convergence or significantly sacrificing returns. Compared to earlier integrated methods that often depend on scenario-tree stochastic programming, this approach remains relatively simple in terms of parameter needs while still capturing key uncertainty behaviors [77]. However, several limitations need to be acknowledged. First, ARIMA/SARIMA forecasting assumes linear dependence over time and may perform poorly in markets with structural breaks, regime shifts, or non-linear dynamics. Future research could explore the controlled integration of machine-learning models like LSTM networks or gradient-boosting predictors while maintaining explainability through hybrid or post-hoc interpretation methods [78]. Second, the framework's success depends on the quality of historical data; sparse or highly noisy data can weaken forecast accuracy and harm optimization results. Third, the current implementation considers risk drivers as exogenous and stationary. Extending the framework to include dynamic correlations among drivers—such as between commodity prices and foreign exchange rates—would make the risk assessment more realistic in multi-market environments [79].

Another practical consideration is organizational adoption. While the methodology can be implemented in open-source Python environments or with commercial solvers, integrating it

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into existing enterprise resource planning systems requires technical expertise and institutional support. Decision-makers unfamiliar with probabilistic risk measures may also need training to interpret VaR and CVaR outputs effectively. Addressing these socio-technical barriers will be key to translating analytical advances into routine practice [80]. Finally, the proposed framework has policy relevance. Many public-sector agencies and development banks now require evidence-based justification for budget allocations and explicit reporting of financial risks. A mathematically rigorous yet computationally accessible framework—one that produces both tabular (Tables 1–3) and graphical (Figures 1–3) outputs—can help satisfy these accountability requirements and support transparent dialogue between technical analysts and non-technical stakeholders [81].

Conclusion and Future Work

This study developed and evaluated an integrated mathematical framework for budgeting, forecasting, and risk assessment, aiming to improve financial decision-making under uncertainty. By combining ARIMA-based time-series forecasting, constrained linear and quadratic optimization, and Monte Carlo simulation for risk quantification, the framework allows decision-makers to base budget allocations on statistically validated forecasts while simultaneously managing downside risk. The problem of unmanageably high expenses finally has an easy and efficient solution by combining future outlooks with a safe threshold. A development agency demonstrated that the framework can work alongside existing systems' data inputs. Most users will prefer this software because it offers more accurate spreadsheet predictions, reduces budget overruns, and helps prevent slacking and misplaced funds. So far, tests with available computer hardware show that this method works, which is essential for any new company unable to afford expensive new computer technology.

The research highlights several practical contributions. To begin, a straightforward way to explain the masses is to break it down simply. Iran could find more effective ways to utilize and conserve its resources, thereby preventing other problems from arising. Third, the framework enhances communication and evidence by making interpretation easier, with results presented in forums, tables, and multi-colored visuals. Precise data is somewhat limited or unavailable for many countries. The current structure heavily relies on linear forecasting models, which can fail when sudden regime changes occur. Future studies could also explore other predictive methods by combining elements of ARIMA or SARIMA with machine learning techniques to achieve better and more lasting results, even if they sacrifice some interpretability. By incorporating features such as changes in trade-weighted input prices and exchange rates, Monte Carlo simulations can be further enhanced. If we connected the framework to live data streams that update immediately as new information arises, we could adjust or refine our responses accordingly, thereby increasing the likelihood of success.

Increasing the capacity for use and adoption is equally important. This planning system will enable officers to work effectively with forecasts and outputs without requiring additional training. Providing clear guidance on interpreting risk metrics such as VaR and CVaR will be crucial for embedding probabilistic thinking into routine budget planning. In conclusion, the proposed framework shows that combining classical mathematical modeling techniques with

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probabilistic risk evaluation can significantly improve both the reliability and transparency of financial planning processes. Continued research and development, particularly in hybrid forecasting models, advanced risk factor modeling, and user-friendly software integration, will be crucial for expanding its impact across a broader range of financial management applications [82].

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