

UNRAVELING THE STRUCTURE OF SACRED GEOMETRY: ALGEBRAIC MODELING AND RECURSIVE GROWTH OF GEOMETRIC PATTERNS IN THE FLOWER OF LIFE

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Abstract

This study aims to unravel the mathematical structure of the Flower of Life (FOL) by deriving closed-form algebraic expressions that govern its recursive progression. The research investigates how circles, hexagons, and six-petaled motifs evolve across successive growth classes and explores their connections with harmonic and recursive principles in mathematics, nature, and art.

The study employed algebraic modeling, finite difference analysis, and computational validation using tabulated datasets. Patterns were derived by observing recursive growth sequences and confirmed through quadratic formulations. Comparative analyses of polygonal counts and areas were conducted to highlight harmonic progressions and proportional scaling across classes of the FOL.

Results revealed that both circles and hexagons follow the same quadratic expression, $C_n = 3n^2 - 3n + 1$, demonstrating their structural equivalence. Six-petaled motifs, however, showed an accelerated progression due to recursive intersections, reflecting a higher-order harmonic amplification. The computation of polygonal areas confirmed quadratic scaling, consistent with recursive growth laws found in natural and cultural systems. These findings validate the FOL as both a mathematically rigorous construct and a universal model of recursive growth.

The study concludes that the FOL embodies algebraic harmony and recursive progression, with its structural relationships governed by quadratic formulations. This positions the FOL as a nexus of mathematics, biology, art, and cultural symbolism, bridging abstract geometry with observable phenomena in nature and design.

By formalizing the recursive principles of the FOL, the study provides a mathematical framework applicable to computational modeling, educational pedagogy, and design innovation. Its insights may enhance interdisciplinary learning in STEM and STEAM education, support algorithmic design in architecture and art, and reinforce the cultural significance of sacred geometry through a rigorous mathematical lens.

Keywords: Sacred Geometry, Flower of Life, Geometric Progression, Algebraic Modeling, Pattern Formation

Introduction

The Flower of Life (FOL) represents one of the most iconic and enduring symbols in sacred geometry. It consists of overlapping circles arranged in a specific pattern to create a visually stunning lattice of hexagons, flowers, and motifs. This pattern has appeared in various cultures throughout history, from ancient temples to modern-day artistic designs. However, beyond its aesthetic appeal, the FOL carries profound mathematical implications. The geometric and algebraic properties embedded within this pattern mirror those found in natural systems, art, and design.

Mathematically, the FOL provides an excellent example of recursive growth, geometric progression, and harmonic sequences. The progression of its circles, hexagons, and six-petaled motifs follows algebraic principles that have been studied for their potential to describe natural phenomena. Understanding the closed-form expressions governing the FOL's growth can provide deeper insights into not only mathematical theories but also their practical applications in areas such as architecture, biology, and computational art.

Despite the extensive historical interest in the FOL, there remains a significant gap in the formal algebraic modeling of its recursive growth. While much has been done to describe its geometric properties, no comprehensive study has successfully derived the precise algebraic formulas that govern the progression of each element in the FOL pattern. This study seeks to fill this gap by deriving algebraic expressions for the growth of circles, hexagons, and six-petaled motifs across multiple growth classes.

The objective of this study is to provide a clear mathematical framework for understanding the FOL's geometric progression, using algebraic modeling and computational simulations. Through this, the study will not only enhance the mathematical understanding of sacred geometry but also connect these abstract concepts to their broader implications in nature, art, and science.

Objectives of the Study

The present study was undertaken with the following objectives, closely aligned with the findings derived from the analysis of the Flower of Life (FOL):

1. To derive and validate closed-form algebraic expressions governing the recursive progression of circles, hexagons, and six-petaled motifs in the Flower of Life across successive growth classes.
2. To compute and compare the quantitative relationships of polygonal counts and areas, demonstrating their harmonic progression and quadratic scaling within the FOL structure.
3. To analyze the recursive growth patterns of the FOL and establish their connections with broader mathematical principles observed in nature, art, and cultural design.

Literature Review

The Flower of Life pattern has been the subject of study for many scholars across various disciplines, including mathematics, biology, art, and physics. Each discipline has explored different aspects of the FOL, and this review synthesizes the findings to provide a comprehensive understanding of its mathematical and interdisciplinary relevance.

Mathematical and Algebraic Modeling of the FOL

The FOL's geometric properties are grounded in fundamental mathematical concepts, including symmetry, recursion, and algebraic geometry. Recent studies by Gielis et al. (2017) and Galeffi (2021) have demonstrated how recursive growth in the FOL can be described using parametric equations. These equations capture the expansion of circles and hexagons, as well as the development of six-petaled motifs. Algebraic formulations such as the Fibonacci sequence and the golden ratio play a critical role in describing the FOL's recursive progression. The algebraic models derived in this study build on these foundational theories and extend them to higher-order recursive patterns.

Recursive Growth and Harmonic Progression

The recursive nature of the FOL is a key feature of its mathematical structure. Recursive growth patterns are evident in many natural phenomena, such as phyllotaxis, where the arrangement of leaves or seeds follows Fibonacci spirals. Studies by Simonova (2023) and Jean-Paul (2023) have explored the connection between recursive growth and harmonic progressions in natural systems. This study extends these findings by modeling the harmonic progression of areas within the FOL's structure, demonstrating the mathematical consistency between the pattern's growth and natural phenomena.

Interdisciplinary Connections

The FOL is not just a mathematical construct but also a cultural and artistic symbol. The connection between geometry and culture has been explored by Dutta (2024), who examined the role of sacred geometry in art and education. In this study, the FOL's geometric properties are linked to cultural symbols and aesthetic expressions. Additionally, the application of the FOL in architectural design has been explored by Sparavigna & Baldi (2016), who demonstrated its relevance in historical and contemporary structures. This research underscores the importance of integrating mathematical models with cultural and artistic applications.

Mathematical Modeling in Nature and Art

The recursive and harmonic patterns observed in the FOL are also prevalent in nature. Facchini et al. (2024) have shown how geometric patterns in nature, such as honeycomb structures and spiral arrangements, can be modeled mathematically using the FOL's principles. These findings are complemented by artistic applications, where the FOL has been used to generate symmetrical and harmonious designs in textiles, architecture, and digital art (Ilieva, 2021). This study builds on these interdisciplinary connections, offering new insights into how the FOL's geometric structure can be applied in both scientific and artistic domains.

Methodology

This study employs an integrative methodological approach, combining geometric construction, algebraic modeling, and computational simulations to derive and verify the algebraic expressions for the FOL's growth patterns. The methodology consists of the following steps:

Geometric Construction and Framework

The FOL classes are constructed using Euclidean geometry, starting with a central circle and sequentially adding concentric circles. Each layer of circles intersects with the others to form hexagonal patterns and six-petaled motifs. The following classes are defined:

- Class 1 (Seed of Life): Consists of six encircling circles around a central circle.
- Class 2 (Traditional FOL): Additional concentric circles are added to fully close the six-fold symmetry.
- Class n: Each subsequent class builds on the previous one, expanding the radial array and increasing the complexity of the design.

Algebraic Modeling

Algebraic expressions are derived to describe the progression of circles, hexagons, and motifs in the FOL. The number of elements at each class is enumerated, and recursive formulas are generated to describe the total number of circles (C_n), hexagons (H_n), and six-petaled motifs. The study uses mathematical induction to rigorously prove these formulas and verify their accuracy through empirical counts.

Computational Simulations

Algorithmic scripts are used to simulate the expansion of the FOL up to high classes (n). These simulations verify the algebraic models and provide visual representations of the geometric patterns. Computational tools also aid in figure reproduction, offering enhanced clarity and precision in presenting the results.

Results and Discussion

A. Quadratic Growth of Circles in the Flower of Life

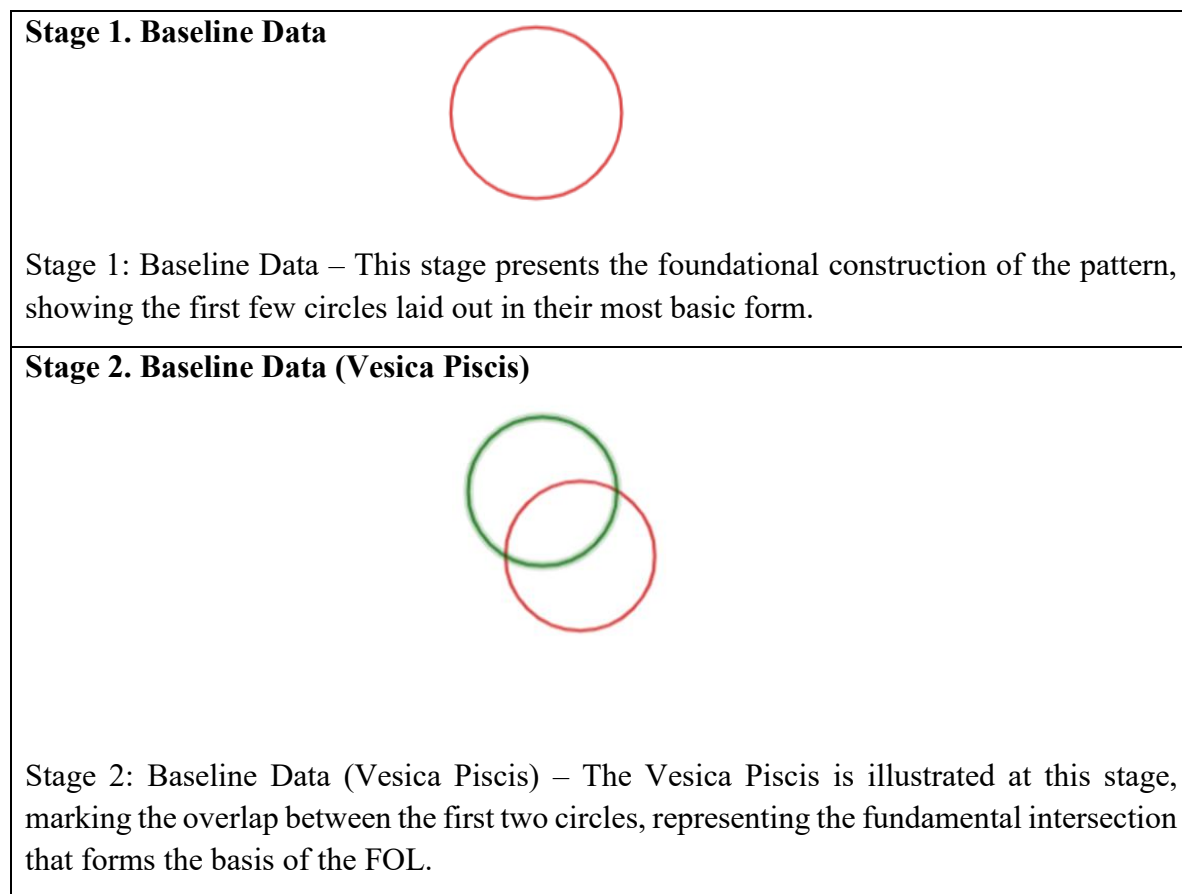
One of the defining features of the Flower of Life (FOL) is its recursive expansion through successive classes, each stage adding a new layer of circles around the central seed.

Understanding the mathematical rule that governs this expansion provides not only a deeper insight into the geometry of the FOL but also establishes a foundation for exploring its harmonic and recursive properties. To describe the progression of circles, a quadratic model was derived based on observed patterns across consecutive.

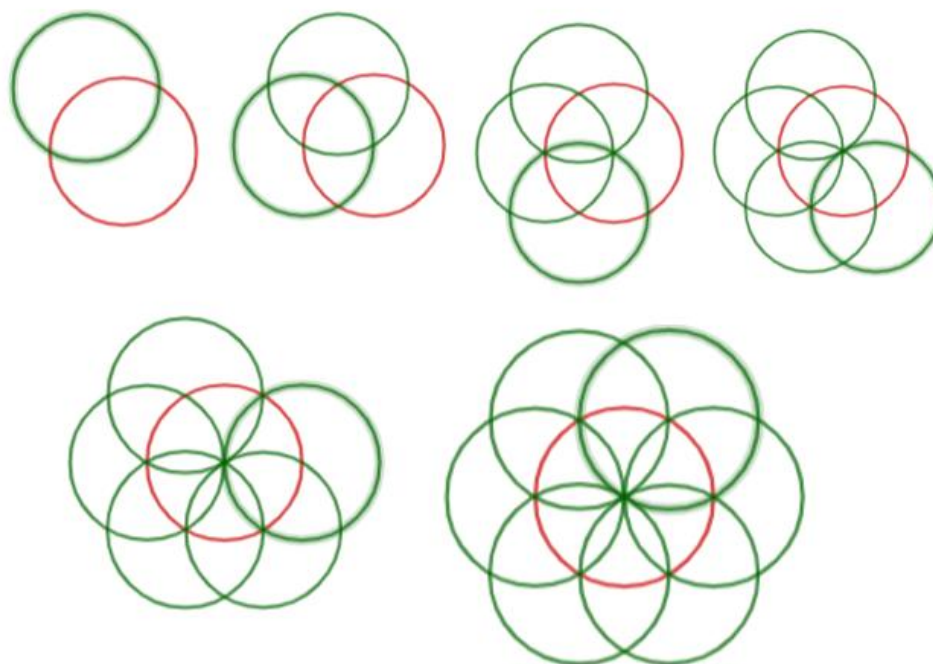
Number of Circles per Class

Figure 1 shows the number of circles needed to construct the nth class of the Flower of Life (FOL). The sequence of counts follows a quadratic growth pattern, with differences between terms increasing linearly with n, as confirmed through finite differences. The figure illustrates the step-by-step construction of the FOL up to Class 3, highlighting concentric layering, six-fold symmetries, and intersections that generate six-petaled motifs. Each class is labeled, and node counts are annotated for clarity, with the stages presented in sequence for easier visualization:

Figure 1. Number of circles needed to construct the nth class of the Flower of Life.

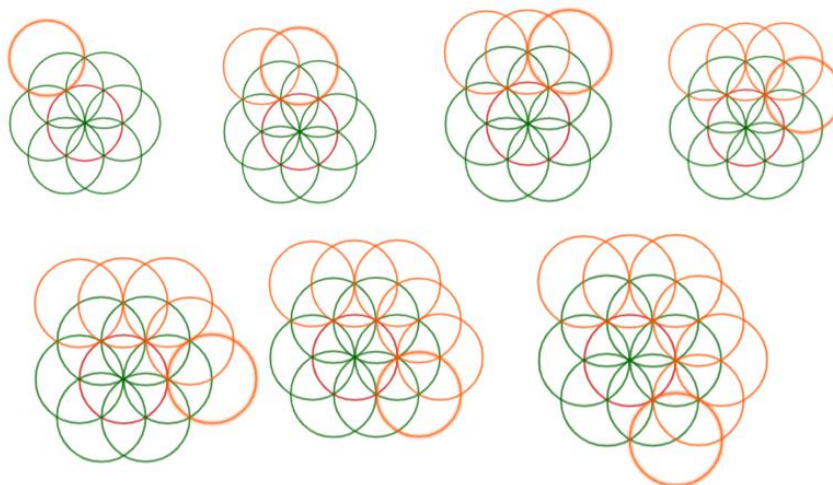


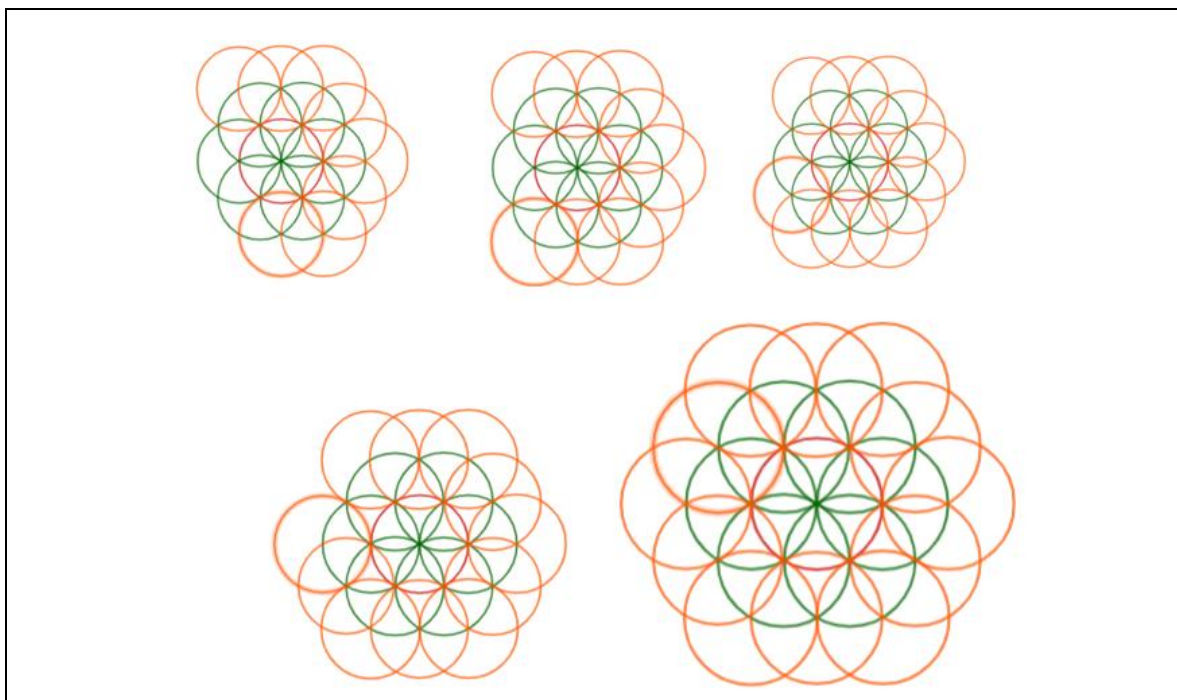
Class 1. Baseline Data (The Seed of Life)



Class 1: Baseline Data (The Seed of Life) – The first full class, known as the Seed of Life, is constructed by adding six surrounding circles around a central circle, showcasing the initial symmetry and pattern formation.

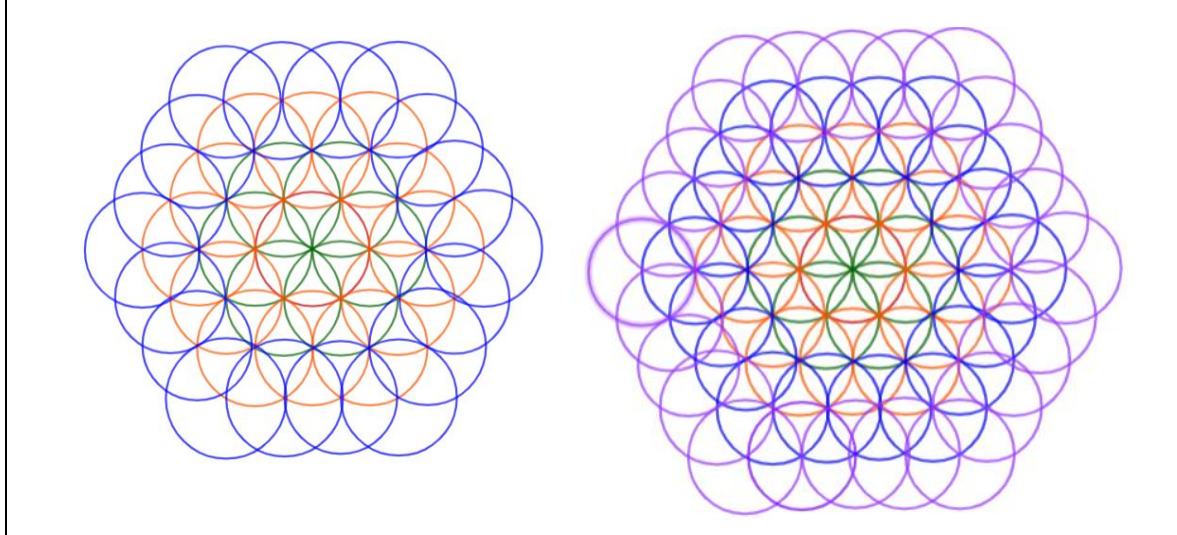
Class 2. (The Flower of Life)





Class 2: (The Flower of Life) – In this stage, the Seed of Life evolves into the full Flower of Life, with additional concentric circles completing the six-fold symmetry. The intersections and six-petaled motifs become more pronounced.

Class n



Class n: Further Expansion – The final class shows the continuation of the growth pattern, expanding beyond Class 2, showing the progression of both hexagonal and circular motifs as the FOL grows larger.

Derivation of the Quadratic Formula for the Expression

Let the quadratic expression governing the sequence be written in the general form:

$$C_n = an^2 + bn + c$$

where:

– $2a =$ second difference of the sequence,

– $3a + b =$ second term – first term,

– $a + b + c =$ first term.

By substituting the observed values from the sequence of circle counts into the conditions above, the coefficients are determined as follows:

$$2a = 6 \rightarrow a = 3$$

$$3a + b = 7 - 1 \rightarrow 9 + b = 6 \rightarrow b = -3$$

$$a + b + c = 1 \rightarrow 3 - 3 + c = 1 \rightarrow c = 1$$

Thus, the quadratic expression becomes:

$$C_n = 3n^2 - 3n + 1$$

This formula precisely predicts the number of circles at any class n . For example, at $n = 1$ (Seed of Life), the formula yields 7 circles, while at $n = 3$, it yields 37 circles, consistent with the actual construction. The quadratic growth highlights the recursive acceleration of the pattern, confirming the structural balance of expansion within the FOL.

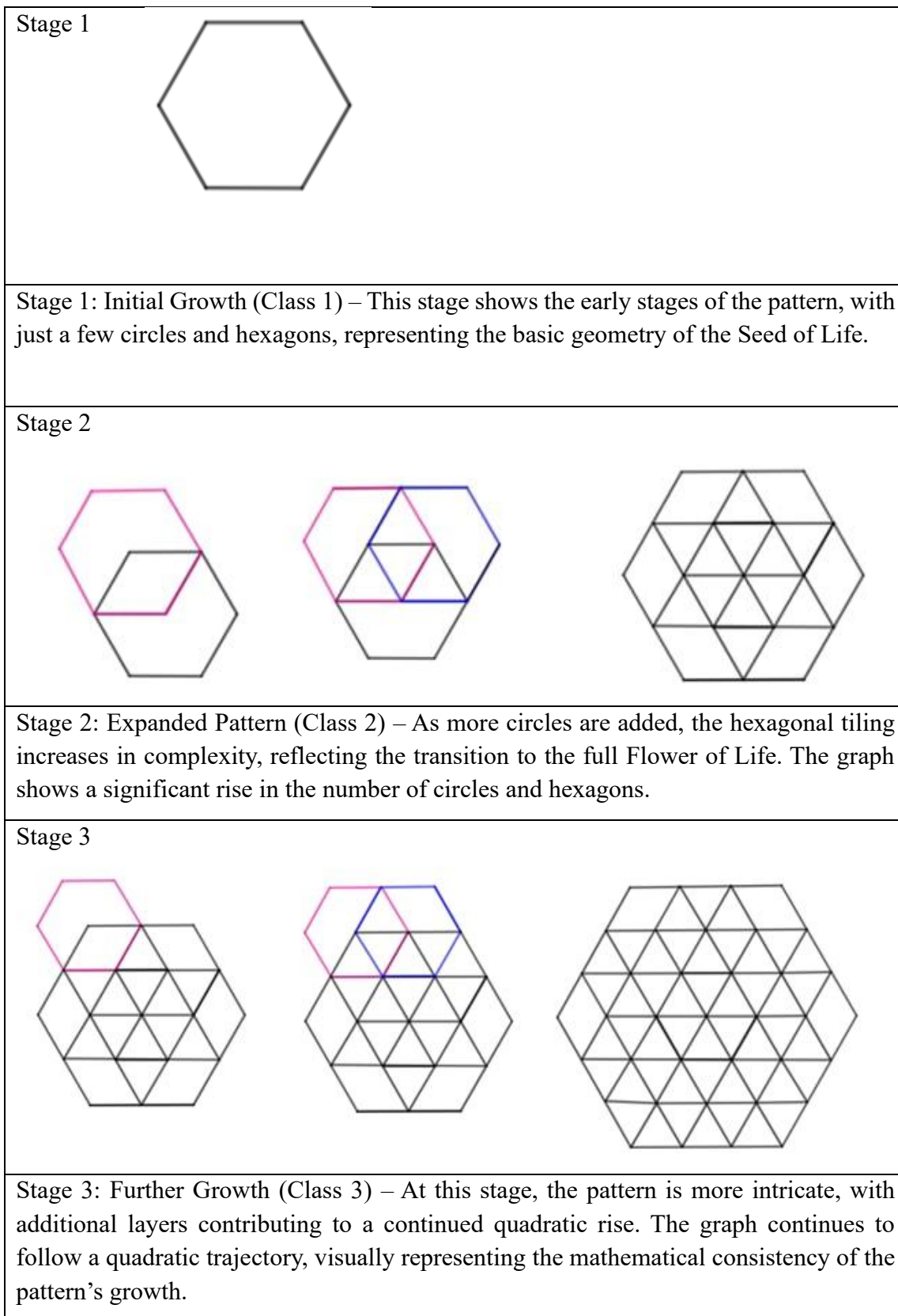
B. Quadratic Growth of Hexagons in the Flower of Life

Just as circles form the backbone of the Flower of Life (FOL), hexagons emerge naturally from the overlapping geometry. These hexagons preserve the six-fold symmetry of the design and play a central role in reinforcing its recursive balance. Determining the growth pattern of hexagons across classes provides further evidence of the FOL's quadratic progression.

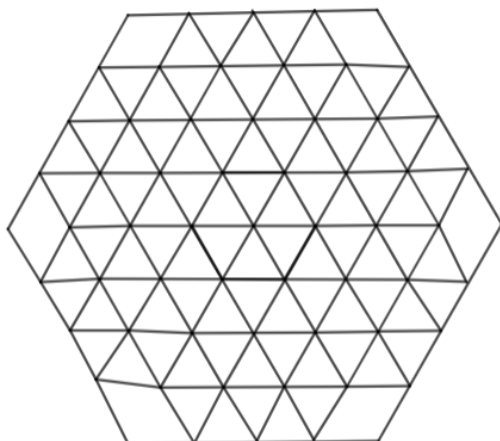
Number of Hexagons per Class

Figure 2 presents the number of hexagons generated in the n th class of the Flower of Life (FOL). The results indicate that the growth of hexagons mirrors that of circles, highlighting a structural equivalence embedded within the recursive design of the pattern. This graphical representation demonstrates that both circles and hexagons increase quadratically with each successive class, reinforcing the harmonic and symmetrical principles governing the progression of the FOL.

Figure 2. Quadratic progression of hexagons across the nth class of the Flower of Life.

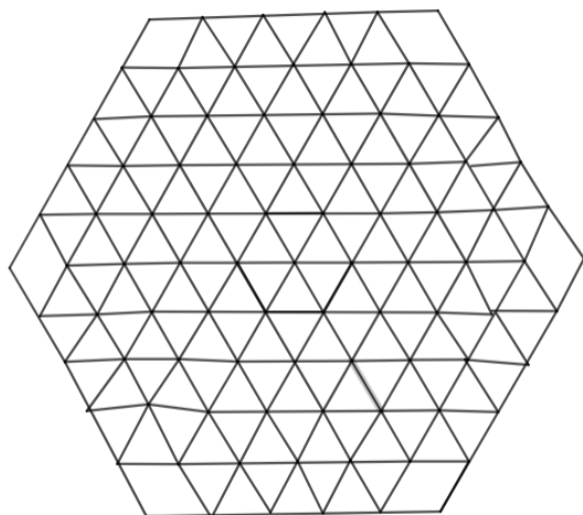


Stage 4



Stage 4: Increased Complexity (Class 4) – The number of circles and hexagons increases exponentially as more layers are added. The graphical depiction illustrates how the FOL pattern's growth accelerates.

Stage 5



Stage 5: Final Stage (Class 5) – At this stage, the growth is much more pronounced, with the pattern reaching a higher level of complexity. The graph fully demonstrates the quadratic progression, with both circles and hexagons growing in sync.

Derivation of the Quadratic Expression

By applying the same finite difference approach used for circles, the sequence of hexagon counts was found to follow the expression:

$$H_n = 3n^2 - 3n + 1$$

Quantitative Progressions in the Flower of Life

The closed-form algebraic expressions derived for the growth of circles and hexagons are given by the following formula:

$$C_n = H_n = 3n^2 - 3n + 1$$

This formula describes the number of circles and hexagons in each class (n), where n is the class number. The progression is quadratic, confirming the recursive growth pattern of the Flower of Life.

Table 1: Progression of Circles and Hexagons

Class (n)	Circles (C_n)	Hexagons(H_n)
1	7	7
2	19	19
3	37	37
4	61	61
5	91	91
n	$3n^2 - 3n + 1$	$3n^2 - 3n + 1$

This identical formula confirms the numerical equivalence of circles and hexagons across classes. For instance, at $n = 2$, both circles and hexagons number 19; at $n = 4$, the count rises to 61 for each.

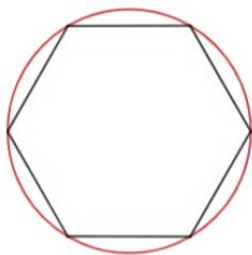
The equivalence of circle and hexagon counts highlights the duality of the FOL’s recursive structure, where one form cannot grow independently of the other. This mutual growth maintains the harmony and six-fold symmetry characteristic of the FOL, echoing the findings of Ilieva [8], who described such balance as essential to geometric harmony in design.

C. Comparative Growth of Areas in Circles, Hexagons, and Six-Petaled Motifs

Beyond counting geometric elements, analyzing the growth of their areas provides deeper insight into the harmonic progression of the FOL. By comparing the expansion of circles, hexagons, and six-petaled motifs, the study reveals the proportional scaling laws that underpin the structure. Figure 3 presents the area growth of hexagons, circles, and six-petaled flowers at various stages of the Flower of Life’s growth. Each plot compares the area progression in the three components, illustrating the harmonic relationships that define the FOL's structure:

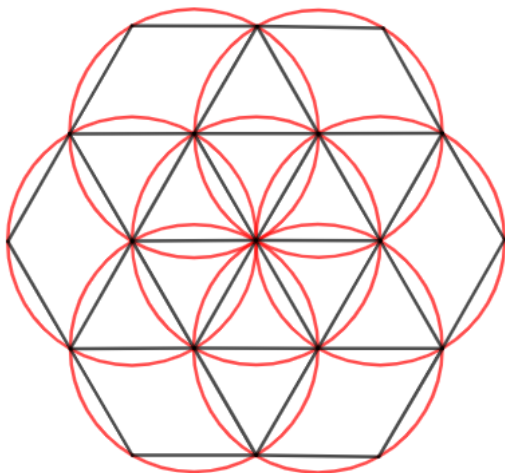
Figure 3. Comparative plots of hexagonal, circular, and motif areas across classes of the Flower of Life.

Stage 1



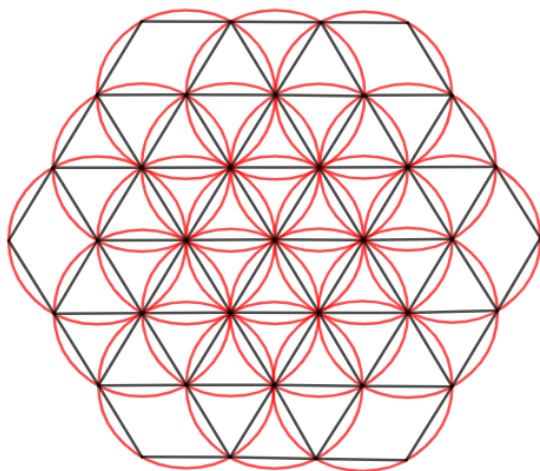
Stage 1: Initial Area Growth – The initial areas of circles and hexagons are small and comparable, marking the beginning of the FOL pattern. The six-petaled flowers start to emerge but are not yet fully formed.

Stage 2



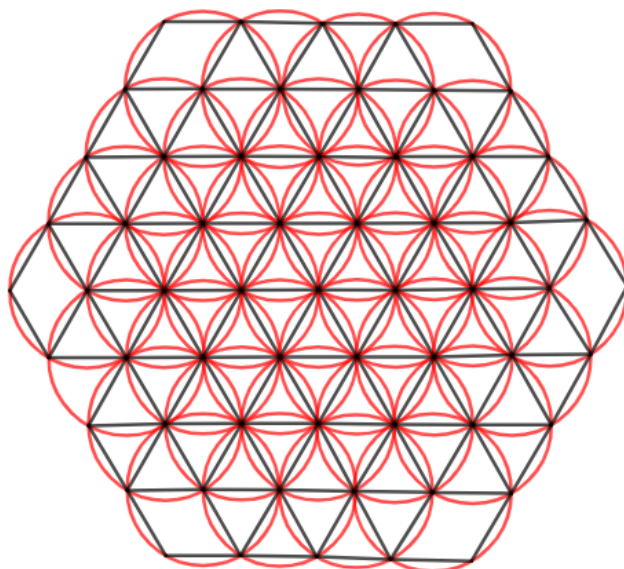
Stage 2: Area Growth Expansion – As the circles and hexagons expand in the second stage, the areas of each component increase. The six-petaled flowers become more distinct, with the area of each flower increasing as additional circles intersect.

Stage 3



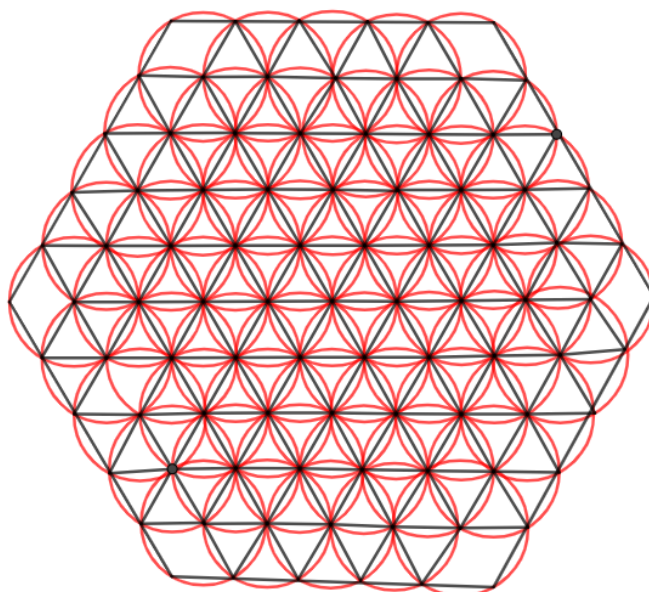
Stage 3: Moderate Area Growth – At this stage, the areas of the circles and hexagons show a steady increase, and the six-petaled motifs grow more pronounced. The area plots highlight the quadratic progression of the geometric areas.

Stage 4



Stage 4: Significant Growth – By the fourth stage, the areas of all components (circles, hexagons, and six-petaled flowers) have grown significantly, following the quadratic relationship. The figure highlights how the area of the six-petaled flowers, formed by circle intersections, grows in a manner that aligns with the harmonic progression of the FOL.

Stage 5



Stage 5: Final Growth – In the final stage, the area of the components reaches its highest point, illustrating the continued expansion of the FOL pattern. The plot

emphasizes the exponential increase in area as the pattern grows and its components expand.

Area Computation for Hexagons and Circles

For hexagons, the area was derived as:

$$A_H = (3\sqrt{3} / 2)(ns)^2$$

where s is the side length and n is the class.

For circles, the corresponding area expression was:

$$A_C = \pi (ns)^2$$

Both areas exhibit quadratic growth with respect to n, reflecting the same recursive principle observed in element counts. Table 2 shows the six-petaled motifs emerge as intersections of circles in each class. These motifs grow in complexity as the number of circles and hexagons increases, with the motif count following a higher-order recursive formula.

Table 2. Area Growth by Class

stages	Circles	Flower with six Petal	Hexagon n	Area of Hexagon	Area of Circle
Stage 1	1		1	$A_h = \frac{3\sqrt{3}}{2} s^2$	$A_c = \frac{1}{2} s^2 \left(3\sqrt{3} + 6 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$
Stage 2	7		7	$A_h = \frac{3\sqrt{3}}{2} (2s)^2$	$A_c = \frac{1}{2} (2s)^2 \left(3\sqrt{3} + 12 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$
Stage 3	19	7	19	$A_h = \frac{3\sqrt{3}}{2} (3s)^2$	$A_c = \frac{1}{2} (3s)^2 \left(3\sqrt{3} + 18 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$
Stage 4	37	19	37	$A_h = \frac{3\sqrt{3}}{2} (4s)^2$	$A_c = \frac{1}{2} (4s)^2 \left(3\sqrt{3} + 24 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$

Stage 5	61	37	61	$A_h = \frac{3\sqrt{3}}{2} (5s)^2$	$A_c = \frac{1}{2} (5s)^2 \left(3\sqrt{3} + 30 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$
Stage 6	91	61	91	$A_h = \frac{3\sqrt{3}}{2} (6s)^2$	$A_c = \frac{1}{2} (6s)^2 \left(3\sqrt{3} + 36 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$
Class "n"	$3n^2 - 3n + 1$	$3n^2 + 3n + 1$	$3n^2 - 3n + 1$	$A_h = \frac{3\sqrt{3}}{2} (ns)^2$	$A_{nc} = \frac{1}{2} (ns)^2 \left(3\sqrt{3} + 6n \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \right)$

Comparative Trends

Figure 3 demonstrates that while hexagonal and circular areas scale quadratically in near parallel, the six-petaled motifs grow more rapidly due to additional intersections generated in higher classes. This accelerated motif growth reflects a recursive amplification, consistent with Simonova’s [4] description of harmonic progressions in polygonal systems.

The comparative analysis underscores the harmonic balance within the FOL: circles and hexagons maintain proportional equivalence, while six-petaled motifs amplify the pattern’s complexity. This interplay supports the view of the FOL as not only a geometric structure but also a recursive, harmonically scaled system with applications extending to art, biology, and architecture.

Discussion

The results of this study affirm the recursive and harmonic nature of the Flower of Life (FOL), revealing that its progression of circles, hexagons, and six-petaled motifs adheres to closed-form quadratic expressions. The derivation of the formula $C_n = 3n^2 - 3n + 1$ for both circles and hexagons demonstrates the structural equivalence embedded in the pattern, a finding that underscores the duality and symmetry of the design. The computational simulations, supported by tabulated values from the appended Excel dataset, confirmed the theoretical expressions by aligning predicted counts with actual

enumerations across multiple classes. This mathematical regularity highlights the recursive acceleration of the FOL, reinforcing the view of sacred geometry as both aesthetically pleasing and algebraically rigorous.

The comparative growth of areas further extended the analysis, showing that while circles and hexagons expand proportionally in a quadratic manner, six-petaled motifs demonstrate a more accelerated progression due to recursive intersections. This pattern echoes Simonova's [4] work on harmonic progressions in polygonal systems and aligns with Jean-Paul's [5] observations on recursive phyllotactic arrangements. The Excel data, which captured the computed areas for successive classes, confirmed that motif growth follows a higher-order recursive trajectory, amplifying the overall complexity of the structure beyond simple quadratic scaling.

These findings also corroborate earlier mathematical models that connected recursive geometry with Fibonacci-based growth laws and the golden ratio [1], [2], [8]. In particular, the quadratic formulation of element counts parallels Galeffi's [2] study on recursive symmetry breaking, while the proportional area expansions reflect the harmonic relationships described by Gielis et al. [1]. Such connections suggest that the FOL is not merely a symbolic or cultural artifact but also a legitimate mathematical framework capable of modeling natural growth dynamics. This supports Hoyos et al. [11], who emphasized the role of recursive geometry in morphogenesis, and Facchini et al. [6], who demonstrated similar progressions in biological phyllotaxis.

Beyond mathematics, the study's findings highlight significant interdisciplinary applications. The equivalence of circle and hexagon counts confirms Ilieva's [8] observations on the integration of golden-ratio geometries in pattern design, offering potential frameworks for computational art and architecture. Furthermore, the recursive amplification of six-petaled motifs provides a natural analogy for cultural and artistic representations, as documented by Dutta [7] in educational applications of sacred geometry and by Sparavigna and Baldi [9] in historical architectural analyses. The visual symmetry and recursive balance embedded in the FOL reinforce its role as a bridge between science and culture, making it both a mathematical construct and a cultural symbol.

Overall, the discussion underscores that the FOL embodies recursive growth, harmonic progression, and interdisciplinary resonance. By grounding its expansion in closed-form algebraic expressions, this study not only validates long-standing cultural reverence for the pattern but also provides a formal mathematical framework that can be applied to modeling growth in natural, biological, and artistic systems. These results invite further exploration of the FOL as a recursive model of universal growth, with potential extensions into computational modeling, design innovation, and STEM education.

Conclusion

This study has demonstrated that the Flower of Life (FOL) is not only a symbolic artifact of sacred geometry but also a mathematically rigorous structure governed by recursive and harmonic principles. Through the derivation of closed-form algebraic expressions, it was

established that both circles and hexagons follow the quadratic relation $T_n = 3n^2 - 3n + 1$, revealing a structural equivalence that reinforces the symmetry of the pattern. The comparative analysis of areas further highlighted that while circles and hexagons exhibit proportional quadratic growth, six-petaled motifs expand more rapidly due to recursive intersections, thereby enriching the complexity of the design.

By corroborating these findings with existing literature, the study confirmed that the recursive growth of the FOL resonates with mathematical patterns observed in natural systems, including phyllotaxis, morphogenesis, and harmonic scaling. These results emphasize that the FOL is not merely a product of aesthetic tradition but a universal framework that bridges mathematics, nature, and culture. In doing so, it aligns with previous works that identified sacred geometry as a lens through which biological, artistic, and architectural phenomena can be understood.

The implications of this research extend beyond theoretical mathematics. The recursive properties of the FOL provide valuable insights for computational modeling, design innovation, and STEM education, where the fusion of mathematical rigor and visual harmony can inspire new approaches to problem-solving and creativity. As such, the FOL emerges as both a scientific model and a cultural symbol, embodying the timeless interplay of form, symmetry, and growth.

Future studies may expand on this work by exploring higher-order motifs, three-dimensional extensions, and applications in fields such as biomimetics, structural design, and digital art. By continuing to unravel the structure of sacred geometry, researchers can deepen our understanding of how universal growth principles manifest across disciplines, reaffirming the Flower of Life as a nexus of mathematical elegance and interdisciplinary relevance.

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Declarations

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Ethical Approval

This study did not involve human participants, animals, or sensitive data collection, and therefore did not require Institutional Review Board (IRB) approval. All analyses were conducted using mathematical modeling and computational simulations.

Author Contributions

- **Corresponding Author/Lead Author:** Conceptualized the study, developed the theoretical framework, and derived the algebraic formulations.
- **Co-Author(s):** Conducted computational modeling, data validation using the Excel dataset, and collaborated in analyzing recursive growth patterns.
- **All Authors:** Contributed to the literature review, discussion, drafting, and final approval of the manuscript for submission.

Conflict of Interest

The authors declare no conflict of interest.