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On the Block Vertex Neighborhood Number of Connected Graphs

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Abstract

The open neighborhood $N(w)$ of a vertex $w \in V$ consists of all vertices adjacent to w in an undirected graph. The closed neighborhood $N[w]$, includes w and all vertices reachable from it. The fragments of subgraphs held together by its cut-vertices are its blocks. Let $B(G)$ denote the set of all blocks in G. A block $b \in B(G)$ bv-covers a vertex v if $v \in \langle N[b] \rangle$, where $\langle N[b] \rangle$ is the subgraph induced by the closed neighborhood of b. A set $S \subseteq B(G)$ is a bv-neighborhood set if every vertex v is bv-covered by some $b \in S$, that is, $G = \bigcup_{b \in B(G)} \langle N[b] \rangle$. The minimum cardinality

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of bv-neighborhood set is called a *block vertex neighborhood number* $n_{bv}(G)$. In this work, we establish upper and lower bounds for $n_{bv}(G)$, characterize extremal graph classes attaining these bounds and compute exact values of $n_{bv}(G)$ for graphs associated with block-related parameters.

Math. Subject Classification: 05C69.

Key Words and Phrases: Block vertex degree, block block degree, block regular graph, semitotal block vertex graph, total block vertex graph and block vertex neighborhood number.

1 Introduction

We will use the foundational graph-theoretic terminology established in standard references such as [5] and [1]. By a graph we mean a connected finite simple graph. A vertex $v \in V$ is a cut-vertex of a graph $G(V, E)$ if $G - v$ is disconnected. A maximal connected subgraph with no cut-vertices is a block of G. A maximal complete subgraph is a clique. Let $B(G)$ and $C(G)$ denote the set of all blocks and cut-vertices of G respectively. A block $b \in B(G)$ and a cut-vertex $c \in C(G)$ are said to incident to each other if b contains c. Two blocks in G are adjacent if there is a common cut-vertex incident on them. A block graph $B_G(G)$ is a graph with vertex set $B(G)$ and any two vertices in $B_G(G)$ are adjacent if and only if the corresponding blocks are adjacent in G. Similarly a cut-vertex graph $C_G(G)$ is defined.

A sequence of blocks and cut-vertices in which all the cut-vertices are distinct is a B-path. Any B-path with m blocks is denoted as B_{P_m} and has $m-1$ cut-vertices. If there exists a unique B-path between any two blocks of a graph then G is a B-tree. In a B-complete graph B_m, any two blocks of G are adjacent. A graph G is a B-star, $B_{m_1,m_2,\ldots,m_k}$ if there exists a block b with k cut-vertices and i^{th} cut-vertex is incident with $m_i + 1$ blocks, $m_i \in N$, $1 \leq i \leq k$. As shown in Figure 1, B-path, B-complete graph, B-star are B-trees.

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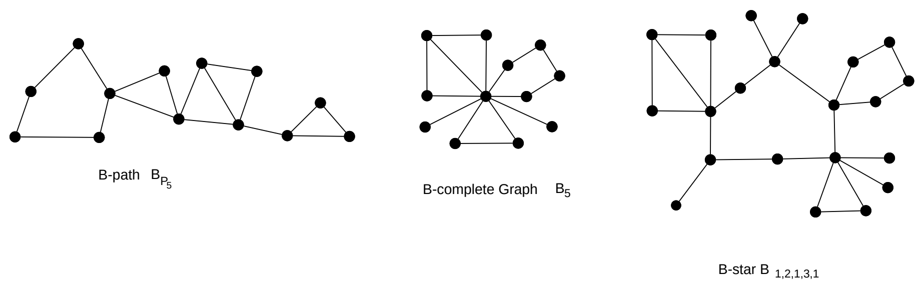

Figure 1: Varieties of block-trees.

The domination number $\gamma = \gamma(G)$ of G is the minimum cardinality of a set of vertices that dominate all the vertices of G. The concept of domination has been studied extensively in [11], [6] and [8]. A block $F \subseteq B(G)$ is said to be block-vertex dominating set if every vertex in G is b-dominated by some block in F. The block-vertex domination number $\gamma_{bv}(G)$ is the minimum cardinality of a block vertex dominating set of G. A set $L \subseteq B(G)$ is said to be bb-dominating set (BBD) if every block in G is bb-dominated by some block in L. The bb-domination number $\gamma_{bb}(G)$ is the minimum cardinality of BBD-set of G. We can also find several mixed domination concepts in graphs that are defined and studied in [4], [3], [9], [10] and [7].

Another notable graph invariant is the neighborhood covering (n-covering) number, introduced by Sampathkumar and Neeralagi [2]. We introduced a novel graph invariant called the block vertex neighborhood number, extending the concept of neighborhood covering. This new invariant investigates the interaction between vertex neighborhoods and blocks, offering a fresh perspective on how these parameters function within different graph families. In essence, this paper highlights the concepts of the block vertex neighborhood number and examines their connections with other established graph invariants.

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2 Block Vertex Neighborhood Number

Definition 2.1. A block $b \in B(G)$ bv-covers a vertex v if $v \in \langle N[b] \rangle$, where $\langle N[b] \rangle$ is the subgraph induced by the closed neighborhood of b. A set $S \subseteq B(G)$ is a bv-neighborhood set if every vertex v is bv-covered by some $b \in S$, that is, $G = \bigcup\limits_{b \in B(G)} \langle N[b] \rangle$. The minimum cardinality of bv-neighborhood set is called a *block vertex neighborhood number* $n_{bv}(G)$.

Example 2.1. *For the graph G of Figure 2, the blocks formed by vertices $\{v_4, v_6, v_7\}$ and $\{v_{10}, v_{11}\}$ form a n_{bv}-set. Therefore $n_{bv}(G) = 2$.*

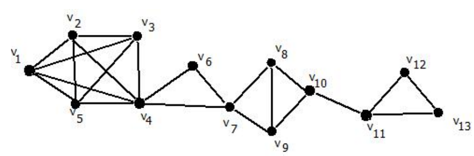

Figure 2: B-path (B_{P_5}).

Proposition 2.2.

- *For a path graph P_n with $n \geq 2$ vertices, $n_{bv}(P_n) = \lceil \frac{n-1}{3} \rceil$.*
- *For B-complete graph with m blocks, $n_{bv}(B_m) = m - r$, where r is the number of blocks which are complete.*
- *For B-complete graph with m blocks, if each block is a cycle C_k with $k \geq 4$, then the set of all blocks in G forms a $n_{bv}(G)$-set.*
- *For B-complete graph with m blocks, if each block is a clique Q_k, then $n_{bv}(G) = 1$.*
- *For B-star graph $B_{m_1,\ldots,m_n}$ with n cut-vertices, if the blocks on the cut-vertices says $m_1, \ldots, m_n$ are complete, then $n_{bv}(B_{m_1,\ldots,m_n}) = 1$.*

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- *For B-complete graph and B-star graph, $n_{bv}(B_G(G)) = 1$.*
- *For any B-path with m blocks, $n_{bv}(\mathrm{B}_{P_m}) \geq m - r$, where r is the number of blocks which are complete.*
- *For a B-path G with m blocks, $n_{bv}(B_G(G)) = \lceil \frac{m-1}{3} \rceil$.*

Let $K(i)$ be a cactus graph that is a B-path, where all blocks are cycles of length i. Let the set of blocks in $K(i)$ be $\{b_1, b_2, \ldots, b_m\}$ as shown in Figure 3.

Figure 3: Cactus K(i).

Proposition 2.3. *A cactus graph is block-regular where all of its blocks are isomorphic to each other and all of its vertices have the same degree. Then, we have the following:*

1. *When $i = 4$,*
   - *$m =$ odd, n_{bv}(cactus $K(i)$) $= \frac{m+1}{2}$.*
   - *$m =$ even, n_{bv}(cactus $K(i)$) $= \frac{m+2}{2}$.*
2. *When $i \geq 4$ then, n_{bv}(cactus $K(i)$) $= \gamma_{bv}$(cactus $K(i)$).*
3. *$n_{bv} = \lceil \frac{m}{3} \rceil$, when all the blocks in cactus $K(i)$ is a clique Q_k.*

The block-vertex degree $d_{bv}(f)$ of a block f is the number of vertices in the block f. A graph G is said to be BVk-regular if $d_{bv}(h) = k$ for every $h \in B(G)$. Here, Figure 4 is a $BV4-$ regular graph.

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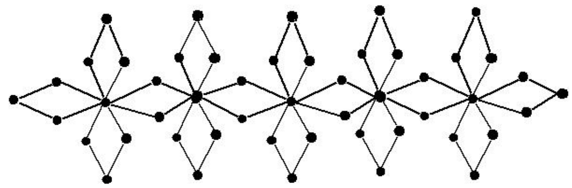

Figure 4: BV4-regular graph.

Proposition 2.4. *Let G be a connected BVk-regular graph with $k \geq 3$ and m blocks. Then*

1. *For $k = 4$, $n_{bv}(G)$-set consist of all pendant blocks if every block of G is a cycle C_4.*
2. *For $k \geq 5$, $n_{bv}(G) = \gamma_{bv}(G)$.*
3. *If every block of G is a clique Q_k then, $n_{bv}(G) = \lfloor \frac{non-pendant\ blocks}{2} \rfloor + 1$.*

The maximum strength of a block (Δ_{sbv}) is the number of vertices in $\langle N[b] \rangle$. Now we obtain a bound for $n_{bv}(G)$ in terms of maximum strength Δ_{sbv}.

Theorem 2.5. *Let G be a graph with m blocks. Then $\frac{n}{\Delta_{sbv}} \leq n_{bv}(G) \leq n - \Delta_{sbv} + 1$.*

Proof. Let $S = \{b_1, b_2, \ldots, b_k\}$ be a n_{bv}-neighborhood set, so $k = n_{bv}(G)$. By definition, $G = \bigcup_{i=1}^{k} \langle N[b_i] \rangle$. The total number of vertices in G is n. Each $\langle N[b_i] \rangle$ can contain at most Δ_{sbv} vertices, that is, $|V(\langle N[b_i] \rangle)| \leq \Delta_{sbv}$ for all i. Since the union of these sets must cover all n vertices, we have: $n = \left| \bigcup_{i=1}^{k} V(\langle N[b_i] \rangle) \right| \leq \sum_{i=1}^{k} |V(\langle N[b_i] \rangle)| \leq \sum_{i=1}^{k} \Delta_{sbv} = k \cdot \Delta_{sbv}$. So, $n \leq n_{bv}(G) \cdot \Delta_{sbv}$, which implies $\frac{n}{\Delta_{sbv}} \leq n_{bv}(G)$. To prove upper bound, choose a block b^* with maximum strength. Consider the vertex not covered by b^* says $V' = V(G) - V(\langle N[b^*] \rangle)$. Bounding the number of blocks needed to cover V'. The size of V' is $n - \Delta_{sbv}$, so these vertices can be covered by at most Δ_{sbv} additional blocks. Adding one for the

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initial block leads to $n - \Delta_{sbv} + 1$.
Therefore, $\frac{n}{\Delta_{sbv}} \leq n_{bv}(G) \leq n - \Delta_{sbv} + 1$. □

Clique-vertex neighborhood number: Let $K(G)$ denote the set of all cliques in a graph G. A clique $k \in K(G)$ is said to cv-covers a vertex v if $v \in \langle N[k] \rangle$, where $\langle N[k] \rangle$ denotes the subgraph induced by the $N[k]$. A set $S \subseteq K(G)$ is a cv-neighborhood set if every vertex v is cv-covered by some $k \in S$, that is, $G = \bigcup\limits_{k \in K(G)} \langle N[k] \rangle$. The minimum cardinality of such a set is the *clique vertex neighborhood number*, $n_{cv}(G)$.

Theorem 2.6. *Let G be a graph with m blocks. Then $n_{bv}(G) \leq n_{cv}(G)$.*

Proof. Let S_K be an n_{cv}-set of G, so $|S_K| = n_{cv}(G)$. This means every vertex in G is cv-covered by at least one clique in S_K. For each clique $k \in S_K$, there exists a block b_k such that $k \subseteq b_k$. Construct a set of blocks $S_B = \{b_k \mid k \in S_K\}$. The size of this set, $|S_B|$, is less than or equal to $|S_K|$, meaning $|S_B| \leq n_{cv}(G)$. Since $k \subseteq b_k$, the closed neighborhood of k is contained within the closed neighborhood of b_k ($N[k] \subseteq N[b_k]$). Therefore, if a vertex is cv-covered by k, it is also bv-covered by b_k. Thus, S_B is a bv-neighborhood set for G. Since $n_{bv}(G)$ is the minimum size of any bv-neighborhood set, $n_{bv}(G) \leq |S_B|$. Combining these, we get $n_{bv}(G) \leq n_{cv}(G)$. □

Theorem 2.7. *Let G be a graph with m blocks. Then $\gamma_{bb}(G) \leq n_{bv}(G) \leq \gamma_{bv}(G)$.*

Proof. To prove $n_{bv}(G) \leq \gamma_{bv}(G)$, here a subset of blocks whose collective bv-coverage spans all vertices in the graph G. Since both $\gamma_{bv}(G)$ and $n_{bv}(G)$ are defined as the minimum cardinality of this same type of set, their values must be equal. Therefore, we have $n_{bv}(G) = \gamma_{bv}(G)$.
From this equality, the inequality $n_{bv}(G) \leq \gamma_{bv}(G)$.
To prove $\gamma_{bb}(G) \leq n_{bv}(G)$, let S be a minimum n_{bv}-set, so $|S| =$

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$n_{bv}(G)$. We need to show that S is also a bb-dominating set. Consider any block $h \in B(G)$. If $h \in S$, it dominates itself. If $h \notin S$, pick any vertex $v \in V(h)$. Since S is a bv-neighborhood set, v must be bv-covered by some block $s \in S$. Since h and s are distinct blocks, this implies they must share a common vertex. Therefore, h is bb-dominated by s. Since every block h is bb-dominated by some $s \in S$, S is a bb-dominating set. Therefore, $\gamma_{bb}(G) \leq |S| = n_{bv}(G)$. Combining both parts, we get $\gamma_{bb}(G) \leq n_{bv}(G) \leq \gamma_{bv}(G)$. □

Point graph is a graph whose vertex set is same as that of G and any two vertices in $P_G(G)$ are adjacent if and only if they are vv-adjacent in G. Point graph of a graph G denoted by $P_G(G)$. Two vertices $\{u, w\} \in V$ are vv-adjacent if they incident on the same block. As shown in the Figure 5, $P_G(G)$ is a point graph of G.

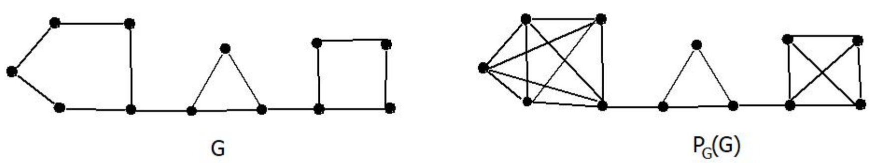

Figure 5: Graph G and point graph of G.

Theorem 2.8. *A graph G has the property that its $P_G(G) \cong G$ if and only if every block in G is a complete graph. Furthermore, $n_{bv}(P_G(G)) = n_{bv}(G)$ if and only if $P_G(G) \cong G$.*

Proof. Let $G = (V, E)$ be a graph such that every block in G is a complete graph.

To prove $P_G(G) \cong G$ by definition, $V(P_G(G)) = V(G)$. To prove isomorphism, we must show $E(P_G(G)) = E(G)$. Let $u, w \in V(G)$ be two distinct vertices. Assume $(u, w) \in E(G)$. Every edge belongs to a block. Thus, there exists a block $b \in B(G)$ such that $(u, w) \in E(b)$. This implies $u, w \in V(b)$. By definition, u and w are vv-adjacent in G, which means $(u, w) \in E(P_G(G))$. Hence, $E(G) \subseteq E(P_G(G))$.

Conversely assume $(u, w) \in E(P_G(G))$. By definition, u and w are

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vv-adjacent in G. This means there exists a block $b' \in B(G)$ such that $u, w \in V(b')$. Since every block in G is complete by hypothesis, b' is a complete graph. As $u, w \in V(b')$ and b' is complete, there must be an edge $(u, w) \in E(b')$. Since $E(b') \subseteq E(G)$, it follows that $(u, w) \in E(G)$. Hence, $E(P_G(G)) \subseteq E(G)$.
From both inclusions, $E(P_G(G)) = E(G)$. Since their vertex sets are identical and their edge sets are identical, $P_G(G) \cong G$ by definition of graph isomorphism.
To prove $n_{bv}(P_G(G)) = n_{bv}(G)$. Since $P_G(G) \cong G$, these two graphs are structurally identical. Graph isomorphism preserves all graph-theoretic properties and invariants, including the set of blocks, block vertex neighborhoods, and consequently, the block vertex neighborhood number. Therefore, $n_{bv}(P_G(G)) = n_{bv}(G)$. □

Theorem 2.9. *Let G be a graph with m blocks. Then $n_{bv}(P_G(G)) \leq n_{bv}(G)$.*

Proof. Let $S_G = \{b_1, \ldots, b_k\}$ be a minimum bv-neighborhood set for G, so $n_{bv}(G) = k$. This means $V = \bigcup_{i=1}^{k} \left(\bigcup_{x \in V(b_i)} N_G[x] \right)$.
For each $b_i \in S_G$, the subgraph $P_G(G)[V(b_i)]$ is a complete graph in $P_G(G)$. As a clique, it is 2-connected (if $|V(b_i)| \geq 2$) and thus contained in some block b'_i of $P_G(G)$, implying $V(b_i) \subseteq V(b'_i)$.
Consider the set $S_{P_G(G)} = \{b'_1, \ldots, b'_k\}$, where each $b'_i \in B(P_G(G))$ contains $V(b_i)$. For any $v \in V$, there exists $b_j \in S_G$ such that $v \in \bigcup_{x \in V(b_j)} N_G[x]$ this implies if $v \in \bigcup_{x \in V(b_j)} N_G[x] \subseteq \bigcup_{x \in V(b_j)} N_{P_G(G)}[x]$ $\subseteq \bigcup_{y \in V(b'_j)} N_{P_G(G)}[y]$. Thus, every vertex $v \in V$ is bv-covered by some $b'_j \in S_{P_G(G)}$. Therefore, $S_{P_G(G)}$ is a bv-neighborhood set for $P_G(G)$ with cardinality k. Since $n_{bv}(P_G(G))$ is the minimum cardinality, $n_{bv}(P_G(G)) \leq |S_{P_G(G)}| = k = n_{bv}(G)$. □

Semitotal block-vertex graph $T_{bv}(G)$ is a graph with vertex set $V(G) \cup B(G)$ and any two vertices in $T_{bv}(G)$ are adjacent if and only if the corresponding vertices are vv-adjacent or the corresponding members are incident. As shown in Figure 6, $T_{bv}(G)$ is a semitotal block-vertex graph of G.

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Total block-vertex graph $T_{BV}(G)$ is a graph with vertex set $V(G) \cup B(G)$ and any two vertices in $T_{BV}(G)$ are adjacent if and only if the corresponding vertices are vv-adjacent or incident. As shown in Figure 6, $T_{BV}(G)$ is a total block-vertex graph of G.

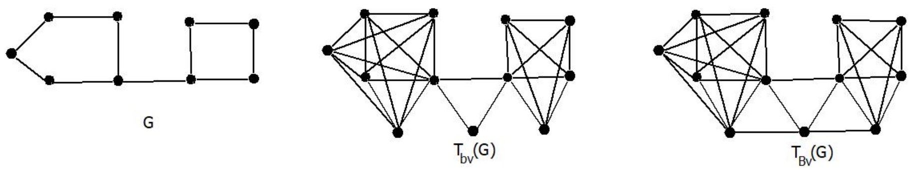

Figure 6: Semitotal and total block-vertex of G.

Theorem 2.10. *For any connected graph G, $T_{BV}(G)$ has no cut-vertices; hence $T_{BV}(G)$ is a single block and $n_{bv}(T_{BV}(G)) = 1$.*

Proof. The graph $T_{BV}(G)$ has a vertex set $V(G) \cup B(G)$, with edges as follows: vv-edges (edges from G), bv-edges (between blocks and their constituent vertices), bb-edges (between blocks that share a cut-vertex).

We will show that removing any vertex does not disconnect the graph.

Case 1: The removed vertex is $v \in V(G)$. If v is a cut-vertex of G, it is contained in multiple blocks, say $\{B_1, \ldots, B_k\}$. These blocks are connected to each other by bb-edges, ensuring that the block-side of the graph remains a single component. Since every vertex in $V(G)$ is connected to a block, all original vertices remain connected to this block component. If v is not a cut-vertex, it belongs to a single block, and its neighbors are still connected through this block.

Case 2: The removed vertex is $B \in B(G)$. If B is the only block, removing it leaves the original graph G, which is connected. If B is not the only block, it must contain a cut-vertex c. The vv-edges within $G[V(B)]$ ensure that all vertices of $V(B)$ remain connected to each other and to c. Since c is also in another block B', the path through B' maintains connectivity to the rest of the graph.

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In all cases, the graph remains connected after a single vertex is removed. Therefore, $T_{BV}(G)$ has no cut-vertices. A connected graph with no cut-vertices is a single block, and a single block trivially covers all vertices, so

$$n_{bv}(T_{BV}(G)) = 1.$$

□

Theorem 2.11. *Let G be a graph with m blocks. Then $n_{bv}(T_{bv}(G)) \leq n_{bv}(G)$.*

Proof. Let $k = n_{bv}(G)$, and $S_G = \{b_1, ..., b_k\}$ be a minimum bv-neighborhood set of G.
We construct a bv-neighborhood set for $T_{bv}(G)$. For each $b_i \in S_G$, consider the 2-connected component in $T_{bv}(G)$ that strongly corresponds to b_i and its vertices in G. Let us denote such a block as B'_i. Generally, $V(B'_i)$ would include $V(b_i)$ and the vertex $b_i \in B(G)$ itself, due to the vv-adjacencies and incidence edges($b_p(G)$) within $T_{bv}(G)$. Let us form the set $S_{T_{bv}} = \{B'_i | b_i \in S_G\}$. The cardinality of this set is $|S_{T_{bv}}| = |S_G| = k$.
Form the set $S_{T_{bv}} = \{B'_i \mid b_i \in S_G\}$. The cardinality of this set is $|S_{T_{bv}}| = |S_G| = k$. We now show $S_{T_{bv}}$ is a bv-neighborhood set for $T_{bv}(G)$:

- For $v \in V(G)$: Since S_G covers $V(G)$, for any $v \in V(G)$, there's a $b_i \in S_G$ such that v is covered by b_i in G. Due to the construction of $T_{bv}(G)$, v is connected to b_i and other vertices of $V(b_i)$ within B'_i. Thus, v is bv-covered by B'_i in $T_{bv}(G)$.

- For $b_j \in B(G)$:

  – If $b_j \in S_G$, then b_j is the block-vertex of $B'_j \in S_{T_{bv}}$, so it's covered by B'_j.

  – If $b_j \notin S_G$, it must still be covered. Every $b_j \in B(G)$ has at least one vertex $v \in V(b_j)$. This vertex v must be covered by some $b_i \in S_G$. Since v is adjacent to

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b_j in $T_{bv}(G)$, this adjacency ensures b_j is part of the closed neighborhood of v, which is covered by B_i'. More precisely, if $v \in V(b_j)$ and v is covered by B_i', and b_j is adjacent to v in $T_{bv}(G)$, then b_j will be included in $\langle N_{T_{bv}(G)}[B_i'] \rangle$.

Since $S_{T_{bv}}$ covers all vertices in $V(G) \cup B(G)$ and has size $k = n_{bv}(G)$, we conclude $n_{bv}(T_{bv}(G)) \leq k = n_{bv}(G)$. □

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