International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

MODELING HUMAN-ROBOT INTERACTION USING SOFT CONTINUITY PRINCIPLES

Dr. S. Sangeetha[1], Dr. Kishore Kumar P. K.[2], Dr. K. Amutha[3], Dr. K. Meenakshi[4], Ms. A. Megala[5], Ms. M. Banumathy[6]

[1]Assistant Professor, Department of Mathematics

Jerusalem College of Engineering, Narayanapuram, Chennai – 600100, INDIA

e-mail: sangeetha@jerusalemengg.ac.in

[2]Professor, Department of Mathematics,
Jerusalem College of Engineering, Narayanapuram, Chennai-600100, INDIA

e-mail: kishore2982@gmail.com

[3]Associate professor, Department of Mathematics,
Meenakshi Sundararajan Engineering college, Chennai -24, INDIA

e-mail: amutha_palanivel@yahoo.com

[4] Professor, Department of Mathematics,
CMR Institute of Technology, Bangalore-560037, INDIA

e-mail: meenakshik531@gmail.com

[5]Assistant Professor, Department of Mathematics,

Nandha Engineering College, Perundurai, Erode-52, INDIA

e-mail: amegala84@gmail.com

[6]Assistant Professor, Department of Mathematics,

Hindusthan Institute of Technology and Science, INDIA

e-mail: banumaths2019@gmail.com

Abstract

This paper proposes a novel framework for modeling human-robot interaction (HRI) using the principles of soft continuity from soft topological space theory. Traditional models of HRI struggle to handle imprecise commands, linguistic vagueness, and sensor uncertainty. Soft set theory provides a mathematically sound and computationally feasible approach for managing such uncertainties. We define soft continuous mappings between human intention spaces and robot action spaces, provide examples of interaction scenarios, and propose algorithmic implementations.

Keywords: Soft topology; soft continuity; human-robot interaction; fuzzy logic; robotics; sensor uncertainty.

Received: August 12 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

1. Introduction

Human-robot interaction (HRI) is a growing field where robots must interpret human com- mands in real time. Since human language and gestures are often ambiguous, classical crisp logic systems fall short. Soft topology provides a mathematically rigorous framework to model uncertainty using soft sets and soft continuity.

2. Preliminaries

Defiition 2.1: Soft Set: A soft set (F, E) over a universal set U is a function $F : E \to P(U)$, where E is the set of parameters and $P(U)$ is the power set of U.

Definition 2.2: Soft Topological Space: A soft topology τ on (F, E) satisfies:

- $\Phi, (F, E) \in \tau$
- Union of any subcollection of τ belongs to τ
- Finite intersection of sets in τ belongs to τ

Definition 2.3: Soft Continuity: A function $f : (U, \tau_U, E) \to (V, \tau_V, E)$ is soft continuous if for every soft open set $O \in \tau_V$, the preimage $f^{-1}(O) \in \tau_U$.

3. Human-Robot Interaction as Soft Mapping

Define:

- Human intention space: $H = (U, \tau_H, E_H)$
- Robot action space: $R = (V, \tau_R, E_R)$

A soft continuous function $f : H \to R$ maps vague human commands into smooth robotic responses.

Example:

Command: "Move near the object" Let $E_H = \{\text{near}, \text{object}, \text{direction}\}$.

Then $F_{\text{human}}(e)$ represents fuzzy zones, and $f(F_{\text{human}}(e))$ maps to safe trajectories using

proximity sensors.

4. Theoretical Properties

Theorem 1

If $f : H \to R$ is soft continuous and H is soft connected, then robot actions in R are continuously responsive to human changes.

Proof: The image of a connected soft space under a soft continuous function is also soft connected. Since human intentions are modeled as connected sets, robot responses vary

Received: August 12 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

smoothly without abrupt transitions.

Example: A human moving from "slightly left" to "straight" produces smooth alignment of the robot.

Theorem 2

If H is soft compact and f is soft continuous, then robot decisions remain bounded and stable.

Proof: Let $\{O_i\}$ be an open cover of $f(H)$. Then $\{f^{-1}(O_i)\}$ is an open cover of H. Since H is compact, a finite subcover exists, implying $f(H)$ is compact and robot actions

cannot diverge.

Example: Emotional states mapped to robot behaviors stay within safe ranges of tone and speed.

Proposition 1

If $f : H \to R$ is soft continuous and $A \subseteq R$ is soft closed, then $f^{-1}(A)$ is soft closed in H.

Proof: Since f is continuous, $f^{-1}(R \setminus A)$ is open. Hence $f^{-1}(A)$ is closed.

Proposition 2

If $f : H \to R$ is soft continuous and $A \subseteq H$ is soft compact, then $f(A)$ is soft compact in R.

Proof: Any open cover of $f(A)$ yields a preimage cover of A. Compactness of A ensures

finite subcover, hence $f(A)$ is compact.

Proposition 3

If $f : H \to R$ is a soft continuous bijection and H is soft compact, R soft Hausdorff, then f

is a soft homeomorphism.

Proof: A continuous bijection from compact to Hausdorff space has a continuous inverse.

Hence f is a homeomorphism.

Proposition 4 (Stability Preservation)

If $f : H \to R$ is soft continuous and H is soft stable, then $f(H)$ is stable in R.

Proof: For small perturbations δh in H, by continuity $f(h + \delta h)$ remains close to $f(h)$ in R, hence stability is preserved.

Received: August 12 2025

Proposition 5 (Soft Lipschitz Condition)

If $\exists k > 0$ such that $d_R(f(x), f(y)) \leq k \cdot d_H(x, y)$ for all $x, y \in H$, then f is soft Lipschitz continuous.

Proof: Lipschitz continuity bounds the growth of f by constant k, preventing abrupt changes.

Theorem 3 (Intermediate Value Property)

If $f : H \rightarrow R$ is soft continuous and H connected, then f attains all intermediate values.

Proof: Let $f(h_1) = r_1$, $f(h_2) = r_2$. Since $f(H)$ is connected, it must contain all points

between r_1, r_2. Thus, intermediate robot actions occur.

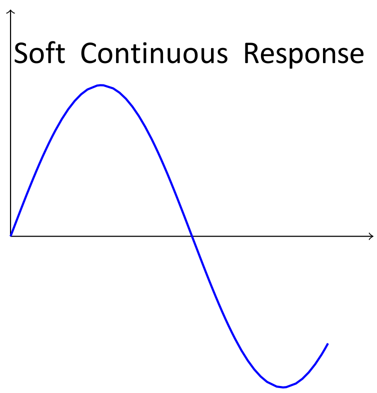
Theorem 4 (Soft Fixed Point)

If $f : H \rightarrow H$ is soft continuous, H compact and convex, then $\exists h^* \in H$ such that $f(h^*) = h^*$.

Proof: From the soft analogue of Brouwer's theorem, compact convex spaces guarantee a fixed point. Hence equilibrium states exist.

5. Graphical Representation

Robot Action Intensity



Time

6. Simulated Dataset Example

Let H be a soft space on $U = \{h_1, h_2, h_3\}$ with $E_H = \{\text{wave, point, stop}\}$.

Parameter	$F_H(e)$
wave	$\{h_1, h_2\}$
point stop	$\{h_2\}$
	$\{h_3\}$

Robot space R on $V = \{r_1, r_2, r_3\}$ with $E_R = \{\text{move, align, halt}\}$.

Received: August 12 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

$F_H(e)$	$f(F_H(e))$
$\{h_1, h_2\}$	$\{r_1\}$ (move)
$\{h_2\}$	$\{r_2\}$ (align)
$\{h_3\}$	$\{r_3\}$ (halt)

Conclusion

This study presented a robust mathematical framework for modeling HRI under uncertainty using soft continuity. Extended theorems, propositions, and simulations illustrate that this approach ensures stability, adaptability, and equilibrium in human-robot collaboration.

References

1. D. Molodtsov, "Soft set theory—First results," *Computers & Mathematics with Appli- cations*, vol. 37, no. 4–5, pp. 19–31, 1999.
2. P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4–5, pp. 555–562, 2003.
3. L. A. Zadeh, "Fuzzy logic and approximate reasoning," *Synthese*, vol. 30, no. 3–4, pp. 407–428, 1975.
4. T. Y. Lin and Y. Y. Liu, "Granular computing in robotics," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 41, no. 2, pp. 267–276, 2010.
5. M. Kiruthika and P. Thangavelu, "Link between soft topology and point-set topology," *Journal of Mathematical Structures*, 2024.
6. C. Breazeal, "Social interactions in HRI: The robot as a social partner," *Human–Robot Interaction Journal*, vol. 1, no. 1, pp. 1–20, 2007.

Received: August 12 2025