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Circular Intuitionistic Fuzzy-TOPSIS method by using the new similarity measure: new hospital location selection

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Abstract

Technique for Order Preference by Similarity (TOPSIS) have been developed. The TOPSIS method also been developed using intuitionistic fuzzy set. The existing method use the Eucledean distance, but in this article we use the new formula. This formula different from the existing method. It is comparable and more transparent with the measure proposed earlier. This formula is used to calculate distance for each alternative according to the positive-ideal solution

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and negative-ideal solution. Clearly this is different from existing CIF-TOPSIS methods. We explain the method step by step. Finally, we presented the illustrative example of this method to select a new hospital location.

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Key Words and Phrases: Fuzzy set; Intuitionistic fuzzy set; Circular intuitionistic fuzzy set; TOPSIS; Circular intuitionistic fuzzy-TOPSIS.

1 Introduction

One of the decision-making problems is related to the selecting the most appropriate option based on criteria determined from several available alternative options. One of the method have developed for decision making is "Multi Criteria Decision Making (MCDM)". Some MCDM techniques include: "Analytic Hierarchy Process (AHP)", "Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)", "Analytic Network Process (ANP)". One technique that is currently widely used is the TOPSIS method. However, the classic TOPSIS method requires exact numerical values, even though in the context or problems in our daily lives it is related to things that are not exact, uncertainty and vagueness.

The "fuzzy" concept provides an alternative solution to overcome the above problems. The concept of fuzzy sets was first introduced by Lotfi Zadeh [1] in 1 965. He introduced degrees of membership to a set which were not just 0 and 1, but were numbers in the interval [0,1]. The concept of fuzzy sets was then expanded with the development of the concept of intuitionistic fuzzy sets by Atanassov in 1986 [2]. In intuitionistic fuzzy sets, an object apart from having a degree of membership, also has a degree of non-membership. The sum of the degrees of membership and non-membership does not have to be 1. The difference b etween t he n umber 1 a nd the number of degrees of membership and non-membership is called "hesitancy" or the degree of doubt. Ones of many application of hesitancy concept was proposed by Asthiani in [3].

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After developing the concept of intuitionistic fuzzy sets, then in 2020 Atanassov developed the concept of Circular Intuitionistic Fuzzy (CIF) set [4]. The set is defined as a set whose universal set elements have a degree of membership and a degree of non-membership with a circle with radius r where the maximum number of degrees of membership and degree of non-membership in the circle is 1. This shows that the "fuzziness" of the membership function is more flexible. Therefore, CIFS becomes more effective for use in MCDM, especially in the TOPSIS method.

There are a lot of research on the application of fuzzy concepts regarding fuzzy TOPSIS. In previous studies, the intuitionistic fuzzy topsis method was introduced by Boran [5], Rouyendegh [6] and [7], Tlig and Rebai [8], and was used by Astuti et al [9] who used Intuitionistic Fuzzy Topsis with Euclidean distance to determine the dominant factors that influence the resilience of Covid-19 patients.

The Circular intuitionistic fuzzy-TOPSIS (CIF-TOPSIS) method was developed by Alkan [10] in 2022. In this article,we propose circular intuitionistic fuzzy-TOPSIS method by using the new distance measure on circular intuitionistic fuzzy. That measure use weighted distance.

Finally, after the modified method has been developed, a case study of the implementation of the method will be provided for decision making in the case of selecting a new hospital location.

2 Circular Intuitionistic Fuzzy (CIF): Basic Concepts

Before proceeding to describe method of CIF-TOPSIS, we introduce concepts of CIF. As a basis for the discussion, this section describes the definition fuzzy sets, Intuitionistic fuzzy set (IFS) then expands them into Circular Intuitionistic Fuzzy (CIF) set. The definition of fuzzy set was first introduced by Zadeh [1] in 1965, then expanded to an intuitionistic fuzzy set by Atanassov [2] in 1986. The following

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describes these two definitions.

Definition 1 . (see [1]). Let $X = \{x_1, x_2, x_3, ..., x_n\}$ be a crisp set. A fuzzy set (FS) of X is defined as a set of ordered pair $B = \{(x, \mu_B(x)) : x \in X\}$,where $0 \leq \mu_B(x) \leq 1$, for all $x \in X$. The function μ_B is called the membership function or grade of membership of x in B.

By the definition, we can see easily that every crisp set X can be represented as fuzzy set $A = \{(x, 1), x \in$X$\}$. The concept of fuzzy set was generalized by Atanasov in 198 [2] by introducing the concept of intuitonistic fuzzy set.

Definition 2 . (see [2]). Let X be a crisp set. Intuitionistic Fuzzy Set (IFS) $\mathcal{A}$ of X is defined as $\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$,where:
$0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1$, and $0 \leq \mu_A(x) + v_A(x) \leq 1$, for all $x \in X$.

The μ_A is called the function of membership and the the v_A is called the function of non-membership. The $\mu_A(x)$ is called the degree of membership of x to the set $\mathcal{A}$, while the $v_A(x)$ is called the degree of non-membership of x to the set $\mathcal{A}$. The $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy or hesitation part, which may cater to either membership value or non-membership value or both.

We can see that every fuzzy set $A = \{(x, \mu_A(x)) : x \in X\}$ can be viewed as intuitionistic fuzzy set $\mathcal{A} = \{(x, \mu_A(x), 1 - \mu_A(x)) : x \in X\}$.

Definition 3 . Let $\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ be an intuitionistic fuzzy set of X, then $A = (\mu_A, v_A)$ is called intuitionistic fuzzy number (IFN).

Definition 4 . Let $A = (\mu_A, v_A)$ and $B = (\mu_B, v_B)$ be two IFNs, then the multiplication operations on these two intuitionistic fuzzy numbers (IFNs) are defined as follows:

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$A \otimes B = (\mu_A \mu_B, v_A + v_B - \mu_A \mu_B)$.

Definition 5. Let $A = (\mu_A, v_A)$be an IFN, then the score function $S(A)$ and accuracy function $H(A)$ of A can be defined as in Eqs. (1) and (2), respectively.

$$S(A) = \mu_A - v_A \tag{1}$$

$$H(A) = \mu_A + v_A \tag{2}$$

Definition 6. Let $A_i = (\mu_{A_i}, v_{A_i})$ be a set of IFNs and $w = (w_1, w_2, ..., w_n)^T$ be weight vector of A_i with $\sum_{i=1}^{n} w_i = 1$, then an intuitionistic fuzzy weighted geometric ($IFWG$) operator is
$IFWG(A_1, A_2, ..., A_n) = (\prod_{i=1}^{n} \mu_{A_i}^{w} i, (1 - \prod_{i=1}^{n} (1 - v_{A_i})^{w_i}))$

Definition 7. Let C_i be an IFS and intuitionictic fuzzy pairs have the form $\{< m_{i1}, n_{i1} >, < m_{i2}, n_{i2} >, ...\}$, where i is the number of IFS C_i, each including k_i intuitionictic fuzzy pairs. The arithmetic average of the intuitionictic fuzzy pairs is calculated as in Eq. (3).

$$< \mu(C_i), v(C_i)) >=< \frac{\sum_{j=1}^{k_i} m_{ij}}{k_i}, \frac{\sum_{j=1}^{k_i} n_{ij}}{k_i} > \tag{3}$$

where k_i is the number of intuitionistic fuzzy pairs in C_i.

The radius of the $< \mu(C_i), v(C_i)) >$ is the maximum of the Euclidean distances given in Eq. (4).

$$r_i = \max_{1 \leq j \leq k_i} \sqrt{(\mu(C_i) - m_{ij})^2 + (v(C_i) - n_{ij})^2}. \tag{4}$$

The concept of intuitionistic fuzzy set was further developed by Atanasov [4] in 2020 by adding one component. The new component is called radius and the new concrpt is Circular Intuitionistic Fuzzy Set. Below is the definition of Circular Intuitionistic Fuzzy Set.

Definition 8. (see [4]). Let X be a crisp set. Circular Intuitionistic Fuzzy (CIF) set $\mathcal{A}$ of X is defined as $\mathcal{A}_\nabla = \{(x, \mu_A(x), v_A(x), r) :$

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$x \in X\}$, where:
$0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1$, $0 \leq \mu_A(x) + v_A(x) \leq 1$, for all $x \in X$, and $r \in [0,1]$ is a radius of the circle around each $x \in X$.

On IFS, each element can be represented by a point, here, each element of CIF represented by a circle in the intuitionistic fuzzy interpretation trianglwith with center ($\mu_A(x)$, $v_A(x)$) and radius r (see [4]). Therefore, CIF can be viewed as an extension of IFS, since every IFS can be expressed as CIF with $r = 0$. Since CIF can be viewed as an extension of IFS, we can still define the degree of of indeterminacy or hesitation part of CIF $\mathcal{A}$ as $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is just like in IFS.

The concept of CIF can be used in Multi Criteria Decision Making (MCDM). Many researchers have used the CIF concept in various applications. Some of them are Esra Cakir and M. Ali Tas [11] proposed application of CIF in MCDM, while C. Kahraman [12] who developed CIF-TOPSIS with vague membership function to select supplier.

3 Similarity Measure of CIF

Before explaining about the measure of CIP, we will first present the some measure of IFS that have been developed by researchers. The definition of many researchers is explained as follows.
Let $X = \{x_1, x_2, x_3, ..., x_n\}$. $\mathcal{A}, \mathcal{B} \in IFS(X)$ with
$\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ and $\mathcal{B} = \{(x, \mu_B(x), v_B(x)) : x \in X\}$.

1. The similarity measure formula of Chen [13] was constructed in 1995 as:

   . $\mathrm{S}_C(AB) = 1 - \frac{1}{2n} \sum_{j=1}^{n} |\mu_A(x_i) - v_A(x_i) - (\mu_B(x_i) - v_B(x_i))|$

2. Hong and Kim [14] in 1999 proposes the following to overcome the deficiency that Chen has proposed.

   . $\mathrm{S}_H(AB) = 1 - \frac{1}{2n} \sum_{j=1}^{n} |\mu_A(x_i) - \mu_B(x_i) - (v_A(x_i) - v_B(x_i))|$

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3. Wang and Xin [15] constructed the similarity measure:

$$S_W(AB) = 1 - \frac{1}{4n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |(v_A(x_i) - v_B(x_i))| \\ - \frac{1}{2n}\sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|\,, |v_A(x_i) - v_B(x_i)|\}.$$

Many other researchers have developed similarity measurement formulas, namely: Chuntian [16], Garg and Rani [17], Liang and Shi [18], Mitchell [19], Hung and Yang [20], Ye [21], and Boran and Akay [22]. But, most of them have counterintuitive results. Therefore, Yafi Song et.al. [23] proposed the new formula of similarity measure. They constructed the new formula of similarity measure of IFS as shown below:

$$\frac{1}{2n}\sum_{j=1}^{n} w_i \Big(\sqrt{\mu_A(x_i)\,\mu_B(x_i)} + 2\sqrt{v_A(x_i)\,v_B(x_i)} \\ + \sqrt{\pi_A(x_i)\,\pi_B(x_i)} + \sqrt{(1 - v_A(x_i))(1 - v_B(x_i))}$$

where $w_i \in [0, 1]$, $\sum_{j=1}^{n} w_i = 1$. The w_i denotes the weight factor of the features x_i. This formula of similarity measure is comparable and more transparent with the measure proposed earlier.

Based on these considerations, we propose the CIF-TOPSIS method using the semilarity measurement formula that Yafie Song has developed. This formula is used to calculate distance for each alternative according to the positif-ideal solution and negative-ideal solution on step 13. Clearly this is different from existing CIF-TOPSIS methods.

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4 CIF-TOPSIS method: Using the new similarity measure

In this section, the CIF-TOPSIS method is introduced. The steps of the proposed CIF-TOPSIS method by using the new formula similarity measure will be described below.

Step 1. Determine all of the alternatives, the relevant criteria, and the decision makers (DM). Let $A = \{ A_1, A_2, ..., A_m\}$ denote set of alternatives, $C = \{ C_1, C_2, ..., C_n\}$ denote set of criteria, and the $w = \{ w_1, w_2, ..., w_n\}$ be the vector set used for defining the weights of criteria, where $w_i \in [0, 1]$ and $\sum_{k=1}^{n} w_k = 1$.

Step 2. Constuct the matrices that the entries consisting of linguistic terms with respect to opinion of DM by using the scale given in Table 1.

Table 1: IF linguistic scale for rating of Alternatives

Linguistic terms	IFN's for alternatives)
	m ; n
Certainly High Value-(CHV)	0.9 ; 0.1
Very High Value-(VHV)	0.8 ; 0.15
High Value-(HV)	0.7 ; 0.25
Above Average Value-(AAV)	0.6 ; 0.35
Average Value-(AV)	0.5 ; 0.45
Under Average Value-(UAV)	0.4 ; 0.55
Low Value (LV)	0.3 ; 0.65
Very Low Value-(VLV)	0.2 ; 0.75
Certainly Low Value-(CLV)	0.1 ; 0.9

Step 3. Convert the linguistic data to their corresponding IFNs using Table 1. Through the scale, the decision matrix consisting of the intuitionistic fuzzy pairs (D_k) with respect to DM k is given in Table 2. Here $\bar{D}_k = (\bar{d}_{ijk})_{n\times m}$ in which $\bar{d}_{ijk} = (m_{ijk}, n_{ijk})$ is

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constructed by utilizing the IFNs given in Table 1. Accordingly, $\bar{d}_{ijk}$ indicates the performance of alternative A_i in terms of criterion C_j of k^{th} DM.

Table 2: Decision matrix based on intuitionistic fuzzy numbers with respect to DM$_k$

Criteria	Alternatives
	A_1 ; A_2 ; ... ; A_m
C_1	(m_{11k}, n_{11k}) ; (m_{12k}, n_{12k}); ... ; (m_{1mk}, n_{1mk})
C_2	(m_{21k}, n_{21k}) ; (m_{22k}, n_{22k}); ... ; (m_{2mk}, n_{2mk})
...	. . .
C_n	(m_{n1k}, n_{n1k}) ; (m_{n2k}, n_{n2k}); ... ; (m_{nmk}, n_{nmk})

Step 4. Determine the matrix of the aggregated intuitionistic fuzzy decision using the decision matrices consisting of intuitionistic fuzzy pairs. The intuitionistic fuzzy pairs in individual decision matrices are converted to the aggregated intuitionistic fuzzy numbers using Eq. (3).

Step 5. Based on the decision matrices of all DMs using Eq. (4), Calculate the maximum radius lengths $R^D = (r^d_{ij})_{n\times m}$. The maximum radius lengths are found by considering the radius lengths obtained by each DM evaluating the i-th alternative according to the j-th criterion. Then, the circular intuitionistic fuzzy decision matrix $(\tilde{D})$ is constructed as in Table 3. The matrix $\tilde{D} = (\tilde{d}_{ij})_{n\times m}$ in which $\tilde{d}_{ij} = (\mu_{ij}, v_{ij}, r_{ij})$ is used to indicate the circular intuitionistic fuzzy number of the i-th alternative with respect to the j-th criterion.

Step 6. Determine the the optimistic decision matrix $(Q^{O_d} = (q^{O_d}_{ij})_{m\times n})$ and pessimistic decision matrix $(Q^{P_d} = (q^{P_d}_{ij})_{m\times n})$ by using Eq. (5) and Eq. (6) respectively. The intuitionistic fuzzy decision matrix is performed as conversion from the circular intuitionistic fuzzy decision matrix by utilizing the membership functions and radius given in Table 3. Here, according to the attitude of the decision-maker to be optimistic or pessimistic, two different de-

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Table 3: Circular intuitionistic fuzzy decision matrix

Criteria	Alternatives
	A_1 ; A_2 ; ... ; A_m
C_1	$(\mu_{11}, v_{11}, r_{11})$; $(\mu_{12}, v_{12}, r_{12})$; ... ; $(\mu_{1m}, v_{1m}, r_{1m})$
C_2	$(\mu_{21}, v_{21}, r_{21})$; $(\mu_{22}, v_{22}, r_{22})$; ... ; $(\mu_{2m}, v_{2m}, r_{2m})$
...	. . .
C_n	$(\mu_{n1}, v_{n1}, r_{n1})$; $(\mu_{n2}, v_{n2}, r_{n2})$; ... ; $(\mu_{nm}, v_{nm}, r_{nm})$

cision matrices are obtained, namely the optimistic decision matrix and the pessimistic decision matrix, respectively.

$$Q^{O_d} = \begin{bmatrix} (\mu_{11}+r_{11}, v_{11}-r_{11}) & (\mu_{12}+r_{12}, v_{12}-r_{12}) & ... & (\mu_{1m}+r_{1m}, v_{1m}-r_{1m}) \\ (\mu_{21}+r_{21}, v_{21}-r_{21}) & (\mu_{22}+r_{22}, v_{22}-r_{22}) & ... & (\mu_{2m}+r_{2m}, v_{2m}-r_{2m}) \\ .. & .. & .. & .. \\ (\mu_{n1}+r_{n1}, v_{n1}-r_{n1}) & (\mu_{n2}+r_{n2}, v_{n2}-r_{n2}) & ... & (\mu_{nm}+r_{nm}, v_{nm}-r_{nm}) \end{bmatrix}, \tag{5}$$

$$Q^{P_d} = \begin{bmatrix} (\mu_{11}-r_{11}, v_{11}+r_{11}) & (\mu_{12}-r_{12}, v_{12}+r_{12}) & ... & (\mu_{1m}-r_{1m}, v_{1m}+r_{1m}) \\ (\mu_{21}-r_{21}, v_{21}+r_{21}) & (\mu_{22}-r_{22}, v_{22}+r_{22}) & ... & (\mu_{2m}-r_{2m}, v_{2m}+r_{2m}) \\ .. & .. & .. & .. \\ (\mu_{n1}-r_{n1}, v_{n1}+r_{n1}) & (\mu_{n2}-r_{n2}, v_{n2}+r_{n2}) & ... & (\mu_{nm}-r_{nm}, v_{nm}+r_{nm}) \end{bmatrix}. \tag{6}$$

Step 7. Determine the weights consisting of intuitionistic fuzzy pairs of criteria for each DM (W_k) using the scale given in Table 4. Here, $W_k = (w_{jk})_{1\times n}$ indicates intuitionistic fuzzy pairs of the k-th DM with respect to the j-th criterion.

Step 8. Obtain the aggregated intuitionistic fuzzy criteria weight matrix using the individual criteria weight matrices consisting of intuitionistic fuzzy pairs. The intuitionistic fuzzy pairs in the individual criteria weight matrices are converted to the aggregated intuitionistic fuzzy numbers of criteria weights using the IFWG operator given in Definition (6).

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Table 4: Linguistic scale for weighting the criteria

Linguistic terms	IFN's for criteria
	m ; n
Certainly High Importance -(CHI)	0.9 ; 0.1
Very High Importance-(VHI)	0.8 ; 0.15
High Importance-(HI)	0.7 ; 0.25
Above Average Importance-(AAI)	0.6 ; 0.35
Average Importance-(AI)	0.5 ; 0.45
Under Average Importance-(UAI)	0.4 ; 0.55
Low Importance (LV)	0.3 ; 0.65
Very Low Importance-(VLV)	0.2 ; 0.75
Certainly Low Importance-(CLV)	0.1 ; 0.9

Step 9. Calculate the maximum radius lengths $R^W = (r_j^w)_{1\times n}$ based on the criterion weight matrices of all DMs using Eq. (4). The maximum radius lengths are found by considering the radius lengths obtained by each DM with respect to the j-th criterion. Then, the circular intuitionistic fuzzy criteria weight matrix (W) is constructed as in Table 5.

Table 5: Circular intuitionistic fuzzy criterion weight matrix

Criteria	Circular intuitionistic fuzzy criteria weights
C_1	(μ_1, v_1, r_1)
C_2	(μ_2, v_2, r_2)
...	. . .
C_n	(μ_n, v_n, r_n)

Step 10. Determine the optimistic criterion weight matrix $(Q^{O_w} = (q_j^{O_w})_{n\times 1})$ by using Eq. (5) and the pessimistic criterion weight matrix $(Q^{P_w} = (q_j^{P_w})_{n\times 1})$ using Eq. (6).

The conversion of the circular intuitionistic fuzzy criterion weight matrix into the intuitionistic fuzzy criterion weight matrix is per-

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formed by utilizing the membership functions and radius given in Table 5. Here, according to the attitude of the decision maker to be optimistic and pessimistic, two different criteria weight matrices are obtained, namely the optimistic criterion weight matrix and the pessimistic criterion weight matrix, respectively.

Step 11. Obtain the weighted optimistic decision matrix ($\psi^O = (\psi_{ij}^O)$) and the weighted pessimistic decision matrix ($\psi^P = (\psi_{ij}^P)$) by using Eqs. (7) and (8), respectively.

$$\psi_{ij}^O = q_j^{O_w} \otimes q_{ij}^{O_d}, \tag{7}$$

$$\bar{\psi}_{ij}^P = q_j^{P_w} \otimes q_{ij}^{P_d}. \tag{8}$$

Step 12. Determine the positive ideal solution $\mathcal{X}_{\mathcal{O}}^*$ and negative ideal solution $\mathcal{X}_{\mathcal{O}}^-$ based on the optimistic matrix and positive ideal solution $\mathcal{X}_{\mathcal{P}}^*$ and negative ideal solution $\mathcal{X}_{\mathcal{P}}^-$ based on the pessimistic matrix by using score function and accuracy function given in Eqs. (1) and (2), respectively. The positive ideal solution and negative ideal solution based on the optimistic matrix are as in Eqs. (9) and (10), respectively.

$$\mathcal{X}^{\mathcal{O}^*} = \{\psi_1^{O+}, \psi_2^{O+}, ..., \psi_n^{O+}\}^T, \tag{9}$$

$$\mathcal{X}^{\mathcal{O}^-} = \{\psi_1^{O-}, \psi_2^{O-}, ..., \psi_n^{O-}\}^T. \tag{10}$$

The positive ideal solution and negative ideal solution based on pessimistic matrix are as in Eqs. (11) and (12), respectively.

$$\mathcal{X}^{\mathcal{P}^*} = \{\psi_1^{P+}, \psi_2^{P+}, ..., \psi_n^{P+}\}^T, \tag{11}$$

$$\mathcal{X}^{\mathcal{P}^-} = \{\psi_1^{P-}, \psi_2^{P-}, ..., \psi_n^{P-}\}^T. \tag{12}$$

where $\psi_j^{O+} = (\mu_j^{O+}, v_j^{O+})$,$\psi_j^{P+} = (\mu_j^{P+}, v_j^{P+})$ are the maximum IFN with the highest score value among alternatives for the j-th criterion and $\psi_j^{O-} = (\mu_j^{O-}, v_j^{O-})$,$\psi_j^{P-} = (\mu_j^{P-}, v_j^{P-})$ are the minimum IFN with the lowest score value among alternatives for the j-th criterion.

Step 13. Obtain the separation measures by calculating the distances for each alternative according to the positive-ideal solutions ($\mathcal{X}^{O^*}$), ($\mathcal{X}^{P^*}$) and negative-ideal solutions ($\mathcal{X}^{O^-}$), ($\mathcal{X}^{P^-}$). The

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distances to the positive ideal solution and negative ideal solution based on the optimistic matrix are given by Eqs. (13) and (14), respectively.

$$D_i^{O^*} = \frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\mu_{ij}(x_j)\,\mu_j^{O+}(x_j)} + 2\sqrt{v_{ij}(x_j)\,\mu_j^{O+}(x_j)}\right) +$$

$$\frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\pi_{ij}(x_j)\,\pi_j^{O+}(x_j)} + \sqrt{(1-v_{ij}(x_j))(1-v_j^{O+}(x_j))}\right). \tag{13}$$

$$D_i^{O^-} = \frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\mu_{ij}(x_j)\,\mu_j^{O-}(x_j)} + 2\sqrt{v_{ij}(x_j)\,\mu_j^{O-}(x_j)}\right) +$$

$$\frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\pi_{ij}(x_j)\,\pi_j^{O-}(x_j)} + \sqrt{(1-v_{ij}(x_j))(1-v_j^{O-}(x_j))}\right). \tag{14}$$

The distances to positive ideal solution and negative ideal solution based on the pessimistic matrix are given by Eqs. (15) and (16), respectively.

$$D_i^{P^*} = \frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\mu_{ij}(x_j)\,\mu_j^{P+}(x_j)} + 2\sqrt{v_{ij}(x_j)\,\mu_j^{P+}(x_j)}\right) +$$

$$\frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\pi_{ij}(x_j)\,\pi_j^{P+}(x_j)} + \sqrt{(1-v_{ij}(x_j))(1-v_j^{P+}(x_j))}\right). \tag{15}$$

$$D_i^{P^-} = \frac{1}{2n}\sum_{j=1}^{n} w_i\left(\sqrt{\mu_{ij}(x_j)\,\mu_j^{P-}(x_j)} + 2\sqrt{v_{ij}(x_j)\,\mu_j^{P-}(x_j)}\right) +$$

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$$\frac{1}{2n}\sum_{j=1}^{n} w_i \left(\sqrt{\pi_{ij}(x_j)\, \pi_j^{P-}(x_j)} + \sqrt{(1 - v_{ij}(x_j))(1 - v_j^{P-}(x_j))} \right). \tag{16}$$

This formula diffrent from the existing methode. The existing methode use the eucledean distance, but in this article we use the new formula. Ths formula is comparable and more transparent with the measure proposed earlier.

Step 14. Calculate the relative closeness coefficient based on the optimistic matrix (CC_i^O) and the relative closeness coefficient based on the pessimistic matrix (CC_i^P) of alternatives using Eqs. (17) and (18) , respectively.

$$CC_i^O = \frac{D_i^{O-}}{D_i^{O-} + D_i^{O*}}. \tag{17}$$

$$CC_i^P = \frac{D_i^{P-}}{D_i^{P-} + D_i^{P*}}. \tag{18}$$

Step 15. Obtain the composite ratio CR score to determine the rankings between alternatives considering the scores obtained with both the optimistic approach and the pessimistic approach as given in Eq. (19).

$$CC_i^{CR} = \lambda \times CC_i^O (1 - \lambda) \times CC_i^P), \tag{19}$$

where λ is the weight of DM's optimistic attitude and $(1 - \lambda)$ is the weight of DM's pessimistic attitude.

Step 16. Rank the alternatives according to final scores. The best alternative(s) are selected based on the descending order of the values of the relative closeness coefficient CC_i^{CR}.

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5 Application: simulation for selection the new hospital location

To provide an overview of the application of this method, below we present a simulation in determining the location of a new hospital. Assume that there are four candidate locations for a new hospital available. We will determine the most suitable location based on several predetermined criteria. Let, the four candidate locations are locations-1 (A_1), locations-2 (A_2), locations-3 (A_3), and locations-4 (A_4).

To evaluate the new location of hospital, a variety of criteria have been determined based on a comprehensive literature review and experts' opinions. There are six criteria used, namely Cost (C_1), Demographics (C_2), Environmental (C_3), Transportation (C_4), Healthcare and Medical Practices (C_5) and Infrastructure (C_6). The description for each criterion is as Table 6 follows.

In the following have been presented the proposed methode to determine the solution of this problem.

Step 1. The proposed methode is applied to the best new location hospital selection amoung four alternatives. According four criteria determind based on literature review and experts opinion , these alternatives are evaluated. A team consist of the five experts has been formed to evaluate the locations using the proposed methode. Five decision-makers (DM), consisting of academics who are experts on multi-criteria decision making in a fuzzy environment have been selected, and are abbreviated as DM_1, DM_2, DM_3, DM_4 , and DM_5.

Step 2. The five experts evaluates the candidate of locations in line with the defined objectives and criteria based on the intuitionistic fuzzy linguistic scale given in Table 1. The linguistic decision matrix created based on the assessments of experts is presented in Table 7.

Step 3 and 4. Linguistic variables are converted to their corresponding IFNs by utilizing the scale given in Table 1. Then, the

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Table 6: Description of criteria

Criteria	The criteria description
Cost	These are the costs that may occur during the establishment of the hospital such as land acquisition, construction and labor costs.
Demographics	It is the determination of the demand for medical services depending on the population number, the population density and the population age profile in the hospital region
Enviromental	It is the determination of the location of the hospital, taking into account the traffic density which causes difficult access to the region and the possibility of being affected by noise and pollution sources.
Transportation	Providing medical equipment support from other hospitals in emergencies Opportunities and in case of emergency patients with infection in other hospital, the existence and number of transportation mean that will facilitate access to the hospital and the existence and suitability of the roads providing access to the hospital should be taken into consideration in the selection of the hospital location.
Healthcare and Medical Practices	The number of family health centers and hospitals in the region, the number Medical of physicians on duty and the pharmaceutical industry.
Infrastructure	It indicates the availability ofrequirements for electricity, water and waste management, the parking capacity in and around the possible hospital project and the potential for future expansion.

individual decision matrices consisting of intuitionistic fuzzy pairs are combined to obtain the aggregated intuitionistic fuzzy decision matrix as in Table 7 using Equation (3).

Step 5. Calculate the maximum radius among all experts is obtained as seen in Table 8 using Equation (4). Then, the circular

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intuitionistic fuzzy decision matrix is constructed as in Table 9 by utilizing the radius lengths obtained.

Step 6. By utilizing the circular intuitionistic fuzzy decision matrix based on the membership functions and radius lengths obtained in Step 5 using Eq. (5) and Eq. (6), the optimistic and pessimistic decision matrices are created as in Tables 10 and 11.

Step 7 and 8. The linguistic evaluations of the criteria assigned by DMs using the scale given in Table 5 can be shown in Table 12.

Step 9 and 10. Construct the circular intuitionistic fuzzy criteria weight matrix by utilizing the maximum radius lengths as in Table 13.

Step 11. By utilizing the circular intuitionistic fuzzy criterion weight matrix based on the membership functions and radius lengths obtained in Step 9, the optimistic and pessimistic criteria weight matrices are constructed as in Table 14 using Equations (7) and Equation (8), respectively.

Step 12. The weighted optimistic and pessimistic decision matrices are created based on the decision matrices obtained in Step 6 and the weight vectors obtained in Step 11 by utilizing Equations (7) and (8), respectively. The weighted optimistic and pessimistic decision matrices are as in Tables 15 and 16.

After score values for each IFN in the weighted optimistic and pessimistic decision matrices are calculated using Equation (7) and (8), while the positive and negative ideal solutions based on the optimistic decision matrix are obtained by using Equations (9) and (10), the positive and negative ideal solutions based on the pessimistic decision matrix are found by using Equations (11) and (12). The obtained positive and negative ideal solutions are as in Table 17.

Step 13. The separation measure are obtained by calculating the distance for each alternatives according to the positive and negative ideal solutions on optimistic and pessimistic matrices. While the separation measures based on the optimistic matrix are presented as in Table 18 using Equations (13) and (14), the separation measures based on the pessimistic matrix are created as in Table

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19 using Equations (15) and (16).

Step 14. The closeness coefficient of each alternative is calculated for optimistic and pessimistic matrices. While the closeness coefficients of each alternative and ranks of the alternatives based on the optimistic are presented as given in Table 20 using Equation (17), the closeness coefficient of each alternative and ranks of the alternatives based on the pessimistic are shown as given in Table 21 using Equation (18).

Step 15. The composite ratio (CR) score of CC_i^O and CC_i^P is calculated to determine the rankings between alternatives by considering the scores obtained with both the optimistic approach and the pessimistic approach using Equation (19) for $\lambda = 0.5$ as given in Table 22. Here, the weights of DM's optimistic and pessimistic attitudes are considered equal.

Step 16. The combined ratio scores indicate that the ranking order of the alternatives are A_2, A_3, A_1, A_4.

6 Conclusion

The TOPSIS method been developed using intuitionistic fuzzy set, but the existing method use the Eucledean distance. In this article we proposed the new formula for similarity measure of intuitionistic fuzzy set. This formula different from the existing method. It is comparable and more transparent with the measure proposed earlier. This formula is used to calculate distance for each alternative according to the positive-ideal solution and negative-ideal solution. The proposed method can be use to select a new hospital location with six criteria, namely: Cost, Demographics, Environmental, Transportation, Healthcare and Medical Practices, and Infrastructure.

For further study, different methods can be developed by developed another formula of similarity measure. Different MCDM methods based on Circular Intuitionistic Fuzzy Set can be developed to compare to our proposed approach.

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Table 7: Linguistic decision matrix for each expert

Criteria	DMs	A_1	A_2	A_3	A_4
C_1	DM_1	HV	LV	AV	VLV
	DM_2	HV	UAV	AV	LV
	DM_3	AAV	LV	AAV	LV
	DM_4	HV	LV	AV	LV
	DM_5	AAV	LV	AAV	VLV
C_2	DM_1	AAV	VHV	VHV	VHV
	DM_2	AAV	HV	HV	VHV
	DM_3	AV	VHV	HV	CHV
	DM_4	AV	HV	HV	CHV
	DM_5	HV	HV	HV	CHV
C_3	DM_1	LV	AV	UAV	UAV
	DM_2	LV	AV	UAV	UAV
	DM_3	UAV	AAV	AV	LV
	DM_4	LV	AV	UAV	LV
	DM_5	UAV	AV	AV	UAV
C_4	DM_1	HV	AV	AAV	UAV
	DM_2	VHV	AV	HV	UAV
	DM_3	CHV	AV	AAV	AV
	DM_4	HV	AV	HV	AV
	DM_5	CHV	AAV	HV	UAV
C_5	DM_1	AAV	AV	AAV	AV
	DM_2	AAV	AV	AAV	AV
	DM_3	HV	UAV	HV	AAV
	DM_4	HV	AAV	HV	AV
	DM_5	HV	AV	AAV	AAV
C_6	DM_1	UAV	AV	AAV	CLV
	DM_2	UAV	AAV	AAV	CLV
	DM_3	LV	HV	HV	VLV
	DM_4	LV	HV	HV	CLV
	DM_5	UAV	AV	AV	VLV

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Table 8: Agregared Intuitionistic Fuzzy decision Matrix

	A_1	A_2	A_3	A_4
C_1	(0.66,0.29)	(0.32,0.63)	(0.54,0.41)	(0.26,0.69)
C_2	(0.58,0.37)	(0.74,0.21)	(0.72,0.23)	(0.86,0.12)
C_3	(0.34,0.61)	(0.52,0.43)	(0.44,0.51)	(0.36,0.59)
C_4	(0.8,0.17)	(0.52,0.43)	(0.66,0.29)	(0.44,0.51)
C_5	(0.66,0.29)	(0.5,0.45)	(0.64,0.31)	(0.54,0.41)
C_6	(0.36,0.59)	(0.6,0.35)	(0.62,0.33)	(0.14,0.84)

Table 9: Maximum radius lengths based on desicion matrices

	A_1	A_2	A_3	A_4
C_1	0.085	0.113	0.085	0.085
C_2	0.170	0.085	0.113	0.067
C_3	0.085	0.113	0.085	0.085
C_4	0.128	0.113	0.085	0.085
C_5	0.085	0.141	0.085	0.085
C_6	0.085	0.141	0.170	0.108

Table 10: Optimistic desicion matrix

	A_1	A_2	A_3	A_4
C_1	(0.745,0.205)	(0.433,0.517)	(0.625,0.325)	(0.345,0.605)
C_2	(0.75,0.2)	(0.825,0.125)	(0.833,0.117)	(0.927,0.053)
C_3	(0.425,0.525)	(0.633,0.317)	(0.525,0.425)	(0.445,0.505)
C_4	(0.928,0.042)	(0.633,0.317)	(0.745,0.205)	(0.525,0.425)
C_5	(0.745,0.205)	(0.641,0.309)	(0.725,0.225)	(0.625,0.325)
C_6	(0.445,0.505)	(0.741,0.209)	(0.79,0.16)	(0.248,0.732)

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Table 11: Pessimectic desicion matrix

	A_1	A_2	A_3	A_4
C_1	(0.575,0.375)	(0.207,0.743)	(0.455,0.495)	(0.175,0.775)
C_2	(0.41,0.54)	(0.655,0.295)	(0.607,0.343)	(0.793,0.187)
C_3	(0.255,0.695)	(0.407,0.543)	(0.355,0.595)	(0.275,0.675)
C_4	(0.672,0.298)	(0.407,0.543)	(0.575,0.375)	(0.355,0.595)
C_5	(0.575,0.375)	(0.359,0.591)	(0.555,0.395)	(0.455,0.495)
C_6	(0.275,0.675)	(0.459,0.491)	(0.45,0.5)	(0.032,0.948)

Table 12: Linguistic evaluations of criteria for each DM

	C_1	C_2	C_3	C_4	C_5	C_6
DM_1	AI	VHI	AAI	HI	LI	LI
DM_2	AI	VHI	HI	HI	UAI	UAI
DM_3	AAI	HI	HI	VHI	UAI	UAI
DM_4	AI	HI	AAI	VHI	LI	LI
DM_5	AAI	VHI	AI	HI	UAI	LI

Table 13: Maximum radius lengths based on decision matrices and circular intuitionistic fuzzy criteria weight matrix

Criterion	Maximum Radius	Criteria weight martix
C_1	0.085	(0.54,0.41)
C_2	0.085	(0.76,0.19)
C_3	0.170	(0.62,0.33)
C_4	0.085	(0.74,0.21)
C_5	0.085	(0.36,0.59)
C_6	0.085	(0.34,0.61)

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Table 14: Optimistic and pessimistic criteria weight matrices

Criterion	Optimistic criteria weights	Pessimestic criteria weights
C_1	(0.625,0.325)	(0.455,0.495)
C_2	(0.845,0.105)	(0.675,0.275)
C_3	(0.79,0.16)	(0.45,0.5)
C_4	(0.825,0.125)	(0.655,0.295)
C_5	(0.445,0.505)	(0.275,0.675)
C_6	(0.425,0.525)	(0.255,0.695)

Table 15: Weighted optimistic decision Matrix

	A_1	A_2	A_3	A_4
C_1	(0.466,0.464)	(0.271,0.674)	(0.39,0.545)	(0.215,0.734))
C_2	(0.633,0.284)	(0.697,0.217)	(0.704,0.21)	(0.783,0.153)
C_3	(0.336,0.601)	(0.5,0.426)	(0.414,0.517)	(0.351,0.584)
C_4	(0.766,0.162)	(0.522,0.402)	(0.614,0.305)	(0.433,0.497)
C_5	(0.331,0.607)	(0.285,0.658)	(0.322,0.617)	(0.278,0.666)
C_6	(0.189,0.765)	(0.315,0.624)	(0.336,0.601)	(0.105,0.873)

Table 16: Weighted pessimistic decision Matrix

	A_1	A_2	A_3	A_4
C_1	(0.262,0.684)	(0.094,0.87)	(0.207,0.745)	(0.08,0.886)
C_2	(0.277,0.666)	(0.442,0.489)	(0.41,0.524)	(0.535,0.411)
C_3	(0.115,0.847)	(0.183,0.771)	(0.16,0.797)	(0.124,0.837)
C_4	(0.44,0.505)	(0.267,0.678)	(0.377,0.559)	(0.233,0.714)
C_5	(0.158,0.797)	(0.099,0.867)	(0.153,0.803)	(0.125,0.836)
C_6	(0.07,0.901)	(0.117,0.845)	(0.115,0.847)	(0.008,0.984)

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Table 17: Positive and negative ideal solutions based on the optimistic decision matrix

Optimistic martix			Pessimistic matrix	
	Positive Ideal Solution	Negative Ideal solution	Positive Ideal solution	Negative Ideal solution
C_1	(0.215,0.734)	(0.466,0.464)	(0.08,0.886)	(0.262,0.684)
C_2	(0.783,0.153)	(0.633,0.284)	(0.535,0.411)	(0.277,0.666)
C_3	(0.5,0.426)	(0.336,0.601)	(0.183,0.771)	(0.115,0.847)
C_4	(0.766,0.162)	(0.433,0.497)	(0.44,0.505)	(0.233,0.714)
C_5	(0.331,0.607)	(0.278,0.666)	(0.158,0.797)	(0.099,0.867)
C_6	(0.336,0.601)	(0.105,0.873)	(0.117,0.845)	(0.008,0.984)

Table 18: Separation measures of the alternatives based on the optimistic matrix

	A$_1$	A$_2$	A$_3$	A$_4$
D_i^{O*}	0.014	0.008	0.007	0.022
D_i^{O-}	0.013	0.015	0.014	0.009

Table 19: Separation measures of the alternatives based on the pessimestic matrix

	A_1	A_2	A_3	A_4
D_i^{P*}	0.014	0.008	0.007	0.022
D_i^{P-}	0.013	0.015	0.014	0.009

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Table 20: Closeness coefficient and ranks of the alternatives based on the optimistic desision matrix

	A_1	A_2	A_3	A_4
CC_i^O	0.486	0.661	0.658	0.286
Rank	3	1	2	4

Table 21: Closeness coefficient and ranks of the alternatives based on the pessimistic desision matrix

	A_1	A_2	A_3	A_4
CC_i^O	0.387	0.780	0.721	0.501
Rank	4	1	2	3

Table 22: Composite ratio scores and ranks of the alternatives

	A_1	A_2	A_3	A_4
CC_i^{CR}	0.436	0.721	0.689	0.394
Rank	3	1	2	4

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