International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

GRAPHICAL MODELLING FOR FUZZY ANALYTICAL HIERARCHY PROCESS BASED COMPLEXITY OF INVESTMENT CASTING

Divya Mobarsa[1], Dr. Minal Shukla[2*], Dr. Nikunj Maheta[3], Dr. Amit Sata[4]

[1]Research Scholar, Department of Mathematics, Marwadi University, Rajkot

[2]Associate Professor, Department of ASH (Mathematics), PIET, Parul University, Vadodara, *minal.vadhel42536@paruluniversity.ac.in*

[3]Assistant Professor, Department of Mechanical Engineering, Marwadi University, Rajkot

[4]Professor, Department of Mechanical Engineering, Marwadi University, Rajkot

Abstract

The decision to produce any industrial component through the investment casting process primarily depends on the manufacturing complexity of that specific component. Three factors primarily influence the complexity of the industrial investment casting: geometry, desired features, and manufacturability. These three factors are additionally influenced by 19 elements, 52 attributes, and 212 meta-attributes. The complexity value based on the relative weightage of these variables is computed using one of the widely accepted multi-criteria decision-making methods, such as the fuzzy analytical hierarchy process. The cumulative complexity value assists in identifying the impact of any specific variable as well as the dependency of the variable on the others. However, it can be challenging to represent the complexity value in a way that allows for straightforward interpretation. challenging.

This work proposes a method for describing the complexity index in graph-based systems by utilizing an adaptive consensus mechanism derived from graph theory and graph coloring. The suggested method ensures efficient node representations and is sufficiently resilient to ascertain the maximum and least complexity values of specific parameters on the graph. Proper coloring minimizes collection conflict and enhances node distinctiveness, facilitating the easy identification of essential network components. The adaptive consensus mechanism technique efficiently adjusts to a fluctuating graph topology to enhance scalability and performance.

Keywords: Graph Theory, Graph Adaptive Consensus Mechanism, Investment Casting, Complexity Index, Fuzzy Analytical Hierarchy Process

Introduction

Investment casting is a widely acknowledged manufacturing process used for producing complex industrial castings with precise dimensional and accurate surface quality for various industrial sectors related to aerospace, automobiles, biomedicine, chemicals, defense, etc. [1]. The process includes various steps linked to the development of die and wax patterns, coating of ceramic material to form a shell, dewaxing of the pattern, shell baking of ceramic coating,

Received: August 04 2025

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

and pouring of molten metal. Additionally, the castings undergo finishing and testing to ensure required quality standards. Figure 1 illustrates the progressive steps of the investment casting process. Various variables, such as geometry, desired features, and manufacturability, significantly influence these sub steps, contributing to the complexity of investment casting [2]. A specific range of values dominates these variables [3].

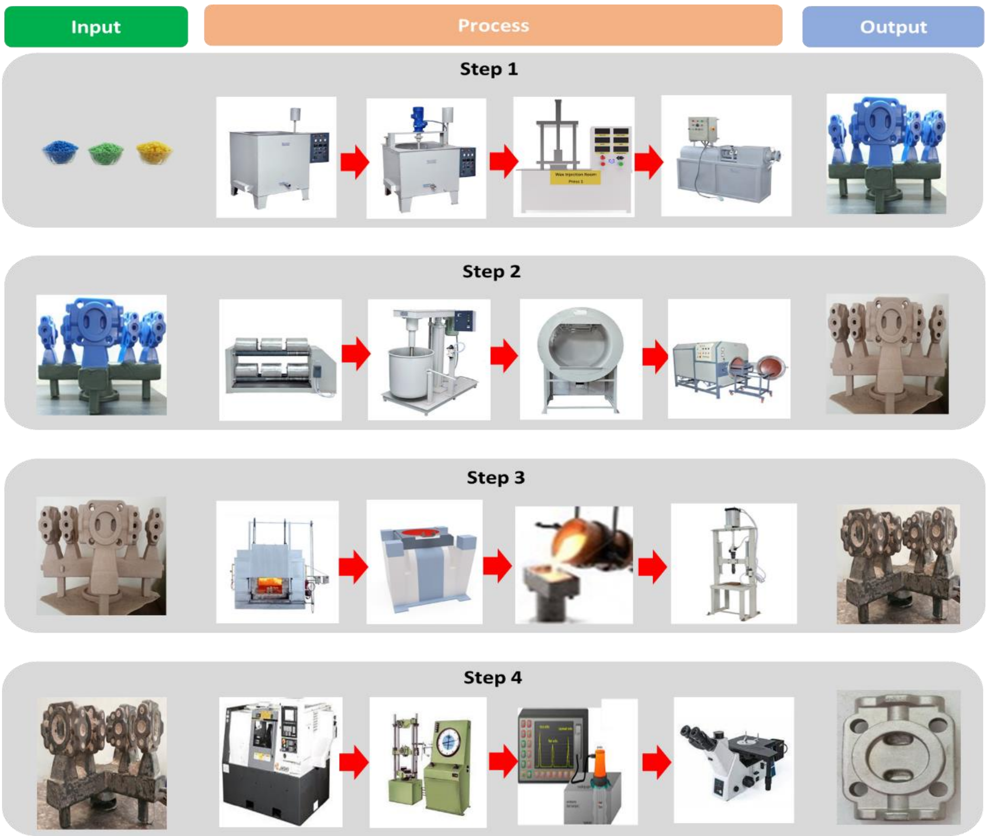

Figure 1: Various steps of the investment casting process

To identify the various variables that affect the complexity of the investment casting process, an industrial survey was conducted in one of the important investment casting clusters of India. After recognizing the variable, it has been organized into a hierarchical structure including the cumulative complexity index, factors (an extensive category of variables), elements (characteristics of the factors), attributes (possible variations of elements), and meta-attributes (values of each variation for attributes). It was observed that the cumulative complexity index of investment casting is primarily determined by 3 broad categories of factors linked to geometry, features, and manufacturability. Furthermore, the geometry factor is driven by two key elements related to characteristics of dimension and surface. These elements are further determined by seven attributes linked to thickness, height, length, width, flatness, curvature, and slant. These attributes are additionally classified into 26 meta-attributes that account for the range of each specific attribute [4].

Likewise, the factor associated with desired features comprises eight elements that are further categorized into 29 attributes and 136 meta-attributes. Additionally, the manufacturability factor encompasses nine elements, 16 attributes, and 50 meta-attributes.

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Overall, 3 factors, 19 elements, 52 attributes, and 212 meta-attributes define the cumulative complexity of the investment casting process [5]. Representing the complexity values for any component manufactured through the investment casting process becomes challenging when dealing with a large number of variables, emphasizing the need for a more effective approach to precisely capture and communicate the intricacies.

Graph theory forms an important realm of discrete mathematics that studies graphs and mathematical objects that consist of vertices (or nodes) and edges (or lines). As such, graph theory is relevant in many fields, such as computer science, engineering, social sciences, and operations research, thus representing an important study area in both the theoretical and applied senses. Graph theory, a fascinating area of mathematics, originated from the pioneering work of Leonhard Euler in 1736. Euler's resolution of the renowned "Seven Bridges of Königsberg" problem laid the groundwork for Eulerian graphs, establishing a key turning point in the field. The key areas of graph theory are graph coloring, graph labeling, graph domination, graph energy, graph decompositions, etc. [6].

All these areas have numerous applications in various fields of real-world problems in scientific and engineering domains. Graph coloring is used in scheduling problems, wireless network frequency assignment, and register allocation in compilers that ensures no two adjacent vertices share the same color [7-9]. Graph labeling has a significant role in x-ray crystallography, DNA sequencing, social network analysis, etc., and is solved by assigning labels to vertices or edges under certain constraints [10-12]. Domination of graphs that can be used in Wireless Sensor Networks (WSNs), cybersecurity, optimal facility location, etc. because of its property of definition, which is a subset of vertices such that every other vertex is adjacent to at least one member of this subset [13-15].

Graph energy is derived from the eigenvalues of the adjacency matrix of the graph. Thus, problems such as chemical graph theory, quantum mechanics, social network dynamics, etc., are solved by it [16-18]. In the decomposition of graphs, the graphs break down (i.e., subgraphs) into smaller components that are useful in parallel computing, protein interaction networks, transportation networks, etc. [18-21]. As provided information, the fields of graph theory play a crucial role in the Industrial Internet of Things (IIoT) by optimizing network performance, enhancing security, and improving data processing efficiency.

As previously mentioned, graph coloring is crucial in many practical scenarios of IIoT using its color pattern, and also it is a challenging and remarkable topic in graph theory that involves assigning colors to graph nodes according to specific criteria. In this paper, graph theory is implemented for the representation of the complexity index of the investment casting process. The complexity value is calculated based on the relative weightage of factors, elements, attributes, and meta-attributes using a Fuzzy Analytical Hierarchy Process (FAHP).

Prior work

The above section concisely discusses prior research work on graph coloring in IIoT applications. In addition. The various aspects of graph coloring techniques, such as heuristic

Received: August 04 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

and metaheuristic methods, have been explored to improve computational efficiency, and the hybrid models integrating graph theory that emerged with machine learning have been developed for predictive maintenance and anomaly detection in the Internet of Things (IoT). In conclusion, these advancements in graph coloring play a pivotal role in ensuring reliable and scalable IIoT systems.

In the Internet of Things (IoT), a dynamic hypergraph coloring approach is used for Unmanned Aerial Vehicle (UAV)-assisted emergency communications within social IoT, aiming to simulate cumulative interference and maintain mutual interference at a minimal level by analyzing a limited number of vertices during dynamic graph updates.

The recent literature has discussed the use of graph coloring to improve communication in IIoT. Graph coloring was used to create energy-efficient routing in IIoT applications, demonstrating that this technique reduces energy consumption while maintaining the integrity of data transmission [22]. It can attain a superior balance among the velocity of information dispersion, the cost of channel switching, and complexity [23]. IoT devices have surged in popularity owing to their capacity for connectivity and collaboration. Graph Neural Networks (GNNs) effectively encapsulate intricate interactions in sensor networks, achieving superior outcomes in IoT applications. This paper provides a comprehensive analysis of GNN implementation in IoT [24]. The research presents a 5G network framework that enhances job distribution across IoT devices and Fog Access Points (FAPs) using a graph-coloring approach. It illustrates the influence of factors on device functionality and latency across diverse contexts, with prospects for future use [25].

A review of the literature published in the direction of graph theory indicates that the theory is compatible with representing complex problems. However, no literature was found that employed graph theory to characterize the complexity index of the investment casting process. This paper presents the application of Graph Adaptive Consensus Mechanisms (GACM) that integrates graph theory with adaptive consensus to represent the complexity index for investment casting. The subsequent sections outline the computation of relative weightage and complexity values for hierarchical structures linked to factors, elements, attributes, and meta-attributes using the FAHP along with a thorough explanation of the steps for employing GACM.

Our work

Fuzzy Analytical Hierarchy Process (FAHP)

The FAHP combines fuzzy logic to address uncertainty and ambiguity in decision-making. FAHP helps decision makers to efficiently handle linguistic expression and subjective opinions. The pairwise comparisons in FAHP are performed with linguistic variables expressed by upper, middle, and lower numbers of triangle membership functions. FAHP can be applied to decision-making problems using several methods, including Buckley's geometric mean, Cheng's extent analysis, and Yager's weighted goal. However, Buckley's geometric mean approach is selected for this specific work because of its general popularity [26]. The flow diagram to compute relative weightage using FAHP is represented in Figure 2.

Received: August 04 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

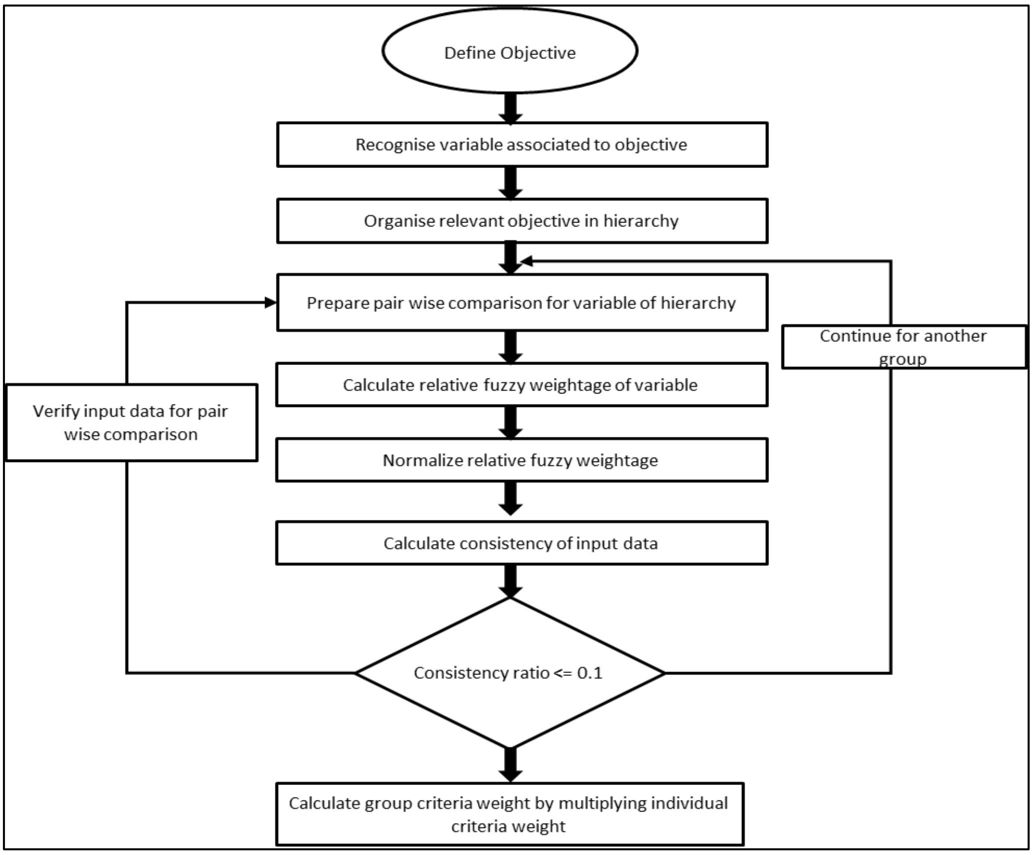

Figure 2: Flow diagram to compute the relative weightage using FAHP

The relative weightage computation using FAHP comprises mainly four stages related to describing the objective as well as preparing the hierarchy, developing a pairwise comparison matrix, calculating relative weightage, and computing the consistency of the pairwise matrix. The first steps involve defining the objective linked to the problem of consideration (e.g., calculating complexity index) as well as identifying the criteria that influence the overall objective and structuring them in the hierarchy (e.g., similar meta-attributes are grouped in attributes, and similar attributes are clustered in elements). The hierarchy in Multi-Criteria Decision-Making (MCDM) represents the structured framework of the decision-making problem. Once the hierarchy is established, the second step is to assign relative importance to each criterion compared to others and create a pairwise comparison matrix. The pairwise comparisons are performed at each hierarchical level, evaluating both individual criteria within each group and between different groups at the same level using the triangular membership function [27-29]. Overall, 17 pairwise comparison matrices have been prepared for the mentioned objective, and that will be a square matrix with a size of (where n signifies the number of variables for each comparison matrix). Moreover, each criterion will be given relative importance as per the fuzzy triangular membership scale by industry experts.

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The third step involves the computation of relative weightage for different criteria. This step includes the calculation of the geometric mean of the pairwise comparison matrix and computes the relative fuzzy weightage of every criterion by multiplying each row with the inverse power of vector summation of the respective row. After calculating relative fuzzy weightage, the next step is defuzzifying relative weightage using the center of area method followed by normalization of defuzzified weightage. The fourth step focuses on ensuring the reliability of the inputs provided by industry experts and identifying any potential bias. This stage is important since it directly affects the accuracy of the weightage calculated in the third step. The transposed transformed pairwise matrix is thus multiplied by the original matrix with dimensions of resulting in a matrix of size that is required for the next calculation. This matrix then is divided by the transposed transformed matrix and provides the maximum eigenvalue. It is vital to note that it always holds a positive value, and if it were to become negative, that would indicate the presence of bias in the inputs provided by industry experts. This calculation is further used to calculate the consistency index.

The consistency index value is then further divided by the random index value to compute the consistency ratio. The random index value is taken based on the size of the pairwise comparison matrix. A consistency ratio value below 0.10 indicates the consistency in the input given by the decision maker. The value above 0.10 indicates that the judgments made during the pairwise comparisons may lack coherence or logical consistency. It suggests that the pairwise comparisons may be unreliable or the judgment of the decision maker may be inconsistent or influenced by biases [30]. The relative weightage and complexity value for each criterion are represented in Table 1.

Table 1: Relative weightage and complexity value of each criterion

<table>
<tr><th>Cumulative Complexity Index</th><th>Complexity Index</th><th>Elements</th><th>Attributes</th></tr>
<tr><td rowspan="7">Cumulative Complexity Index
(100)</td><td rowspan="7">Geometry
(RW = 42)
(CV = 42)</td><td rowspan="4">Dimension Characteristics
(RW = 23)
(CV = 9.66)</td><td>Overall Length (RW = 25) (CV = 2.41)</td></tr>
<tr><td>Overall Height (RW =22) (CV = 2.12)</td></tr>
<tr><td>Overall Width (RW =11) (CV = 1.06)</td></tr>
<tr><td>Average Thickness (RW =42) (CV = 4.05)</td></tr>
<tr><td rowspan="3">Surface Characteristics
(RW = 77)
(CV = 32.34)</td><td>Flat (RW =29) (CV =9.37)</td></tr>
<tr><td>Slanted (RW =33) (CV = 10.67)</td></tr>
<tr><td>Curved (RW =38) (CV = 12.28)</td></tr>
</table>

Received: August 04 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Cumulative Complexity Index	*Complexity Index*	*Elements*	*Attributes*
	Features (RW = 30) (CV = 30)	*Hole (RW = 13) (CV = 3.9)*	*Number of Holes (RW =11) (CV = 0.42)*
			Maximum Length of Hole (RW =17) (CV = 0.66)
			Maximum Diameter of Hole (RW =15) (CV = 0.58)
			Contour of Hole (RW =11) (CV = 0.42)
			Open to Surface (RW =16) (CV = 0.62)
			Type of Hole (RW =30) (CV = 1.17)
		Slot (RW = 14) (CV = 4.2)	*Number of Slots (RW =21) (CV = 0.88)*
			Maximum Length of Slot (RW =18) (CV = 0.75)
			Maximum Width of Slot (RW =21) (CV = 0.88)
			Maximum Depth of Slot (RW =40) (CV = 1.68)
		Groove (RW = 30) (CV = 9)	*Number of Grooves (RW =22) (CV = 1.98)*
			Maximum Length of Groove (RW =15) (CV = 1.35)
			Maximum Width of Groove (RW =20) (CV = 1.8)
			Maximum Depth of Groove (RW =43) (CV = 3.87)
		Fillet (RW = 4) (CV = 1.2)	*Number of Fillets (RW =54) (CV = 0.64)*
			Maximum Radius of Fillet (RW =46) (0.552)
		Chamfer	*Number of Chamfers (RW =32) (CV*

Received: August 04 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Cumulative Complexity Index	***Complexity Index***	***Elements***	***Attributes***
		(RW = 4) *(CV = 1.2)*	*= 0.38)*
			Chamfer Angle or Distance (RW =40) (CV = 0.48)
			Distance (RW =28) (CV = 0.33)
		Hollow Region *(RW = 22)* *(CV = 6.6)*	*Number of Hollow Regions (RW =48) (CV = 3.16)*
			Maximum Length of Hollow Region (RW =28) (CV =1.84)
			Maximum Diameter of Hollow Region (RW =24) (CV = 1.58)
		Rib *(RW = 7)* *(CV = 2.1)*	*Number of Ribs (RW =17) (CV = 0.35)*
			Maximum Length of Rib (RW =34) (CV =0.714)
			Maximum Thickness of Rib (RW =27) (CV = 0.56)
			Maximum Width of Rib (RW =22) (CV = 0.46)
		Boss *(RW = 6)* *(CV = 1.8)*	*Number of Bosses (RW = 37) (CV = 0.66)*
			Maximum Diameter of Boss (RW =23) (CV = 0.41)
			Maximum Thickness of Boss (RW =40) (CV = 0.72)
	Manufacturability *(RW = 28)* *(CV = 28)*	*Types of Alloys* *(RW = 8)* *(CV = 2.24)*	*Alloying Elements (RW =100) (CV =2.24)*
		Requirement of Property *(RW = 11)* *(CV = 3.08)*	*Hardness (RW =100) (CV =3.08)*

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Cumulative Complexity Index	***Complexity Index***	***Elements***	***Attributes***
		Bulkiness (RW = 6) (CV = 1.68)	*Relative Density (RW =100) (CV = 1.68)*
		Solidification Aids (RW = 13) (CV = 3.64)	*Insulating Sleeve (RW =30) (CV =1.09)*
			Chills (RW =50) (CV =1.82)
			Exothermic Powder (RW =20) (CV =0.72)
		Melting Aids (RW = 13) (CV = 3.64)	*Degassing (RW = 36) (CV =1.31)*
			Filler (RW =45) (CV =1.63)
			Slag Powder (RW =19) (CV =0.69)
		Supplementary Aids (RW = 16) (CV = 4.48)	*Heat Treatment (RW =17) (CV = 0.76)*
			Machining (RW =35) (CV = 1.56)
			Destructive Testing (RW = 22) (CV = 0.98)
			Non-Destructive Testing (RW = 26) (CV =1.16)
		Meltability (RW = 9) (CV = 2.52)	*Melting Temperature (RW =100) (CV = 2.52)*
		Valuation (RW = 17) (CV = 4.76)	*Application (RW = 100) (CV = 4.76)*
		Batch Size (RW = 7) (CV = 1.96)	*Quantity (RW = 100) (CV = 1.96)*

Graph Adaptive Consensus Mechanisms

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

GACM establishes a framework for managing resources in a network utilizing proper coloring to assign a unique resource to each node to reduce interference to neighboring nodes. GACM also aids in ensuring communications for devices beginning from topologically different locations so that users can connect and disconnect as time goes on and still communicate without contention for their particular channels. The system is dynamic; GACM continually checks resources and reassigns them to provide optimal performance. Being a dynamic, scalable framework, it is also somewhat ubiquitous (can be applied for both small and large networks). GACMs combine the flexibility of graph theory with adaptive consensus protocols to improve network communication and decision-making in dynamic systems. These strategies provide distributed processes for the consensus of agency nodes, irrespective of changes in network design or the failure or disconnection of some nodes.

GACM algorithms in graph theory have been shown to be useful for distributed optimization problems, load balancing, and dynamic resource allocation. [31, 32]. GACM algorithms' main benefit is their ability to adapt to graph complexity. As the ultimate graph topology changes, the synthesizing process may adapt such that nodes can work together toward reaching a consensus on the solution. This technique is advantageous in a decentralized network such that the graph topology may change due to the mobility of nodes or different communication patterns [33]. This study proposes the GACM combined with a proper coloring approach in the FAHP to effectively classify data into a set of complexity values. According to the proper coloring, no two adjacent nodes receive the same colors, which regulates interference and increases the overall efficiency of the process. When FAHP is incorporated into the process, it can successfully prioritize data into higher and lower complexity categories, thus allowing for more precise decision-making in cases of complex networks. In addition to its effectiveness in resource optimization, this approach automatically adapts to network conditions even with a growing set of data complexity. As a combination of GACM, proper coloring, and FAHP, this solution has the properties and processes to effectively and accurately manage data in large sets, thus being very useful for IIoT application networks, blockchain networks, and large computational systems. The steps for implementation of GACM are outlined as follows.

Step 1: Graph representation of the network

A graphical visual representation of the hierarchy of the Cumulative Complexity Index (CCI) is shown in figure 3. The hierarchy has four levels composed of different levels of complexity assessment as exploratory value is also shown in Table 1.

1. Cumulative Complexity Index (CCI)
2. Factor
3. Elements
4. Attributes

Each of these factors is divided into both elements and attributes as shown in Table 1. The graph representation of the complexity relationship is represented as a weighted graph

Received: August 04 2025

International Journal of Applied Mathematics
Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

G(V,E), where nodes (V) are representative of factors, elements, and attributes. Edges (E) are representative of the connectiveness between the nodes, showing relationships and dependency between factors, elements, and attributes.

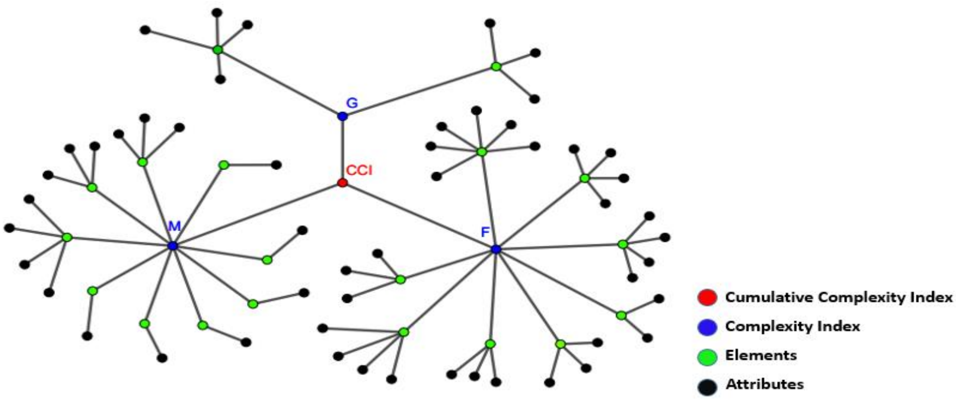

Fig. 3. Graphical representation of Cumulative complexity index

Step 2: Employing the shortest path algorithm

Dijkstra's algorithm identifies the greatest and lowest complexity values according to their complexity index for determining the shortest path. Figure 4 illustrates that the geometry factor has two elements: surface characteristics and dimensional characteristics, with the shortest route (8.4) being a subset of the dimensional characteristics determined by its complexity value. Consequently, the properties of dimensional characteristics are less complex than those of surface characteristics. The route delineates the precise complexity level of each part, indicating that the curved (12.64) is more complicated than the slanted (10.11) and the flat (8.84). The flat exhibits the lowest complexity level, making it a less complex component. The average thickness (3.52) represents the highest complexity number among the elements of dimension characteristics, while the overall width (0.92) is the lowest. The complexity level of the remaining attributes indicates a more diverse and intricate part of the system.

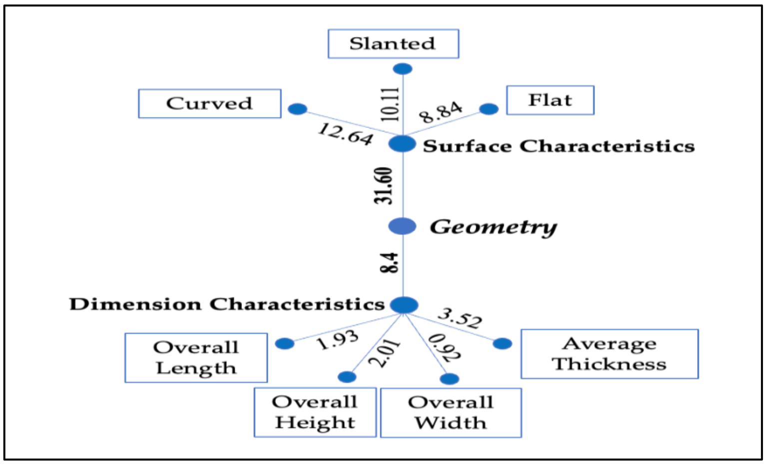

Fig. 4. Graphical representation of complexity values of elements for geometry factor

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Figure 5 represents the complexity values for elements and attributes of the feature factor as paths, and table 2 gives the data of the labelled that indicates the various elements and attributes. Now, as mentioned previously, the shortest paths from feature to elements are fillet (1.32) and chamfer (1.32). That means both are the least complex among all elements, and the highest complexity value is the slot (4.95) (i.e., more complicated than others). The attributes also show their complexity level based on their values. For example, in the attributes of fillet and chamfer, the most intricate parts are the "number of fillets (0.69)" and "chamfer angle or distance (0.50)," respectively. The lowest complexity levels are "maximum radius of fillet (0.62)" and "distance (0.36)," respectively. Similarly, the highest and lowest complex parts in all attributes of various elements are;

1. Hole:

· The highest is the "number of holes (1.17)"

· The lowest is the "open the surface & type of holes (0.42)"

2. Slot:

· The highest is the "Maximum Depth of Slot (1.98)"

· The lowest is the "Maximum Length of Slot (0.84)"

3. Groove:

· The highest is the "Maximum Depth of Groove (3.69)"

· The lowest is the "Maximum Length of Groove (1.47)."

4. Fillet:

· The highest is the "Number of Fillets (0.69)"

· The lowest is the "Maximum Radius of Fillet (0.62)"

5. Chamfer:

· The highest is the "Chamfer Angle or Distance (0.50)"

· The lowest is the "Distance (0.36)"

6. Hollow region:

· The highest is the "Number of Hollow Regions (2.97)"

· The lowest is the "Maximum Diameter of Hollow Region (1.65)"

7. Rib:

· The highest is the "Maximum Length of Rib (0.84)"

· The lowest is the "Number of Ribs (0.50)"

8. Boss:

· The highest is the "Maximum Thickness of Boss (0.92)"

Received: August 04 2025

International Journal of Applied Mathematics
Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Additionally, the "Maximum Diameter of Boss (0.60)" is the lowest.

Table 2: Label of elements and attributes for features factor

	Labels	***Elements***	***Labels***	***Attributes***
Features	*A*	*Hole*	v_1	*Number of Holes*
			v_2	*Maximum Length of Hole*
			v_3	*Maximum Diameter of Hole*
			v_4	*Contour of Hole*
			v_5	*Open to Surface*
			v_6	*Type of Hole*
	B	*Slot*	v_7	*Number of Slots*
			v_8	*Maximum Length of Slot*
			v_9	*Maximum Width of Slot*
			v_{10}	*Maximum Depth of Slot*
	C	*Groove*	v_{11}	*Number of Grooves*
			v_{12}	*Maximum Length of Groove*
			v_{13}	*Maximum Width of Groove*
			v_{14}	*Maximum Depth of Groove*
	D	*Fillet*	v_{15}	*Number of Fillets*
			v_{16}	*Maximum Radius of Fillet*
	E	*Chamfer*	v_{17}	*Number of Chamfers*
			v_{18}	*Chamfer Angle or Distance*
			v_{19}	*Distance*
	F	*Hollow Region*	v_{20}	*Number of Hollow Regions*
			v_{21}	*Maximum Length of Hollow Region*
			v_{22}	*Maximum Diameter of Hollow Region*
	G	*Rib*	v_{23}	*Number of Ribs*
			v_{24}	*Maximum Length of Rib*
			v_{25}	*Maximum Thickness of Rib*

Received: August 04 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

			v_{26}	*Maximum Width of Rib*
	H	*Boss*	v_{27}	*Number of Bosses*
			v_{28}	*Maximum Diameter of Boss*
			v_{29}	*Maximum Thickness of Boss*

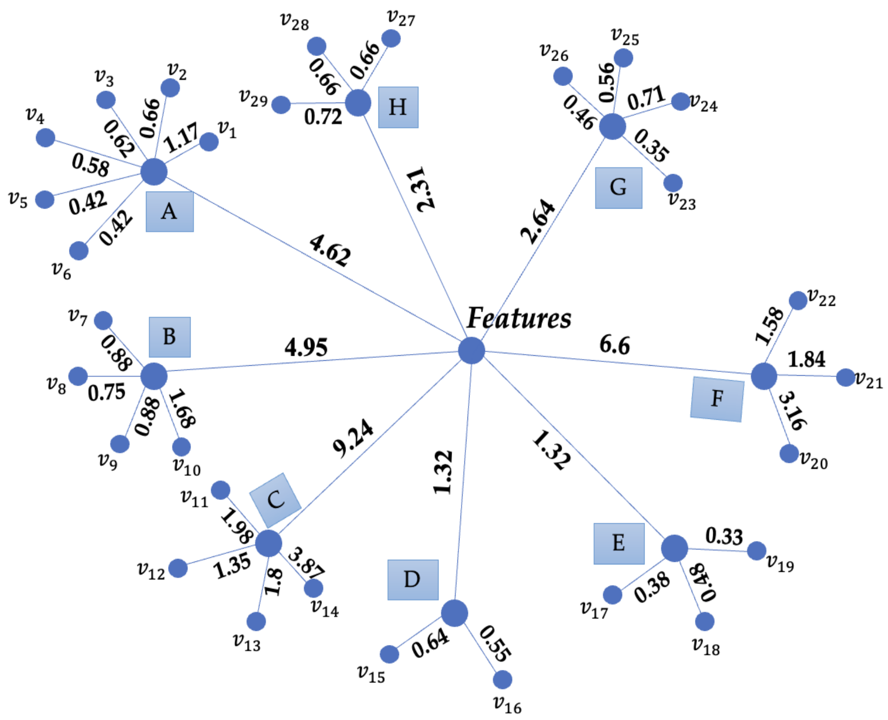

Fig. 5. Graphical representation of complexity values of elements for the feature factor

Similarly, for manufacturability factor the highest complexity level of the manufacturability factor is valuation (4.59) while the lowest is bulkiness (1.35) as represented in figure 6. The valuation attribute is particularly complex within the manufacturability factor compared to the least intricate level of the bulkiness attribute. The valuation and bulkiness possess a singular attribute; therefore, their complexity level is contingent upon their complexity values. Additional elements and attributes exhibit a variety of complex levels in their values. The representation of attributes and elements is also illustrated in figure 6, as detailed in table 3.

Table 3: Label of elements and attributes for manufacturability factor

	Labels	***Elements***	***Labels***	***Attributes***
Manufacturability	*J*	*Types of Alloys*	u_1	*Alloying*
	K	*Requirement of Property*	u_2	*Hardness*

Received: August 04 2025

International Journal of Applied Mathematics
Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

	L	*Bulkiness*	u_3	*Relative Density*
	M	*Solidification Aids*	u_4	*Insulating Sleeve*
			u_5	*Chills*
			u_6	*Exothermic Powder*
	N	*Melting Aids*	u_7	*Degassing*
			u_8	*Filler*
			u_9	*Slag Powder*
	O	*Supplementary Aids*	u_{10}	*Heat Treatment*
			u_{11}	*Machining*
			u_{12}	*Destructive Testing*
			u_{13}	*Non-Destructive Testing*
	P	*Meltability*	u_{14}	*Melting Temperature*
	Q	*Valuation*	u_{15}	*Application*
	R	*Batch Size*	u_{16}	*Quantity*

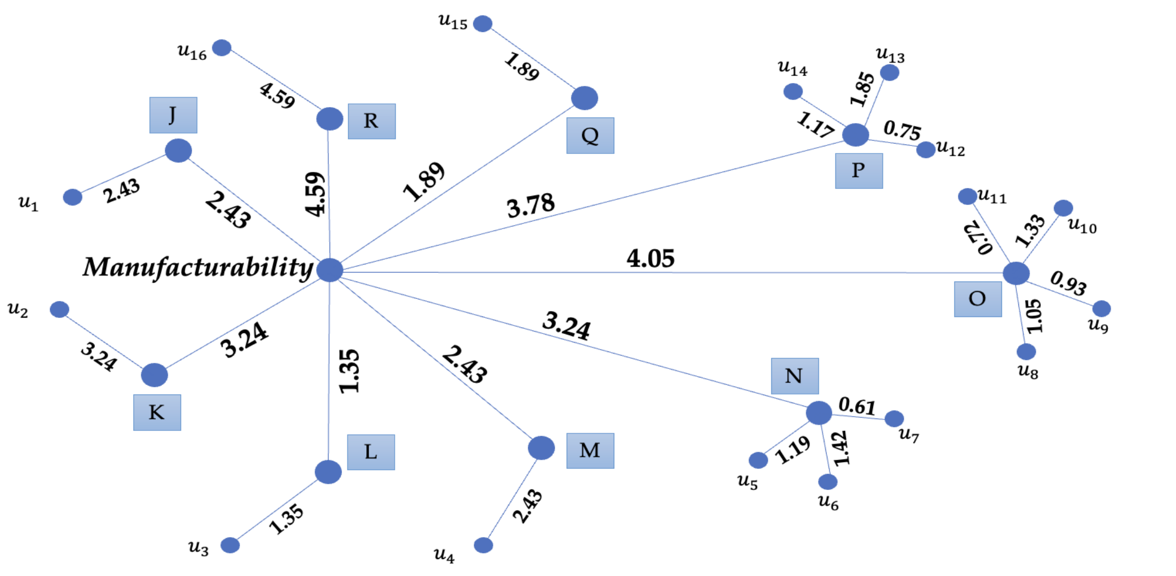

Fig. 6: Graphical representation of complexity values of elements for manufacturability factor

Step: 3 The proper coloring for visualize complex attributes of complexity index

Figures 7, 8, and 9 show the coloring used to represent the trajectories of maximum and minimum complexity values, along with the selected path for complex attributes. Figure 7

Received: August 04 2025

International Journal of Applied Mathematics
Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

depicts the appropriate coloration assigned to the complexity values of elements in relation to geometric factors. The nodes reflecting attributes of components such as surface characteristics and dimensional characteristics are designated with "blue" colors signifying the maximum complexity values, while "red" colors signify the lowest complexity values of the complexity index. Furthermore, various colors of characteristics denote distinct intermediate complexity values for attributes.

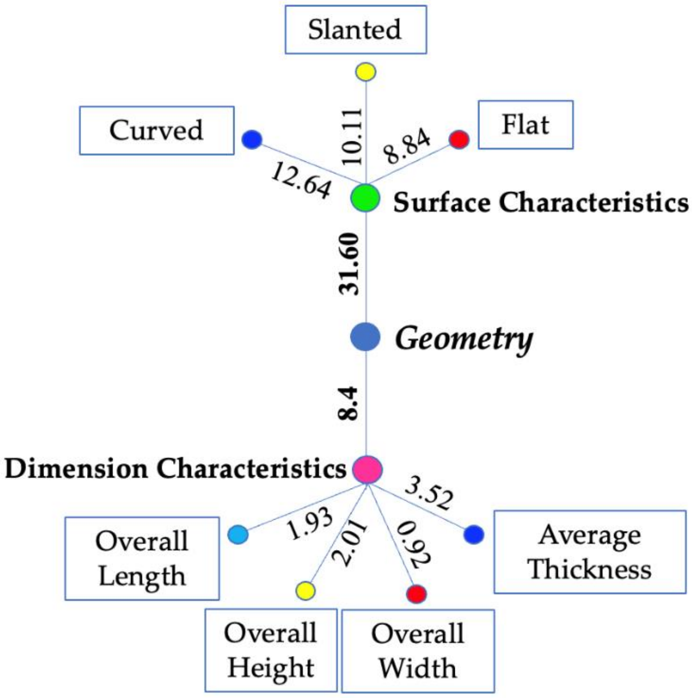

Fig. 7: Proper coloring of complexity values of elements for geometry factor

For the purpose of classifying the complexity of various elements and characteristics according to the complexity value of such elements and attributes, proper coloring is utilized. The attributes that have the blue color are the ones that signify the attribute with the highest complexity value for a certain element. Furthermore, there are some attributes that share the same colors, which indicates that they have the same complexity value and contribute in a comparable manner to the total complexity of the system. Therefore, the proper coloring pattern makes it simpler to understand how the many components of a system interact with one another.

In the coloring pattern, red denotes the element with the least complexity for a certain attribute and those factors. This also suggests that the single property or element in question has a modest impact on the overall complexity of the system. On the other hand, different colors are allocated to different properties of elements that represent a discrete range of complexity values. In addition, the presence of several colors draws attention to the more varied and sophisticated components of the system. In general, the injective coloring provides an efficient method for visualizing the ways in which various elements and qualities contribute to complexity. It assists in the rapid identification of the components or

Received: August 04 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

characteristics of a system that are more or less complicated as well as the recognition of patterns and the comprehension of the links between them.

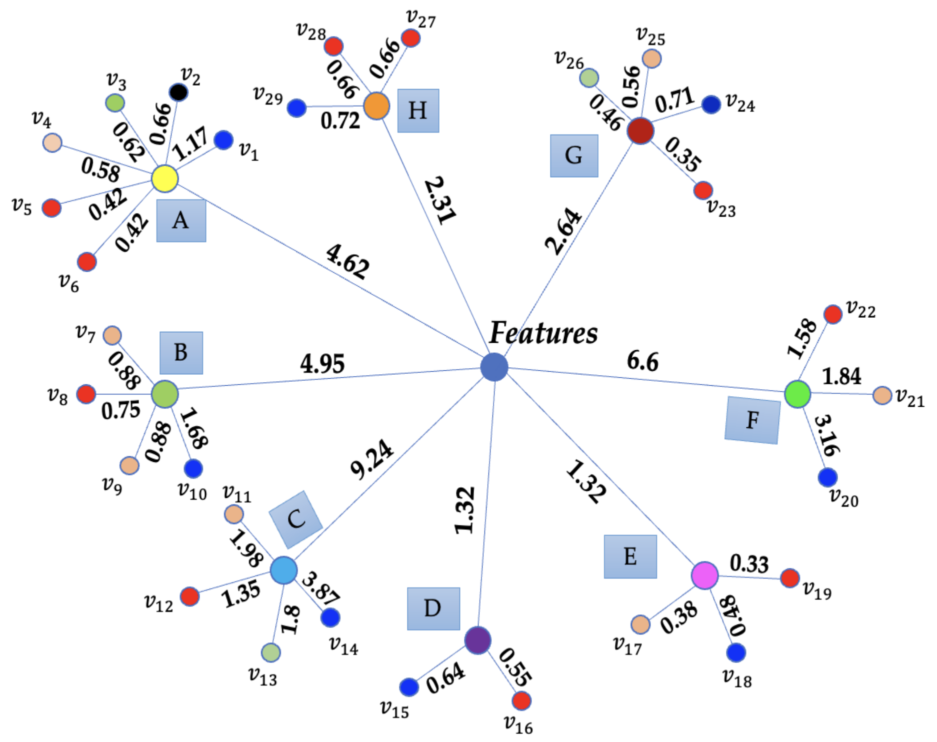

Fig. 8: Proper coloring of complexity values of elements for the Feature factor

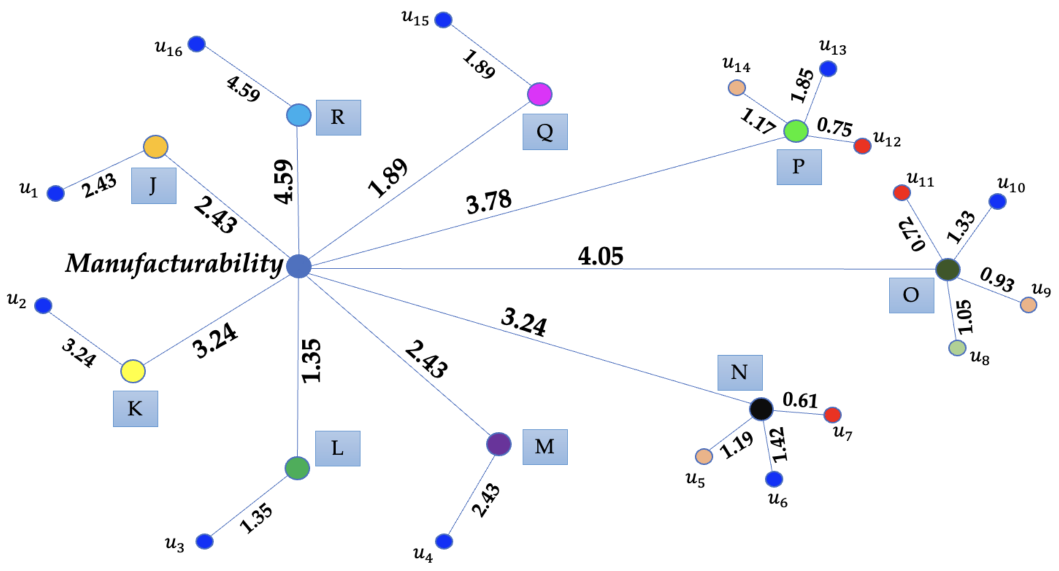

Fig. 9: Proper coloring of complexity values of elements for manufacturability factor

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Conclusion

The investment casting process is one of the important and recognized manufacturing processes to produce complex industrial castings with exceptional quality for various industrial sectors. The complexity of the investment casting process is influenced by three factors: 19 elements, 52 attributes, and 212 meta-attributes. Effectively representing these complexity values becomes increasingly challenging, especially when managing a large number of variables. This highlights the need for a novel approach to convey the intricacies of the industrial component manufactured through the investment casting process. This paper has explored the role of graph theory for the representation of FAHP-based complexity involved in manufacturing industrial components using the investment casting process. This provides better support for decision-making in contexts that involve multi-criteria and complex scenarios. Because of the structural properties of graph theory, this method presents a very systematic way of treating uncertainty and imprecision when carrying out multi-criteria decision-making approaches. The results indicate the potential for graph-based representations to support clear and straightforward visualizations of hierarchical relationships that may assist with identifying dependencies and incongruities in pairwise comparisons. Graph theory also offers a means of extending the fuzzy logic, thus allowing decision-makers to articulate preferences in a less artificial and therefore more human way, which indirectly advances the FAHP to alleviate its deficiencies.

The combination of the two encourages the reliability and precision of decision-making in cases of qualitative assessments and expert judgment. This perspective indicates there is potential for broader dissemination and utility for the graph-theoretic FAHP across several contexts, beyond possible continued use in supply chain management, risk assessments, project prioritization, and engineering design as in the case study context. Future work could also look to hybridize this approach with other advanced computational approaches, such as artificial intelligence and machine learning, to further enhance decision-making processes. Furthermore, empirical investigations of actual use cases would enhance comprehension of the framework's practical viability and scalability. Overall, the added value of combining graph theory and FAHP is that it provides a powerful decision-support mechanism for creating tools for hierarchical decision analysis that are easier to interpret, better able to adapt, and more accurate. Decision-making will continue to evolve in an increasingly complex world, where this framework offers an opportunity for decision-making in a more informed and transparent and, at the same time, more effective way.

References

[1] Nikunj Mehta, Dr Amit Sata 5 Cs of Investment Casting Foundries in Rajkot Cluster – An Industrial Survey, Archives of Foundry Engineering, 21(3), 101–107 (2021). https://doi.org/10.24425/afe.2021. 138672.

[2] Sata Amit, Sutaria M, Scope of Investment Castings Supported by Survey of Foundries in Rajkot Cluster. Indian Foundry Journal, Volume 60(6), pp. 42-46, 2014.

Received: August 04 2025

International Journal of Applied Mathematics

Volume 38 No. 8s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

[3] Maheta N, Sata A. Implementing multi criteria decision making methods for computing complexity involved in industrial investment castings. Mechanical Engineering Advances. 2025; 3(1): 1962.https://doi.org/10.59400/mea1962.

[4] Nikunj Mehta, Dr Amit Sata Systematic Development of Cumulative Complexity Index for Investment Casting, Journal of Advanced Manufacturing System 22(02), 323–338 (2023) https://doi.org/10.1142/S0219686723500166.

[5] Nikunj Maheta, Dr Amit Sata Development of a Novel Complexity Index for Investment Casting, International Journal of Metalcasting, 18, 2165–2180 (2024). https://doi.org/10.1007/s40962-023-01151-1.

[6] West, D. B. (2001). Introduction to graph theory (Vol. 2). Upper Saddle River: Prentice Hall.

[7] Burke, E. K., McCollum, B., Meisels, A., Petrovic, S., & Qu, R. (2007). A graph-based hyper-heuristic for educational timetabling problems. European Journal of Operational Research, 176(1), 177-192.

[8] Hale, W. K. (1980). Frequency assignment: Theory and applications. Proceedings of the IEEE, 68(12), 1497-1514.

[9] Chaitin, G. J. (1982). Register allocation & spilling via graph coloring. ACM Sigplan Notices, 17(6), 98-101.

[10] Rosa, A. (1967). On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966).

[11] Sharma, S., & Sharma, D. (2018). Intelligently applying artificial intelligence in chemoinformatic. Current topics in medicinal chemistry, 18(20), 1804-1826.

[12] Aggarwal, C. C. (2011). An introduction to social network data analytics (pp. 1-15). Springer US.

[13] Dai, F., & Wu, J. (2004). An extended localized algorithm for connected dominating set formation in ad hoc wireless networks. IEEE transactions on parallel and distributed systems, 15(10), 908-920.

[14] Khalid, M. N. A., Al-Kadhimi, A. A., & Singh, M. M. (2023). Recent developments in game-theory approaches for the detection and defense against advanced persistent threats (APTs): a systematic review. Mathematics, 11(6), 1353.

[15] Slater, P. J. (2001). Domination and location in graphs. Networks, 38(2), 81-99.

[16] Gutman, I., & Furtula, B. (2019). Graph energies and their applications. Bulletin (Académie serbe des sciences et des arts. Classe des sciences mathématiques et naturelles. Sciences mathématiques), (44), 29-45.

[17] Randić, M., Zupan, J., & Novic, M. (2001). On 3-D graphical representation of proteomics maps and their numerical characterization. Journal of chemical information and computer sciences, 41(5), 1339-1344.

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

[18] Estrada, E. (2006). Spectral scaling and good expansion properties in complex networks. Europhysics Letters, 73(4), 649.

[19] Faik, S. L., Parashar, M., Taylor, V. E., & Varela, C. A. (2006). Scientific Computation. Parallel Processing for Scientific Computing, 20, 33.

[20] Barabasi, A. L., & Oltvai, Z. N. (2004). Network biology: understanding the cell's functional organization. Nature reviews genetics, 5(2), 101-113.

[21] Ji, Y., & Geroliminis, N. (2012). On the spatial partitioning of urban transportation networks. Transportation Research Part B: Methodological, 46(10), 1639-1656.

[22] Sharma, N., Agarwal, U., Shaurya, S., Mishra, S., & Pandey, O. J. (2023). Energy-efficient and QoS-aware data routing in node fault prediction based IoT networks. IEEE Transactions on Network and Service Management, 20(4), 4585-4599.

[23] Wang, B., Sun, Y., Sun, Z., Nguyen, L. D., & Duong, T. Q. (2020). UAV-assisted emergency communications in social IoT: A dynamic hypergraph coloring approach. IEEE Internet of Things Journal, 7(8), 7663-7677.

[24] Dong, G., Tang, M., Wang, Z., Gao, J., Guo, S., Cai, L., ... & Boukhechba, M. (2023). Graph neural networks in IoT: A survey. ACM Transactions on Sensor Networks, 19(2), 1-50.

[25] Pratap, A., Gupta, R., Nadendla, V. S. S., & Das, S. K. (2019, June). On maximizing task throughput in IoT-enabled 5G networks under latency and bandwidth constraints. In 2019 IEEE International Conference on Smart Computing (SMARTCOMP) (pp. 217-224). IEEE.

[26] J. J. Buckley, “Ranking alternatives using fuzzy numbers,” Fuzzy Sets Syst, vol. 15, no. 1, pp. 21–31, 1985, doi: 10.1016/0165-0114(85)90013-2.

[27] A. Sharma, V. M. S. Hussain, P. Abhishek Kumar, and M. Pandit, “Prioritization of forging die design criteria based on failure analysis using fuzzy analytic hierarchy process (FAHP),” Mater Today Proc, no. xxxx, 2022, doi: 10.1016/j.matpr.2022.11.329.

[28] M. B. Ayhan, “A Fuzzy Ahp Approach For Supplier Selection Problem: A Case Study In A Gearmotor Company,” International Journal of Managing Value and Supply Chains, vol. 4, no. 3, pp. 11–23, 2013, doi: 10.5121/ijmvsc.2013.4302.

[29] M. S. F. Soberi and R. Ahmad, “Application of fuzzy AHP for setup reduction in the manufacturing industry,” Journal of Engineering Research and Education, vol. 8, no. January, pp. 73–84, 2016.

[30] T. L. Saaty, “How to make a decision: The analytic hierarchy process,” Eur. J. Oper. Res., vol. 48, no. 1, pp. 9–26, 1990, doi: 10.1016/0377-2217(90)90057-I.

[31] Su, W., Xie, L., & He, J. (2016). Distributed Consensus and Optimization in Graph Networks. Springer.

Received: August 04 2025

Volume 38 No. 8s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

[32] Zhao, M., Zhang, X., & Liu, Z. (2018). Graph Coloring and Consensus Mechanisms in Blockchain. Journal of Computer Networks, 67(4), 231–243.

[33] Xie, M., Liu, J., Chen, S., & Lin, M. (2023). A survey on blockchain consensus mechanism: research overview, current advances and future directions. International Journal of Intelligent Computing and Cybernetics, 16(2), 314-340.

Received: August 04 2025