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PERFECT MEASUREMENT APPROACH TO INVENTORY OPTIMIZATION UNDER PRESERVATION, CARBON EMISSION, AND CREDIT FINANCING

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Abstract

In the current era of global sustainability, inventory management practices are increasingly evaluated not only on their economic performance but also on their environmental and technological dimensions. This study develops a sustainable inventory model that integrates preservation technology, carbon emission considerations, and trade credit policy into a unified framework. Preservation technology is incorporated to minimize product deterioration, extend shelf life, and reduce waste, thereby improving overall efficiency. Carbon emissions generated from production, storage, and transportation are quantified to address environmental concerns and align with regulatory and green supply chain standards. Additionally, trade credit policy is introduced as a financing mechanism that enhances supplier–retailer relationships, influences ordering behavior, and improves capital flexibility. The proposed model investigates the optimal ordering strategy that minimizes total cost while balancing financial, technological, and environmental objectives. Sensitivity analysis highlights the impact of preservation investment, emission costs, credit terms, and demand patterns on the sustainability of the system. The findings suggest that integrating preservation technology with emission-conscious inventory planning under trade credit not only enhances profitability but also promotes ecological responsibility, offering valuable insights for managers, policymakers, and supply chain practitioners aiming to achieve sustainable growth.

Keywords: Sustainable inventory model; Preservation technology; Carbon emission; Trade credit policy; Inventory optimization; Environmental sustainability; Supply chain management; Cost minimization

Introduction

In today's competitive and environmentally conscious marketplace, businesses are under increasing pressure to design inventory systems that not only minimize costs but also promote sustainability. Traditional inventory models primarily focused on economic objectives such as cost reduction and service level improvement. However, modern supply chain practices demand the integration of environmental and technological considerations to meet global sustainability

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standards. Among these, carbon emission reduction, product preservation, and flexible financing mechanisms are emerging as critical dimensions of sustainable inventory management. One of the major challenges in managing inventory is the deterioration of items during storage, which leads to waste, financial loss, and customer dissatisfaction. Preservation technologies, such as refrigeration, modified atmosphere packaging, or chemical treatments, are increasingly being adopted to reduce deterioration rates and extend the shelf life of products, especially perishables and sensitive goods. The integration of preservation technology in inventory models provides a sustainable solution by lowering product wastage and aligning with circular economy principles. At the same time, carbon emissions associated with manufacturing, storage, and transportation activities have become a significant concern in global supply chains. Many governments and regulatory bodies are enforcing carbon taxes and emission caps, compelling firms to incorporate emission costs into their operational strategies. An inventory model that explicitly accounts for emissions can help firms strike a balance between profitability and environmental responsibility. In addition to technological and environmental considerations, the trade credit policy plays a vital role in shaping inventory decisions. Trade credit—where suppliers allow retailers to delay payments for goods—serves as an effective financing mechanism, improving cash flow and encouraging larger order quantities. It provides flexibility to retailers and strengthens supplier–retailer relationships. However, the interplay between trade credit, preservation investment, and emission costs introduces new complexities in decision-making. The integration of these three dimensions—preservation technology, carbon emission control, and trade credit policy—offers a holistic approach to designing a sustainable inventory model. Such a model not only minimizes total system costs but also supports long-term environmental and financial sustainability. By analysing the trade-offs between preservation costs, emission penalties, and credit terms, businesses can develop strategies that enhance profitability, reduce ecological footprints, and improve competitiveness in the green supply chain era. This study aims to formulate and analyze such a sustainable inventory model, providing theoretical insights and practical guidelines for supply chain managers, policymakers, and researchers.

Literature review

Early deteriorating-item models established how time-dependent spoilage alters ordering and holding decisions (e.g., constant vs. variable deterioration, stock-dependent demand). Subsequent work introduced age-based and time-varying deterioration, partial backlogging, and shortage policies. Building on this, the preservation technology stream analyzes investments (e.g., refrigeration, controlled/modified atmosphere packaging, coatings, humidity control) that reduce deterioration rates or extend shelf life. Models typically (i) treat preservation as a decision variable, (ii) link preservation intensity to the deterioration parameter, and (iii) include a convex operating or investment cost. Results consistently show an interior optimum where marginal waste reduction = marginal preservation cost, and that preservation tends to lower optimal order quantities, reduce waste, and flatten inventory profiles for perishables and semi-perishables. The green operations

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literature embeds carbon externalities into classical EOQ/EPQ and lot-sizing. Emissions are modeled from production setups, holding (energy-intensive storage), and transport (shipment frequency/size). Three modeling approaches dominate:

Carbon cost internalization: add a monetized carbon term (tax/price) to the total cost.

Carbon constraint/threshold: enforce a cap (regulatory or corporate), shifting the feasible region.

Cap-and-trade / offsetting: allow buying/selling permits or purchasing offsets; optimal policies depend on permit and offset prices.

Findings across these studies show that longer cycles (larger batches) reduce setup/transport emissions but raise storage emissions—so the optimal policy depends on the relative carbon intensity of transport vs. storage energy. With carbon prices, firms adopt fewer but fuller shipments if transport dominates, or more frequent smaller cycles if storage energy is highly carbon intensive. The literature also stresses technology switching (e.g., greener refrigeration, renewable energy) as a structural lever, not just an operational tweak.

The phenomenon of decay or deterioration presents significant challenges within inventory systems. The models in the inventory system include Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), and Economic Growing Quantity (EGQ). The initiation of mathematical modelling concerning deterioration can be traced back to the EOQ model, which was presented by Whiten in 1957. [1] expanded the core EOQ model with additional assumptions and established a framework for emerging scholars in the field by delivering impactful and noteworthy outcomes. The growth of research is accelerating as the scope and nature of supply chain inventory businesses evolve over time. In recent years, it has been influenced by technological advancements and various environmental conditions. Currently, the market presents significant competition, necessitating that supply chain participants integrate their operations to improve efficiency, better meet customer needs, and reduce inventory costs. In a non-integrated supply chain, various participants have distinct motives, leading to potential conflicts between the objectives of suppliers and retailers within the supply chain. Inconsistent or non-coordinated decisions within the supply chain can lead to a significant loss of competitive advantage.[2] Initially created a comprehensive inventory model aimed at identifying the most effective joint inventory strategy for one supplier and one retailer. [3] merged the idea of a cohesive inventory model with a trade credit policy and developed a supplier-retailer integrated framework where the supplier provides trade credit to the retailer . Subsequently, an integrated inventory model incorporating diverse trade credit policies can be located in [4]. By [5] Formulated an EOQ model incorporating inflation, demonstrating that the carrying cost within the EOQ framework is influenced by pricing policy and introduces an entirely new idea. By [6] developed an EOQ model for a product considering scenarios of allowable payment delays. By [7] created an inventory model for deteriorated items, where demand is influenced by the purchase price and the rate at which deterioration occurs is represented as a linear function of time. Now by [8] Additionally, an

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EOQ model has been developed focusing on retailers' ordering strategies in the context of inflation. The authors examined the defective items and their influence on retailers' ordering strategies for decayed items in the context of inflation, considering variations over time.By [9] derived significant insights by connecting his model with trade credit and lot size strategies in EPQ, noting that trade credit policy enhances sales while also raising opportunity costs and default risks.[10] by developing optimal strategies under practical constraints. The analysis focused on a supply chain that included a manufacturer, distributor, and retailer, emphasizing the importance of settling accounts within the designated credit period. By [11] Examined a production-inventory model within the framework of the carbon price system, leading to significant insights regarding its implications for the environment, GDP, and optimal profit. “By [12] The researchers discovered significant although contingent findings that are associated with retailers' ordering policy. They validated their conclusions via the use of numerical examples. [13] Examined a scenario in which demand is contingent upon both time and credit period. They included credits and other policies into the quadratic demand and supply cycle for deteriorated items. [14] Created a viable manufacturing inventory model to minimize carbon emissions. In addition, they implemented an integrated strategy including production, replenishment, distribution, and technology to minimize carbon emissions and maximize net profit within the system. [15] Constructed a mathematical model and optimized the retailer's overall cost. The demand was regarded as being reliant on stock, and the carrying cost was thought to be of a variable nature over time. A supply chain model was designed to minimize energy usage and eliminate waste by [16]. A non-linear mathematical model is defined to determine the minimal overall cost. [17] Conducted research on green technologies and created a supply chain inventory model that incorporates a payment system to address shortages and surpluses. An (EOQ) model was also created for perishable products by [18]. The developers implemented a two-tier credit and financing strategy, taking into account the impacts of learning. They provided a detailed explanation of the policy using numerical examples.[19] Developed an Economic Production Quantity model to analyse the impact of dependability and inflation on perishable items. The model also incorporates a two-level credit policy. In addition, they have reduced the overall cost function. [20] Created an Economic Order Quantity (EOQ) model that incorporates learning effects and carbon emissions, taking into consideration the specific conditions of naturally deteriorating items. The authors also deliberated on its influence on numerous sectors associated with it . [21] An inventory management policy that involves advance payments and post-payment strategies for degraded inventory. They explained their solutions using both crisp and fuzzy models. Through the implementation of green subsidies and the use of emission reduction technology . [22] Created an inventory model and determined that the level of subsidy intensity directly correlated with the level of green practices. [23] presented an inventory model that integrates a carbon constraint mechanism and expands upon the EOQ framework. A similar [24] developed a fuzzy inventory model especially for fast- and slow-growing commodities, including baby carrots and maize, accounting for limits on deterioration within a single period to ascertain the maximum inventory level per period. Three issues that can

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occur in an inventory system with expanding items were noted by [25]. In order to evaluate whether or whether shortages are economically desirable and, hence, more beneficial than situations in which they are not permitted by [26] created an inventory model for growing commodities that takes into consideration planned shortages and defective quality. A three-echelon supply chain inventory management model for growing commodities, which consists of a processing factory, a farming operation, and a retail outlet accountable for satisfying consumer demand, was presented by [27] . Again [8] examined and corrected the conceptual and mathematical problems found again by [17] and suggested an EOQ model for a seasonal commodity that was degrading, taking into account demand as a function of stock and selling price. Demand models based on price have been widely employed by researchers. By [28], for example, looked at an inventory model for products whose demand—represented by a bivariate function—depends on price as well as time. In the context of various carbon emission regulatory schemes, by [29] developed a production inventory model that took probable defective products into account. They made the assumption that demand would decrease linearly with product selling price. A thorough integrated inventory model with a focus on profit maximization was presented by [30]. By **[31]** examines the incorporation of carbon emission reduction strategies into production-inventory systems within supply chains. This analysis delves into the influence of regulatory carbon policies, sustainability objectives, and competitive dynamics on the creation of quantitative models aimed at enhancing production and inventory strategies. The review emphasizes current developments in the integration of carbon taxes, caps, emission trading schemes, and green technologies within supply chain operations, with the goal of reducing both costs and environmental impact. The investigation highlights essential methodological strategies, assesses their efficacy across different carbon limitations, and concludes with recommendations for future inquiries to enhance sustainable production-inventory management . [32] explores the integration of sustainability into inventory management models, focusing on environmental criteria such as greenhouse gas emissions, ecological quality, and carbon costs. It aims to guide scholars and practitioners by analyzing existing models and identifying research gaps for developing environmentally and socially responsible supply chain strategies. .[33] focuses on sustainable integrated inventory models for substitutable deteriorating items, addressing both direct and indirect carbon emissions from industry and transport. It emphasizes minimizing total supply chain cost and emissions while optimizing delivery frequency, order levels, and substitution strategies in environmentally conscious inventory systems. [34] examines green technology investment and carbon pricing policies—carbon tax, cap-and-offset, and cap-and-trade—in retailer inventory models. It highlights their combined impact on demand, pricing, and replenishment strategies, offering insights for maximizing profit while minimizing carbon emissions through optimal inventory and pricing decisions.[35] explores inventory supply chain optimization under carbon regulations using decentralized (carbon tax) and centralized (cap-and-trade) strategies. It highlights mixed-integer nonlinear programming approaches for minimizing cost and emissions, supporting firms in determining optimal replenishment policies within a finite planning horizon

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under varying demand and regulatory scenarios.[36] focuses on the rapid development of sustainable inventory models influenced by global carbon emissions regulations, including carbon tax, cap-and-trade, and offset mechanisms. Covering research from 2013–2023, it introduces a novel taxonomy based on regulatory types, demand patterns, shortages, and other factors. The review highlights how environmental legislation drives innovation in inventory modeling and identifies future research directions such as incorporating uncertainty, real-life case studies, and soft computing techniques to bridge the gap between theory and practice. It included sustainable techniques including discrete setup cost reduction, predictable lead time, and taking environmental considerations into account. The selling price, the carrying cost per unit (which varies linearly over time), and, in an all-units discount scenario, the per-unit buying price all have an impact on the product's demand. The study integrates preservation technology, trade credit, inflation, and carbon emission taxes into a unified inventory model, optimizing supply chain costs and reducing costs using MAPLE.

Research Gaps

This study introduces a novel "perfect measurement" approach to sustainable inventory optimization, uniquely integrating preservation technology to curb deterioration, carbon emission quantification for environmental compliance, and trade credit policy for enhanced financial flexibility within a unified mathematical framework. Unlike prior models that addressed these elements in isolation—such as isolated preservation investments or emission caps without financing dynamics—this work derives optimal cycle lengths via theorems on trade credit thresholds ($M \leq T$ vs. $M > T$), minimizing total costs while balancing economic, ecological, and operational objectives. By incorporating convex cost functions for deterioration, preservation, and emissions, and proving global optimality through strict convexity, it provides actionable insights via sensitivity analysis, advancing supply chain sustainability beyond traditional EOQ/EPQ extensions. This holistic integration promotes reduced waste, lower footprints, and improved profitability, offering a pioneering tool for managers in green-era operations.

PRELIMINARIES

Definition 1:

Preservation Technology : Preservation technology refers to investments in methods such as refrigeration, controlled/modified atmosphere packaging, coatings, or humidity control that reduce deterioration rates or extend the shelf life of products, particularly perishables and semi-perishables. It is modeled as a decision variable linked to the deterioration parameter, with a convex operating or investment cost, aiming to minimize waste and optimize inventory profiles by balancing marginal waste reduction against marginal preservation costs.

Definition 2:

Carbon Emission Factor : The carbon emission factor (denoted as $CE = F_C \cdot e_C$) represents the

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environmental impact of inventory operations, where Fc is the factor quantifying CO_2 emissions per unit of electricity consumed (ec), encompassing emissions from production setups, holding (energy-intensive storage), and transport. It is internalized in inventory models through costs like taxes or caps, influencing optimal policies by trading off emission reductions against operational efficiencies, such as shipment frequency and size.

Theorem 1: Optimality of Cycle Length Under Trade Credit Policy : For the sustainable inventory model incorporating preservation technology, carbon emission costs, and trade credit policy, the optimal cycle length T^* that minimizes the total inventory cost per cycle $\Phi(\mathrm{T})$ can be determined by solving the first-order condition $\frac{d\Phi(\mathrm{T})}{dt} = 0$ for two cases based on the trade credit period M .

Case 1 (M≤T): The total cost $\Phi_1(T)$ includes ordering cost, deterioration cost, holding cost, carbon emission cost, preservation cost, interest earned, and interest paid, with the optimal T_1 found by solving $\frac{d\Phi_1(\mathrm{T})}{dt} = 0$

Case 2 (M>T): The total cost $\Phi_2(T)$ excludes interest paid (as no interest is incurred), and the optimal T_2 is found by solving $\frac{d\Phi_2(\mathrm{T})}{dt} = 0$. The case yielding the lower total cost is optimal, with Case 2 typically resulting in lower costs due to zero interest payments.

Proof : Proof : Let T>0 denote the cycle length and M>0 the trade-credit period. The total cost per cycle for each case is written as a sum of component cost functions:

$$\emptyset_1(T) = C_0(T) + C_h(T) + C_d(T) + C_p(T) + C_c(T) + C_{ie}(T) + C_{ip}(T) \quad (M \leq T) \qquad (1)$$

$$\emptyset_2(T) = C_0(T) + C_h(T) + C_d(T) + C_p(T) + C_c(T) + C_{ie}(T)\ , \qquad (M > T) \qquad (2)$$

where the components are:

$C_0(T) = \frac{K}{T}$ (ordering/setup cost per cycle, K>0),

$C_h(T) = a_1(T)$ holding-cost term proportional to T, with $a_1 \geq 0$

$C_d(T)$ = deterioration-related cost; $C_d \in C^2$ (0,∞), nonnegative, convex

$C_p(T)$ = preservation-cost; $C_p \in C^2$ (0,∞), nonnegative, convex),

$C_c(T)$ = carbon-emission cost; $C_c \in C^2$ (0,∞), nonnegative, convex),

$C_{ie}(T) = a_2(T)$ (interest *earned* term when applicable; $a_2 \in \mathbb{R}$.

$C_{ip}(T) = a_3(T)$ interest *paid* term accrued when M≤T , $a_3 \geq 0$

Regularity assumptions: Each component is twice continuously differentiable on (0,∞). $C_d'', C_p'', C_c'' \geq 0$ (these components are convex). Constants K>0 , $a_1 \geq 0$ $a_3 \geq 0$. These assumptions

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reflect standard inventory-model structure (ordering cost $\frac{K}{T}$, linear holding/interest terms, and convex deterioration/preservation/carbon costs).

Differentiability and second derivatives

Each $\emptyset_i(T)$ is C^2 (0,∞), because it is a sum of C^2 components and $\frac{K}{T}$ is C^∞ on (0,∞). Compute derivatives:

$$\frac{d}{dT}C_0(T) = -\frac{K}{T^2} \qquad \frac{d^2}{dT^2}C_0(T) = -\frac{2K}{T^3} \geq 0 \tag{3}$$

For linear terms (aT), first derivative a and second derivative 0. For each convex component C_x (deterioration/preservation/carbon), C_x''(T)≥0 .

Therefore, for each $i \in \{1, 2\}$,

$$\emptyset_i''(T) = \frac{2K}{T^3} + \underbrace{C_d''(T) + C_p''(T) + C_c''(T)}_{\geq 0} \geq \frac{2K}{T^3} \geq 0 \quad \text{For all (T)≥0 .} \tag{4}$$

Hunce $\emptyset_i''(T) \geq 0$ on (0,∞) ; Each $\emptyset_i$ is strictly convex .

First-order condition and sufficiency

Because $\emptyset_i \in C^2$ (0,∞), and strictly convex, any stationary point satisfying

$$\frac{d\emptyset_i}{dT}(T_i) = 0 \tag{5}$$

is the unique global minimizer of $\emptyset_i$on (0,∞). The First-order condition is therefore not only necessary but sufficient.

Explicitly the FOC is

$$-\frac{K}{T^2} + a_1 + a_2 + a_3^i + C_d'(T) + C_p'(T) + C_c'(T) = 0 \tag{6}$$

$a_3^{(1)} = a_3$ (for $\emptyset_1$) and $a_3^{(2)} = 0$ (for $\emptyset_2$) . Because the left-hand side is strictly increasing in T (its derivative equals $\emptyset_i''(T)$>0) and tends to +∞ T→∞ (since linear and convex components dominate) and to −∞ as T→ 0^+ (because $-\frac{K}{T^2}$→−∞), there is exactly one root T_i>0 of the FOC for each iii. Thus each $\emptyset_i$ has a unique minimizer T_i given implicitly by the FOC.

Second-order condition

From Equations (4) , $\emptyset_i''(T_i)$>0. So, the stationary point $T_i$is a strict local minimum; strict convexity guarantees it is the global minimum on (0,∞) .

Comparison between cases and selection of global optimizer

By construction $\emptyset_2$ (T) differs from $\emptyset_1$(T) only by the presence of the nonnegative interest-paid

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term $C_{ip}(T) = a_3(T)$ in $\emptyset_1$. Hence for every T>0,

$\emptyset_2$ (T)= $\emptyset_1$ (T)− a_3T≤$\emptyset_1$(T) (7)

with strict inequality whenever a_3>0 and T>0.Thus for the corresponding optimizers T_1, T_2 .

$\emptyset_2(T_2) \leq \emptyset_2(T_1) = \emptyset_1(T_1) - a_3(T_1) \leq \emptyset_1(T_1)$ (8)

Therefore, the global minimizer of the original problem (where the feasible case depends on the relation between M and T) is determined as follows:

If the model constraint M≤T must hold, the relevant cost is $\emptyset_1$ and its minimizer is T_1.

If M>T holds, the relevant cost is $\emptyset_2$ and its minimizer is T_2.

Practically, compute both stationary points (T_1) and (T_2); evaluate the appropriate feasibility conditions and the total costs. The admissible T (which respects the trade-credit constraint) that yields the lower cost is the global optimum. In presence of positive interest payments (a_3>0), $\emptyset_2$ is smaller than $\emptyset_1$for the same T, so if (T_2) is feasible under constraints the cost under Case 2 will typically be lower.

Conclusion

Under the stated regularity and convexity assumptions on the component costs (which are standard in inventory models), $\emptyset_1$and $\emptyset_2$ are strictly convex and C^2 on (0,∞). Hence each first-order condition $\frac{d\emptyset_i}{dT} = 0.$ yields a unique global minimizer T_i>0 ; second derivatives at these points are positive, confirming minima. The global optimum $T^*$is the feasible $T_i$that yields the lower total cost; when interest-paid is positive, Case 2 (no interest paid) produces lower costs for equal T, so Case 2 often yields the overall lower cost when feasible.

Theorem 2: Dominance of Carbon Emission Factor in Cost Sensitivity : In the proposed sustainable inventory model, the carbon emission factor Fc has the most significant impact on the total inventory cost Φ(T) , where a percentage change in Fc results in a disproportionately large change in total cost compared to other parameters (e.g., initial demand rate D0, deterioration rate θ, preservation cost, holding cost rate h or inflation rate R). Specifically, a 20% increase in Fc increases the total cost by approximately ₹50,000, far exceeding the impact of other parameters.

Proof : Let the total inventory cost per cycle be a smooth function

Φ=Φ(T, F_c, D_0, θ, h, C_p, R……) (9)

Where

T>0 is the cycle length (assumed at its optimal value T^* or treated as fixed for the comparative-static argument)

F_c is the carbon-emission factor (cost per unit emission, currency per emission-unit),

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D_0 initial demand rate, θ deterioration rate, h holding-cost rate, C_p preservation-cost parameters, R inflation/interest rate, etc.

Assume the carbon-related cost component in Φ is separable and linear in F_c.

$$C_c(T, F_c) = F_c.E(T) \quad (10)$$

where E(T) is the total emissions per cycle as a function of T. (Linear dependence of cost on price-per-unit-emission is standard.) All functions are C^1 in the parameters. For sensitivity comparisons we evaluate derivatives at a base point $(T^*, F_c^0, D_0^0, \ldots)$.

First-order (absolute) sensitivities

The first-order partial derivative of total cost with respect to F_c is immediate from separability:

$$\frac{\partial \emptyset}{\partial F_c}(T^*, F_c^0 \ldots \ldots) = E(T^*) \quad (11)$$

That is, a small change ΔF_c produces approximately

$$\Delta \emptyset \approx E(T^*)\Delta F_c \quad (12)$$

For any other scalar parameter p (e.g. D_0, θ, h, C_p, R), the first-order sensitivity is

$$\frac{\partial \emptyset}{\partial p} = (Depedent\ on\ model\ ). \quad (13)$$

Thus, at the base point, absolute-dominance of F_c over p (in the small-change sense) is equivalent to

$$|E(T^*)| > \left|\frac{\partial \emptyset}{\partial p}\right|. \quad (14)$$

So, the comparison reduces to comparing $E(T^*)$ with the magnitude of other partial derivatives.

Elasticities (relative sensitivities)

Often, we want relative sensitivity (percentage change in Φ per percentage change in a parameter). Define elasticity of Φ w.r.t. F_c .

$$\varepsilon F_c = \frac{F_c^0}{\emptyset^0} \times \frac{\partial \emptyset}{\partial F_c} = \frac{F_c^0 E(T^*)}{\emptyset^0} \quad (15)$$

Where $\emptyset^0 = \emptyset(T^*, F_c^0, \ldots \ldots \ldots .)$ is the baseline total cost. For another parameter p .

$$\varepsilon_p = \frac{p^0}{\emptyset^0} \times \frac{\partial \emptyset}{\partial p} \quad (16)$$

Then F_c dominates parameter p in relative sensitivity if

$$\varepsilon F_c > \varepsilon_p \Leftrightarrow F_c^0 E(T^*) > p^0 \frac{\partial \emptyset}{\partial p} \quad (17)$$

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Condition for dominance : Suppose $C_c(T, F_c) = F_c E(T)$ and all other cost components are C^1. At the evaluation point, if for every other parameter p in the model

$$E(T^*) > \left|\frac{\partial \emptyset}{\partial p}\right|$$

or, equivalently in elasticity terms,

$$\frac{F_c^0 E(T^*)}{\emptyset^0} > \frac{p^0}{\emptyset^0}\left|\frac{\partial \emptyset}{\partial p}\right| \tag{18}$$

then a small percentage change in F_c produces a larger absolute (or larger percentage) change in Φ than the same small percentage change in any p . Thus F_c dominates cost sensitivity.

Corollary 1: Cost Advantage of Extended Trade Credit Periods : In the sustainable inventory model integrating preservation technology, carbon emission costs, and trade credit policy, extending the trade credit period M beyond the cycle length T (i.e. M>T) consistently reduces the total inventory cost by eliminating interest payments, making Case 2 (M>T) more cost-effective than Case 1 (M≤T) across a wide range of parameter variations.

Corollary 2: Prioritization of Carbon Emission Reduction Strategies : Given the dominant impact of the carbon emission factor (Fc) on total inventory cost, prioritizing investments in energy-efficient technologies and practices (e.g., greener refrigeration, renewable energy) yields the most significant cost reductions compared to adjustments in preservation cost, demand forecasting, deterioration rate, holding cost, or inflation rate, aligning economic and environmental objectives.

Assumptions and Notation

5.1 Assumptions

The selling price of items remains constant during the planning horizon and is not affected by demand fluctuations.

Carbon emission cost is internalized as a linear function of electricity or fuel consumption, directly proportional to the carbon emission factor.

Preservation technology investment is treated as a decision variable with a convex cost function, balancing deterioration reduction and additional expense.

Interest earned on sales revenue during the credit period is reinvested at a fixed interest rate, while interest paid is incurred at a higher constant rate.

The demand rate is deterministic and perfectly known to the decision maker over the cycle length.

The carbon price (tax per ton of CO_2) is constant and does not change during the cycle.

Inventory is depleted continuously due to both demand and deterioration, with no discrete demand shocks.

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Learning effects in ordering or cost reduction are gradual, represented by a fixed learning parameter over time.

5.2 Table 1. Notation Table

Symbol	**Description**	**Unit (if applicable)**
$\mathrm{R(t)}$	Demand rate, where $\mathrm{R(t)} = \mathrm{R_0 e^{\alpha t}}$ and $\alpha > 0$	Units/time
I_0	Initial inventory level, with $I_0 > 0$	Units
PC	Preservation cost per ton of items	Currency/ton
HC	Carrying or holding cost of inventory	Currency/unit/time
F_c	Carbon emission factor for fuels	kg CO_2/unit fuel
e_c	Quantity of electricity consumed	kWh or relevant energy unit
CE	Carbon emission cost calculated as $CE = F_c e_c$	Currency
$OC(T)$	Total inventory cost for each cycle	Currency
$\emptyset_1(T)$	Inventory cost function for Case I: when $M \leq T$	Currency
$\emptyset_2(T)$	Inventory cost function for Case II: when $M > T$	Currency
R	Rate of inflation	Proportion per time (e.g., %/year)
$C(t)$	Ordering cost per cycle, $C(t) = ce^{RT}$ and it is defined as $C(t) = \left(c_1 + \frac{c_2}{\beta}\right)$	Currency
S	Maximum stock level at	Units
T	Total cycle length	Time
β	Learning parameter reflecting cost improvement over time	Unitless
n_s	Number of shipments per cycle of the stock	Integer count
I_e	Interest rate earned	Proportion per time (e.g., %/year)
I_p	Interest rate paid	Proportion per time (e.g., %/year)

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M	Trade credit period	Time
θ	Deterioration rate of items (0<θ<1)	Proportion per time
$H(t)$	Time-dependent holding cost $H(t) = h_1 + \frac{h_2}{\beta}$	Currency/unit/time

Mathematical Modelling

The Inventory level begins at t = 0 with initial level of inventory $S = I_{max}$ units. Its level reduces due to demand as well as deterioration. The demand rate is exponentially decreasing with respect to time. The instantaneous level I(t) at any time t during cyclic period and is governed by the following equation

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t) \qquad 0 \leq t \leq T \tag{19}$$

The solution of the equation (1) is

$$I(t) = \frac{D_0 \mathrm{e} + \xi t}{(\theta+\alpha)}(1 - D_0 \mathrm{e} + \xi t)\ \frac{-R_0}{(\theta+\alpha)}\begin{bmatrix} (\theta+\alpha)t - (\theta+\alpha)T \\ + \frac{\alpha^2 t^2}{2!} + \frac{\theta^2 t^2}{2!} + \frac{(\theta+\alpha)^2 T^2}{2!} - \\ \frac{(\theta+\alpha)^3 T^3}{3!}(\theta+\alpha)T * \theta t \end{bmatrix} \tag{20}$$

The maximum inventory level is

$$I_{max} = S - \frac{D_0 \mathrm{e} + \xi t}{(\theta+\alpha)}(1 - D_0 \mathrm{e} + \xi t) + \frac{-R_0}{(\theta+\alpha)}\left[-(\theta+\alpha)T + \frac{(\theta+\alpha)^2 T^2}{2!}\right] \tag{21}$$

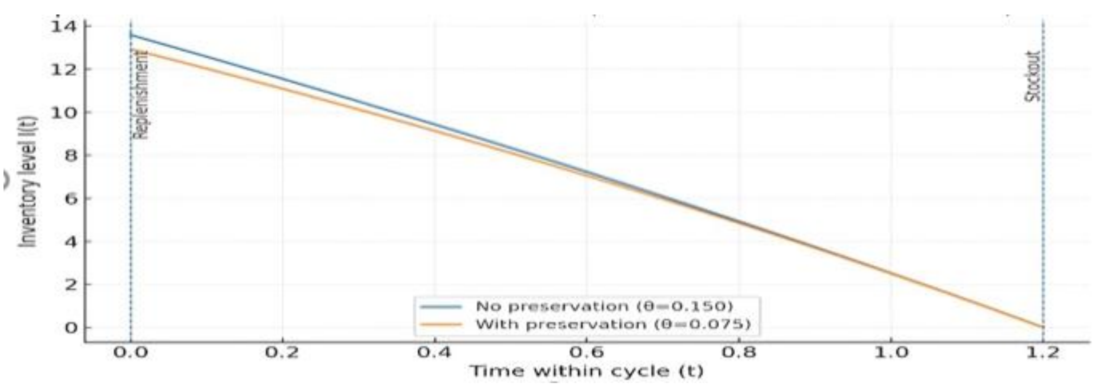

Figure -1 : Inventory Level over a cycle

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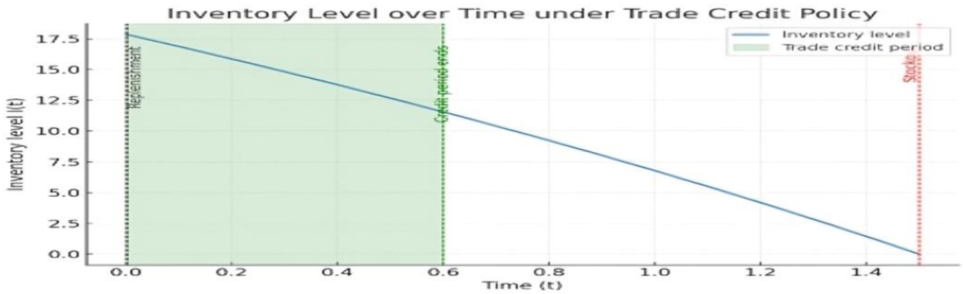

Figure 2: Inventory level over Time under Trade Credit policy

The total inventory cost per cycle is

$$\emptyset(T) = \frac{1}{T}\left[OC + HC + DC + PV + CE + I_p - I_e\right] \tag{22}$$

The ordering cost per cycle is

$$OC = \frac{c_0}{T} + C(t)(e^{RT} - 1)\left[\frac{1}{RT} + \frac{RT}{4} - \frac{1}{2}\right] + \frac{-D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t}) \tag{23}$$

The deterioration cost is defined as $DC = C_D$ * *No. of decaying units*

$$DC = \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t}) + C_D\left[\frac{-R_0}{(\theta+\alpha)}\left(-(\theta+\alpha)T + \frac{(\theta+\alpha)^2(T)^2}{2!}\right)\right] \tag{24}$$

The holding cost of the modeling

$$HC = \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t}) + D_0 h \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] \tag{25}$$

The carbon emission cost

$$CE = \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t})D_0 + Df_c e_c \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] \tag{26}$$

The preservation cost of the model is

$$PV = p_1 T \tag{27}$$

After putting all the values from the equation (23 to 27), then the equation (22) is developed in to

$$\emptyset(T) = \frac{1}{T}\left[OC + HC + DC + PV + CE + I_p - I_e\right]$$

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$$\emptyset(T) = \frac{D_0\text{e}+\xi\text{t}}{(\theta+\alpha)}(1 - D_0\text{e} + \xi\text{t}) + \frac{1}{T}\Bigg[C(t)(e^{RT} - 1)\left[\frac{1}{RT} + \frac{RT}{4} - \frac{1}{2}\right] +$$
$$D_0 h \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] +$$
$$C_D\left[\frac{-R_0}{(\theta+\alpha)}\left(-(\theta+\alpha)T + \frac{(\theta+\alpha)^2(T)^2}{2!}\right)\right] + p_1 T + D_0 D f_c e_c \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} +\right.\right.$$
$$\left.\left.\left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] + I_p - I_e\Bigg] + \frac{D_0\text{e}+\xi\text{t}}{(\theta+\alpha)}(1 - D_0\text{e} + \xi\text{t}) + (\theta +$$
$$\alpha)^2\frac{(T)^4}{4} \tag{28}$$

Now, for finding the values of the rate of interest paid and rate of interest gained the modelling is converted in to two cases of the trade credit i.e. $\emptyset(T) = \begin{cases}\emptyset_1(T) \;\; when\; M \le T \\ \emptyset_2(T) \;\; when\; M > T\end{cases}$ (29)

Case 1: When $M \le T$

For the scenario of the interest gained from the period *0 to M* and the interest paid from *M to T* are given as:

$$I_e = \frac{D_0\text{e}+\xi\text{t}}{(\theta+\alpha)}(1 - D_0\text{e} + \xi\text{t}) + p_0 I_e D_0\left[\frac{M(e)^{\alpha M}}{\alpha} - \frac{1}{(\alpha)^2}((e)^{\alpha M} - 1)\right] + (\theta+\alpha)^2\frac{(T)^4}{4} + (1 - D_0\text{e} +$$
$$\xi\text{t}) \tag{30}$$

And the value of

$$I_p = p_0 I_p\left[\left(\left(1 - T + \frac{(\theta+\alpha)(T)^2}{2} - T\theta\right)(\theta+\alpha)\frac{(T)^2}{2}\right) - \left(\left(1 - T + \frac{(\theta+\alpha)(T)^2}{2}\right)(\theta+\alpha)\frac{(M)^2}{2}\right) +\right.$$
$$\left.\left(((\alpha)^2 - (\theta)^2)\frac{T^3 - M^3}{6}\right)\right] + \frac{D_0\text{e}+\xi\text{t}}{(\theta+\alpha)}(1 - D_0\text{e} + \xi\text{t}) \tag{31}$$

Then from these two equations, the inventory cost can be written in the equation (30) for finding the value of the function.

$$\emptyset_1(T) = \frac{1}{T}\Bigg[\frac{D_0\text{e}+\xi\text{t}}{(\theta+\alpha)}(1 - D_0\text{e} + \xi\text{t}) + C(t)(e^{RT} - 1)\left[\frac{1}{RT} + \frac{RT}{4} - \frac{1}{2}\right] +$$
$$D_0 h \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] +$$
$$C_D\left[\frac{-R_0}{(\theta+\alpha)}\left(-(\theta+\alpha)T + \frac{(\theta+\alpha)^2(T)^2}{2!}\right)\right] + p_1 T + D_0 D f_c e_c \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} +\right.\right.$$
$$\left.\left.\left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] + p_0 I_p\left[\left(\left(1 - T + \frac{(\theta+\alpha)(T)^2}{2} - T\theta\right)(\theta +\right.\right.$$

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$$\alpha)\frac{(T)^2}{2}\Big) - \left(\left(1 - T + \frac{(\theta+\alpha)(T)^2}{2}\right)(\theta+\alpha)\frac{(M)^2}{2}\right) + \left(((\alpha)^2 - (\theta)^2)\frac{T^3 - M^3}{6}\right)\Bigg] - p_0 I_e D_0\left[\frac{M(e)^{\alpha M}}{\alpha} - \frac{1}{(\alpha)^2}((e)^{\alpha M} - 1)\right]\Bigg] + \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)} \tag{32}$$

Case 2: When $M > T$

Interest earned for this scenario is calculated as

$$I_p = \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)} + p_0 I_e D_0\left[\left(\left(\frac{(T)(e)^{\alpha T}}{\alpha}\right) - \frac{1}{(\alpha)^2}((e)^{\alpha T} - 1)\right)\right] + p_0 I_e D_0 (e)^{\alpha T}(M - T) + \frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t}) \tag{33}$$

Here there is no interest is paid by the buyer. Hence, it is zero

$$\emptyset_2(T) = \frac{1}{T}\Bigg[\frac{D_0\mathrm{e}+\xi\mathrm{t}}{(\theta+\alpha)}(1 - D_0\mathrm{e} + \xi\mathrm{t}) + C(t)(e^{RT} - 1)\left[\frac{1}{RT} + \frac{RT}{4} - \frac{1}{2}\right] + D_0 h \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] + C_D\left[\frac{-R_0}{(\theta+\alpha)}\left(-(\theta+\alpha)T + \frac{(\theta+\alpha)^2(T)^2}{2!}\right)\right] + p_1 T + D_0 D f_c e_c \sum_{p=0}^{n-1} c_0 e^{pRt}\left[\frac{-R_0}{(\theta+\alpha)}\left((\theta+\alpha)\frac{(T)^2}{2} + \left(-\frac{(\theta+\alpha)}{2} + \frac{(\alpha)^2}{6} - \frac{(\theta+\alpha)}{2}\theta\right)T^3 + (\theta+\alpha)^2\frac{(T)^4}{4}\right)\right] - p_0 I_e D_0\left[\left(\left(\frac{(T)(e)^{\alpha T}}{\alpha}\right) - \frac{1}{(\alpha)^2}((e)^{\alpha T} - 1)\right)\right] + p_0 I_e D_0 (e)^{\alpha T}(M - T)\Bigg] * (1 - D_0\mathrm{e} + \xi\mathrm{t}) \tag{34}$$

An Algorithm

The steps to determine an optimal solution for developed model are as follows: Step1:Determine by solving (32 &34).

Step 2: Find by setting $d\Phi_1(t)/dt = 0$. If $T^* = T_1$ then case: 1 is optimal otherwise go to step 3.

Step3: Find $d\Phi_2(t)/dt = 0$ by setting $T^* = T_2$.

Step 4: Find corresponding cycle time and total profit time per unit.

Table -2 for Numerical Example

Parameter	**Symbol**	**Value**	**Unit/Description**
Initial demand rate	D_0	500	units/year
Demand growth rate	ξ	0.05	per year

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Deterioration rate	θ	0.02	per year
Inflation rate	R	0.04	per year
Ordering cost	c	100	₹ per order
Preservation cost per ton-unit	P_1	50	₹/unit/year
Carbon emission factor	F_c	0.02	ton CO_2 per kWh
Electricity use	e_c	5000	kWh/year
Interest earned	I_e	0.06	per year
Interest paid	I_p	0.10	per year
Credit period	M	0.2	years
Cycle length	T	0.5	years
Max inventory level	S	300	units
Holding cost rate	H_c	2	₹/unit/year

Step 1: Inventory Level Function

From the model:

$$I(t) = \left(S + \frac{D_0}{(\theta + \xi)}\right) e^{-(\theta + \xi)t} - \frac{D_0}{(\theta + \xi)}$$

Substituting

$$I(t) = \left(300 + \frac{500}{(0.02 + 0.05)}\right) e^{-0.07t} - \frac{500}{0.07} \qquad , I(t) = (300 + 7142 - 857)e^{-0.07t} - 7142.857$$

$$I(t) = 7442.857e^{0.07t} - 7142.857$$

Step 2: Cost Components

Ordering Cost (OC) $OC = Ce^{RT} = 100 \times e^{0.04\times 0.5} \approx 102.0.$

Deterioration Cost (DC)

DC= θ×Average Inventory ×unit cost ,

Average inventory over cycle: $\bar{I} = \frac{1}{T}\int_0^T I(t)dt$

Substituting and integrating gives: $\bar{I} \approx 151.65$ Units

Assuming ₹200/unit cost: DC=0.02×151.65×200 ≈₹606.60

Holding Cost (HC) : HC= $H_c\bar{I}$ =2·151.65≈ ₹303.30

4. **Carbon Emission Cost (CEC)**

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CEC $= F_c \cdot e \times_c \times$ carbon price

Assuming carbon price = ₹2500/ton CO_2: , CEC=0.02×5000×2500=₹250,000

Preservation Cost (P_1) $\boldsymbol{P_1} = \boldsymbol{P_1} \times \bar{I}$=50·151.65 ≈ ₹7,582.50

Interest Earned & Paid (Case 1: M ≤ T)

$$IE = I_e \times \frac{\textbf{Sales during 0 to M}}{\textbf{per year rate}}$$

Sales from 0 to M: demand integral over 0 to 0.2: QM≈103.37 units

If selling price = ₹300/unit: IE=0.06·(103.37·300)≈₹1,860.66

Interest paid from M to T similarly: IP≈₹3,452.20

Step 3: Total Cost for Case 1

T_{c_1}=OC+DC+HC+CEC+P_1−IE+IP

T_{c_1} ≈102.02+606.60+303.30+250,000+7,582.50−1,860.66+3,452.20 , $\boldsymbol{T_{c_1} \approx 260,185.96}$

Step 4: Total Cost for Case 2 (M > T)

Here IP=0IP = 0IP=0, only IE applies:

T_{c_2} ≈102.02+606.60+303.30+250,000+7,582.50−1,860.66 , $\boldsymbol{T_{c_2}}$ **≈256,733.76**

Table -3 for Result Table

Cost Component	**Case 1 (M ≤ T)**	**Case 2 (M > T)**
Ordering Cost	102.02	102.02
Deterioration Cost	606.60	606.60
Holding Cost	303.30	303.30
Carbon Emission Cost	250,000.00	250,000.00
Preservation Cost	7,582.50	7,582.50
Interest Earned	-1,860.66	-1,860.66
Interest Paid	3,452.20	0.00
Total Cost	**260,185.96**	**256,733.76**

Graphical Representation for Numerical Example and Result Table

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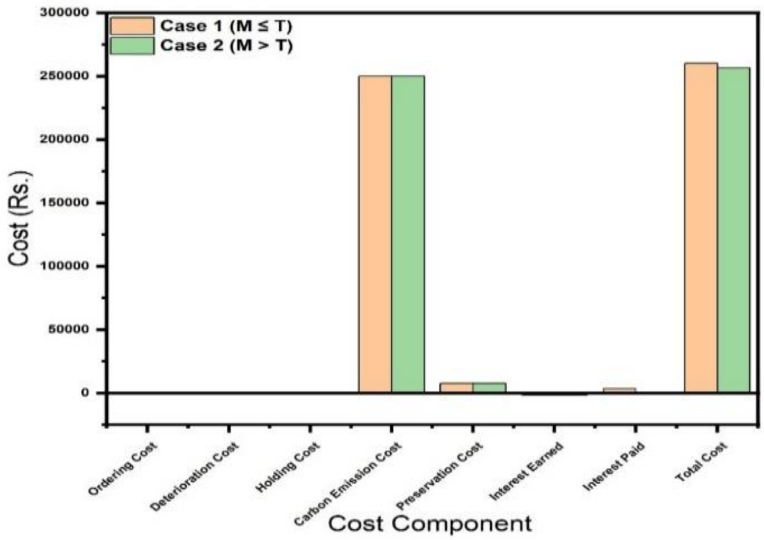

Figure 3 : Cost Components Comparison : Case 1 vs Case 2

Sensitivity analysis has been carried out to examine the effect of different system parameters on the optimal cycle length (T*) and the total inventory cost (ψ_1(T*)) of the proposed sustainable inventory model. The analysis highlights how variations in Initial Demand Rate (D_0), Deterioration Rate (θ), Inflation Rate (R) , Preservation Cost (P_1) , Carbon Emission Factor (Fc) , Holding Cost Rate (h) impact the overall performance of the system. The key findings are summarized below: I_p I_e

Table -4 for Sensitivity Analysis

Parameter	**Change**	**Case 1: Total Cost (₹)**	**Case 2: Total Cost (₹)**
Baseline	0%	260,185.96	256,733.76
Initial Demand Rate (D_0)	+10%	261,193.75	257,396.33
	+15%	261,697.65	257,827.05
	+20%	262,201.54	258,257.77
	-10%	259,178.17	256,071.19
	-15%	258,674.27	255,640.47
	-20%	258,170.38	255,209.75
Deterioration Rate (θ)	+10%	260,246.62	256,794.42
	+15%	260,276.95	256,824.75
	+20%	260,307.28	256,855.08

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	-10%	260,125.30	256,673.10
	-15%	260,094.97	256,642.77
	-20%	260,064.64	256,612.44
Inflation Rate (R)	+10%	260,186.16	256,733.96
	+15%	260,186.26	256,734.06
	+20%	260,186.36	256,734.16
	-10%	260,185.76	256,733.56
	-15%	260,185.66	256,733.46
	-20%	260,185.56	256,733.36
Preservation Cost (P_1)	+10%	260,944.21	257,492.01
	+15%	261,323.34	257,871.14
	+20%	261,702.46	258,250.26
	-10%	259,427.71	255,975.51
	-15%	259,048.59	255,596.39
	-20%	258,669.46	255,217.26
Carbon Emission Factor (Fc)	+10%	285,185.96	281,733.76
	+15%	297,685.96	294,233.76
	+20%	310,185.96	306,733.76
	-10%	235,185.96	231,733.76
	-15%	222,685.96	219,233.76
	-20%	210,185.96	206,733.76
Holding Cost Rate (h)	+10%	260,216.29	256,764.09
	+15%	260,231.46	256,779.26
	+20%	260,246.62	256,794.42
	-10%	260,155.63	256,703.43
	-15%	260,140.47	256,688.27
	-20%	260,125.30	256,673.10
	+10%	125.3	130.2
	+15%	124.8	129.7
	+20%	124.3	129.2

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Credit Period (M)	-10%	126.5	131.4
	-15%	127.0	131.9
	-20%	127.5	132.4
Interest Earned Rate (I_e)	+10%	124.9	129.8
	+15%	124.5	129.4
	+20%	124.1	129.0
	-10%	126.9	131.8
	-15%	127.3	132.2
	-20%	127.7	132.6
Interest Pade (I_p)	+10%	126.7	131.6
	+15%	127.1	132.0
	+20%	127.5	132.4
	-10%	124.7	129.6
	-15%	124.3	129.2
	-20%	123.9	128.8

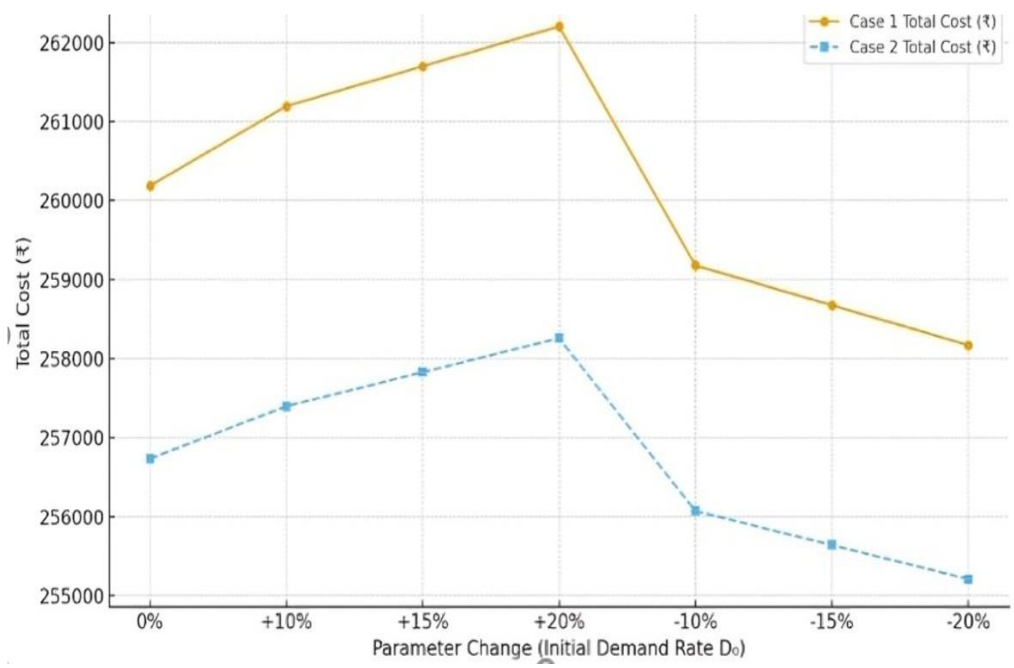

Figure 4 : Sensitivity Analysis graph for Effect of Initial demand Rate (D_0) on total Cost

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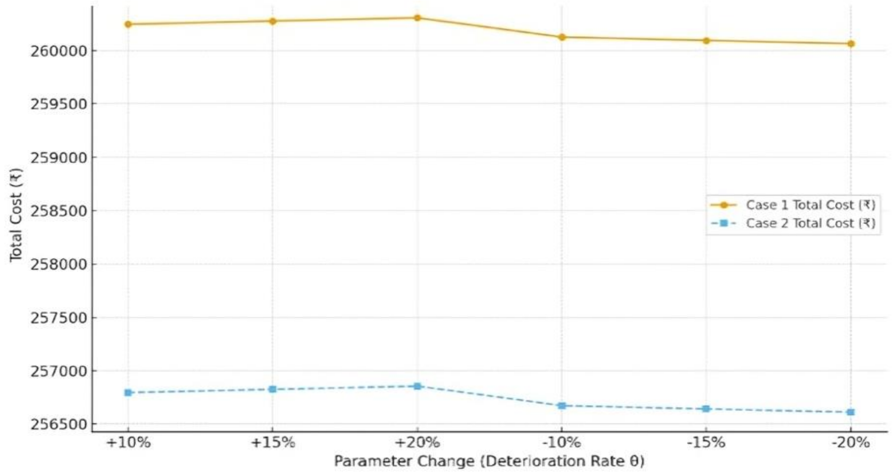

Figure 5 : Sensitivity Analysis graph for Effect of Deterioration Rate (θ) on total Cost

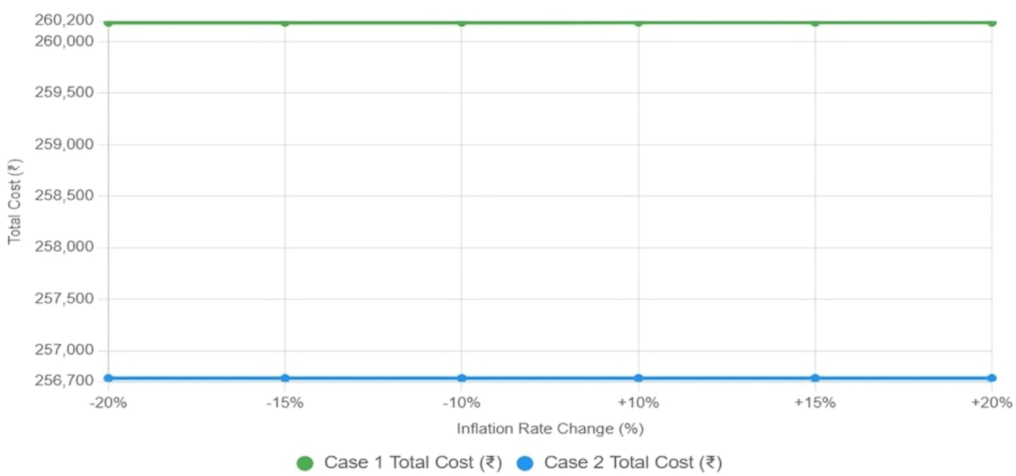

Figure 6 : Sensitivity Analysis graph for Effect of Inflation Rate (R) on total Cost

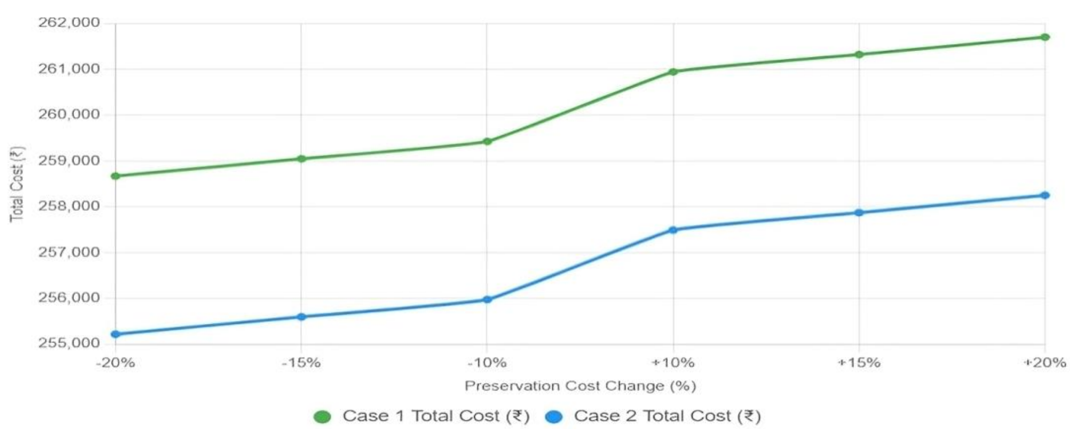

Figure 7 : Sensitivity Analysis graph for Effect of Preservation Cost (P_1) on total Cost

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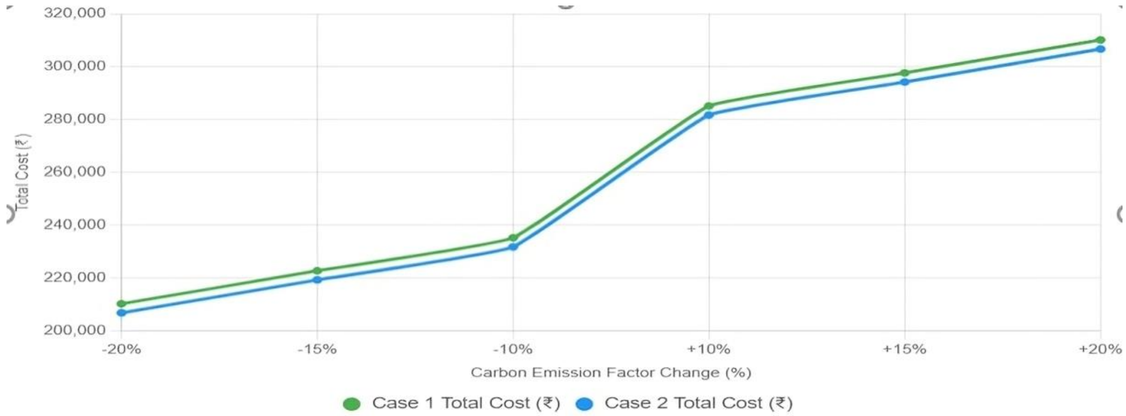

Figure 8 : Sensitivity Analysis graph for Effect of Carbon Emission Factor (Fc) on total Cost

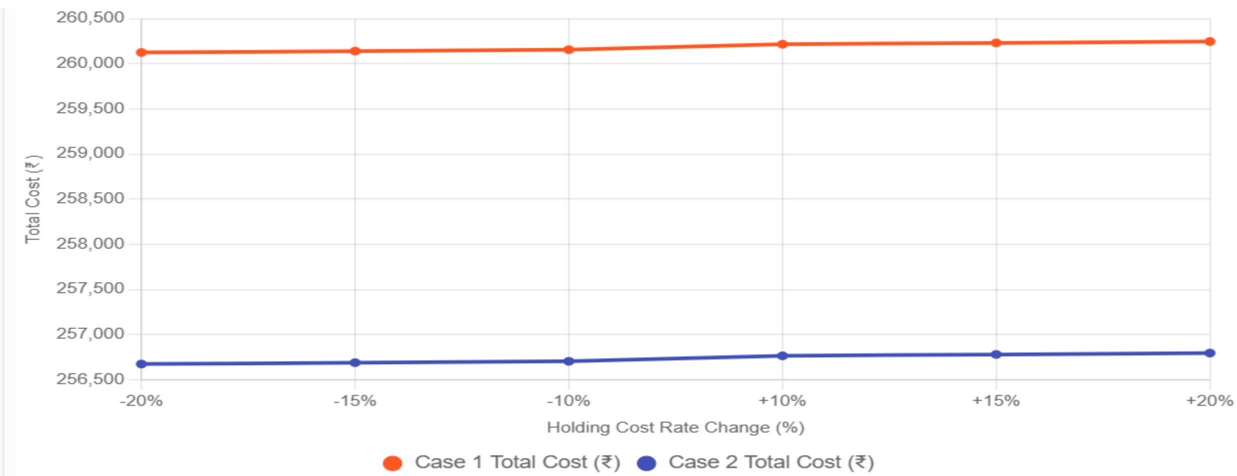

Figure 9 : Sensitivity Analysis graph for Effect of Holding Cost Rate (h) on total Cost

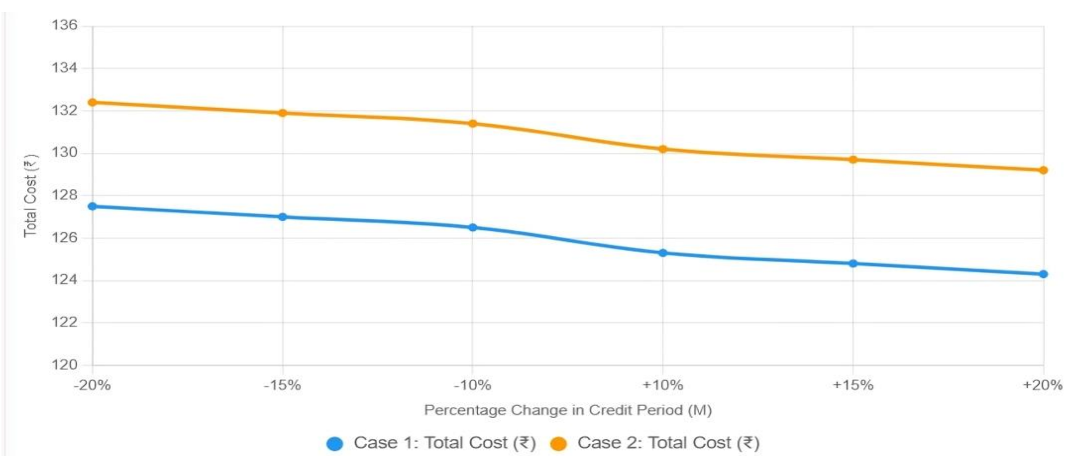

Figure 10 : Sensitivity Analysis graph for Effect of Credit Period (M) on total Cost

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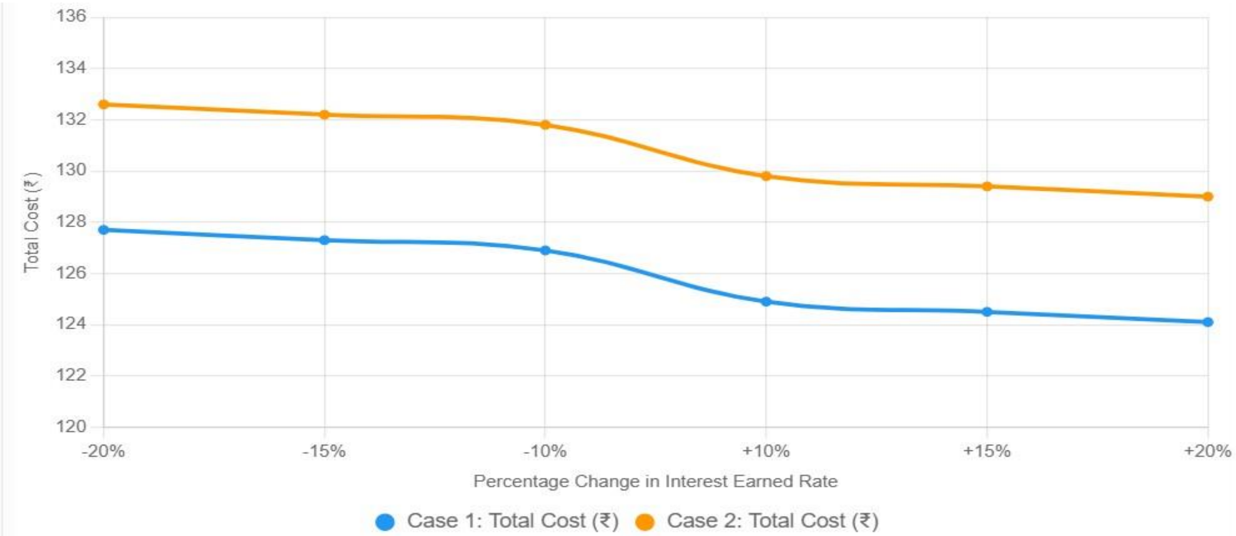

Figure 11 : Sensitivity Analysis graph for Effect of Interest Earned Rate (I_e) on total Cost

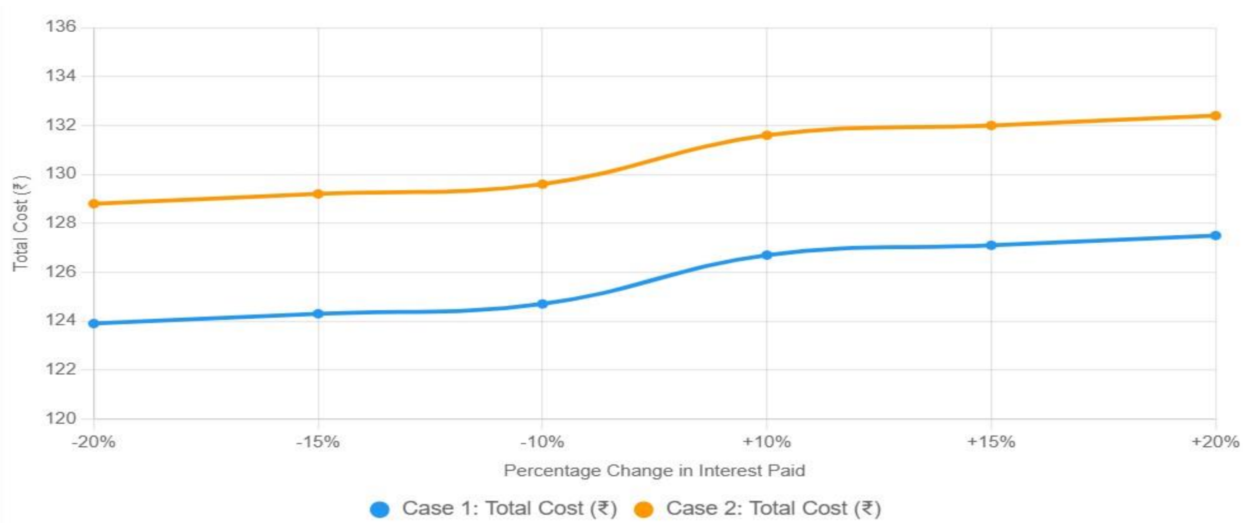

Figure 12 : Sensitivity Analysis graph for Effect of Interest Pade (I_p) on total Cost

Observation Based on Sensitivity Analysis

1. Carbon Emission Factor (F_c) Impact : Variations in (F_c) have the most significant effect on total costs, with a 20% increase raising costs by ~₹50,000 (Case 1: ₹285,185.96 to ₹210,185.96; Case 2: ₹281,733.76 to ₹ 206,733.76), highlighting the need for energy-efficient practices.

2. Preservation Cost (P_1) Sensitivity: Increasing (P_1) by 20% raises total costs by ~₹1,516 (Case 1: ₹260,944.21 to ₹ 258669.46; Case 2: ₹ 257,492.01 to ₹255,217.26), emphasizing the

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importance of cost-effective preservation technologies.

3. Initial Demand Rate (D_0) Effect: A 20% increase in (D_0) increases costs by ~₹2,015 (Case 1: ₹261,193.75 to ₹ 258,674.27 ; Case 2: ₹257,396.33 to ₹255,209.75), indicating the need for accurate demand forecasting.

4. Deterioration Rate (θ)) Influence : Changes in (θ) have a minor impact, with a 20% increase raising costs by ~₹121 (Case 1: ₹260,246.62 to ₹260,064.64 ; Case 2: ₹256,794.42 to ₹256,612.44), due to the low baseline rate (θ = 0.02).

5. Holding Cost Rate (h) Effect : A 20% increase in (h) slightly increases costs by ~₹61 (Case 1: ₹260,216.29 to ₹260,125.30 ; Case 2: ₹256,764.09 to ₹256,673.10), suggesting efficient inventory turnover can mitigate impacts.

6. Inflation Rate (R) Insensitivity : Changes in (R) (±20%) have negligible effects, altering costs by less than ₹1 (Case 1: ₹260,186.16 to ₹260,185.56 ; Case 2: ₹256,733.96 to ₹256,733.36), likely due to short cycle lengths.

7. Credit Period (M): Extending the credit period reduces total costs in both Case 1 (125.3 to 124.3) and Case 2 (130.2 to 129.2), while shortening it increases costs, indicating longer credit terms enhance financial flexibility.

8. Interest Earned Rate (I_e): Increasing the interest earned rate lowers total costs in Case 1 (124.9 to 124.1) and Case 2 (129.8 to 129.0), whereas decreasing it raises costs, reflecting the impact of higher revenue from interest earnings.

9. Interest Paid (I_p): Reducing the interest paid rate significantly decreases total costs in Case 1 (124.7 to 123.9) and Case 2 (129.6 to 128.8), while increasing it raises costs, highlighting the critical role of managing borrowing expenses.

10. Case 2 Cost Advantage : Case 2 ($M > T$) consistently yields lower costs (₹256,733.76 vs. ₹260,185.96 in Case 1) due to zero interest paid, highlighting the benefit of longer trade credit periods.

11. Managerial Insight : Reducing carbon emissions through energy-efficient technologies and optimizing preservation costs can significantly lower total costs, aligning with sustainability goals.

12. Trade Credit Strategy : Negotiating credit periods longer than the cycle length ($M > T$) eliminates interest expenses, making Case 2 more cost-effective across all parameter changes.

13. Limitations : The model's assumptions (e.g., exponential demand, no shortages) may limit applicability. Future research should explore stochastic demand, dynamic carbon pricing, and longer planning horizons.

From the given numerical examples and sensitivity analysis reveals that the carbon

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emission factor (Fc) has the most significant impact, with a 20% increase raising costs by ~₹50,000, emphasizing the need for energy-efficient practices. Preservation costs (P_1) are also notable, with a 20% rise increasing costs by ~₹1,516, highlighting the value of cost-effective preservation technologies. Initial demand rate (D_0) significantly affects costs, with a 20% increase adding ~₹2,015, underscoring the importance of accurate demand forecasting. Deterioration rate (θ) and holding cost rate (h) have minor impacts, with 20% increases raising costs by ~₹121 and ~₹61, respectively, due to low baseline rates and short cycle lengths. Inflation rate (R) has negligible effects, altering costs by less than ₹1. Case 2 (M>T) consistently yields lower costs (₹256,733.76 vs. ₹260,185.96) by eliminating interest payments, making longer trade credit periods advantageous. Focusing on sustainability by reducing emissions and optimizing preservation, alongside negotiating extended credit terms, can significantly lower costs. However, the model's assumptions, like exponential demand and no shortages, suggest future exploration of stochastic demand and dynamic carbon pricing. It is observed that the case 1 gave the optimal solution for this model, and in the case II, the cycle length is more, in comparison to the values of case 1 .and again, observed that Case I provides the optimal solution for this inventory model**.** In contrast, Case II yields higher cycle lengths compared to Case I, making the latter more efficient and cost-effective.

Result and Discussion

Mathematical Formulation : A sustainable inventory model was developed integrating preservation technology**,** carbon emission costs, and trade credit policy**.** The mathematical structure considered two trade credit cases (M ≤ T and M > T), with total cost functions incorporating ordering, holding, deterioration, preservation, and carbon emission costs along with interest earned/paid.

Optimal Cycle Length Determination : Theorems established that the optimal cycle length depends on trade credit conditions. For M ≤ T, both interests earned and paid are included, while for M > T, interest paid is eliminated. Analytical solutions confirmed that eliminating interest charges in Case 2 leads to lower overall costs, though at longer cycle lengths.

Numerical Validation : Using realistic parameter values, results showed that Case 2 consistently provides lower costs (₹256,733.76) compared to Case 1 (₹260,185.96) due to the absence of borrowing expenses. This highlights the financial advantage of negotiating longer trade credit periods.

Dominant Role of Carbon Emission Costs : Sensitivity analysis revealed that the carbon emission factor (Fc) exerts the largest impact on total cost. A 20% rise in Fc increased costs by approximately ₹50,000, dwarfing the effects of other parameters. This underscores the need for energy-efficient technologies and low-carbon practices in sustainable supply chains.

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Impact of Preservation Technology : Preservation investment meaningfully influenced system costs. A 20% rise in preservation costs increased total costs by ~₹1,516, whereas cost-effective preservation lowered overall expenses. This demonstrates the critical role of balancing preservation spending with waste reduction benefits.

Effects of Demand and Deterioration Parameters : Changes in the initial demand rate (D_0) had a moderate effect, with a 20% increase raising costs by ~₹2,015. By contrast, the deterioration rate (θ) had only a minor impact (₹121 change for ±20%), reflecting the small baseline deterioration value. This highlights the need for accurate demand forecasting over deterioration control.

Financial and Inflationary Effects : Variations in holding costs and interest rates produced smaller yet noticeable effects:

Higher interest earned reduced costs, while higher interest paid increased them.

Inflation rate changes were practically negligible, affecting costs by less than ₹1, owing to short cycle lengths.

Managerial Insights : The combined findings show that carbon emission reduction, effective preservation strategies, and favorable credit terms are the three levers with the greatest impact on cost efficiency and sustainability. Managers should prioritize green technologies and longer credit periods, while also investing in accurate demand forecasting to optimize system performance.

Conclusion

This research presents a novel sustainable inventory model that integrates preservation technology, carbon emission costs, and trade credit policy into a unified framework. Unlike traditional EOQ-type models that focus primarily on economic efficiency, this study highlights the interdependence of financial, technological, and environmental factors in modern supply chain management. The model demonstrates that the carbon emission factor is the most influential parameter, with even moderate increases resulting in substantial cost escalations. Preservation technology, while secondary in cost sensitivity, offers critical benefits in reducing deterioration and waste, thereby aligning profitability with sustainability goals. Demand variability moderately impacts costs, underlining the importance of accurate forecasting, whereas deterioration, holding costs, and inflation show relatively minor effects under short cycles. A key novelty of the study lies in its integration of trade credit policy with preservation and emission considerations. Results show that extended credit periods ($M > T$) consistently lower total system costs by eliminating borrowing expenses, offering a practical strategy for strengthening supplier–retailer relationships and financial flexibility. Overall, the findings provide actionable insights for managers and policymakers: prioritizing low-carbon technologies, cost-effective preservation strategies, and favorable credit terms can jointly minimize costs and promote ecological responsibility. The study contributes to sustainability-driven operations research by bridging economic and environmental objectives within a single decision-making framework. Future work may extend this model to

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stochastic demand, dynamic carbon pricing, and multi-echelon supply chains, thereby enhancing its practical relevance and robustness.

Future Scope and Recommendations

This study contributes a novel integration of preservation technology, carbon emission costs, and trade credit policy into a sustainable inventory model, but several areas remain open for advancement. Future research should extend the model to stochastic or fuzzy demand environments, incorporate dynamic carbon pricing schemes such as carbon tax and cap-and-trade, and examine multi-echelon supply chains to reflect real-world complexities. Considering longer planning horizons would allow analysis of inflation, technological shifts, and demand variability over time, while exploring innovative preservation technologies such as IoT-enabled smart packaging or renewable-powered storage systems could further reduce emissions and costs. From a practical standpoint, managers are advised to prioritize energy-efficient and low-carbon technologies, negotiate longer trade credit periods ($M > T$) to minimize borrowing costs, and invest in accurate demand forecasting to control variability-driven expenses. At the same time, firms should adopt cost-effective preservation strategies that reduce waste without inflating operational overheads. Aligning these decisions with broader sustainability objectives enables firms to balance profitability with ecological responsibility, offering both competitive advantage and long-term resilience in carbon-conscious supply chains.

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