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A NEW MULTI-OBJECTIVE ARITHMETIC OPTIMIZATION ALGORITHM

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Abstract:

Today, as engineering problems become more complex in terms of the effective variables in these problems and the range of their changes and their multidimensionality (in terms of not being able to look at a problem from one dimension and its various dimensions must be considered) And the need to trade off on the goals of these issues, especially issues that are in conflict with each other and the need for simultaneous optimization of these goals, the need to use multi-objective optimization methods has become more apparent. For example, in the design of VLSI circuits that have both power and latency, given that these two parameters have opposite behaviors, that is, by increasing one of them, the other decreases, and we need to optimize both of these parameters together. In such problems, using multi-objective optimization methods, the most optimal state for these two parameters can be obtained simultaneously. Since one-objective optimization methods have shown their ability to deal with various engineering problems, so often with the introduction of any one-objective optimization method after a while, the multi-objective optimization method Based on the same method has been presented by researchers to the scientific community to test and evaluate the ability of optimization based on the same single-objective method in the field of multi-objective optimization. For example, the NSGAII multi-objective optimization algorithm is based on the GA single-objective optimization algorithm, the MOPSO algorithm is based on the PSO, the MOGSA algorithm is based on the GSA algorithm, and the MOIPO algorithm is based on the IPO algorithm. Today, more than 350 one-objective methods have been introduced to the scientific community, which are widely used in various researches and examined and tested, and in terms of quantity and quality, the development of these one-purpose methods is a competition field Researchers have provided, and no necessary effort has been made in multi-objective versions, and as many as the single-objective method has been introduced to the scientific community, the multi-objective method has not been introduced based on those methods. Therefore, in this dissertation, we have tried to multi-objective this method by considering a recently introduced one-objective optimization method called Arithmetic Optimization Algorithm (AOA). Add existing multi-objective methods and enable researchers to use the ability of this single-objective method to deal with multi-objective problems. In this regard, in order to multiply the goals of the arithmetic optimization algorithm, the concept of Pareto optimality has been used to detect non dominated solutions and repository to store these solutions. To measure the performance of the Multi-objective Arithmetic Optimization Algorithm (MOAOA), this algorithm is applied to the famous benchmark functions and to compare the performance of this algorithm with

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the famous NSGAII and MOPSO algorithms, spacing and generational distance criteria have been used. Based on the results of these experiments, it was found that this algorithm shows acceptable performance compared to popular optimization algorithms.

Keywords: Meta-heuristic methods, multi-objective optimization, arithmetic optimization algorithm, Pareto optimality.

1- Introduction

The development of multi-objective optimization algorithms has attracted significant attention in recent years, especially due to their applicability in solving complex real-world problems involving conflicting objectives. In [1], the authors introduced a parameter adaptive harmony search algorithm designed for both unimodal and multimodal problems, setting the stage for adaptiveness in modern optimization methods. In [2], Yang emphasized the role of metaheuristics in solving complex optimization problems where deterministic methods fail. This foundational understanding was expanded in [3] and [4], where genetic algorithms were explained in the context of nature-inspired optimization frameworks. These algorithms serve as precursors for more advanced techniques that deal with multiple objectives.

Evolutionary concepts continue to underpin most optimization techniques. In [5], Daniel discussed how evolutionary strategies have been used to mimic biological evolution to solve engineering problems. Likewise, [6] addressed the use of optimization in energy management, emphasizing the relevance of algorithms that can handle multiple objectives such as cost, quality, and reliability. A similar theme was explored in [7], where voltage stability and contingency analysis were optimized simultaneously in power systems using multi-objective methods. Particle Swarm Optimization (PSO), an early population-based method, was reviewed in [8], showing its strengths and weaknesses in diverse applications.

The introduction of new physics-based algorithms such as the Gravitational Search Algorithm (GSA) in [9] further diversified the optimization toolkit. However, a significant leap came with the Arithmetic Optimization Algorithm (AOA) introduced in [10], which drew inspiration from mathematical operators to guide the search process. This algorithm demonstrated strong performance in high-dimensional and complex problems. According to [11], computational mechanics also benefits from such innovations, where precision and efficiency are vital.

Further progress in multi-objective optimization included biclustering in bioinformatics as presented in [12], and the integration of swarm intelligence and fuzzy systems in [13], which improved the adaptability of algorithms in uncertain environments. Hardware-level implementations of evolvable systems, as shown in [14], opened new avenues for real-time optimization. Meanwhile, [15] and [16] highlighted the role of computer-aided design and eco-friendly innovations, respectively, reinforcing the need for multi-objective solutions in sustainable engineering.

NSGA-II, one of the most widely used multi-objective evolutionary algorithms, was reviewed in [17] for optimizing machining parameters. In [18], multi-objective spectral

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unmixing in hyperspectral imaging demonstrated the critical role of multi-criteria optimization in remote sensing. Similarly, [19] applied a co-evolutionary PSO to weapon-target assignment, showcasing multi-population strategies for better convergence. Workload efficiency enhancement using multi-objective methods was explored in [20], demonstrating benefits in energy-aware computing systems.

An overview of evolutionary algorithms in multi-objective problems was given in [21], while early methods like vector optimization using evolution strategies and genetic algorithms were detailed in [22] and [23], respectively. In [24], Ishibuchi et al. further refined these approaches by addressing multi-criterion optimization with more structured evaluation strategies. Early applications in fault-tolerant design were discussed in [25], showing the importance of robustness alongside optimality.

Recent advancements include improvements to classic algorithms, such as the modified conjugate gradient method in [26] and its hybrid adaptations in [27], which show how traditional mathematical techniques can evolve into competitive metaheuristics. In [28], an archive-based multi-objective arithmetic optimization algorithm was introduced, enhancing AOA to handle conflicting objectives by storing elite solutions. The integration of arithmetic and geometric means into moth-flame optimization, as in [29], exemplifies hybridization trends for better balance between exploration and exploitation.

In [30], a multi-objective version of AOA incorporating random search strategies was proposed to solve the combined economic emission dispatch problem, which reflects the algorithm's effectiveness in handling real-world industrial constraints. Finally, in [31], a novel metaphor-free optimizer called the Multi-Objective Geometric Mean Optimizer (MOGMO) was developed, emphasizing simplicity and performance over conceptual metaphors. Together, these contributions represent a growing shift toward efficient, mathematically grounded, and problem-specific multi-objective optimization techniques, with the Arithmetic Optimization Algorithm and its variants playing a central role in this evolution.

2- Problem statement

Many real-world engineering problems involve optimizing multiple conflicting objectives simultaneously. Traditional single-objective algorithms, while effective for scalar optimization, fall short when dealing with multidimensional problems that require trade-offs among objectives. For example, in VLSI circuit design, optimizing both power consumption and latency presents a conflicting challenge, as improving one often degrades the other. To address such complexity, multi-objective optimization (MOO) techniques have become essential.

While the field of single-objective metaheuristic algorithms has seen rapid and extensive development, with over 350 such methods introduced, the multi-objective counterparts have not kept pace in either quantity or diversity. Most of the well-known multi-objective algorithms, such as NSGA-II [17] and MOPSO [19], are extensions of foundational single-objective methods like Genetic Algorithm (GA) and Particle Swarm Optimization (PSO),

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respectively. However, despite the proven effectiveness of newer single-objective algorithms like the Arithmetic Optimization Algorithm (AOA), there has been limited effort to explore their potential in the multi-objective domain.

This gap highlights an important research need: to develop a robust and effective Multi-Objective Arithmetic Optimization Algorithm (MOAOA) that leverages the strengths of AOA for solving problems involving multiple, often conflicting, objectives. This paper introduces MOAOA by integrating the concept of Pareto optimality and a dynamic external repository to store non-dominated solutions.

The main contributions of this work are as follows:

1. Extension of AOA to Multi-Objective Domain: The Arithmetic Optimization Algorithm, originally designed for single-objective problems, is extended to support multi-objective optimization by incorporating Pareto dominance principles and maintaining a repository of non-dominated solutions.

2. Repository-Based Solution Management: The algorithm uses hypercube-based region management and elite solution repositories to preserve diversity and guide the search toward the true Pareto front.

3. Comprehensive Benchmarking: The MOAOA is evaluated on three widely used multi-objective benchmark functions—Fonseca-Fleming [21], Kursawe [22], and Schaffer [23]. Its performance is compared with established algorithms like NSGA-II [17] and MOPSO [19] using standard metrics such as Generational Distance (GD) and Spacing (SP).

4. Parameter Sensitivity Analysis: The paper investigates the impact of critical algorithmic parameters (e.g., population size, repository size, α, and μ) on performance, providing valuable insights into the tuning and adaptability of the algorithm.

The experimental results demonstrate that MOAOA performs competitively with or even surpasses traditional algorithms in terms of convergence and distribution quality. This confirms the algorithm's potential as a promising tool for solving complex multi-objective optimization problems in various domains.

3- Proposed method

In this part, we first briefly review the arithmetic optimization algorithm, then we examine the multi-objective arithmetic optimization algorithm, and finally, the results obtained from evaluating this algorithm on benchmark functions are compared with the results obtained by the famous MOPSO and NSGA-ll algorithms.

3-1- Overview of the Algorithm: AOA

The Arithmetic Optimization Algorithm (AOA) is an optimization algorithm based on four basic mathematical operations that was published in 2020 [10]. and its steps are as follows.

$$\alpha = 5 \quad (1)$$

$$\mu = 0.5 \quad (2)$$

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Here, α is a sensitive parameter that determines the exploitation accuracy across iterations, and μ is a control parameter for regulating the search process. The values of these parameters are determined based on empirical experiments reported in this study.

2. Initialization of the Population:

The initial population of candidate solutions is generated randomly.

$$X = initialization(N.Dim.UB.LB) \tag{3}$$

Where N is the number of candidate solutions, LB and UB are the lower and upper bounds of the problem's variables respectively, and Dim is the dimensionality of the problem.

3. Fitness Evaluation and Update:

The fitness of the candidate solutions is evaluated and updated within a for-loop.

$$Ffun(1.i) = F\_obj(X(i.:)) \tag{4}$$

Where Ffun represents the fitted (feasible) solutions, and F_obj is the fitness function.

4- We update the MOP and MOA coefficients using the following relations.

$$MOP = 1 - (\frac{(c_{Iter})^{\frac{1}{\alpha}}}{(M_{Iter})^{\frac{1}{\alpha}}}) \tag{5}$$

$$MOA = MOP_{Min} + C_{Iter} \times \left(\frac{MOM_{Max} - MOM_{Min}}{M_{Iter}}\right) \tag{6}$$

Using the coefficients r_1, r_2, and r_3 which are random coefficients between zero and one, and the multiplication and division relations (7) for the exploration stage and the addition and subtraction relations (8) for the exploitation or deep exploration of the search space stage, the optimal responses are calculated in each iteration.

$$x_{i.j}(\boldsymbol{C_{Iter}}) = \begin{cases} best(x_j) \div (MOP + \varepsilon) \times \left((UBj - LB_j) \times \mu + LB_j\right), & r_2 < 0.5 \\ best(x_j) \times \left((UBj - LB_j) \times \mu + LB_j\right), & otherwise \end{cases} \tag{7}$$

$$x_{i.j}(\boldsymbol{C_{Iter}} + \mathbf{1}) = \begin{cases} best(x_j) - MOP \times \left((UBj - LB_j) \times \mu + LB_j\right), & r_3 < 0.5 \\ best(x_j) + MOP \times \left((UBj - LB_j) \times \mu + LB_j\right), & otherwise \end{cases} \tag{8}$$

5- From step 3, this process continues until one of the stopping conditions is reached.

6- End.

3.2- Introduction to the MOAOA Method

The MOAOA method (Multi objective arithmetic optimization algorithm) identifies non-dominated responses through the beam front and uses a repository to store these responses.

In this method, first random responses are generated and then their fitness value is determined by the fitness function, the best responses are selected and stored in the

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repository, and then at each stage of the algorithm's computational process, the responses stored in the repository are updated.

MOAOA Algorithm steps:

1- Definition of the parameters of the algorithm α and μ.

$$\alpha = 3 \tag{9}$$

$$\mu = 0.52 \tag{10}$$

α is a sensitive parameter and determines the accuracy of the operation in the iterations, and μ is a control parameter for adjusting the search process. The values of these parameters are obtained based on practical experiments.

2- Create a random population of responses (*pop*), with a number of (*Npop*).

3- Separate the non-dominated members of the population and store them in the cache.

4- Generate hyper-cubes in the reviewed regions of the response space and place the non-dominated responses inside these hyper-cubes.

5- We update the MOP and MOA coefficients using the following relationships.

$$MOP = 1 - ((it)^\wedge(1 / \alpha) / (Maxlt)^\wedge(1 / \alpha)) \tag{11}$$

$$MOA = MOP\_Min + it \times \left((MOP_{Minax} - MOP_{Min}) / (Maxlt\right) \tag{12}$$

6- Each of the reservoir responses is updated using coefficients r1, r2, and r3, which are random coefficients between zero and one, and the relationships related to the exploration phase and the exploitation phase of MOAOA.

$$x_{i.j}(\boldsymbol{C_{Iter}}) = \begin{cases} best(x_j) \div (MOP + \varepsilon) \times \left((UBj - LB_j) \times \mu + LB_j\right), & r_2 < 0.5 \\ best(x_j) \times \left((UBj - LB_j) \times \mu + LB_j\right), & otherwise \end{cases} \tag{13}$$

$$x_{i.j}(\boldsymbol{it+1}) = \begin{cases} \frac{rep_{h(j)}}{MOP+\in} \times \left((VarMax - VarMin) \times \mu + VarMin\right) & r_3 < 0.5 \\ rep_{h(j)} \times MOP \times \left((VarMax - VarMin) \times \mu + VarMin\right) & otherwise \end{cases} \tag{14}$$

where $rep_{h(j)}$ is the position of the best response in the pool and $VarMin$ is the lower bound and $VarMax$ is the upper bound of the problem variables.

7- The non-dominated members of the current population are added to the pool. 8- The defeated members of the pool are removed.

9- If the number of members in the pool exceeds the specified capacity, we remove the excess members.

10- If the termination condition is not met, we return to 3, otherwise end.

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4- Results and Analysis

4-1- Performance measurement and comparison of results:

In this section, the results obtained from measuring the multi-objective arithmetic optimization algorithm on three well-known benchmark functions [21], [22] and [23] are presented. In the following, we will introduce these three functions. also, the results obtained from the performance of this algorithm have been compared with the results obtained from the performance of two famous algorithms, MOPSO and NSGA-II, on these three functions.

Different metrics are used to quantitatively compare the performance of optimization algorithms. Here, the GD (Generational distance) and SP (Spacing) metrics, which are explained below, are used.

4.1.1- GD Index

This index evaluates the quality of a set of obtained responses in comparison with a set of predetermined reference points. This index is based on the distance between a response and a reference point. [24]

In other words, the GD criterion is an index for calculating the distance between the beam front and the True pareto front. This criterion is expressed as follows:

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{15}$$

where $d_i$is the Euclidean distance between the non-dominated members of the population and the closest member of the optimal pareto set obtained in the target space, and n is the number of non-dominated members of the population.

4-1-2-: SP Index

This index is used to measure the dispersion of non-dominated responses along the pareto beam front [25]. It is calculated from the following equation:

$$SP = \frac{\sqrt{\sum_{i=1}^{n} \left(\bar{d} - d_i\right)^2}}{n-1} \tag{16}$$

Where n is the number of non-dominated responses and $d_i$is obtained from the following equation.

$$d_i = min_j\left(\left|f_1^i(x) - f_1^j(x)\right| + \left|f_2^i(x) - f_2^j(x)\right|\right). \qquad i = 1.\dots.n \tag{17}$$

And $\bar{d}$ is the average of d_i.

If SP is zero, it indicates that all members of the pareto front are equally spaced.

4-1-3- Introducing the test functions and examining the results

In this section, while introducing the test functions, we compare the results obtained from applying the MOAOA algorithm to these functions with the results obtained from applying

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the MOPSO and NSGAII algorithms to these functions. It is important to remember that the results obtained from the experiments are the result of 1000 iterations of the algorithm loop and 20 independent iterations of the algorithm.

A - Test function Fonseca-Fleming:

This function is defined as follows:

$$\min \quad f_1(\vec{x}) = 1 - \exp\left(-\sum_{i=1}^{5}\left(x_i - \frac{1}{\sqrt{5}}\right)^2\right)$$
$$\min \quad f_2(\vec{x}) = 1 - \exp\left(-\sum_{i=1}^{5}\left(x_i + \frac{1}{\sqrt{5}}\right)^2\right) \tag{18}$$

In this function, the range of change of x is $-4 < x_1 . x_2 . x_3 . x_4 . x_5 < 4$.

Fig. (1) shows the real pareto front of this test function and the pareto front generated by the MOPSO, NSGAII, and MOAOA algorithms.

Table (1) shows the values of the GD and SP criteria related to the results of applying the three algorithms MOPSO, NSGAII, and MOAOA to the Fonseca test function.

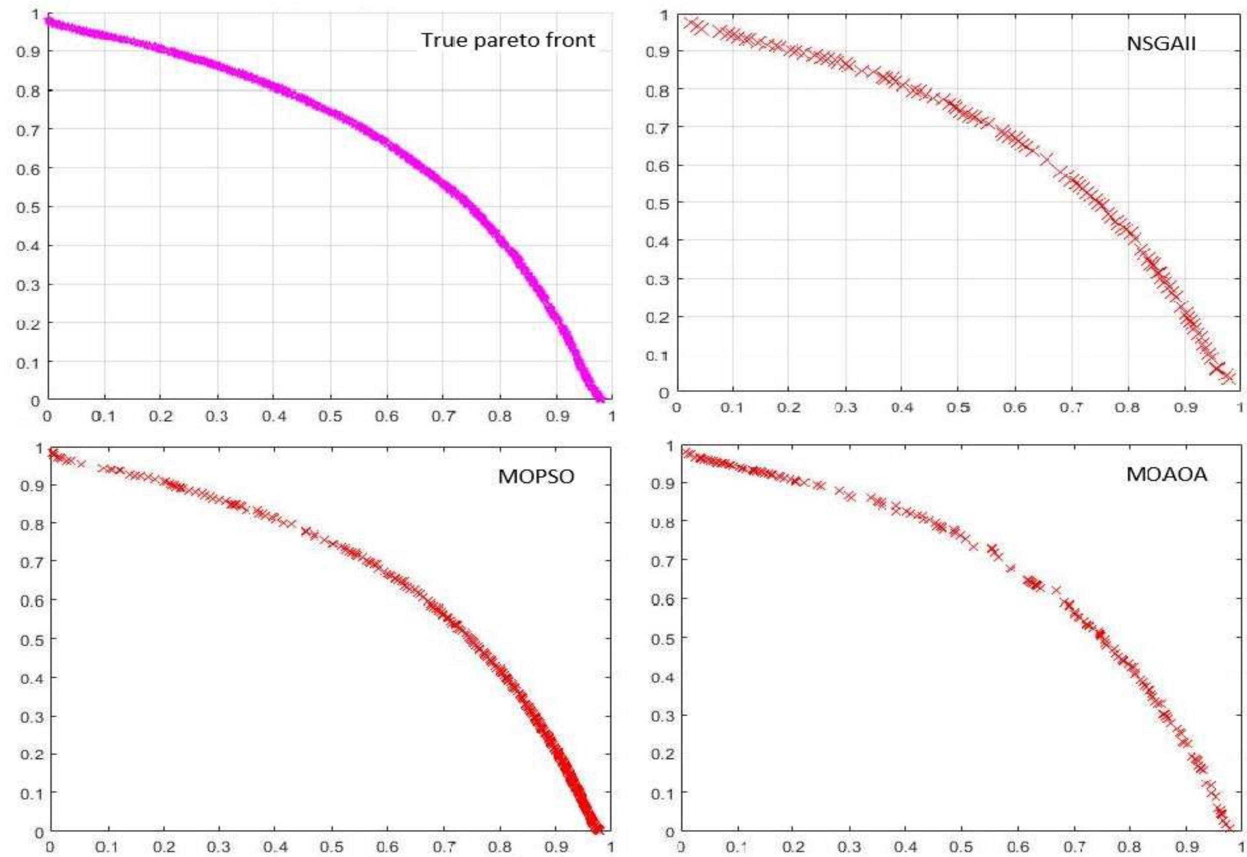

Fig. (1) The true pareto front of the Fonseca test function and the pareto front generated by the three algorithms NSGAII, MOPSO, and MOAOA for this test function.

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Table (1): GD and SP criteria values related to the results of applying the three algorithms MOPSO, NSGAII and MOAOA on the test Fonseca function.

Algorithm		MOAOA	MOPSO [19]	NSGAII [17]
GD	Min	0.00007	0.0005	0.00010
	Max	0.00040	0.00040	0.00020
	Average	0.00014	0.00015	0.00013
	Median	0.00010	0.00009	0.00010
SP	Min	0.0034	0.0012	0.0047
	Max	0.0086	0.0090	0.0065
	Average	0.0057	0.0040	0.0055
	Median	0.0061	0.0030	0.0055

According to the above table, it can be seen that the results of calculating the GD and SP criteria related to the MOAOA algorithm are very close to the MOPSO and NSGAII algorithms.

B- Kursawe Test function:

This function is defined as follows:

$$\min \quad f_1(\vec{x}) = \sum\nolimits_{i=1}^{2}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right)$$
$$\min \quad f_2(\vec{x}) = \sum\nolimits_{i=1}^{3}\left(|x_i|^{0.8} + 5\sin\,(x_i)^3\right) \tag{19}$$

In this function, the range of change of x is $-5 < x_1 . x_2 . x_3 < 5$.

In Fig. (2), the real beam front of this test function and the pareto front generated by the MOPSO, NSGAII and MOAOA algorithms are shown.

Table (2) shows the values of the GD and SP criteria related to the results of applying the three algorithms MOPSO, NSGAII, and MOAOA to the Kursawe test function.

Table (2): GD and SP criterion values related to the results of applying the three algorithms MOPSO, NSGAII and MOAOA on the Kursawe test function.

Algorithm		MOAOA	MOPSO [19]	NSGAII [17]
GD	Min	0.0012	0.0005	0.0012
	Max	0.0209	0.00040	0.0026
	Average	0.0104	0.00015	0.0017

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	Median	0.0076	0.00009	0.0016
SP	Min	0.0438	0.0663	0.0571
	Max	0.0725	0.0997	0.0915
	Average	0.0585	0.0869	0.0839
	Median	0.0567	0.0883	0.0864

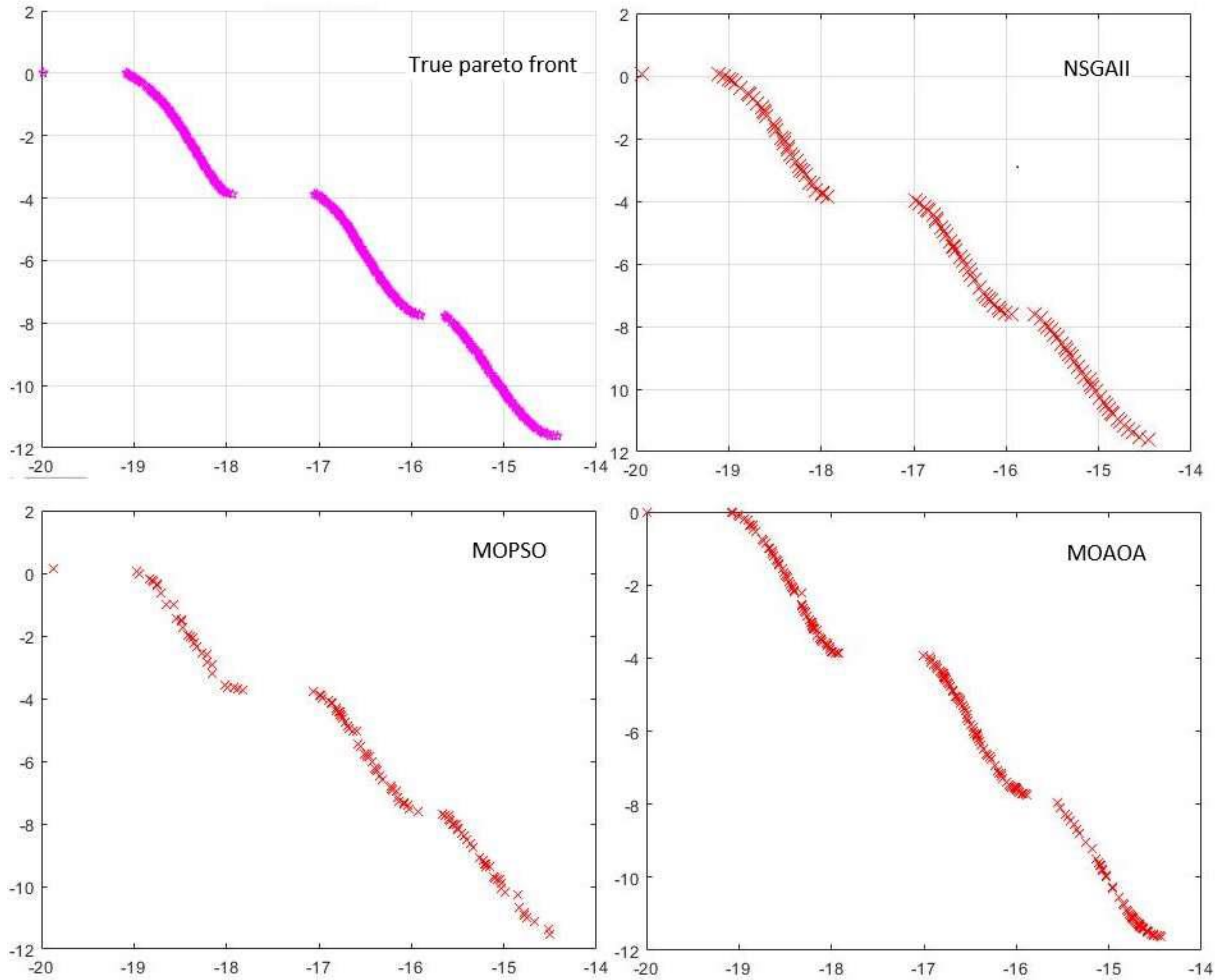

Fig. (2): The real beam front of the Kursawe test function and the pareto front generated by the three algorithms NSGAII, MOPSO, and MOAOA for this test function.

According to the above table, it can be seen that the results of calculating the GD criterion related to the MOAOA algorithm are close to the MOPSO and NSGAII algorithms and the results of calculating the SP criterion for the MOAOA algorithm are better than the results of these two algorithms.

C- Schaffer Test function:

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This function is defined as follows:

$$\begin{aligned} \min \quad & f_1(x) = x^2 \\ \min \quad & f_2(x) = (x-2)^2 \end{aligned} \tag{20}$$

In this function, the range of change of x is 1000 < x < -1000.

In Fig. (3), the true pareto front of this test function and the pareto front generated by the MOPSO, NSGAII and MOAOA algorithms are shown.

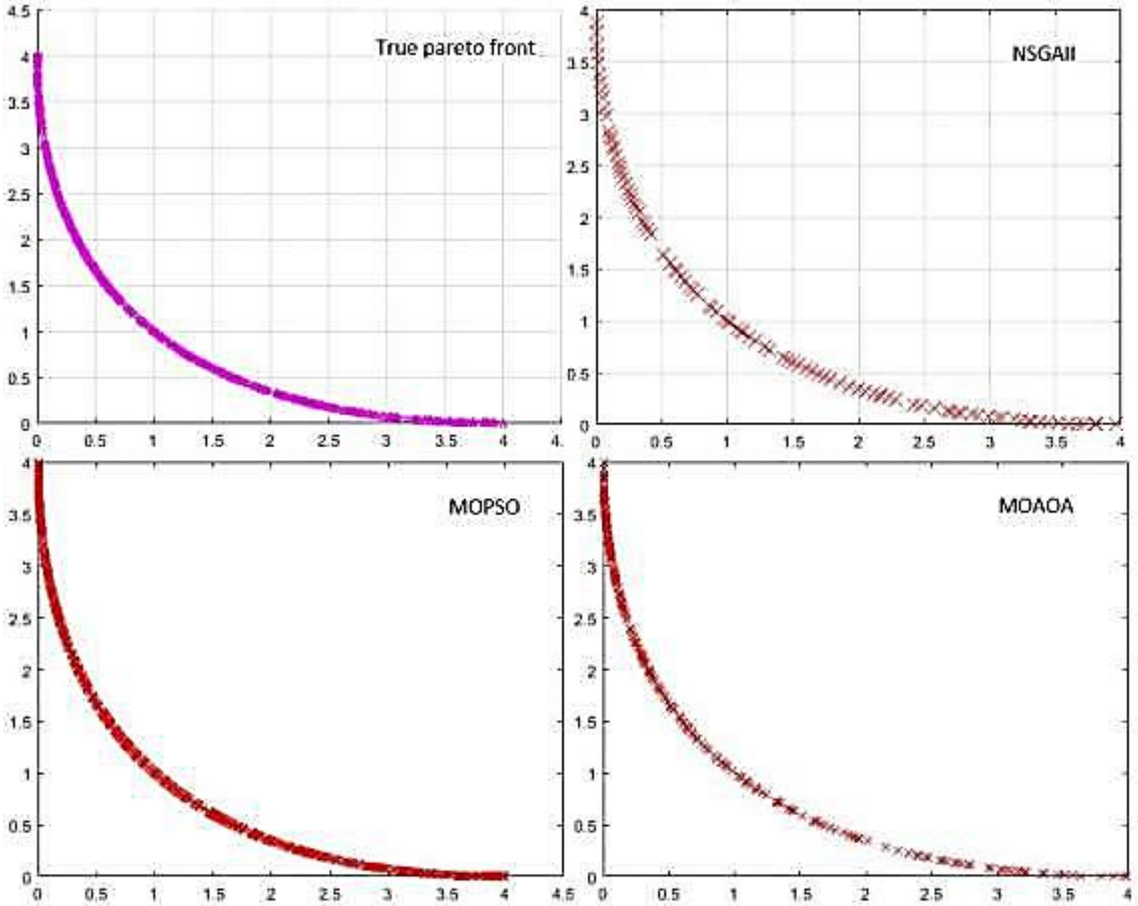

Table (3): Values of GD and SP criteria related to the results of applying the three algorithms MOPSO, NSGAII and MOAOA on the Schaffer test function.

According to the above table, it can be seen that the results of calculating the GD and SP criteria related to applying the MOAOA algorithm to the Schaffer test function are close to the results obtained from applying the MOPSO and NSGAII algorithms to this test function.

4-1-4- Examining the performance of the MOAOA algorithm by changing its parameters:

In this section, we will examine the effect of the values of the parameters Npop, Nrep, α, and μ. Npop is the initial population size, Nrep is the size of the reservoir, α is a sensitive parameter and specifies the accuracy of exploitation in iterations, and μ is a control parameter for adjusting the search process.

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It is necessary to remember that the results obtained from the experiments are the result of 1000 iterations of the algorithm loop and 20 independent iterations of the algorithm.

A) Change in the initial population size (Npop):

To examine the effect of changing the initial population size, other parameters are considered as follows.

$$Nrep = 100 \;,\; \alpha = 3 \,,\; \mu = 0.520$$
(21)

In Tables (4), (5) and (6), the effect of changing the initial population size has been measured for three test functions in four cases, with respect to the GD and SP indices.

Table (4): Results of GD and SP indices with different npop values for test function (a).

Npop		50	100	150	200
GD	Min	0.00008	0.00006	0.0012	0.00006
	Max	0.0006	0.0003	0.0026	0.0008
	Average	0.00016	0.0010	0.0017	0.00019
	Median	0.0001	0.00009	0.0016	0.0001
SP	Min	0.0035	0.0035	0.0571	0.0033
	Max	0.0163	0.0083	0.0915	0.0183
	Average	0.0087	0.0055	0.0839	0.0069
	Median	0.0082	0.0055	0.0864	0.0050

Table (5): Results of GD and SP indices with different npop values for test function (b).

Npop		50	100	150	200
GD	Min	0.0010	0.0010	0.0007	0.0005
	Max	0.1517	0.1020	0.0818	0.0602
	Average	0.0529	0.0247	0.0197	0.0205
	Median	0.0458	0.0081	0.0140	0.0166
SP	Min	0.0294	0.0324	0.0210	0.0197
	Max	0.1920	0. 1422	0.0672	0.0787
	Average	0.0964	0.0635	0.0467	0.0468
	Median	0.0974	0.0568	0.0483	0.0468

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Table (6): Results of GD and SP indices with different npop values for test function (c).

Npop		50	100	150	200
GD	Min	0.0005	0.0003	0.00037	0.0001
	Max	0.0121	0.0020	0.0009	0.0020
	Average	0.0021	0.0007	0.0004	0.0005
	Median	0.0008	0.0005	0.0004	0.0004
SP	Min	0.0028	0.0139	0.0000	0.0068
	Max	0.2948	0. 0457	0.0329	0.0479
	Average	0.0667	0.0257	0.0164	0.0171
	Median	0.0472	0.0261	0.0140	0.0139

B) Tank size change (Nrep):

To examine the effect of tank size change, other parameters are considered as follows:

$Npop = 100$, $\alpha = 3$, $\mu = 0.520$
(22)

In Tables (7), (8) and (9), the effect of tank size change has been measured for three test functions in four cases, with respect to GD and SP indices.

Table (7): Results of GD and SP indices with different Nrep values for test function (a).

Npop		50	100	150	200
GD	Min	0.0006	0.0002	0.0001	0.0001
	Max	0.0018	0.0007	0.0011	0.0011
	Average	0.0011	0.0004	0.0003	0.0003
	Median	0.0011	0.0003	0.0002	0.0002
SP	Min	0.0190	0.0105	0.0065	0.0041
	Max	0.0426	0. 0178	0.0452	0.0123
	Average	0.0284	0.0132	0.0128	0.0076
	Median	0.0274	0.0131	0.0094	0.0076

Table (8): Results of GD and SP indices with different values of Nrep for test function (b).

Npop		50	100	150	200

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GD	Min	0.0025	0.0013	0.0011	0.0010
	Max	0.1457	0.0839	0.0624	0.0822
	Average	0.0325	0.0279	0.0209	0.0182
	Median	0.0213	0.0260	0.0056	0.0081
SP	Min	0.0916	0.0508	0.0424	0.0296
	Max	0.5522	0. 1402	0.1707	0.2976
	Average	0.2187	0.0956	0.0848	0.0679
	Median	0.1923	0.0975	0.0788	0.0524

Table (9): Results of GD and SP indices with different values of Nrep for test function (c).

Npop		50	100	150	200
GD	Min	0.0008	0.0006	0.0005	0.0002
	Max	0.0065	0.0043	0.0062	0.0039
	Average	0.0019	0.0014	0.0015	0.0010
	Median	0.0013	0.0009	0.0007	0.0006
SP	Min	0.0905	0.0336	0.0283	0.0254
	Max	0.2107	0. 2057	0.1620	0.0950
	Average	0.1355	0.0753	0.0590	0.0425
	Median	0.1398	0.0639	0.0514	0.0355

According to Tables (7), (8) and (9), it is observed that with increasing the size of the reservoir, the values of the GD and SP indices for the three test functions become closer to zero. As a result, increasing the size of the reservoir leads to achieving optimal responses.

c) Changing the parameter value α:

To examine the effect of changing the parameter α, the other parameters are considered as follows:

$$Npop = 100 \;.\; Nrep = 100 \;.\quad \mu = 0.520 \quad (23)$$

In Tables (10), (11) and (12), the effect of changing the parameter α has been measured for three test functions in four cases, with respect to the GD and SP indices.

Table (10): Results of GD and SP indices with different values of α for test function (a).

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α		1	3	5	9
GD	Min	0.0002	0.0002	0.0002	0.0003
	Max	0.0015	0.0012	0.0015	0.0016
	Average	0.0005	0.0004	0.0005	0.0007
	Median	0.0003	0.0003	0.0004	0.0006
SP	Min	0.0095	0.0068	0.02092	0.0086
	Max	0.0159	0. 0204	0.0171	0.0178
	Average	0.0125	0.0119	0.0129	0.0124
	Median	0.0119	0.0115	0.0126	0.0122

Table (11): Results of GD and SP indices with different values of α for test function (b).

α		50	100	150	200
GD	Min	0.0015	0.0014	0.0011	0.0014
	Max	0.0777	0.1042	0.2710	0.2392
	Average	0.0291	0.0271	0.0503	0.0470
	Median	0.0219	0.0214	0.0296	0.0373
SP	Min	0.0571	0.0603	0.0612	0.0571
	Max	0.2497	0.1247	0.2458	0.1719
	Average	0.1149	0.0884	0.1016	0.1013
	Median	0.1046	0.0925	0.0932	0.0928

Table (12): Results of GD and SP indices with different values of α for test function (c).

α		1	3	5	9
GD	Min	0.0006	0.0005	0.0005	0.0005
	Max	0.0144	0.0047	0.0057	0.0035
	Average	0.0033	0.0012	0.0014	0.0012
	Median	0.0009	0.0008	0.0009	0.0009
SP	Min	0.0522	0.0226	0.0388	0.0427
	Max	0.3218	0. 1142	0.0887	0.0878

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	Average	00940	0.0602	0.0651	0.0620
	Median	0.0828	0.0581	0.0624	0.0603

As can be seen in Tables (10), (11) and (12), GD and SP criteria have better values at α = 3.

d) Parameter change μ:

To examine the effect of changing the parameter μ, other parameters are considered as follows:

$Npop = 100 \ . \ \ Nrep = 100 \ . \ \ \alpha = 3$
(24)

In Tables (13), (14) and (15), the effect of changing the parameter with respect to the GD and SP indices in four cases has been measured for three test functions.

Table (13): Results of GD and SP indices with different values of μ for test function (a).

μ		0.510	0.515	0.520	0.525
GD	Min	0.0004	0.0002	0.0002	0.0002
	Max	0.0018	0.0009	0.0008	0.0029
	Average	0.0009	0.0005	0.0004	0.0005
	Median	0.0009	0.0004	0.0003	0.0003
SP	Min	0.0072	0.0086	0.0100	0.0095
	Max	0.0190	0. 0178	0.0166	0.0178
	Average	0.0125	0.0132	0.0121	0.0125
	Median	0.0126	0.0129	0.0122	0.0121

Table (14): Results of GD and SP indices with different values of μ for test function (b).

μ		0.510	0.515	0.520	0.525
GD	Min	0.0014	0.0013	0.0013	0.0016
	Max	0.1071	0.0584	0.0834	0.0983
	Average	0.0323	0.0245	0.0214	0.0239
	Median	0.0232	0.0267	0.0212	0.0160
SP	Min	0.0518	0.0474	0.0665	0.0594
	Max	0.2503	0.1891	0.2458	0.7614
	Average	0.1051	0.0992	0.0963	0.1533

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	Median	0.1915	0.0988	0.0947	0.0901

Table (15): Results of GD and SP indices with different values of μ for test function (c).

μ		0.510	0.515	0.520	0.525
GD	Min	0.0004	0.0003	0.0004	0.0005
	Max	0.0022	0.0047	0.0045	0.0114
	Average	0.0010	0.0013	0.0010	0.026
	Median	0.0009	0.0009	0.0010	0.0013
SP	Min	0.0495	0.0424	0.0012	0.0315
	Max	0.1012	0. 1076	0.0906	0.1110
	Average	0.0662	0.0662	0.0491	0.0641
	Median	0.0624	0.0597	0.0492	0.0623

According to Tables (13), (14) and (15), it is observed that GD and SP criteria have better values at μ= 0.52.

5- Conclusion

In this paper, first, a review of the most important metaheuristic algorithms was conducted, and then a new metaheuristic method for solving multi-objective optimization problems, called the multi-objective arithmetic optimization algorithm, was introduced for the first time. According to the results obtained from the performance of this algorithm on three benchmark functions, it can be seen that this algorithm has acceptable performance compared to famous multi-objective algorithms, and this new method can provide a new arithmetic-based approach to solving optimization problems. Multi-objective arithmetic optimization algorithm can provide a new method for solving optimization problems for researchers and the scientific community. Such as multi-objective classification and multi-objective clustering, etc. It is proposed to apply this algorithm to other criterion functions and compare its results with the results of well-known algorithms. Also, considering that this algorithm, like the MOPSO algorithm, uses an external repository to store optimal responses, it is proposed to multi-objective the arithmetic optimization algorithm using the approach used in NSGAII.

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