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A NOVEL APPROACH TO SOLVING TIME FRACTIONAL ADVECTION-DISPERSION EQUATIONS USING TRIPLE LAPLACE TRANSFORM IN CONJUNCTION WITH THE ADOMIAN DECOMPOSITION METHOD

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Abstract

This paper presents a novel analytical method for solving Time-fractional Advection–dispersion equations (TFADEs) by integrating the Triple Laplace Transform (TLT) with the Adomian Decomposition Method (ADM). Time-fractional partial differential equations are widely used to model complex transport phenomena that exhibit memory and hereditary properties, especially in Engineering, solid mechanics, biomedical sciences, environmental modeling and physics. The proposed method transforms the original fractional model into an algebraic form using Triple Laplace transform (TLT). The resulting equation is then solved using the Adomian decomposition technique, allowing the construction of a rapidly convergent series solution. Illustrative examples are provided to demonstrate the accuracy and efficiency of the method. The results validate that the Triple Laplace Transform Adomian Decomposition Method (TLTADM) approach is a powerful and flexible tool for solving a broad class of time-fractional transport models. Compared to purely numerical approaches, the TLTADM framework maintains analytical transparency while capturing the complex behavior of fractional order systems in two spatial dimensions. We examined the impact of the fractional order parameter. Moreover, 2D and 3D graphs representing the solutions for some values of fractional order parameter have been displayed using the MATLAB software.

Keywords - *Triple Laplace Transform, Adomian Decomposition method, Time-fractional Advection – dispersion equations, Partial differential equation, Caputo fractional derivative, Inverse Triple Laplace Transform*

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1. Introduction

One of the most used mathematical models for studying transport processes is the Advection-Dispersion Equation (ADE), which describes the combined effects of dispersion and advection. It is essential for simulating heat transfer in porous media, solute migration, pollutant transport, and groundwater pollution. The anomalous diffusion seen in complicated physical systems like biological tissues, fractal porous structures, and heterogeneous aquifers is frequently missed by the classical ADE, which assumes that the transport mechanism is Markovian and controlled by integer-order derivatives. The time-fractional advection-dispersion equation (TFADE) was created by incorporating fractional calculus into the ADE in order to get around these restrictions. Subdiffusive behaviors in real-world transport processes are better described by the TFADE, which incorporates memory effects and nonlocal dynamics by substituting a Caputo fractional derivative of order $0< \alpha \leq 1$ for the first-order time derivative.

In this work, we present an analytical method to solve the two-dimensional time-fractional advection–dispersion equation by combining the Adomian Decomposition Method (ADM) with the Triple Laplace Transform (TLT). The governing partial differential equation is transformed into an algebraic equation in the transform domain by applying the Triple Laplace Transform to the three independent variables of time (t) and space (x, y). The simultaneous handling of fractional derivatives and spatial derivatives is made simpler by this transformation. The Adomian Decomposition Method, which efficiently manages nonlinearities by expressing the nonlinear terms as quickly convergent sequence of Adomian polynomials.

A space-time spectral collocation method was introduced by Rupali Gupta and Sushil Kumar [1] to solve the two-dimensional variable-order space-time fractional advection-diffusion problem. In order to make the governing equations more reflective of actual transport events in heterogeneous media, the authors tackled the computational difficulties presented by fractional derivatives whose orders fluctuate with space and time. The technique outperformed conventional finite difference methods by utilizing orthogonal polynomial bases in the spectral collocation framework, which resulted in high-order accuracy and fast convergence. The study showed through a number of numerical experiments that the suggested method efficiently manages fractional derivatives of both constant and variable order, preserving stability and accuracy even in challenging situations. Using sophisticated computational methods based on the Caputo fractional derivative, Alshehry, Yasmin, and Mahnashi [2] investigated fractional advection–dispersion equations (ADEs). The goal of the study was to document the memory-dependent and nonlocal behavior of transport processes that are frequently seen in porous or heterogeneous surfaces. The Mohand Transform Iterative Method (MTIM) and the Mohand Residual Power Series Method (MRPSM), two semi-analytical techniques that yield precise and effective approximations of fractional-order solutions, were used by the authors to solve these equations. They proved their approaches' excellent precision and quick convergence over current analytical and numerical techniques through comparative numerical tests. The findings demonstrated the physical significance of fractional modeling by highlighting the effects of changing the fractional order on dispersion and advection properties. Fareed, Hammad, and Elbarawy [3] used the double and triple Laplace Adomian decomposition techniques to solve fractional diffusion equations. According to their research, these hybrid analytical techniques provide very accurate and efficient answers for complicated fractional diffusion issues by producing quickly convergent series solutions. The space–time fractional advection–diffusion equation involving Caputo time and Riesz space derivatives was studied by F. Liu et al. [4] using finite difference methods.To precisely estimate the fractional derivatives, they created both explicit and implicit numerical systems. A thorough stability and convergence study was presented in the publication, demonstrating the implicit scheme's unconditional stability. The accuracy and effectiveness of the suggested algorithms were confirmed by numerical examples. All things considered, the research laid the theoretical groundwork for accurate numerical modeling of fractional advection-diffusion systems. The space time fractional advection-dispersion equation was numerically solved by Shaher Momani and Zaid Odibat [5] using the Variational Iteration Method (VIM) and ADM. They demonstrated that both methods can effectively handle fractional time and space derivatives without requiring discretization or linearization. Jannelli Alessandra, Marianna Ruggieri and Maria Paola Speciale [6] investigated a fractional advection-diffusion-reaction equation in which both time and space derivatives are of fractional order, thereby capturing memory and spatial non-locality effects characteristic of anomalous transport. They first explore reductions via Lie symmetry transformations although the full time-space fractional equation could not be fully reduced to an ordinary fractional differential equation via these symmetries, the authors draw upon previously-obtained analytical and numerical results for the purely time-fractional and purely space-fractional cases to build an alternative solution strategy. Then they present semi-analytical and numerical solutions of the combined model, demonstrating the practicality and high precision of their approach under several test cases. The numerical experiments show how varying the fractional-orders and reaction/advection coefficients affect the transport-reaction behavior, thereby offering insight into the dynamics of fractional transport reaction systems. Huang, Fenghui, and Fawang Liu [7] derived the fundamental solution of a space – time fractional advection – dispersion equation in which the time derivative is replaced by a Caputo derivative of order $0 < \alpha \leq 1$ and the spatial second-order derivative is replaced by a Riesz-Feller fractional derivative of order $0 < \beta \leq 2$. Using the Fourier–Laplace transform technique, they obtain explicit integral-representations for the Green function, which can be interpreted as a probability density function evolving in space and time. The paper demonstrates how the fractional orders α and β control anomalous transport behaviour and reduce to the classical advection – dispersion equation when $\alpha = 1$ and $\beta = 2$. Finally, the solution is expressed in terms of special functions like Mittag-Leffler, Wright and H-functions and is shown applicable for modeling solute transport in heterogeneous media where standard Gaussian diffusion fails. Mirza, Itrat Abbas, and Dumitru Vieru [8] considered the advection – diffusion equation in two spatial dimensions but replace the usual first-order time derivative with the Caputo–Fabrizio derivative a fractional derivative operator without a singular kernel to model anomalous temporal dynamics. They employ Laplace transforms in time and Fourier transforms in space to derive explicit fundamental solutions for both the Dirichlet problem and the source-term problem in a half-plane domain. The analysis shows that, when the fractional parameter α lies in (0,1), the resulting solutions generalize the classical advection–diffusion fundamental solution and exhibit slower spread characteristics due to memory effects. Importantly, because the Caputo–Fabrizio operator leads to kernels without singularities, the authors find their fundamental solutions expressed in terms of elementary and Bessel functions rather than more complex Mittag-Leffler or Fox-H functions simplifying computation. The paper also includes numerical illustrations showing how varying the fractional parameter affects concentration profiles, thereby underscoring the physical relevance of choosing fractional order models in transport phenomena. Tlili, Iskander, Nehad Ali Shah, Saif Ullah, and Humera Manzoor [9] developed one-dimensional advection–diffusion equation with a time-fractional derivative of the Abdon Atangana–Dumitru Baleanu (AB) type, specifically the Atangana–Baleanu fractional derivative with a Mittag-Leffler kernel, to account for memory and non-local temporal effects in transport processes. They apply the Laplace transform in time and a finite sine-Fourier transform in space to derive an analytical expression for the concentration profile under a time-dependent boundary source condition. Their results show that for the AB time-fractional model, the solute concentration evolves more slowly that is, the peak concentration is lower compared with the Caputo derivative case, as the fractional parameter α increases. Through graphical illustrations they examine the influences of the drift velocity, diffusion coefficient and fractional order on the solution, and highlight that the non-singular kernel of the AB derivative offers an alternative modelling framework that can better represent anomalous transport mechanisms in heterogeneous media. Partohaghighi, Mohammad, Mustafa Inc, Mustafa Bayram, and Dumitru Baleanu. [10] considered a one-dimensional advection diffusion equation in which the time derivative is replaced by the Atangana-Baleanu-Caputo derivative a fractional derivative operator with a non-singular Mittag-Leffler kernel to better capture memory and non-local temporal effects in transport phenomena. They introduce a novel numerical approach by transforming the original fractional PDE into an equivalent functional PDE by introducing a fictitious time. 11. Salim, Tariq O., and Ahmad El-Kahlout [11] extends classical transport models to the fractional time domain, offering closed-form analytical expressions and demonstrating how the fractional order influences dispersion or advection behavior. Overall, it contributes a clear analytical framework for solving time-fractional advection–dispersion problems using integral-transform methods. Eltayeb, H. and Bachar, I. [12] applied a hybrid method combining the triple Laplace transform with the Adomian Decomposition Method (ADM) to tackle both regular and singular two-dimensional time-fractional coupled Burgers' equations. Overall, the paper contributes a useful semi-analytic methodology for multidimensional fractional nonlinear PDEs, particularly where singular factors in the spatial terms complicate standard approaches.

The classical advection dispersion equation (ADE) in one spatial dimension is given by[1],

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} - v\frac{\partial u(x,t)}{\partial x}$$

(1)

where u(x,t) represents the concentration of the transported substance, D is the dispersion coefficient, and v is the advection velocity. To capture memory and hereditary properties observed in real-world transport processes, the time derivative is replaced by a fractional derivative of order $0 < \alpha \leq 1$ in the Caputo sense. The time-fractional advection dispersion equation (TFADE) in one dimension is then expressed as[5]:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = D\frac{\partial^2 u(x,t)}{\partial x^2} - v\frac{\partial u(x,t)}{\partial x} \qquad , 0 < \alpha \leq 1 \qquad (2)$$

where $\frac{\partial^\alpha T}{\partial t^\alpha}$ is the Caputo fractional derivative of order α with respect to time.

Extending the model to two spatial dimensions (x,y), the governing equation becomes,

$$\frac{\partial^\alpha u(x,y,t)}{\partial t^\alpha} = D\left[\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}\right] - \upsilon\left[\frac{\partial u(x,y,t)}{\partial x} + \frac{\partial u(x,y,t)}{\partial y}\right], \ 0 < \alpha \le 1 \quad (3)$$

Where $D > 0$ the dispersion coefficient along directions, and $\upsilon > 0$ are the advection velocity in their respective directions.

Research AimS

To develop a novel analytical framework for Time-Fractional Advection–Dispersion Equations (TFADEs) by combining the Triple Laplace Transform (TLT) with the Adomian Decomposition Method (ADM).

To use the Adomian decomposition technique to create an analytical series solution that swiftly converges.

To use illustrative examples to confirm the correctness and efficacy of the suggested TLTADM approach.

3. BASIC EQUATIONS

Definition 1: The triple Laplace transform of function u(x,y,z) is represented as [12],

$$L_x L_y L_z[u(x,y,z)] = U(p,q,s) = \int_0^\infty \int_0^\infty \int_0^\infty e^{-(px+qy+sz)}\, u(x,y,z)\, dxdydz \quad (4)$$

where U(p, q, s) represents the triple Laplace transform of u(x, y, z).

Definition 2: The inverse triple Laplace transform (ITLT) is represented as [12],

$$L_3^{-1}\{U(p,q,s)\} = u(x,y,z) =$$

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{px} dp \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{qy} dq \frac{1}{2\pi i}\int_{e-i\infty}^{e+i\infty} e^{sz} F(p,q,s)\, ds \quad (5)$$

where $u(x,y,z)$ represents ITLT of $U(p,q,s)$ and $\mathrm{Re}\,(p) \ge c$, $\mathrm{Re}\,(q) \ge d$, $\mathrm{Re}\,(s) \ge e$, for some c, d, e are real constants to be chosen suitably.

Theorem: Following Eltayeb and Bachar [12], for any continuous function u(x, y, z) for y, z ≥ 0. Then

$$L_x L_y L_z\left[\frac{\partial^n}{\partial z^n}u(x,y,z)\right] = s^n U(p,q,s) - \sum_{i=0}^{n-1} s^{n-i-1}\frac{\partial^i}{\partial z^i}U(p,q,0)$$

$$L_x L_z L_y\left[\frac{\partial^n}{\partial y^n}u(x,y,z)\right] = q^n U(p,q,s) - \sum_{i=0}^{n-1} q^{n-i-1}\frac{\partial^i}{\partial z^i}U(p,0,s) \quad (6)$$

$$L_z L_y L_x\left[\frac{\partial^n}{\partial x^n}u(x,y,z)\right] = p^n U(p,q,s) - \sum_{i=0}^{n-1} p^{n-i-1}\frac{\partial^i}{\partial z^i}U(0,q,s)$$

4. METHODOLOGY

The methodology combines the Triple Laplace Transform (TLT) with the Adomian Decomposition Method (ADM) to solve Time-Fractional Advection–Dispersion Equations (TFADEs). The TLT reduces the fractional PDE into an algebraic form, and ADM is then applied to construct a rapidly convergent series solution. The approach is validated through illustrative examples and graphical results in MATLAB, highlighting its accuracy and efficiency.

Consider equation (3),

$$\frac{\partial^\alpha u(x,y,t)}{\partial t^\alpha} = D\left[\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}\right] - v\left[\frac{\partial u(x,y,t)}{\partial x} + \frac{\partial u(x,y,t)}{\partial y}\right], \; 0 < \alpha \le 1 \quad (7)$$

with initial condition

$u(x,y,0) = f(x,y)$, (8)

where f(x, y) is any continuous function.

Applying Triple Laplace Transform to equation (7), one gets with [6],

$$s^\alpha L_x L_y L_t[u(x,y,t)] - s^{\alpha-1}U(p,q,0) = D L_x L_y L_t\left[u_{xx} + u_{yy}\right] -$$

$\nu L_x L_y L_t \left[u_x + u_y\right]$(9)

By using equation (8), one gets

$$s^{\alpha} U(p,q,s) - s^{\alpha-1} F(p,q) = D L_x L_y L_t \left[u_{xx} + u_{yy}\right] - \nu L_x L_y L_t \left[u_x + u_y\right] \quad (10)$$

$$U(p,q,s) = \frac{F(p,q)}{s} + \frac{D}{s^{\alpha}} L_x L_y L_t \left[u_{xx} + u_{yy}\right] - \frac{\nu}{s^{\alpha}} L_x L_y L_t \left[u_x + u_y\right] \quad (11)$$

operating inverse triple Laplace transform to equation (11), one obtains

$$u(x,y,t) = L_3^{-1}\left[\frac{F(p,q)}{s} + \frac{D}{s^{\alpha}} L_x L_y L_t \left[u_{xx} + u_{yy}\right] - \frac{\nu}{s^{\alpha}} L_x L_y L_t \left[u_x + u_y\right]\right] \quad (12)$$

The solution in equation (7) having series as

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n \quad (13)$$

By substituting equations (13) in equation (12), one obtains

$$\sum_{n=0}^{\infty} u_n = L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{F(p,q)}{s^{\alpha}}\right] + D L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{L_x L_y L_t [\sum_{n=0}^{\infty} u_{xx_n} + \sum_{n=0}^{\infty} u_{yy_n}]}{s^{\alpha}}\right] - \nu L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{L_x L_y L_t \left[u_{x_n} + u_{y_n}\right]}{s^{\alpha}}\right] \quad (14)$$

Comparing both sides of the equation (14), one gets an iterative algorithm as

$$u_0(x,y,t) = L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{F(p,q)}{s^{\alpha}}\right] \quad (15)$$

$$u_{n+1}(x,y,t) = D L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{L_x L_y L_t \left[u_{xx_n} + u_{yy_n}\right]}{s^{\alpha}}\right] -$$

$$\nu L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{L_x L_y L_t\left[u_{x_n} + u_{y_n}\right]}{s^{\alpha}}\right], \quad n \geq 0 \quad (16)$$

Therefore, the series solution by using equations (15) − (16) is given as

$$u = u_0 + u_1 + u_2 + \ldots$$

5. ILLUSTRATIVE EXAMPLE

This section demonstrates some examples to show the applicability of suggested framework

defined

in the previous section.

We assume D = 1, ν = 1.2, Then equation (3) becomes

$$\frac{\partial^{\alpha} u(x,y,t)}{\partial t^{\alpha}} = \left[\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2}\right] - 1.2\left[\frac{\partial u(x,y,t)}{\partial x} + \frac{\partial u(x,y,t)}{\partial y}\right], \quad 0 < \alpha \leq 1 \quad (17)$$

with initial condition

$$u(x, y, 0) = x^3 y^3 \quad (18)$$

By using equation (15), one obtains

$$u_0 = L_p^{-1} L_q^{-1} L_s^{-1}\left[\frac{36}{s^2 p^4 q^4}\right] = x^3 y^3 \quad (19)$$

By using equation (16), one obtains

$$u_1(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left[\frac{L_xL_yL_t\left[u_{xx_0}+u_{yy_0}\right]}{s^\alpha}\right] - 1.2L_p^{-1}L_q^{-1}L_s^{-1}\left[\frac{L_xL_yL_t\left[u_{x_0}+u_{y_0}\right]}{s^\alpha}\right]$$

$$=\frac{6t^\alpha}{\Gamma(\alpha+1)}[-0.6x^2y^3-0.6y^2x^3+xy^3+yx^3] \quad (20)$$

$$u_2(x,y,t) = L_p^{-1}L_q^{-1}L_s^{-1}\left[\frac{L_xL_yL_t\left[u_{xx_1}+u_{yy_1}\right]}{s^\alpha}\right] - 1.2L_p^{-1}L_q^{-1}L_s^{-1}\left[\frac{L_xL_yL_t\left[u_{x_1}+u_{y_1}\right]}{s^\alpha}\right]$$

$$=\frac{43.2t^{2\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha)}\left[0.2yx^3+0.2xy^3+0.6x^2y^2-\frac{y^3}{6}-\frac{x^3}{6}-\frac{x^2y}{2}-\frac{y^2x}{2}\right]+$$

$$\frac{36t^{2\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha)}\left[2xy-0.2x^3-0.2y^3-0.6xy^2-0.6x^2y\right], \quad (21)$$

and so on.

Therefore, the series solution by using equations (19) − (21) is given as

$$u=u_0+u_1+u_2+\ldots=x^3y^3+\frac{6t^\alpha}{\Gamma(\alpha+1)}[-0.6x^2y^3-0.6y^2x^3+xy^3+yx^3]+$$

$$\frac{43.2t^{2\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha)}\left[0.2yx^3+0.2xy^3+0.6x^2y^2-\frac{y^3}{6}-\frac{x^3}{6}-\frac{x^2y}{2}-\frac{y^2x}{2}\right]+$$

$$\frac{36t^{2\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha)}\left[2xy-0.2x^3-0.2y^3-0.6xy^2-0.6x^2y\right]+\ldots \quad (22)$$

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6.RESULTS AND DISCUSSIONS

Figure 1, Figure 2, Figure 3 and Figure 4 shows the nature of the 3D plots of equation (22) for distinct values of fractional order $\alpha = 0, 0.5, 0.8, 1$ by fixing $x = 0.5$ respectively. Figure 5 and Figure 6 demonstrate the nature of the 2D plots of equations (22) for distinct values of order $\alpha = 0, 0.5, 0.8, 1$ by fixing $\nu = 1.2$, $x = y = 0.5$ and $\nu = 1.2$, $x = 0.5$, $t = 0.2$ respectively.

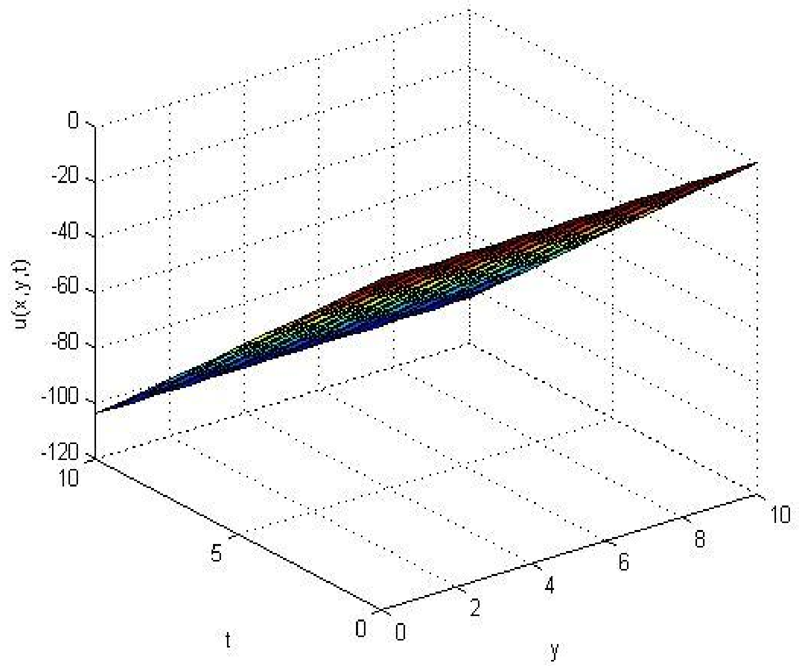

Figure 1: 3D graphs of equation (22) for $\alpha = 0$

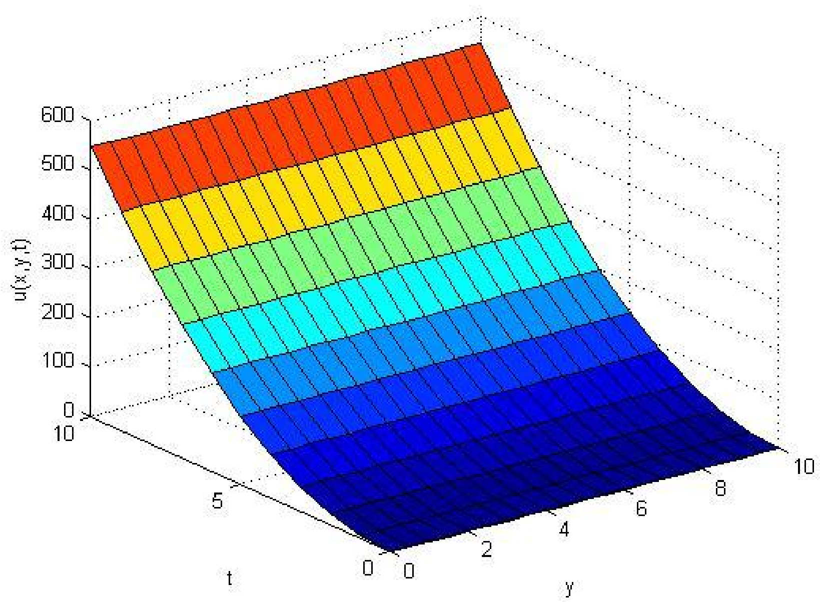

Figure 2: 3D graphs of equation (22) for α = 0.5

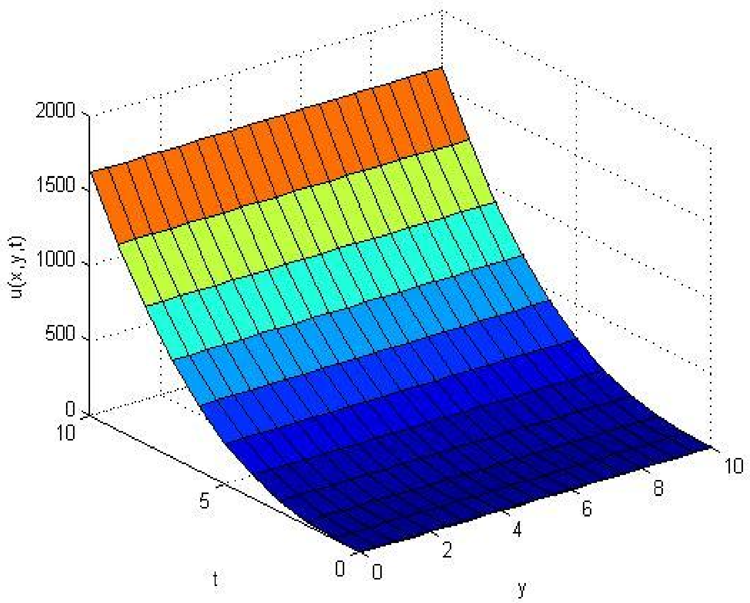

Figure 3: 3D graphs of equation (22) for α = 0.8

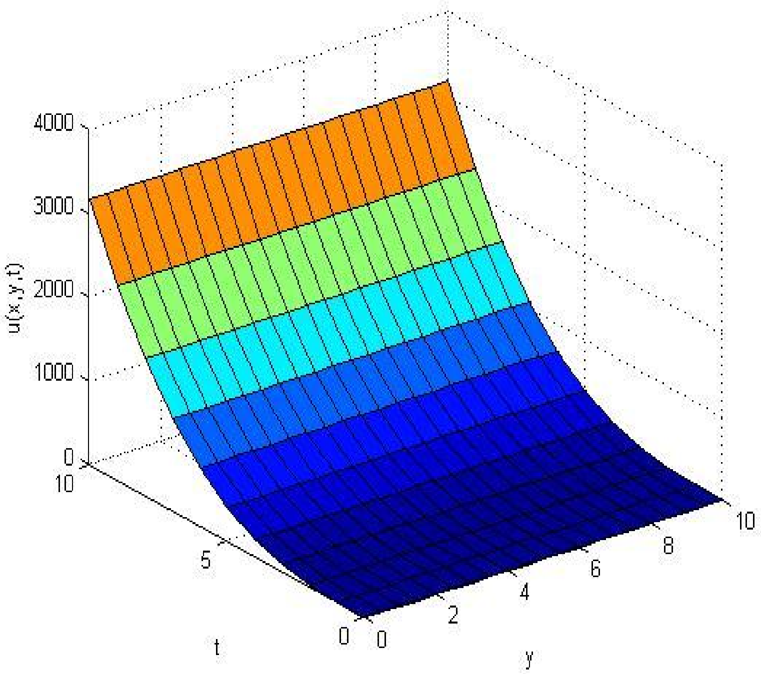

Figure 4: 3D graphs of equation (22) for α = 1

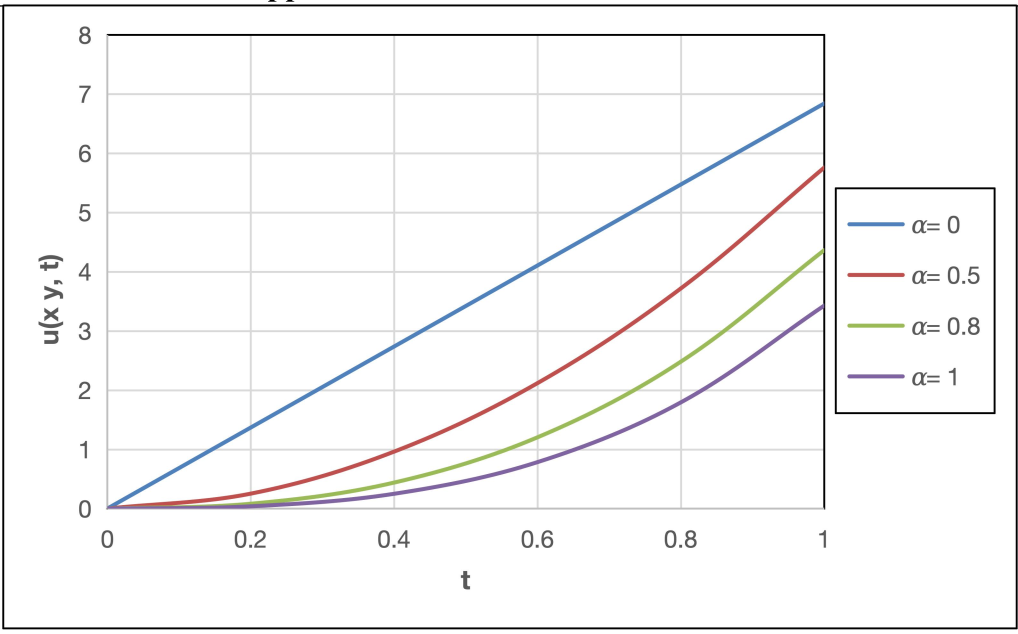

Figure 5: 2D graphs of equation (22) for $\nu = 1.2$, x = y = 0.5

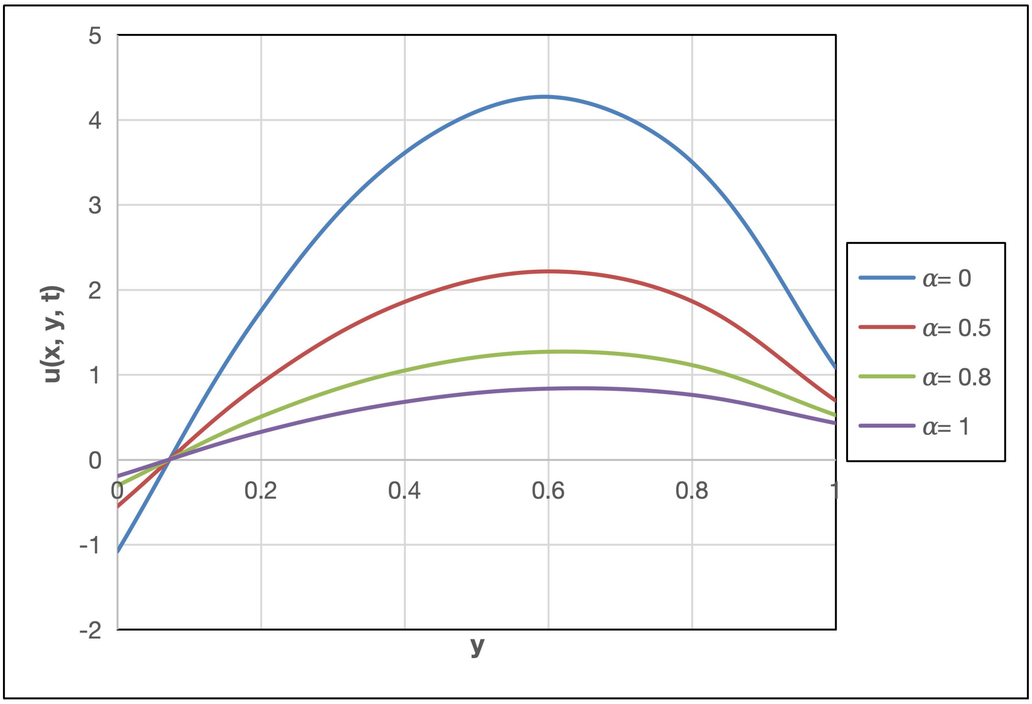

Figure 6: 2D graphs of equation (22) for $\nu = 1.2$, x = 0.5, t = 0.2

7. Conclusions

The integration of the Triple Laplace Transform (TLT) with the Adomian Decomposition Method (ADM) provides an effective analytical framework for solving Time-Fractional Advection–Dispersion Equations (TFADEs).

The proposed TLTADM approach successfully transforms complex fractional partial differential equations into simpler algebraic forms, enabling the construction of rapidly convergent series solutions.

It is evident how the fractional order parameter affects the dynamics of transport models, providing a better understanding of fractional system behavior.

The method's correctness, stability, and efficiency are confirmed by the 2D and 3D MATLAB simulations.

8. LIST OF ABBREVIATIONS

PDE: Partial Differential Equation

ADE: Advection-Dispersion Equation

TFADE: Time Fractional Advection-Dispersion Equation

TLT: Triple Laplace Transform

ADM: Adomian Decomposition Method

TLTADM: Triple Laplace Transform Adomian Decomposition Method

ITLT: Inverse Triple Laplace Transform

VIM: Variational Iteration Method.

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