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SIGNED PLATT NUMBER IN SIGNED NETWORKS

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Abstract

In 1947, J. R. Platt introduced the concept of the Platt number to model the physical properties of alkanes. The Platt number can also be used to analyze other real-world social networks. This paper extends the notion of Platt number to signed Platt number in signed graphs. Some of the characteristics of the signed Platt number relevant to the network analysis perspective are investigated. The bonding quotient of a signed network is defined using the signed Platt number to analyze different networks. An application in network analysis is also given for informed decision-making and optimization strategies.

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Key Words and Phrases: Signed graph, signed degree, net-degree, signed Platt number, bonding quotient, network analysis, graph products

1. Introduction

In Graph theory, a signed graph is a mathematical structure where each edge is associated with a positive or negative sign, indicating a positive or negative relationship between adjacent vertices[23]. Introduced by Frank Harary [8] in 1953, signed graphs provide a framework to model and analyze relationships with both favorable and unfavorable interactions, offering a meticulous perspective compared to traditional unsigned graphs. This concept has applications in various fields such as social networks, biological networks, etc. [6, 7, 9, 21].

In 1947, J. R. Platt introduced the concept of the Platt number [14]. The concept of the Platt number is extended to signed graphs in [4], emphasizing the positive interactions associated with edges, and measures the cumulative influence of positive edges within a

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signed graph. Negative edges significantly affect the centrality and stability, leading to different structural and behavioral outcomes of a system represented by the signed network. The signed degree of a vertex [3] gives a comprehensive framework to analyze the intricate relationships within signed graphs. The term net-degree is also used in literature to refer to a signed degree [11, 17, 15]. A plethora of research has been dedicated to studying the signed degree of vertices in signed graphs[2, 12, 13, 15, 17, 22]. In this paper, the concept of the signed degree of vertices is extended to edges. The signed degree of an edge enables a more subtle representation of interactions within a signed network. From this perspective, the signed Platt number is introduced, contributing to a more comprehensive understanding of the interplay between positive and negative relationships. Some of the properties of this signed Platt number in signed networks are analyzed from the network analysis perspective. The bonding quotient of a signed network is also defined in this paper using the signed Platt number to analyze different networks. In this paper, only simple signed graphs are considered. Also, the terms signed graphs and signed networks are used interchangeably.

2. Preliminaries

The basic definitions needed for the development of concepts and results of this research are discussed in this section.

Definition 2.1 [1] The Platt number of a graph is defined as the sum of the degrees of the edges in a graph.

Definition 2.2 [3, 13] The signed degree of a vertex in a signed graph is defined as the number of positive edges incident on it minus the number of negative edges incident on it. Thus, the signed degree of a vertex v is given by $\mathrm{sdeg}(v) = d^{+}(v) - d^{-}(v)$ where the number of positive edges incident on v is denoted by $d^{+}(v)$ and the number of negative edges incident on v is denoted by $d^{-}(v)$.

Definition 2.3 [18] The positive(negative) edge degree of an edge e in a signed graph is the total number of positive(negative) edges adjacent to it.

Definition 2.4 [20] For signed graphs $\Gamma_1(G_1, \sigma_1)$ and $\Gamma_2(G_2, \sigma_2)$, the positive join (resp. the negative join) is the signed graph obtained by adding a positive (resp. a negative) edge between each vertex of $\Gamma_1(G_1, \sigma_1)$ and each vertex of $\Gamma_2(G_2, \sigma_2)$.

Definition 2.5 [5] Let $\Gamma_1(G_1, \sigma_1)$ and $\Gamma_2(G_2, \sigma_2)$ be two signed graphs. The Cartesian product of Γ_1 and Γ_2, denoted by $\Gamma_1 \square \Gamma_2$, is the signed graph defined as follows:

- $V(\Gamma_1 \square \Gamma_2) = V(G_1) \times V(G_2)$,
- the positive (resp. negative) edges are the pairs $\{(u_1, u_2), (v_1, v_2)\}$ such that:
  - $u_1 = v_1$ and u_2v_2 is a positive (resp. negative) edge of $\Gamma_2(G_2, \sigma_2)$, or
  - $u_2 = v_2$ and u_1v_1 is a positive (resp. negative) edge of $\Gamma_1(G_1, \sigma_1)$

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Definition 2.6 [10, 19] Let $\Gamma_1(G_1, \sigma_1)$ and $\Gamma_2(G_2, \sigma_2)$ be two signed graphs. The tensor product of Γ_1 and Γ_2 is the signed graph $\Gamma_1 \otimes \Gamma_2(G, \sigma)$ with the vertex set $V(G_1) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) are adjacent if and only if u_1 is adjacent to v_1 in G_1 and u_2 is adjacent to v_2 in G_2, and $\sigma\big((u_1, u_2), (v_1, v_2)\big) = \sigma_1(u_1 v_1)\sigma_2(u_2 v_2)$.

3. The signed degree or net-degree of an edge

Here, the concept of the signed degree of a vertex in signed graphs [3] is extended to the edges, and some of its properties are analyzed.

3.1 Definition

Consider a signed graph $\Gamma(G, \sigma)$ with the underlying graph $G(V, E)$ and the signature function $\sigma: E \to \{+1, -1\}$. Let $e = uv \in E$ where $u, v \in V$. The positive edge degree of e is defined as the number of positively labelled edges adjacent to it and is denoted by $d^+(e)$. Similarly, the negative edge degree of e is defined as the number of negatively labelled edges adjacent to it and is denoted by $d^-(e)$.

3.2 Definition

Consider a signed graph $\Gamma(G, \sigma)$ with the underlying graph $G(V, E)$ and the signature function $\sigma: E \to \{+1, -1\}$. Let $e = uv \in E$ where $u, v \in V$. The signed degree or net-degree of an edge e is defined as the number of positive edges adjacent to e less the number of negative edges adjacent to e and is denoted by $sdeg(e)$. Thus

$$sdeg(uv) = \begin{cases} sdeg(u) + sdeg(v) - 2, & if \sigma(uv) = +1, \\ sdeg(u) + sdeg(v) + 2, & if \sigma(uv) = -1 \end{cases} \tag{3.1}$$

Hence,

$$sdeg(uv) = sdeg(u) + sdeg(v) - 2\sigma(uv) \tag{3.2}$$

Also,

$$sdeg(e) = d^+(e) - d^-(e) \tag{3.3}$$

3.3 Definition

Consider a signed graph $\Gamma(G, \sigma)$ with the underlying graph $G(V, E)$ and the signature function σ. The signed degree or net-degree of a signed graph is defined as the total number of positive edges minus the total number of negative edges in it. Thus, if m_Γ^+ denotes the total number of positive edges and m_Γ^- denotes the total number of negative edges in a signed graph Γ, then

$$sdeg(\Gamma) = m_\Gamma^+ - m_\Gamma^- = \sum_{uv \in E} \sigma(uv) \tag{3.4}$$

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Also,

$$\sum_{uv \in E} sdeg(uv) = 2(m_\Gamma^+ - m_\Gamma^-) = 2sdeg(\Gamma) \tag{3.5}$$

3.4 Illustration

Consider the signed graph given below.

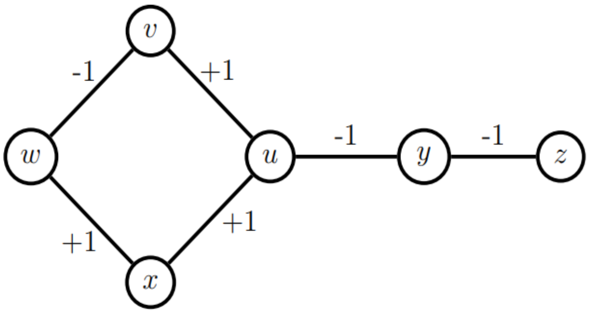

Figure 1: Example

Here, $sdeg(uv) = -1$, $sdeg(vw) = 2$, $sdeg(wx) = 0$, $sdeg(xu) = 1$, $sdeg(uy)$=1, and $sdeg(yz) = -1$. Also, $sdeg(\Gamma) = 0$.

Theorem 3.1 *For an edge e of a signed graph $\Gamma(G, \sigma)$ of order $n \geq 2$,*

$-2(n-2) \leq sdeg(e) \leq 2(n-2)$

Proof. Consider a signed graph $\Gamma(G, \sigma)$ of order $n \geq 2$.The maximum possible edges adjacent to an edge e is $2(n-2)$. The signed degree of this edge is maximum when all these edges are labelled positive. Hence, $sdeg(e) \leq 2(n-2)$. The signed degree of e is minimum when all these $2(n-2)$ edges adjacent to it are labelled negative. Hence, $-2(n-2) \leq sdeg(e)$. This completes the proof. □

Remark 3.1 The bounds are sharp in the above theorem. The edges of a signed complete graph, with all edges labelled positive, attain the upper bound of $sdeg$. Similarly, the edges of a signed complete graph, with all edges labelled negative, attain the lower bound of $sdeg$.

4. Signed Platt number

In this section, the notion of positive Platt number and negative Platt number is introduced. Also, analogously to the notion of the Platt number [1] in unsigned graphs, the signed Platt number of the signed graph is defined using the signed degree of edges. To compare and analyze various networks, a normalized version of the signed Platt number is required. Motivated by this, the bonding quotient is also defined.

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4.1 Definition

In a signed graph $\Gamma(G,\sigma)$ with the underlying graph $G(V,E)$ and the signature function σ, the positive Platt number is defined as the sum of the positive edge degrees of the edges and is denoted by $F^{+}(\Gamma)$.

4.2 Definition

In a signed graph $\Gamma(G,\sigma)$ with the underlying graph $G(V,E)$ and the signature function σ, the negative Platt number is defined as the sum of the negative edge degrees of the edges and is denoted by $F^{-}(\Gamma)$.

4.3 Definition

In a signed graph $\Gamma(G,\sigma)$ with the underlying graph $G(V,E)$ and the signature function σ, the signed Platt number is defined as the sum of the signed degrees of the edges and is denoted by $F^{s}(\Gamma)$. Thus

$$F^{s}(\Gamma) = \sum_{e\in E(\Gamma)} sdeg(e) \tag{4.1}$$

Remark 4.1 From the definition, the signed Platt number of a signed graph may be positive, zero, or negative.

4.4 Example

The signed Platt number of the graph in Figure. 1 is as follows.

$$F^{s}(\Gamma) = sdeg(uv) + sdeg(vw) + sdeg(wx) + sdeg(xu) + sdeg(uy) + sdeg(yz) = 2$$

As the maximum possible signed degree of an edge in a signed graph of order n and size m is $2(n-2)$, the maximum possible value of the signed Platt number is $2m(n-2)$. Based on this, the definition of the bonding quotient is given as follows.

4.5 Definition

In a signed graph $\Gamma(G,\sigma)$ with the underlying graph $G(V,E)$ and the signature function σ, the bonding quotient is defined as

$$BQ(\Gamma) = \frac{F^{s}(\Gamma)}{2m(n-2)} \tag{4.2}$$

where n is the order and m is the size of $\Gamma(G,\sigma)$.

Remark 4.2 In a signed graph $\Gamma(G,\sigma)$, $-1 \leq BQ(\Gamma) \leq 1$. The bounds are sharp. The upper bound is attained by a signed complete graph of order n in which all edges are labelled positive. The lower bound is attained by a signed complete graph of order n in which all edges are labelled negative.

Remark 4.3 In a signed social network, a high bonding quotient indicates that the group is close-knit and supportive, while a low one suggests that the relationships may be weak or

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superficial. The bonding quotient helps to understand how well a group works together and how much they can rely on each other.

Theorem 4.1 *For any signed graph* $\Gamma(G,\sigma)$, $F^{s}(\Gamma) = F^{+}(\Gamma) - F^{-}(\Gamma)$

Proof. Consider a signed graph $\Gamma(G,\sigma)$. Let e be an edge in Γ.

$$\begin{aligned} \mathrm{F}^{s}(\Gamma) &= \sum_{e\in E(\Gamma)} sdeg(e) \\ &= \sum_{e\in E(\Gamma)} (d^{+}(e) - d^{-}(e)) \\ &= \sum_{e\in E(\Gamma)} d^{+}(e) - \sum_{e\in E(\Gamma)} d^{-}(e) \\ &= F^{+}(\Gamma) - F^{-}(\Gamma) \end{aligned}$$

This completes the proof. □

Theorem 4.2 *In a signed graph* $\Gamma(G,\sigma)$ *of order* n, $F^{s}(\Gamma) \leq n(n-1)(n-2)$

Proof. In a signed graph $\Gamma(G,\sigma)$, the maximum $sdeg$ of an edge is $2(n-2)$. In a signed graph of order n, the maximum possible edges are $\frac{n(n-1)}{2}$. Here, the maximum value of the signed Platt number is $n(n-1)(n-2)$. Hence,

$F^{s}(\Gamma) \leq n(n-1)(n-2)$ in any signed graph $\Gamma(G,\sigma)$. □

Remark 4.4 In the above theorem, the sharp bound is attained by the signed complete graph of order n with all edges labelled positive.

Theorem 4.3 *In a signed graph* $\Gamma(G,\sigma)$ *of size* m, $F^{s}(\Gamma) \leq m(m-1)$

Proof. In a signed graph with m edges, the maximum $sdeg$ of an edge is $m-1$. Hence, $F^{s}(\Gamma) \leq m(m-1)$ in any signed graph Γ of size m. □

Remark 4.5 The signed star graph $K_{1,n}$ with all edges labelled positive attains the upper bound in the above theorem.

Theorem 4.4 $F^{s}(T_n) \leq (n-1)(n-2)$ *in a signed tree* T_n *of order* $n \geq 3$.

Proof. The maximum signed degree of an edge is $n-2$ in a signed tree T_n on n vertices. The signed Platt number attains the maximum in this case and is given by $(n-1)(n-2)$. Hence, $F^{s}(T_n) \leq (n-1)(n-2)$ in any signed tree T_n where the order $n \geq 3$.

□

Remark 4.6 The signed tree $K_{1,n-1}$ with all edges labelled positive attains the sharp bound in the above theorem.

Remark 4.7 Signed graphs with all edges having a constant sign can be treated as the usual unsigned graph. Hence, in signed graphs with all positive edges, the definition of the signed Platt number coincides with the definition of the Platt number given in [1].

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5. Effect of edge/vertex addition on the signed Platt number

A network's information or resource distribution can become more efficient or uncover hidden patterns when a new edge or vertex is added. From this perspective, this section analyses the effect of adding an edge or vertex to a signed network in the signed Platt number.

Theorem 5.1 *Let $\Gamma(G,\sigma)$ be a signed graph with at least three vertices, and let u be a vertex in Γ with degree t and signed degree k. Then $F^s(\Gamma+e)-F^s(\Gamma)=k\pm t$ when a pendant edge e is added to Γ at u*

Proof. Consider the graph $\Gamma+e$ where $e=uw$ is the pendant edge added. Then two cases arise.

Case I: e is labelled positive

In this case $sdeg_{\Gamma+e}(u)=sdeg(u)+1$ and $sdeg_{\Gamma+e}(w)=1$.

Hence, $sdeg_{\Gamma+e}(e)= sdeg_{\Gamma+e}(u)+ sdeg_{\Gamma+e}(w)-2=sdeg(u)=k$. Since $d_\Gamma(u)=t$, the signed degree of the t edges incident with u will be increased by one in $\Gamma+e$. Hence $F^s(\Gamma+e)=F^s(\Gamma)+k+t$

Case II: e is labelled negative

As proceeding in case I, we get

$$sdeg_{\Gamma+e}(e)= sdeg_{\Gamma+e}(u)+ sdeg_{\Gamma+e}(w)+2=sdeg(u)=k$$

and $F^s(\Gamma+e)=F^s(\Gamma)-t+k$

This completes the proof. □

Corollary 5.1 *Let $\Gamma(G,\sigma)$ be a signed graph of order $n\geq 3$. If u is a pendant vertex in Γ, then F^s will be increased by two or decreased by two, or remain unchanged when a pendant edge e is added to Γ at u.*

Proof. The proof follows from the fact that $sdeg(u)=\pm 1$. □

Theorem 5.2 *In a signed graph $\Gamma(G,\sigma)$, let u and v be two non-adjacent vertices of degrees t_0 and t_1, respectively, and signed degrees k_0 and k_1, respectively. If an edge e is added between u and v, then $F^s(\Gamma+e)-F^s(\Gamma)=k_0+k_1\pm(t_0+t_1)$*

Proof. Consider the graph $\Gamma+e$ where $e=uv$ is the edge added. Then two cases arise.

Case I: e is labelled positive

In this case, $sdeg_{\Gamma+e}(u)=sdeg(u)+1$ and $sdeg_{\Gamma+e}(v)=sdeg(v)+1$. Hence $sdeg_{\Gamma+e}(e)= sdeg(u)+ sdeg(v)$. Since $d_\Gamma(u)=t_0$, the signed degree of the t_0 edges incident with u will be increased by one in $\Gamma+e$. Similarly, since $d_\Gamma(v)=t_1$, the signed degree of the t_1 edges incident with v will be increased by one in $\Gamma+e$. Hence $F^s(\Gamma+e)=F^s(\Gamma)+k_0+k_1+t_0+t_1$

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Case II: e is labelled negative

As proceeding in case I, we get $F^s(\Gamma + e) = F^s(\Gamma) + k_0 + k_1 - t_0 - t_1$.

This completes the proof. □

Theorem 5.3. *Let $\Gamma(G, \sigma)$ be a signed graph and e be an edge with degrees of end vertices t_0 and t_1 respectively. If e is subdivided into a path of length two, then either F^s will be increased by two, $2(t_0 - 1)$, or $2(t_1 - 1)$, or F^s will be decreased by $2(t_0 + t_1 - 1)$*

Proof. Consider the graph Γ' obtained by subdividing the edge $e = uv$. Let w be the vertex added between u and v. Also, let $uw = e_1$ and $wv = e_2$. Then two cases arise.

Case I: e is labelled positive

Here again, two subcases arise.

Sub case (i): e_1 and e_2 are labelled positive

In this case, signed degrees of u and v remain unchanged in Γ'. Also, $sdeg_{\Gamma'}(w) = 2$

Hence,

$sdeg_{\Gamma'}(e_1) = sdeg_{\Gamma}(u)$ and $sdeg_{\Gamma'}(e_2) = sdeg_{\Gamma}(v)$.

So $F^s(\Gamma') = F^s(\Gamma) - sdeg_{\Gamma}(uv) + sdeg_{\Gamma'}(e_1) + sdeg_{\Gamma'}(e_2) = F^s(\Gamma) + 2$

Sub case (ii): e_1 and e_2 are labelled negative

Here, the signed degrees of u and v will be decreased by 2. Also, $sdeg_{\Gamma'}(w) = -2$

Hence, $sdeg_{\Gamma'}(e_1) = sdeg_{\Gamma}(u) - 2$ and $sdeg_{\Gamma'}(e_2) = sdeg_{\Gamma}(v) - 2$. Also, by adding the negative edges e_1 and e_2, the signed degrees of the $d_{\Gamma}(u) - 1$ and $d_{\Gamma}(v) - 1$ edges incident with *u* and *v,* respectively, will be decreased by 2. Hence,

$$\begin{aligned} F^s(\Gamma') &= F^s(\Gamma) - sdeg_{\Gamma}(uv) - 2[d_{\Gamma}(u) - 1] - 2[d_{\Gamma}(v) - 1] + sdeg_{\Gamma'}(e_1) + sdeg_{\Gamma'}(e_2) \\ &= F^s(\Gamma) + 2 - 2(t_0 + t_1) = F^s(\Gamma) - 2(t_0 + t_1 - 1) \end{aligned}$$

Case II: e is labelled negative

In this case, e_1 and e_2 are labelled with opposite signs. First, assume that e_1 is labelled positive and e_2 is labelled negative. Here $sdeg_{\Gamma'}(w) = 0$.

Here, $sdeg_{\Gamma'}(u) = sdeg_{\Gamma}(u) + 2$ and $sdeg_{\Gamma'}(v) = sdeg_{\Gamma}(u)$.

Thus, $sdeg_{\Gamma'}(e_1) = sdeg_{\Gamma}(u)$ and $sdeg_{\Gamma'}(e_2) = sdeg_{\Gamma}(v) + 2$

$$\begin{aligned} \text{Hence } F^s(\Gamma') &= F^s(\Gamma) - sdeg_{\Gamma}(uv) + 2[d_{\Gamma}(u) - 1] + sdeg_{\Gamma}(u) + sdeg_{\Gamma}(v) + 2 \\ &= F^s(\Gamma) + 2(t_0 - 1) \end{aligned}$$

Similarly, if e_1 is labelled negative and e_2 is labelled positive, we get

$$F^s(\Gamma') = F^s(\Gamma) + 2(t_1 - 1)$$

This completes the proof □

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6. Signed Platt number of some products of signed graphs

In network analysis, graph products are crucial because they enable the modelling of multi-layered or multi-dimensional systems by building complex networks from simpler ones. In this section, the signed Platt number of certain products of signed graphs, such as the join, Cartesian product, and tensor product, is studied.

Theorem 6.1 *Let $\Gamma_1(G_1, \sigma_1)$ and $\Gamma_2(G_2, \sigma_2)$ be two signed graphs, with n_1, n_2 and m_1, m_2 being the number of vertices and the number of edges of G_1 and G_2, respectively. Then the signed Platt number of the positive join, $\Gamma_1 \vee^+ \Gamma_2$, of Γ_1 and Γ_2 is given by*

$$F^S_{\Gamma_1 \vee^+ \Gamma_2} = F^S_{\Gamma_1} + F^S_{\Gamma_2} + 2n_2 sdeg_{\sigma_1}(\Gamma_1) + 2n_1 sdeg_{\sigma_2}(\Gamma_2) + n_1 n_2(n_1 + n_2 - 2) + 2(m_1 n_2 + m_2 n_1)$$

Proof. Consider two signed graphs Γ_1 and Γ_2, with the underlying graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$. Let $\Gamma_1 \vee^+ \Gamma_2$ be the positive join of $\Gamma_1$and Γ_2. Also, let $F^S_{\Gamma_1 \vee^+ \Gamma_2}$, $F^S_{\Gamma_1}$, $F^S_{\Gamma_2}$ be the signed Platt number of $\Gamma_1 \vee^+ \Gamma_2$, Γ_1 and Γ_2 respectively. Let $G(V, E)$ be the underlying graph of $\Gamma_1 \vee^+ \Gamma_2$.

Now, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$ where $E_3 = \{uv : u \in V_1, v \in V_2, \ \sigma(uv) = +1\}$

Also,

$$\sigma(uv) = \begin{cases} \sigma_1(uv) & if\ uv \in E_1 \\ \sigma_2(uv) & if\ uv \in E_2 \\ +1 & if\ uv \in E_3 \end{cases}$$

Let $u \in V$. Then

$$sdeg_\sigma(u) = \begin{cases} sdeg_{\sigma_1}(u) + n_2 & if\ u \in V_1 \\ sdeg_{\sigma_2}(u) + n_1 & if\ u \in V_2 \end{cases},$$

Now,

$$\begin{aligned} F^S_{\Gamma_1 \vee^+ \Gamma_2} &= \sum_{uv \in E} sdeg_\sigma(uv) \\ &= \sum_{uv \in E_1} sdeg_\sigma(uv) + \sum_{uv \in E_2} sdeg_\sigma(uv) + \sum_{uv \in E_3} sdeg_\sigma(uv) \\ &= \sum_{uv \in E_1} [sdeg_{\sigma_1}(u) + n_2 + sdeg_{\sigma_1}(v) + n_2 - 2\,\sigma_1(uv)] + \\ &\quad \sum_{uv \in E_2} [sdeg_{\sigma_2}(u) + n_1 + sdeg_{\sigma_2}(v) + n_1 - 2\,\sigma_2(uv)] + \\ &\quad \sum_{uv \in E_3} [sdeg_{\sigma_1}(u) + n_2 + sdeg_{\sigma_2}(v) + n_1 - 2] \end{aligned}$$

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$$= \sum_{uv\in E_1} [\,sdeg_{\sigma_1}(u) + \ sdeg_{\sigma_1}(v) - 2\,\sigma_1(uv)] +$$
$$\sum_{uv\in E_2} [\ sdeg_{\sigma_2}(u) + \ sdeg_{\sigma_2}(v) - 2\,\sigma_2(uv)] +$$
$$\sum_{uv\in E_3} [\,sdeg_{\sigma_1}(u) + \ sdeg_{\sigma_2}(v)] + 2m_1n_2 + 2m_2n_1 + n_1n_2^2 + n_1^2n_2 - 2n_1n_2$$
$$= F_{\Gamma_1}^S + F_{\Gamma_2}^S + 2n_2sdeg_{\sigma_1}(\Gamma_1) + 2n_1sdeg_{\sigma_2}(\Gamma_2) + n_1n_2(n_1 + n_2 - 2) + 2(m_1n_2 + m_2n_1).$$

This completes the proof. □

Theorem 6.2 *Let $\Gamma_1(G_1, \sigma_1)$ and $\Gamma_2(G_2, \sigma_2)$ be two signed graphs, with n_1, n_2 and m_1, m_2 being the number of vertices and the number of edges of G_1 and G_2, respectively. Then the signed Platt number of the negative join, $\Gamma_1 \vee^- \Gamma_2$, of Γ_1 and Γ_2 is given by*

$$F_{\Gamma_1\vee^-\Gamma_2}^S = F_{\Gamma_1}^S + F_{\Gamma_2}^S + 2n_2sdeg_{\sigma_1}(\Gamma_1) + 2n_1sdeg_{\sigma_2}(\Gamma_2) - n_1n_2(n_1 + n_2 - 2) - 2(m_1n_2 - m_2n_1)$$

Proof. Consider two signed graphs Γ_1 and Γ_2 , with the underlying graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Let $\Gamma_1 \vee^- \Gamma_2$ be the negative join of Γ_1 and Γ_2. Also let $F_{\Gamma_1\vee^-\Gamma_2}^S$, $F_{\Gamma_1}^S$, $F_{\Gamma_2}^S$ be the signed Platt number of $\Gamma_1 \vee^- \Gamma_2$, Γ_1 and Γ_2 respectively. Let $G(V, E)$ be the underlying graph of $\Gamma_1 \vee^- \Gamma_2$.

Now, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$ where $E_3 = \{uv : u \in V_1, v \in V_2,\ \sigma(uv) = -1\}$

Also,

$$\sigma(uv) = \begin{cases} \sigma_1(uv) & if\ uv\ \in E_1 \\ \sigma_2(uv) & if\ uv\ \in\ E_2 \\ -1 & if\ uv\ \in\ E_3 \end{cases}$$

Let $u\ \in\ V$. Then

$$sdeg_\sigma(u) = \begin{cases} sdeg_{\sigma_1}(u) - n_2 & if\ u\ \in\ V_1 \\ sdeg_{\sigma_2}(u) - n_1 & if\ u\ \in\ V_2 \end{cases}$$

Now,

$$F_{\Gamma_1\vee^-\Gamma_2}^S = \sum_{uv\in E} sdeg_\sigma(uv)$$
$$= \sum_{uv\in E_1} sdeg_\sigma(uv) + \sum_{uv\in E_2} sdeg_\sigma(uv) + \sum_{uv\in E_3} sdeg_\sigma(uv)$$
$$= \sum_{uv\in E_1} [sdeg_{\sigma_1}(u) - n_2 + \ sdeg_{\sigma_1}(v) - n_2\ - 2\,\sigma_1(uv)] +$$

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$$\sum_{uv\in E_2}[sdeg_{\sigma_2}(u)-n_1+\ sdeg_{\sigma_2}(v)-n_1-2\,\sigma_2(uv)]+$$
$$\sum_{uv\in E_3}[sdeg_{\ \sigma_1(u)}-n_2+\ sdeg_{\ \sigma_2}(v)-n_1+2]$$
$$=\sum_{uv\in E_1}[\ sdeg_{\ \sigma_1}(u)+\ sdeg_{\ \sigma_1}(v)-2\,\sigma_1(uv)]+$$
$$\sum_{uv\in E_2}[sdeg_{\ \sigma_2}(u)+\ sdeg_{\ \sigma_2}(v)-2\,\sigma_2(uv)]+$$
$$\sum_{uv\in E_3}[\ sdeg_{\ \sigma_1}(u)+\ sdeg_{\ \sigma_2}(v)]-2m_1n_2-2m_2n_1-n_1n_2^2-n_1^2n_2+2n_1n_2$$
$$=F_{\Gamma_1}^S+F_{\Gamma_2}^S+2n_2sdeg_{\sigma_1}(\Gamma_1)+2n_1sdeg_{\sigma_2}(\Gamma_2)-n_1n_2(n_1+n_2-2)$$
$$-2(m_1n_2+m_2n_1).$$

This completes the proof. □

Theorem 6.3 *For the signed graphs $\Gamma_1(G_1,\sigma_1)$ and $\Gamma_2(G_2,\sigma_2)$, let n_1,n_2 and m_1,m_2 be the number of vertices and the number of edges of G_1 and G_2, respectively. Then the signed Platt number of the Cartesian product, ($\Gamma_1 \square \Gamma_2$)$(G,\sigma)$, of Γ_1 and Γ_2 is given by*

$$F_{\Gamma_1\square\Gamma_2}^S=n_2F_{\Gamma_1}^S+n_1F_{\Gamma_2}^S+4\left(m_2sdeg_{\sigma_1}(\Gamma_1)+m_1sdeg_{\sigma_2}(\Gamma_2)\right)$$

Proof. Consider the Cartesian product $(\Gamma_1 \square \Gamma_2)(V,E)$, with the underlying graph $G(V,E)$, of the signed graphs $\Gamma_1$and Γ_2. Let n_1, n_2 and m_1,m_2 be the number of vertices and the number of edges of G_1 and G_2, respectively. Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be the underlying graphs of Γ_1 and Γ_2 respectively. Let $(u,v)\in V(\Gamma_1 \square \Gamma_2)$. Since, as in [16], the neighbours of (u_1,u_2) are the neighbours of u_1 when u_2 is fixed, and the neighbours of u_2 when u_1 is fixed, $sdeg_\sigma(u_1,u_2)=sdeg_{\sigma_1}(u_1)+sdeg_{\sigma_2}(u_2)$

Let $((u_1,u_2)(v_1,v_2))\in E(\Gamma_1 \square \Gamma_2)$. Then,

$$sdeg_\sigma\big((u_1,u_2)(v_1,v_2)\big)=sdeg_\sigma(u_1,u_2)+sdeg_\sigma(v_1,v_2)-2\sigma\big((u_1,u_2)(v_1,v_2)\big).$$

Hence,

$$sdeg_\sigma\big((u_1,u_2)(v_1,v_2)\big)$$
$$=\begin{cases}sdeg_{\sigma_1}(u_1v_1)+2sdeg_{\sigma_2}(u_2) & when\ u_1v_1\in E(G_1)\ and\ u_2=v_2\\ sdeg_{\sigma_2}(u_2v_2)+2sdeg_{\sigma_1}(u_1) & when\ u_1=v_1\ and\ u_2v_2\in E(G_2)\end{cases}$$

Then,

$$F_{\Gamma_1\square\Gamma_2}^S=\sum_{((u_1,u_2)(v_1,v_2))\in E}sdeg_\sigma((u_1,u_2)(v_1,v_2))$$

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$$= \sum_{\substack{u_1=v_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_2}(u_2v_2) + 2\, sdeg_{\sigma_1}(u_1)] + \sum_{\substack{u_1v_1\in E_1\\ u_2=v_2}} [sdeg_{\sigma_1}(u_1v_1) + 2\, sdeg_{\sigma_2}(u_2)]$$

$$= n_1F^S_{\Gamma_2} + n_2F^S_{\Gamma_1} + 2m_2 \sum_{u_1\in V_1} sdeg_{\sigma_1}(u_1) + 2m_1 \sum_{u_2\in V_2} sdeg_{\sigma_2}(u_2)$$

$$= n_2F^S_{\Gamma_1} + n_1F^S_{\Gamma_2} + 4m_2 sdeg_{\sigma_1}(\Gamma_1) + 4m_1 sdeg_{\sigma_2}(\Gamma_2)$$

This completes the proof. □

Theorem 6.4 *For the signed graphs $\Gamma_1(G_1,\sigma_1)$ and $\Gamma_2(G_2,\sigma_2)$ with the underlying graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively, the signed Platt number of the tensor product of Γ_1 and Γ_2 is given by $F^S_{\Gamma_1\otimes\Gamma_2} = 2T_1T_2 - 2sdeg_{\sigma_1}(\Gamma_1)sdeg_{\sigma_2}(\Gamma_2)$ where $T_1 = \sum_{u_1\in V_1} d(u_1)\, sdeg_{\sigma_1}(u_1), T_2 = \sum_{u_2\in V_2} d(u_2)\, sdeg_{\sigma_2}(u_2)$*

Proof. Consider the tensor product $\Gamma_1\otimes\Gamma_2$ of the signed graphs $\Gamma_1$and Γ_2. As in [16], since all edges incident with u_1 and u_2 contribute to the signed degree of (u_1,u_2), $sdeg_\sigma(u_1,u_2) = sdeg_{\sigma_1}(u_1)sdeg_{\sigma_2}(u_2)$

Now,

$$F^S_{\Gamma_1\otimes\Gamma_2} = \sum_{((u_1,u_2)(v_1,v_2))\in E} [sdeg_\sigma(u_1,u_2) + sdeg_\sigma(v_1,v_2) - 2\sigma((u_1,u_2)(v_1,v_2))]$$

$$= \sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_1}(u_1)sdeg_{\sigma_2}(u_2) + sdeg_{\sigma_1}(v_1)sdeg_{\sigma_2}(v_2) - 2\sigma_1(u_1v_1)\sigma_2(u_2v_2)]$$

$$= \sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_1}(u_1)sdeg_{\sigma_2}(u_2)] + \sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_1}(v_1)sdeg_{\sigma_2}(v_2)] - 2\sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [\sigma_1(u_1\,v_1)\sigma_2(u_2v_2)]$$

$$= 2\sum_{u_1v_1\in E_1} sdeg_{\sigma_1}(u_1) \sum_{u_2v_2\in E_2} sdeg_{\sigma_2}(u_2) - 2\sum_{u_1v_1\in E_1} \sigma_1(u_1v_1) \sum_{u_2v_2\in E_2} \sigma_2(u_2v_2)$$

$$= 2\sum_{u_1\in V_1} d(u_1)\, sdeg_{\sigma_1}(u_1) \sum_{u_2\in V_2} d(u_2)\, sdeg_{\sigma_2}(u_2) - 2sdeg_{\sigma_1}(\Gamma)sdeg_{\sigma_2}(\Gamma_2)$$

$$= 2T_1T_2 - 2sdeg_{\sigma_1}(\Gamma)sdeg_{\sigma_2}(\Gamma_2)$$

where

$$T_1 = \sum_{u_1\in V_1} d(u_1)\, sdeg_{\sigma_1}(u_1), T_2 = \sum_{u_2\in V_2} d(u_2)\, sdeg_{\sigma_2}(u_2)$$

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This completes the proof. □

Theorem 6.5 *Consider the signed graphs $\Gamma_1(G_1,\sigma_1)$ and $\Gamma_2(G_2,\sigma_2)$ with the underlying graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Then the signed Platt number of the tensor product of Γ_1 and Γ_2 in terms of the signed Platt numbers $F^S_{\Gamma_1}$ and $F^S_{\Gamma_2}$ is given by*

$$F^S_{\Gamma_1\otimes\Gamma_2} = F^S_{\Gamma_1}F^S_{\Gamma_2} + 2F^S_{\Gamma_2}sdeg_{\sigma_1}(\Gamma_1) + 2F^S_{\Gamma_1}sdeg_{\sigma_2}(\Gamma_2) + 2sdeg_{\sigma_1}(\Gamma_1)sdeg_{\sigma_2}(\Gamma_2) + T_1T_2 \text{ , where}$$

$$T_1 = \sum_{u_1\in V_1} d(u_1)\, sdeg_{\sigma_1}(u_1), T_2 = \sum_{u_2\in V_2} d(u_2)\, sdeg_{\sigma_2}(u_2)$$

Proof. Consider the tensor product $\Gamma_1\otimes\Gamma_2$ of the signed graphs Γ_1 and Γ_2 with the underlying graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Let $(u_1,u_2)\in V(\Gamma_1\otimes\Gamma_2)$. As in [16], since all edges incident with u_1 and u_2 contribute to the signed degree of (u_1,u_2), $sdeg_\sigma(u_1,u_2) = sdeg_{\sigma_1}(u_1)sdeg_{\sigma_2}(u_2)$.

Consider $\big((u_1,u_2)(v_1,v_2)\big)\in E(\Gamma_1\otimes\Gamma_2)$. Then,

$$sdeg_\sigma\big((u_1,u_2)(v_1,v_2)\big) = sdeg_{\sigma_1}(u_1,u_2) + sdeg_{\sigma_2}(v_1,v_2) - 2\sigma\big((u_1,u_2)(v_1,v_2)\big).$$

As in the proof of Theorem 6.4,

$$F^S_{\Gamma_1\otimes\Gamma_2} = \sum_{((u_1,u_2)(v_1,v_2))\in E} [sdeg_{\sigma_1}(u_1)sdeg_{\sigma_2}(u_2) + sdeg_{\sigma_1}(v_1)sdeg_{\sigma_2}(v_2) - 2\sigma\big((u_1,u_2)(v_1,v_2)\big)].$$

Now,

$$F^S_{\Gamma_1}F^S_{\Gamma_2} = \sum_{u_1v_1\in E_1}\big[sdeg_{\sigma_1}(u_1) + sdeg_{\sigma_1}(v_1) - 2\sigma_1(u_1v_1)\big]\sum_{u_2v_2\in E_2}\big[sdeg_{\sigma_2}(u_2) + sdeg_{\sigma_2}(v_2) - 2\sigma_2(u_2v_2)\big]$$

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$$= \sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_1}(u_1)\, sdeg_{\sigma_2}(u_2) + sdeg_{\sigma_1}(v_1) sdeg_{\sigma_2}(v_2) - 2\sigma\big((u_1,u_2)(v_1,v_2)\big)]$$
$$- 2\sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} \sigma_1(u_1v_1)[sdeg_{\sigma_2}(u_2) + sdeg_{\sigma_2}(v_2) - 2\sigma_2(u_2v_2)]$$
$$- 2\sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} \sigma_2(u_2v_2)[sdeg_{\sigma_1}(u_1) + sdeg_{\sigma_1}(v_1) - 2\sigma_1(u_1v_1)]$$
$$- 2\sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [\sigma_1(u_1v_1)\, \sigma_2(u_2v_2)] + \sum_{\substack{u_1v_1\in E_1\\ u_2v_2\in E_2}} [sdeg_{\sigma_1}(u_1)\, sdeg_{\sigma_2}(v_2)$$
$$+ sdeg_{\sigma_1}(v_1) sdeg_{\sigma_2}(u_2)]$$
$$= F^S_{\Gamma_1\otimes\Gamma_2} - 2F^S_{\Gamma_2} sdeg_{\sigma_1}(\Gamma_1) - 2F^S_{\Gamma_1} sdeg_{\sigma_2}(\Gamma_2) - 2sdeg_{\sigma_1}(\Gamma_1) sdeg_{\sigma_2}(\Gamma_2) + 2T_1T_2$$

where

$$T_1 = \sum_{u_1\in V_1} d(u_1)\, sdeg_{\sigma_1}(u_1), T_2 = \sum_{u_2\in V_2} d(u_2)\, sdeg_{\sigma_2}(u_2)$$

This completes the proof. □

7. Application in network analysis

Modelling social networks using signed graphs provides a convenient and powerful framework to understand the multifaceted nature of social relationships and interactions, offering insights into the underlying mechanisms shaping the structure and dynamics of human social systems. From the definition itself, the signed Platt number in social network analysis can be used to describe the strength of relationships within a group of people. It shows how much they trust, support, and care for one another. This is crucial to improve the overall functionality of the network representing a group. Comparing different networks is as essential as analysing a particular network in social network analysis. It helps to understand the strengths and weaknesses of different networks and provides valuable insights for optimization and decision-making. The bonding quotient, which is the normalized version of the signed Platt number, helps to make a comparison regarding the cohesion and bonding of different networks. Greater teamwork and efficiency is reflected in an organizational network with a higher bonding quotient because it promotes smoother communication and stronger interpersonal bonds. A high bonding quotient in crisis response networks facilitates quicker coordination and more consistent support between members.

8. Conclusion

In this paper, the signed Platt number is introduced, and some network properties are analysed. In social network analysis, the signed Platt number could offer a refined perspective on the overall sentiment or tone of interactions, considering both positive and negative edges. The signed Platt number and the bonding quotient can contribute to a more nuanced

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interpretation of the networks' structure and dynamics. From the perspective of social networks, all these concepts help organizations, communities, and policymakers strategically allocate resources to nurture meaningful relationships and cultivate a supportive environment conducive to individual well-being and collective empowerment.

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