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DETERMINATION OF THE CRITICAL SIZE OF THE CLOUD OF INERT PARTICLES FOR INTERRUPTION OF PROPAGATION OF DETONATION WAVES IN GASEOUS FUEL SUSPENSIONS

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Abstract

The article presents the results of numerical modeling of detonation wave propagation in gas suspensions with transversely non-uniform distribution of inert solid particles. It is established that the passage of a detonation wave through a layer of spatially non-uniform particles can lead to both its complete suppression and restoration depending on the geometric and physical parameters of the medium. It is shown that an increase in the length of the solid particle layer helps to reduce the critical mass content required to interrupt detonation. Additionally, the effect of the transverse non-uniformity height and particle diameter on the stability and nature of wave propagation is analyzed. The obtained results are important for the development of passive methods for detonation suppression and improving the safety of process systems operating with flammable gas mixtures.

Key Words: mathematical model, detonation wave, combustion front of particle, unitary fuel, solid particles, pipeline, mass content of particles.

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1. Introduction

In recent years, many studies have been conducted on the weakening of detonation wave propagation in gas suspensions with solid particles. The studies have shown that the introduction of solid particles into the gas medium can significantly affect the propagation of the detonation wave. The size, concentration and distribution of particles play a decisive role in this process. For example, numerical modeling of the passage of hydrogen detonation through a curtain of inert particles revealed three behavior modes: complete passage of detonation, partial attenuation with subsequent renewal and complete attenuation. The critical thickness of the curtain, at which attenuation occurs, depends on the concentration and size of the particles.

The work [1] is devoted to the numerical modeling of the interaction of a cellular detonation wave with inert particles. The results of the study show that inert particles have a significant effect on the propagation of a detonation wave. In particular, particles can lead to a slowdown or even complete attenuation of the wave. These results are important for improving safety and developing explosion control strategies. In [2], it is shown that when a detonation wave collides with a wall, the pressure increases sharply, and a multicomponent particle cloud is effective for reducing it. In [3], the effect of the concentration and size of coal dust particles on the propagation of a methane detonation wave is investigated. It is found that under certain conditions, detonation attenuates, followed by its re-emergence due to the combustion of coal dust in the subsonic zone.

In [4], a mathematical modeling of the explosion of a $H_2/O_2/N_2$ mixture and its extinguishing with solid particles was performed. The model took into account shock waves, energy transfer, and chemical reactions. When solid particles were loaded, combustion was slowed down due to the energy transfer from the gas to the particles, the shock wave was weakened, and the explosion was suppressed. The density, concentration, and diameter of the solid particles had a strong effect on the result. In [5], it was found that if the particle cloud meets critical conditions, it weakens the blast wave. In [6], the experimental results obtained the conditions for termination of detonation and combustion by a layer of solid particles. In [7], the studies of the propagation of shock and detonation waves in CH_4/H_2/air and CH_4/H_2/air/coal mixtures are presented. It was found that the detonation wave parameters are mainly influenced not by the coal dust suspension, but by methane and hydrogen. In [8], the effects of injection of solid particles into a stoichiometric hydrogen-air mixture at velocities of 0.1–1.0 of the Chapman – Jouguet detonation velocity were studied. It was found that an increase in the particle temperature causes ignition of the mixture, and an increase in their velocity initiates a detonation wave. Critical detonation conditions are determined by the concentration, size and velocity of particles. Detonation initiation scenarios are described, including multiple sources, and flow regime maps are constructed by the parameters of the injection velocity and particle temperature.

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In [9], models are proposed that describe the weakening and suppression of detonation in hydrogen–oxygen, methane–oxygen, and silane–air mixtures in the presence of inert micro- and nanoparticles. Dependences of the detonation velocity deficit on the particle size and concentration are found. Three detonation modes are distinguished: weakened stationary, galloping, and complete destruction. It is established that the suppression efficiency does not increase significantly upon transition from microparticles to nanoparticles. In [10], the issue of damping detonation waves propagating in gas suspensions using a layer of inhomogeneous solid particles is considered. The influence of the layer diameter, length, and distribution pattern of solid particles on the process of detonation wave attenuation is studied. It is shown that a layer with a linearly decreasing concentration of solid particles weakens detonation waves better than layers with a uniform or linearly increasing concentration.

2. Formulation of the problem

In a cylindrical tube filled with a mixture of gas and particles, an inert gas (region $(0 \le z \le z_f)$) enters from the left boundary into the region with particles of unitary fuel $(z_f < z < \infty)$, which contains a layer of solid particles in its cross section (region $(z_l \le z \le z_r,\ z_l > z_f)$), a triangular shock wave propagates (Fig. 1). It is necessary to study the effect of the law of transverse spatial non-uniformity of the distribution of the concentration of solid particles on the interruption of the propagation of detonation waves.

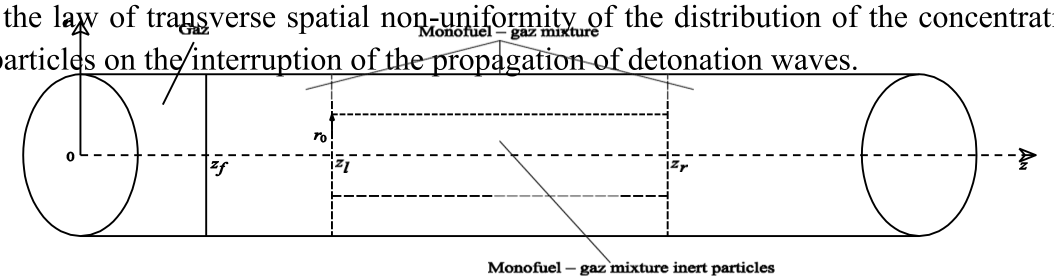

Fig. 1. Geometrical representation of the problem. z_l and z_r – left and right boundaries of the screening layer of solid particles.

3. Statement of the problem

Within the framework of the accepted assumptions, the system of differential equations of two-dimensional axisymmetric non-stationary motion of a dispersed three-phase reacting mixture of gas, particles of unitary fuel and solid particles can be written in the following form [11, 12, 13]:

Equations of conservation of mass of gas mixture, particles of unitary fuel and solid particles

$$\frac{\partial \rho_1}{\partial t} + \frac{1}{r}\frac{\partial (\rho_1 v_{1,r} r)}{\partial r} + \frac{\partial (\rho_1 v_{1,z})}{\partial z} = J,$$

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$$\frac{\partial \rho_2}{\partial t} + \frac{1}{r}\frac{\partial(\rho_2 v_{2,r} r)}{\partial r} + \frac{\partial(\rho_2 v_{2,z})}{\partial z} = -J,$$

(1)

$$\frac{\partial \rho_3}{\partial t} + \frac{1}{r}\frac{\partial(\rho_3 v_{3,r} r)}{\partial r} + \frac{\partial(\rho_3 v_{3,z})}{\partial z} = 0,$$

Equations for conservation of mass of inert gas and gaseous combustion products of a gas mixture

$$\frac{\partial \rho_{11}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_{11} v_{1r} r)}{\partial r} + \frac{\partial(\rho_{11} v_{1z})}{\partial z} = 0,$$

(2)

$$\frac{\partial \rho_{12}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_{12} v_{1r} r)}{\partial r} + \frac{\partial(\rho_{12} v_{1z})}{\partial z} = J,$$

$$\rho_{11} = \rho_{11}^0 \alpha_{11},\ \rho_{12} = \rho_{12}^0 \alpha_{11},\ \rho_1 = \rho_1^0 \alpha_1,\ \rho_1 = \rho_{11} + \rho_{12},\ \rho_2 = \rho_2^0 \alpha_2,\ \rho_3 = \rho_3^0 \alpha_3,$$
$$\alpha_1 = \alpha_{11} = \alpha_{12},\ \rho_1^0 = \rho_{11}^0 + \rho_{12}^0,$$

Equations of conservation of the number of dispersed particles

$$\frac{\partial n_2}{\partial t} + \frac{1}{r}\frac{\partial(n_2 v_{2r} r)}{\partial r} + \frac{\partial(n_2 v_{2z})}{\partial z} = 0,$$

(3)

$$\frac{\partial n_3}{\partial t} + \frac{1}{r}\frac{\partial(n_3 v_{3,r} r)}{\partial r} + \frac{\partial(n_3 v_{3,z})}{\partial z} = 0,$$

Equations of conservation of phase momentum

$$\frac{\partial(\rho_1 v_{1r})}{\partial t} + \frac{1}{r}\frac{\partial(\rho_1 v_{1r} v_{1r} r)}{\partial r} + \frac{\partial(\rho_1 v_{1r} v_{1z})}{\partial z} + \frac{\partial p}{\partial r} = -F_{2r} - F_{3r} + J v_{2r},$$

(4)

$$\frac{\partial(\rho_1 v_{1z})}{\partial t} + \frac{1}{r}\frac{\partial(\rho_1 v_{1r} v_{1z} r)}{\partial r} + \frac{\partial(\rho_1 v_{1z} v_{1z})}{\partial z} + \frac{\partial p}{\partial z} = -F_{2z} - F_{3z} + J v_{2z},$$

$$\frac{\partial \rho_2 v_{2r}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_2 v_{2r} v_{2r} r)}{\partial r} + \frac{\partial(\rho_2 v_{2r} v_{2z})}{\partial z} = F_{2r} - J v_{2r},$$

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$$\frac{\partial \rho_2 v_{2z}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_2 v_{2r} v_{2z} r)}{\partial r} + \frac{\partial(\rho_2 v_{2z} v_{2z})}{\partial z} = F_{2z} - J v_{2z},$$

$$\frac{\partial \rho_3 v_{3r}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_3 v_{3r} v_{3r} r)}{\partial r} + \frac{\partial(\rho_3 v_{3r} v_{3z})}{\partial z} = F_{3r},$$

$$\frac{\partial \rho_3 v_{3z}}{\partial t} + \frac{1}{r}\frac{\partial(\rho_3 v_{3r} v_{3z} r)}{\partial r} + \frac{\partial(\rho_3 v_{3z} v_{3z})}{\partial z} = F_{3z},$$

Equations of heat flows to particle phases

$$\frac{\partial(\rho_2 e_2)}{\partial t} + \frac{1}{r}\frac{\partial(\rho_2 e_2 v_{2r} r)}{\partial r} + \frac{\partial(\rho_2 e_2 v_{2z})}{\partial z} = Q_{12}\eta(-J) - J e_2,$$

(5)

$$\frac{\partial(\rho_3 e_3)}{\partial t} + \frac{1}{r}\frac{\partial(\rho_3 e_3 v_{3r} r)}{\partial r} + \frac{\partial(\rho_3 e_3 v_{3z})}{\partial z} = Q_{13},$$

Equations of conservation of total energy of the mixture

$$\sum_{i=1}^{3}\left[\frac{\partial(\rho_i E_i)}{\partial t} + \frac{1}{r}\frac{\partial(\rho_i E_i + \alpha_i p) v_{ir} r}{\partial r} + \frac{\partial(\rho_i E_i + \alpha_i p) v_{iz}}{\partial z}\right] = 0,$$

(6)

$$v_i^2 = v_{ir}^2 + v_{iz}^2,\ \alpha_j = \frac{1}{6}\pi d_j^3 n_j,\ \alpha_1 + \alpha_2 + \alpha_3 = 1,\ E_i = e_i + 0.5 v_i^2,\ (i = 1; 2; 3).$$

The indices “1”, “2” and “3” at the bottom refer to the parameters of the gas and two dispersed phases;

In equations (1) – (6), ρ_{11}, ρ_{12} and ρ_{11}^0, ρ_{12}^0 are the reduced and true densities of the gas phase components; ρ_i, ρ_i^0, v_i, α_i, e_i, E_i are, respectively, the average and true densities, mass velocity, volume content, specific internal and total energies of the *i*-th phase; v_{ir} and v_{iz} are the components of the velocity v_i; n_2 is the number of particles of unitary fuel per unit volume of the mixture; n_3 is the number of solid particles per unit volume of the mixture; d_2 is the diameter of the particles of unitary fuel and J is the diameter of the solid particles; d_3 is the pressure of the gas mixture; F_{jr} and F_{jz} are the components of the interphase friction force; Q_{1j} is the intensity of heat exchange between the gas and dispersed phases; *J* is the intensity of interphase mass transfer; η is the unit Heaviside function.

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The equations of state of an ideal calorically perfect gas and incompressible solid particles are used [10, 13]:

$$p_{1k} = \rho_{1k}^0 \, R_{1k} \, T_1 \, , \; p = \sum_{k=1}^{2} p_{1k} = \rho_1^0 \, R_1 T_1, \quad \rho_j^0 = const$$

(7)

where p_{1k} , R_{1k} is the partial pressure and gas constant of the k-th gas component; T_1 is the gas temperature.

The system of equations (2)-(6) and (7) is closed by the laws of interphase force and thermal interaction of the gas and dispersed phases [10, 13]:

$$F_{jl} = \frac{1}{2} C_{d_j} \, \rho_1^0 \left| v_1 - v_j \right| (v_{1l} - v_{jl}) \, \frac{\pi d_j^2}{4} n_j \, ; \quad (j = 2,3; \quad l = r, z),$$

$$Q_{1j} = \pi d_j \, \lambda_1 \, Nu \, (T_1 - T_j) \, n_j, \, C_{d_j} = \frac{24}{\mathrm{Re}_{1j}} + \frac{4}{\sqrt{\mathrm{Re}_{1j}}} + 0.4,$$

(8)

$$\mathrm{Re}_{1j} = \frac{\rho_1^0 \left| v_1 - v_j \right| d_j}{\mu_1}, Nu_j = 2 + 0,6 \mathrm{Re}_{1j}^{1/2} \, \mathrm{Pr}^{1/3}, \quad \mathrm{Pr} = \frac{c_{p1} \, \mu_1}{\lambda_1},$$

$$\left| v_1 - v_j \right| = \left[\, (v_{1z} - v_{jz} \,)^2 + (v_{1r} - v_{jr} \,)^2 \right]^{1/2},$$

$$c_{p1} = \sum_{k=1}^{2} x_{1k} c_{p1k} \, , \qquad \lambda_1 = \sum_{k=1}^{2} x_{1k} \lambda_{1k} \, , \qquad \mu_1 = \sum_{k=1}^{2} x_{1k} \mu_{1k} \, .$$

Here C_{d_i} is the aerodynamic drag coefficient of a solid spherical particle; Re_{1i}, Nu_i, Pr are the Reynolds, Nusselt, and Prandtl numbers; c_{p1} and c_{p1k} , μ_1 and μ_{1k} , λ_1 and λ_{1k} are the specific heat capacities, dynamic viscosities, and thermal conductivity coefficients of the gas mixture as a whole and its components.

The intensity of formation of gaseous combustion products of unitary fuel particles is specified according to the layer-by-layer combustion model [12]:

$$J = \pi \rho_2^0 d_2^2 n_2 v_s \left(\frac{p}{p_0} \right)^{\varphi} \eta(T_2 - T_s) \tag{10}$$

where T_s is the decomposition temperature of the unitary fuel, ν_s and ϕ are combustion constants, $\eta(T_2 - T_s)$ is the Heaviside function equal to 0 at $(T_2 - T_s) < 0$ and 1 at $(T_2 - T_s) \geq 0$

The initial and boundary conditions of the problem are set similarly to the work [12].

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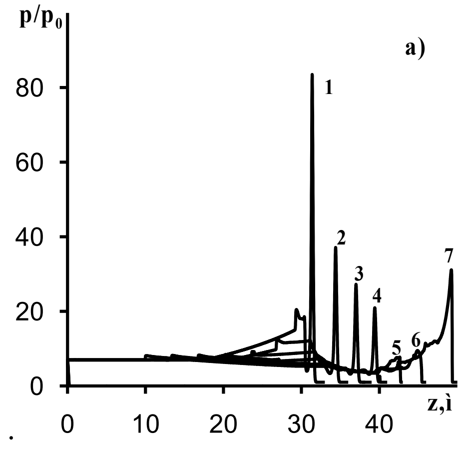
The problem was solved numerically using the large particle method [14].

4. Calculation results

Fig. 2 shows the calculated pressure profiles of the gas mixture during the passage of a heterogeneous detonation wave through a transversely inhomogeneous layer of inert particles on the axis of symmetry for different values of the length of the layer of solid particles l_S. Curves 1–7 in Fig. 2 correspond to the moments of time *t*=25.9, 30.6, 35.4, 40.1, 44.8, 49.6 and 54.3 ms and the phase parameters $m_{20}=1$; $d_{20}=30$ μm; $m_{30}=50$; $d_{30}=15$ μm (at *t*=0). The calculations were carried out for the length of a layer of solid particles with an extension of $l_s=10$ m (Fig. 2(a)), i.e. $z_l=30$ m; $z_r=40$ m and C m (Fig. 2(b)), i.e. $z_l=30$ m; $z_r=45$ m. The value of $A=0.4$, the radius of the pipeline $r=0.2$ m. At the initial stage of interaction (curves 1 in Fig. 2(a, b)) an increase in pressure is observed in the passing wave, which is caused by the deceleration of the oncoming flow in a dense layer of solid particles. As the wave moves forward, it reaches the right boundary of the layer, where it splits into a passing shock wave propagating into the unitary fuel region $(z>z_r)$ and a rarefaction wave directed into the three-phase mixture zone $(z\le z_r)$. The rarefaction wave contributes to a partial weakening of the decaying combustion wave (curves 2–4 in Fig. 2(a, b)). However, after its reflection from the left boundary of the solid particle cloud, a reflected compression wave is formed, returning to the inert gas suspension region, which leads to a temporary increase in particle combustion (Fig. 2(a)). It was found that with a solid particle layer length of 10 m, the detonation wave retains the ability to propagate. At the same time, with an increase in the layer length to 15 m, the reflected compression wave is insufficient to re-initiate combustion, and a gradual attenuation of the detonation process is observed (curves 5–8 in Fig. 2(b)). This indicates the presence of a critical layer length, above which the propagation of detonation is effectively suppressed.



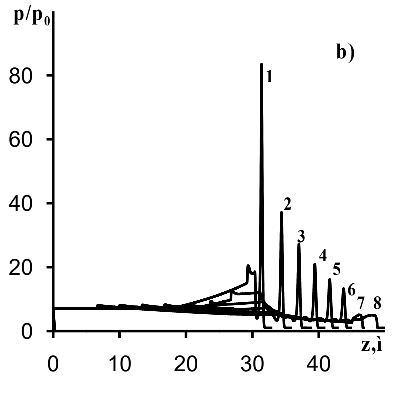


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Fig. 2. Pressure profiles of the gas mixture during the passage of a heterogeneous detonation wave through a layer of inhomogeneous inert particles on the symmetry axis of the pipeline at moments in time t=25.9, 30.6, 35.4, 40.1, 44.8, 49.6 and 54.3 ms. $m_{20} = 1$; $d_{20} = 30$ μm; $m_{30} = 50$; $d_{30} = 15$ μm; $l_s = 10$ m (a); $l_s = 15$ m (b); $A = 0.6$; $r = 0.2$ m

Fig. 3 shows the integral dependence of the minimum initial relative mass content of solid particles with a diameter of $d_{30} = 3$ μm on the length of the layer of inert particles l_S. To determine the critical size of the cloud of inert particles required to interrupt the propagation of detonation waves, a series of calculations was performed for different values of the height of the transverse spatial inhomogeneity. Curve 1 in Fig. 3 corresponds to the height of the transverse layer of solid particles A = 0.2, curve 2 - A = 0.4. Analysis of the results shows that with a decrease in the height of the transverse layer of solid particles, the minimum required mass content of solid particles for effective detonation suppression increases. This is explained by the fact that with a decrease in the layer height, the degree of interaction of the detonation wave front with inert particles decreases, which leads to less intense absorption of the front energy and a lesser slowdown in chemical reactions due to reduced heat exchange and impulse transfer. On the other hand, increasing the length of the inert particle layer has the opposite effect: as the cloud of inert particles lengthens, the time of interaction of the detonation wave with the particles increases, resulting in more efficient heat and mass transfer, as well as absorption of the kinetic and thermal energy of the wave front. As a consequence, with an increase in the layer length, a lower initial relative mass content of inert particles is required to achieve detonation suppression.

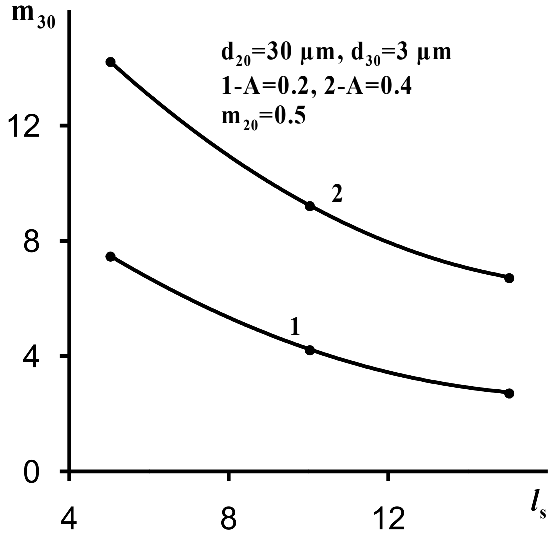

Fig. 3. The influence of the length of the layer of inert particles l_S on the minimum relative mass content of inert particles m_{30} required to quench heterogeneous detonation

Fig. 4 shows the integral dependence of the minimum initial relative mass content of solid particles on the layer length for different values of the mass content of unitary fuel

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particles. Curve 1 corresponds to the mass content of unitary fuel particles $m_{20} = 0.5$, and curve 2 to the mass content of $m_{30} = 1$. According to the presented data, an increase in the mass content of unitary fuel particles leads to an increase in the minimum required mass content of solid inert particles to suppress detonation. This effect is explained by the fact that an increase in the amount of combustible particles in the gas suspension leads to an increase in the energy release and an increase in the intensity of chemical reactions, which, in turn, increases the energy of the detonation wave front and its resistance to the effects of inert particles. Thus, with an increase in the concentration of unitary fuel, inert particles must absorb more energy and participate more actively in heat and mass transfer processes to reduce the temperature and chemical activity of the detonation front. Consequently, an increase in the total mass content of inert particles is required for successful and effective suppression of detonation processes. The presented results confirm the importance of a detailed analysis of the composition and concentrations of gas suspensions when designing measures to control and suppress detonation in various industrial and technological systems.

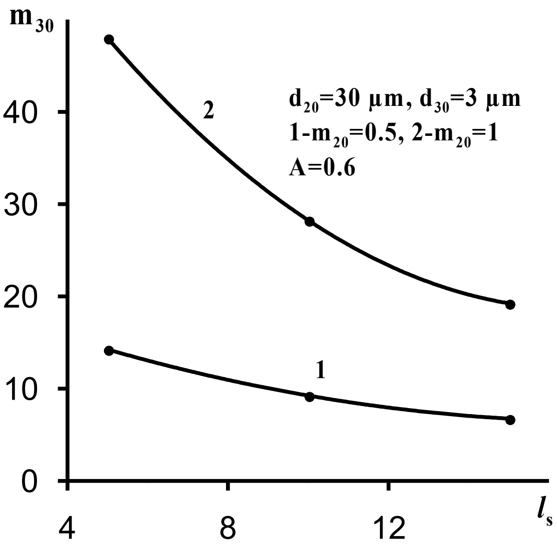

Fig. 4. The influence of the length of the layer of inert particles l_s on the minimum relative mass content of inert particles m_{30} required to quench heterogeneous detonation.

Fig. 5 shows the integral dependence of the minimum initial relative mass content of solid inert particles on the layer length for different particle diameters. Numerical studies were conducted to determine the critical size of the inert particle cloud required to effectively interrupt the propagation of detonation waves at a fixed height of the transverse layer of solid particles. Curve 1 corresponds to solid particles with a diameter of $d_{30} = 3$ µm, and curve 2 – with a diameter of $d_{30} = 60$ µm. The calculation results show that a decrease in the diameter of solid particles helps to reduce the minimum required mass content of inert particles to suppress detonation waves. This is explained by a significant increase in the total surface area of fine particles with a decrease in their size, which significantly enhances heat and mass

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transfer between the detonation wave front and the particles. Smaller particles absorb the energy of the wave front more efficiently due to an increase in the heat exchange rate, thereby intensifying the cooling processes and reducing the temperature of the reaction zone, which leads to a slowdown in chemical reactions and subsequent attenuation of detonation. Thus, the presented dependencies clearly demonstrate the importance of taking into account the size of inert particles when designing protective systems aimed at controlling and suppressing the spread of detonation processes in technological and industrial systems.

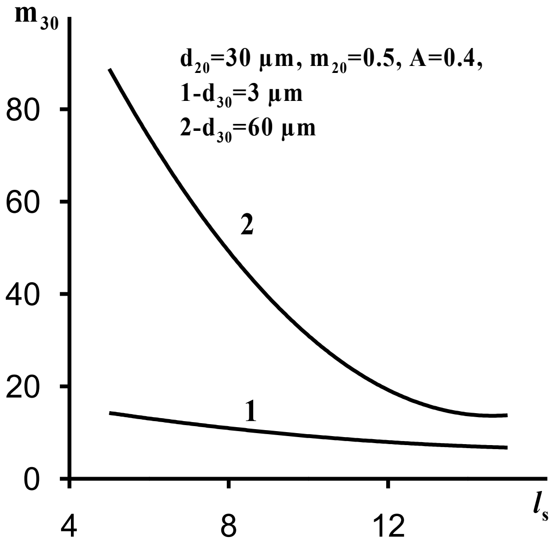

Fig. 5. The influence of the length of the layer of inert particles l_s on the minimum relative mass content of inert particles m_{30} required to quench heterogeneous detonation

5. Conclusion

The conducted numerical studies show a significant influence of the size and concentration of inert solid particles on the propagation of detonation waves in gas suspensions. Small particles contribute to a faster attenuation of detonation compared to large ones. There are critical values of concentration, particle size and particle layer thickness at which detonation attenuation occurs. On the other hand, an increase in the mass content of unitary fuel increases the requirements for the mass content of inert particles to achieve detonation attenuation. The obtained results are of practical importance for the development of safety systems and methods for passive suppression of detonation processes in industrial installations.

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