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PRICING, SALES-EFFORT AND REWARD OFFERING STRATEGY IN A CLOSED LOOP DUAL CHANNEL SUPPLY CHAIN WITH CUSTOMER'S FREE RIDING BEHAVIOR AND RETURN-REFUND POLICY

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Abstract

Over the years, manufacturing industries have evolved due to growing environmental awareness, resource scarcity, and, more recently, shifts in purchasing behavior driven by internet-enabled technologies. These factors have contributed to the rise of Closed-Loop Dual-Channel Supply Chains (CLDSCs). Additionally, challenges such as Free-Riding Behavior (FRB)—where consumers utilize offline services but make purchases online and generous return-refund policies have further complicated supply chain coordination and profitability.

In this research, we develop a Closed-Loop Dual-Channel Supply Chain (CLDSC) consisting of an offline retail channel and an online direct channel incorporating both FRB and a full refund return policy. In this model, the manufacturer acquires used products from customers for remanufacturing by offering a monetary reward. The core objective of this study is to evaluate optimal strategies for pricing, sales effort, and reward levels under two scenarios: a decentralized scenario and a centralized scenario. A numerical analysis with sensitivity investigations is also conducted to examine how increasing return rates impact coordination and profitability, and how FRB should guide strategic adjustments to enhance overall system efficiency.

Math. Subject Classification: MSC90B06, MSC91A10, MSC91A12

Key Words and Phrases: Dual channel, recycling, free riding behaviour, sales-effort, coordination

1. Introduction

Over the past two decades, used product recycling has been widely adopted in supply chains, not only to protect the environment but also for its economic benefits. The remanufacturing process maximizes the residual value of used products, thereby reducing waste and conserving energy, water, and raw materials. This, in turn, proportionally reduces carbon emissions. According to Liu et al. [5], remanufacturing is estimated to reduce CO_2 emissions by 10 million ton annually. An enormous collection of articles on CLSCs can

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be found in Ostlin et al. [8], Khalili et al. [9], Gaur et al. [7], Battini et al. [6], Asghari et al. [10], and Goli [11], wherein the forward supply chain entails the flow of products from upstream manufacturers to end consumers, while the reverse supply chain facilitates the return of used products from customers to upstream entities for remanufacturing purposes.

At the same time, the widespread accessibility of the Internet has prompted companies to increasingly adopt hybrid distribution models that integrate traditional retail outlets with online e-commerce platforms. As a result, sustainable supply chains incorporating reverse logistics activities—particularly through the lens of digitalization—have attracted increasing interest within the academic community, prompting focused investigations into the fundamental principles of Closed-Loop Dual-Channel Supply Chains (CLDSCs). Although, an extensive research on CLDSCs exists (e.g., Pathak et al. [12], Arshad et al. [13], Taleizadeh et al. [14], Kaoud et al. [15], Liu et al. [16], Shi et al. [17]), many studies assume that customers return used products without receiving any incentives. However, in practice, collection rates are significantly influenced by monetary rewards. Industries such as automotive, electronics, and furniture commonly adopt reward-based return policies to increase return volumes—for example, car trade-in programs and return incentives offered by brands like Staples and HP. Only a limited number of researchers have exam- ined a CLDSC model where the return rate is modeled as a function of the reward value offered for acquiring used products (Saha et al. [19] and Pal [18]).

In real-world business scenarios, it is often challenging for customers to evaluate the quality or value of a product in an online marketplace. In con- trast, offline stores can attract more customers by offering value-added services such as product samples, trials, and detailed explanations—features that are typically unavailable online. However, when a manufacturer adopts a dual- channel strategy, some customers may first take advantage of the offline re- tailer's services to experience the product, but ultimately make their purchase through the online channel. This behavior leads to the phenomenon known as free-riding behavior (FRB) by the manufacturer (Pu et al. [1], Tian et al. [2], Zhou et al. [3]). He et al. [4] proposed a CLDSC model that considers carbon emissions, aiming to investigate scenarios where consumers either engage in or refrain from free riding, and to analyze how consumer free riding impacts carbon emissions. Moreover, to remain competitive, many companies allow customers to return products if they are not satisfied. The introduction of full refund policies has contributed to an increase in the number of product returns. However, many retailers implement these policies to foster customer loyalty, enhance customer satisfaction, boost sales, and/or comply with national regulations (Radhi and Zhang [20]). Xu et al. [21] proposed a dual-channel supply chain model to investigate the impacts of channel preference, FRB, sales effort, and cross-channel returns. To the best of our knowledge, existing studies on Closed-Loop Dual-Channel Supply Chains (CLDSC) rarely consider scenarios where used products are bought back from customers, and where the collection rate depends on the incentives or rewards offered by manufacturers. Moreover, limited attention has been given to examining the impacts of Full Refund Behavior (FRB) and return-refund

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policies in such contexts.

Therefore, this paper explores the optimal pricing strategies, sales effort levels, and reward mechanisms for collecting used products under both centralized and decentralized decision-making frameworks within a CLDSC. The study aims to address the following key questions: How does return rate influence coordination mechanisms and the profitability of supply chain members? And, as the FRB of customer increases, how should the players adjust their optimal strategies to enhance overall system efficiency?

2. Model Development

This paper investigates a closed-loop dual-channel supply model in which a manufacturer produces a product through two methods: manufacturing from raw materials at a unit cost of c_m, and remanufacturing used products at a lower unit cost of c_r, where $c_r < c_m$. The manufacturer sells the product to a retailer at a wholesale price w, and the retailer, in turn, offers it to customers through the offline channel at a retail price p_r. Simultaneously, the manufacturer operates an online direct channel with customer by selling the same product at a price p_m, alongside the offline retail channel. Customers may return the product after purchase if they are not satisfied. It is assumed that the return rate is the same for both channels and is denoted by σ $(0 \leq \sigma \leq 1)$. Upon return, customers receive a full refund. As a result, both the manufacturer and the retailer incur a unit processing cost t for handling returned products across both channels. At the end of the sales cycle, the returned products have a residual value s, which is managed by the manufacturer and the retailer.

The retailer exerts a sales effort level denoted by e, which incurs a cost of $\frac{\eta e^2}{2}$. The parameter α represents the sensitivity of market demand in the retail channel to the retailer's sales effort. Meanwhile, γ $(0 \leq \gamma \leq \alpha)$ denotes the extent of customer free-riding behavior, capturing the proportion of customers who, after benefiting from the promotional sales services offered by the retail channel, choose to purchase the product through the online channel.

In this research, following Xu et al. [21], we assume a linear demand structure that depends on both price and sales effort for the online direct (D_m) and traditional offline retail channels (D_r), as described by the following equations:

$$D_m = \rho a + \gamma e - \beta p_m + \delta p_r \quad (1)$$

$$D_r = (1-\rho)a + (\alpha - \gamma)e - \beta p_r + \delta p_m \quad (2)$$

Where, a $(a > 0)$ represents the total market potential, and $\rho \in [0,1]$, denotes the market share of the direct channel. Accordingly, the demands for the direct and retail channels are ρa and $(1-\rho)a$, respectively. The parameters β and δ represent the own-price sensitivity and cross-price sensitivity of demand. Since a channel's demand is more sensitive to its own price than to the competing channel's price, we assume $0 < \delta < \beta < 1$.

The manufacturer collects used products directly from customers through a reward-based return policy. The return rate is modeled as a function of the reward amount r, denoted

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by $f(r)$, where $0 < f(r) < 1$. Consistent with real-world observations, we assume that the return rate is a monotonically increasing function of r, expressed as:

$$f(r) = 1 - \lambda^r \quad \text{where} \quad 0 < \lambda < 1$$

Hence, the profit functions of the manufacturer (π_m) and the retailer (π_r) are as follows

$$\pi_m = (1-\sigma)p_m D_m + w\, D_r - c_m(D_m + D_r) + (c_m - c_r - r)f(r)(D_m + D_r) + (s - t)\sigma D_m \quad (3)$$

$$\pi_r = (1-\sigma)p_r D_r - w\, D_r - \frac{\eta e^2}{2} + (s-t)\sigma D_r \quad (4)$$

In the next subsection, we derive the expressions of all the decision variables under decentralized and centralized decisions.

2.1.Model analysis under decentralized decision

Under decentralized decision, for given values of sales effort e, and reward amount r, the manufacturer first optimizes the direct channel price and wholesale price, taking into account the retailer's best response in setting the retail price.

Proposition 1: *For given value of sales effort e, the retailer's profit function π_r is strictly concave in p_r and the manufacturer's profit function π_m is strictly concave in p_m and w. In this case, optimal solution of retail price, direct price and wholesale price are*

$$p_r^{d^*}(e) = \frac{[(1-\rho)(3\beta^2-\delta^2)+2\beta\delta\rho]a+[(\alpha-\gamma)(3\beta^2-\delta^2)+2\gamma\beta\delta]e}{4\beta(\beta^2-\delta^2)} + \frac{(\beta+\delta)[c_m-(c_m-c_r-r)f(r)-(s-t)\sigma]}{4\beta(1-\sigma)} \quad (5)$$

$$p_m^{d^*}(e) = \frac{[\beta+(1-\rho)\delta]a+[\gamma\beta+(\alpha-\gamma)\delta]e}{2(\beta^2-\delta^2)} + \frac{[c_m-(c_m-c_r-r)f(r)-(s-t)\sigma]}{2(1-\sigma)} \quad (6)$$

$$w^{d^*}(e) = \frac{(1-\sigma)[(1-\rho)\beta+\delta]a+(1-\sigma)[(\alpha-\gamma)\beta+\gamma\delta]e}{2(\beta^2-\delta^2)} + \frac{[c_m-(c_m-c_r-r)f(r)-(s-t)\sigma]}{2} \quad (7)$$

Proof. From Eq [4], we obtain the value of p_r under decentralized decision strategy by solving $\frac{\partial \pi_r}{\partial p_r} = 0$ and the best response is given by

$$p_r^d(w, p_m, e) = \frac{(1-\sigma)[(1-\rho)a+(\alpha-\gamma)e+\delta p_m]+\beta w - \sigma\beta(s-t)}{2\beta(1-\sigma)}$$

We also find that π_r is strictly concave in p_r as $\frac{\partial^2 \pi_r}{\partial p_r^2} = -2\beta(1-\sigma) < 0$

Substituting the value of $p_r^{d^*}(w, p_m, e)$ into Eq [4], we obtain the values of $p_m^{d^*}(e, r)$ and $w^{d^*}(e)$ (given by Eq [6] and Eq [7]) solving the first order conditions $\frac{\partial \pi_m}{\partial p_m} = 0$ and $\frac{\partial \pi_m}{\partial w} = 0$. Finally, putting the values of $p_m^{d^*}(e)$ and $w^{d^*}(e)$ in $p_r^d(w, p_m, e)$, we obtain $p_r^{d^*}(e)$ as given in Eq [5].

Now, $\frac{\partial^2 \pi_m}{\partial p_m^2} = -\frac{(1-\sigma)(2\beta^2-\delta^2)}{\beta} < 0$ and the Hessian matrix of the profit function π_m is

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$$H = \begin{pmatrix} \frac{\partial^2 \pi_m}{\partial p_m^2} & \frac{\partial^2 \pi_m}{\partial p_m \partial w} \\ \frac{\partial^2 \pi_m}{\partial w\, \partial p_m} & \frac{\partial^2 \pi_m}{\partial w^2} \end{pmatrix} = \begin{pmatrix} \frac{(1-\sigma)\,(2\beta^2-\delta^2)}{\beta} & \delta \\ \delta & -\frac{\beta}{(1-\sigma)} \end{pmatrix}$$

$|H| = 2(\beta^2 - \delta^2) > 0$. Hence, the manufacturer's profit function π_m is strictly concave in p_m and w.

The online and offline channel's profits as functions of sales effort level e are given by

$$\pi_m^{d^*}(e) = \frac{1}{K_1}[I_1\,(c_m - c_r - r)^2 f^2(r) + 2(\beta^2 - \delta^2)(c_m - c_r - r)[(1-\sigma)I_2\, a - (3\beta^2 - 2\beta\delta - \delta^2)[c_m - \sigma(s-t)] + (1-\sigma)I_3\, e]f(r) - (1-\sigma)^2\, I_4\, e^2 - 2(1-\sigma)[(1-\sigma)I_5 a - (\beta^2 - \delta^2)I_3\big[(c_m - \sigma(s-t))\big]e + (1-\sigma)^2 I_6 a^2 - 2(1-\sigma)(\beta^2 - \delta^2)[c_m - \sigma(s-t)]I_2 a + I_1\big[(c_m - \sigma(s-t))\big]\big] \quad (8)$$

$$\pi_r^{d^*}(e) = \frac{1}{K_2}\big[(\beta - \delta)^2\,(c_m - c_r - r)^2 f^2(r) + 2(\beta - \delta)(c_m - c_r - r)[(1-\sigma)(1-\rho)a - (\beta - \delta)[c_m + \sigma(s-t)] + (1-\sigma)(\alpha - \gamma)e]f(r) + (1-\sigma)I_7 e^2 + 2(1-\sigma)(\alpha - \gamma)\big[(1-\sigma)(1-\rho)a - (\beta - \delta)[c_m - \sigma(s-t)]\big]e + (1-\sigma)^2(1-\rho)^2 a^2 - 2(1-\sigma)(1-\rho)(\beta - \delta)[c_m - \sigma(s-t)]a + (\beta - \delta)^2[c_m + \sigma(s-t)]^2\big] \quad (9)$$

where $K_1 = 8\beta(\beta^2 - \delta^2)(1-\sigma)$, $K_2 = 16\beta\,(1-\sigma)$, $I_1 = (\beta - \delta)^2(3\beta^2 + 4\beta\delta + \delta^2)$, $I_2 = [\,(1+\rho)\beta + (1-\rho)\delta)]$, $I_3 = [(\beta+\delta)\alpha + (\beta - \delta)\gamma]$, $I_4 = \big[(\alpha-\gamma)^2\,(\beta^2 + \delta^2) + 2\beta^{2\gamma^2} + 4\beta\delta\gamma(\alpha-\gamma)\big]$, $I_5 = (1-\rho)[(\alpha-\gamma)(\beta^2+\delta^2) + 2\beta\delta\gamma] + 2\rho\beta[(\alpha-\gamma)\delta + \beta\gamma]$, $I_6 = (1-\rho)^2\,(\beta^2+\delta^2) + 4(1-\rho)\rho\beta\delta + 2\beta^2\delta^2$, $I_7 = (1-\sigma)(\alpha-\gamma)^2 - 8\beta\eta$

To investigate the effect of sales effort level on optimal online direct selling price p_m , offline retail price p_r and wholesale price w, the first order partial derivative with respect e are obtained in Corollary 1.

***Corollary 1**: Under the decentralized decision, the effect of e on optimal prices are investigated as follows:*

(1) $\frac{\partial p_m^{d^*}}{\partial e} = \frac{\alpha\delta + (\beta - \delta)\gamma}{2(\beta^2 - \delta^2)} > 0$; *(2)* $\frac{\partial w^{d^*}}{\partial e} = \frac{(1-\sigma)[(\alpha - \gamma)\beta + \delta\gamma]}{2(\beta^2 - \delta^2)} > 0$; *and*

(3) $\frac{\partial p_r^{d^*}}{\partial e} = \frac{(3\beta^2 - \delta^2)(\alpha - \gamma) + 2\beta\delta\gamma}{4\beta(\beta^2 - \delta^2)} > 0.$

From Corollary 1, we observe that sales effort level has positive effects on offline and online channel's selling price and wholesale price.

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Proposition 2*: Under the decentralized decision, the retailer's profit function $\pi_r^{d^*}$ is strictly concave in sales effort level e if $\sigma > 1 - \frac{8\beta\eta}{(\alpha - \gamma)^2} = \sigma_0^d$ (say). In this case, ethe optimal values of sales effort e^{d^*} is*

$$e^{d^*} = \frac{(\alpha-\gamma)[(1-\rho)(1-\sigma)a+(\beta-\delta)[(c_m - c_r - r)f(r) - c_m + \sigma(s-t)]}{8\beta\eta - (\alpha-\gamma)^2(1-\sigma)} \quad (10)$$

Proof. From Eq [9], we obtain the optimal value of e under decentralized decision strategy by solving $\frac{\partial \pi_r^{d^*}}{\partial e} = 0$ and We also find that $\frac{\partial^2 \pi_r^{d^*}}{\partial e^2} = \frac{(\alpha-\gamma)^2(1-\sigma) - 8\beta\eta}{8\beta}$.

Hence, $\pi_r^{d^*}$ is strictly concave in e if $(\alpha - \gamma)^2(1-\sigma) - 8\beta\eta < 0$ i.e. $\sigma > 1 - \frac{8\beta\eta}{(\alpha-\gamma)^2}$.

Substituting the optimal value of e obtained in Eq [10] into the Equations [5], [6] and [7] we have the optimal selling price of the offline channel and online channel and wholesale price. Next, we investigate the effects of manufacturing cost and remanufacturing cost on optimal values of p_m, p_r and w

Corollary 2: *Under the decentralized decision, the effect of c_m and c_r on optimal prices are investigated as follows*

(1) $\frac{\partial p_m^{d^*}}{\partial c_m} > 0,\ \frac{\partial p_m^{d^*}}{\partial c_r} >\ if\ \sigma > \sigma_1^d > \sigma_0^d\ and$
$\frac{\partial p_m^{d^*}}{\partial c_m} < 0,\ \frac{\partial p_m^{d^*}}{\partial c_r} < 0\ if\ \sigma_0^d < \sigma < \sigma_1^d;\ where\ \sigma_1^d = 1 - \frac{8\beta\eta(\beta+\delta)}{(\alpha-\gamma)[2\delta(\alpha-\gamma)+\alpha\beta]}$

(2) $\frac{\partial p_r^{d^*}}{\partial c_m} > 0,\ \frac{\partial p_r^{d^*}}{\partial c_r} > 0\ if\ \sigma > \sigma_2^d > \sigma_0^d\ \ and$
$\frac{\partial p_r^{d^*}}{\partial c_m} < 0,\ \frac{\partial p_r^{d^*}}{\partial c_r} < 0\ if\ \sigma_0^d < \sigma < \sigma_2^d;\ where\ \sigma_2^d = 1 - \frac{4\eta(\beta+\delta)^2}{\alpha-\gamma)(2\beta(\alpha-\gamma)+\alpha\delta)}$

(3) $\frac{\partial w^{d^*}}{\partial c_m} > 0,\ \frac{\partial w^{d^*}}{\partial c_r} > 0\ if\ \sigma > \sigma_3^d > \sigma_0^d\ and$
$\frac{\partial w^{d^*}}{\partial c_m} < 0,\ \frac{\partial w^{d^*}}{\partial c_r} < 0\ if\ \sigma_0^d < \sigma < \sigma_3^d;\ where\ \sigma_3^d = 1 - \frac{8\beta\eta(\beta+\delta)}{(\alpha-\gamma)(2\beta(\alpha-\gamma)+\alpha\delta)}$

(4) $\frac{\partial e^{d^*}}{\partial c_m} < 0,\ \frac{\partial e^{d^*}}{\partial c_r} < 0\ if\ \sigma > \sigma_0^d$

From Corollary 2, we can achieve the following conclusions under decentralized decision-making: (1) The selling prices in the offline and online channels, as well as the wholesale price, do not always increase with rising manufacturing cost (c_m) and remanufacturing cost (c_r). Instead, this relationship depends on the product return rate σ. More specifically, if the return rate exceeds a threshold value σ_1^d, the online channel's selling price increases with increasing values of c_m and c_r. Conversely, if $\sigma_0^d < \sigma < \sigma_1^d$, the opposite trend is observed. Similar behavior is also observed for the offline channel's selling price and the wholesale price, with their respective threshold values denoted as σ_2^d and σ_3^d. (2) The retailer's sales effort level always decreases with increasing values of c_m and c_r.

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2.2.Model analysis under centralized decision

Under the centralized decision, the offline retailer and online manufacturer are integrated as a single entity. In this case, the total profit function of the entire supply chain is given by

$$\pi^c = \pi_m + \pi_r = (1-\sigma)(p_m\, D_m + p_r\, D_r) + (c_m - c_r - r)f(r)(D_m + D_r - [c_m - \sigma(s - t)](D_m + D_r) - \frac{\eta e^2}{2} \qquad (11)$$

Proposition 3*: For given value of sales effort e, the overall supply chain's profit π_c is strictly concave in p_m and p_r. In this case, optimal solution of retail price, direct price and wholesale price are*

$$p_m^{c^*}(e) = \frac{[\beta\rho + (1-\rho)\delta]a + [(\alpha-\gamma)\delta + \beta\gamma]e}{2(\beta^2 - \delta^2)} + \frac{[c_m - (c_m - c_r - r)f(r) - (s-t)\sigma]}{2(1-\sigma)} \qquad (12)$$

$$p_r^{c^*}(e) = \frac{[(1-\rho)\beta + \delta]a + [(\alpha-\gamma)\beta + \delta\gamma]e}{2(\beta^2 - \delta^2)} + \frac{[c_m - (c_m - c_r - r)f(r) - (s-t)\sigma]}{2(1-\sigma)} \qquad (13)$$

Proof. From Eq [11] we obtain the values of p_m and p_r under centralized decision strategy by solving $\frac{\partial \pi^c}{\partial p_m} = 0$ and $\frac{\partial \pi^c}{\partial w} = 0$.

We also find that

Now, $\frac{\partial^2 \pi^c}{\partial p_m^2} = -2\beta(1-\sigma) < 0$

and the Hessian matrix of the profit function π^c is

$$H = \begin{pmatrix} \frac{\partial^2 \pi^c}{\partial p_m^2} & \frac{\partial^2 \pi^c}{\partial p_m \partial p_r} \\ \frac{\partial^2 \pi^c}{\partial p_r\, \partial p_m} & \frac{\partial^2 \pi^c}{\partial w^2} \end{pmatrix} = \begin{pmatrix} -2\beta(1-\sigma) & 2\delta(1-\sigma) \\ 2\delta(1-\sigma) & -2\beta(1-\sigma) \end{pmatrix}$$

$|H| = 4(\beta^2 - \delta^2)(1-\sigma)^2 > 0$. Hence, the overall supply chain's profit π is strictly concave in p_m and p_r .

To investigate the effect of sales effort level on optimal online direct selling price p_m and offline retail price p_r, the first order partial derivative with respect e are obtained in Corollary 3.

Corollary 3: *Under the decentralized decision, the effect of e on optimal prices are investigated as follows*

(1) $\frac{\partial p_m^{c^*}}{\partial e} = \frac{(\alpha-\gamma)\delta + \beta\gamma}{2(\beta^2 - \delta^2)} > 0$; (2) $\frac{\partial p_r^{c^*}}{\partial e} = \frac{(\alpha-\gamma)\beta + \delta\gamma}{2(\beta^2 - \delta^2)} > 0$;

From Corollary 3, we observe that sales effort level has positive effects on offline and online channel's selling price.

Proposition 4: *Under the centralized decision, the overall profit function π^c is strictly concave in sales effort level e if $\sigma < 1 - \frac{2\eta(\beta^2 - \delta^2)}{\alpha^2\beta - 2(\beta-\delta)(\alpha-\gamma)\gamma} = \sigma_0^c$ (say). In this case, the optimal values of sales effort e^{c^*} is*

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$$e^{c*} = \frac{(1-\sigma)[\alpha\beta-(\beta-\delta)(\gamma+\alpha\rho-2\gamma\rho)]\alpha}{2\eta(\beta^2-\delta^2)-(1-\sigma)[\alpha^2\beta-2(\beta-\delta)(\alpha-\gamma)\gamma]} + \frac{\alpha(\beta^2-\delta^2)[(c_m-c_r-r)f(r)-c_m+\sigma(s-t)]}{2\eta(\beta^2-\delta^2)-(1-\sigma)[\alpha^2\beta-2(\beta-\delta)(\alpha-\gamma)\gamma]} \quad (14)$$

Proof. We obtain the optimal value of e under centralized decision strategy by solving $\frac{d\pi^c}{de} = \frac{\partial\pi^c}{\partial p_m}\frac{\partial p_m(e)}{\partial e} + \frac{\partial\pi^c}{\partial p_r}\frac{\partial p_r(e)}{\partial e} + \frac{\partial\pi^c}{\partial e} = 0$ and

We also find that

$\frac{d^2\pi^c}{de^2} = \frac{(1-\sigma)[\alpha^2\beta-2(\beta-\delta)(\alpha-\gamma)\gamma]-2\eta(\beta^2-\delta^2)}{2(\beta^2-\delta^2)}$. Hence, π^c is strictly concave in e if $\frac{(1-\sigma)[\alpha^2\beta-2(\beta-\delta)(\alpha-\gamma)\gamma]-2\eta(\beta^2-\delta^2)}{2(\beta^2-\delta^2)} < 0 \; i.e. \sigma < \sigma_0^c$.

Substituting the optimal value of e obtained in Equation [14] into the Equations [12] and [13] we have the optimal selling price of the offline channel and online channel. Next, we investigate the effects of manufacturing cost and remanufacturing cost on optimal values of p_m and p_r .

Corollary 4: *Under the centralized decision, the effect of c_m and c_r on optimal prices are investigated as follows*

(1) $\frac{\partial p_m^{c^*}}{\partial c_m} > 0,\ \frac{\partial p_m^{c^*}}{\partial c_r} > \ if\ \sigma > \sigma_1^c > \sigma_0^c\ and$

$\frac{\partial p_m^{c^*}}{\partial c_m} < 0,\ \frac{\partial p_m^{c^*}}{\partial c_r} < 0\ if\ \sigma_0^c < \sigma < \sigma_1^c; where\ \sigma_1^c = 1 - \frac{2\eta(\beta^2-\delta^2)}{\alpha^2(\beta+\delta)-2(\beta-\delta)(\alpha\gamma-2\gamma^2)}$

(2) $\frac{\partial p_r^{c^*}}{\partial c_m} > 0,\ \frac{\partial p_r^{c^*}}{\partial c_r} > 0\ if\ \sigma > \sigma_2^c > \sigma_0^c\ \ and$

$\frac{\partial p_r^{c^*}}{\partial c_m} < 0,\ \frac{\partial p_r^{c^*}}{\partial c_r} < 0\ if\ \sigma_0^c < \sigma < \sigma_2^c;\ where\ \sigma_2^c = 1 - \frac{2\eta(\beta^2-\delta^2)}{\alpha^2\beta-(\beta-\delta)(3\alpha-2\gamma)\gamma}$

(3) $\frac{\partial e^{c^*}}{\partial c_m} < 0,\ \frac{\partial e^{c^*}}{\partial c_r} < 0\ if\ \sigma > \sigma_0^c$

Hence, according to Corollary 4, the impact of manufacturing cost and remanufacturing cost on selling prices of the offline and online channel and sales effort level are the same as that under decentralized policy.

3. Numerical Analysis

We now illustrate the above analytical findings numerically. The following parameters are used for illustration throughout the paper: $a = 13000; \alpha = 1; \gamma = .3; \beta = .7; \delta = .5;\ s = 300;\ t = 10;\ c_m = 400;\ c_r = 200; \eta = .9; \rho = .62; \sigma = .9; \lambda = 0.5$. We examine the effect of the reward amount on the manufacturer's profit function under decentralized decision-making and on the overall profit function under centralized decision-making. Figure 1 demonstrates that both π_m^d and π^c are strictly concave in r. We use MATLAB's optimtool to determine the optimal value of r under both centralized and decentralized settings. The corresponding optimal solutions are presented in Table 1. It is observed that the optimal reward amount is the same for both decentralized and centralized decisions. However, the optimal values of the other decision

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variables are higher under the centralized decision, and the overall profit of the supply chain is also greater under the centralized decision.

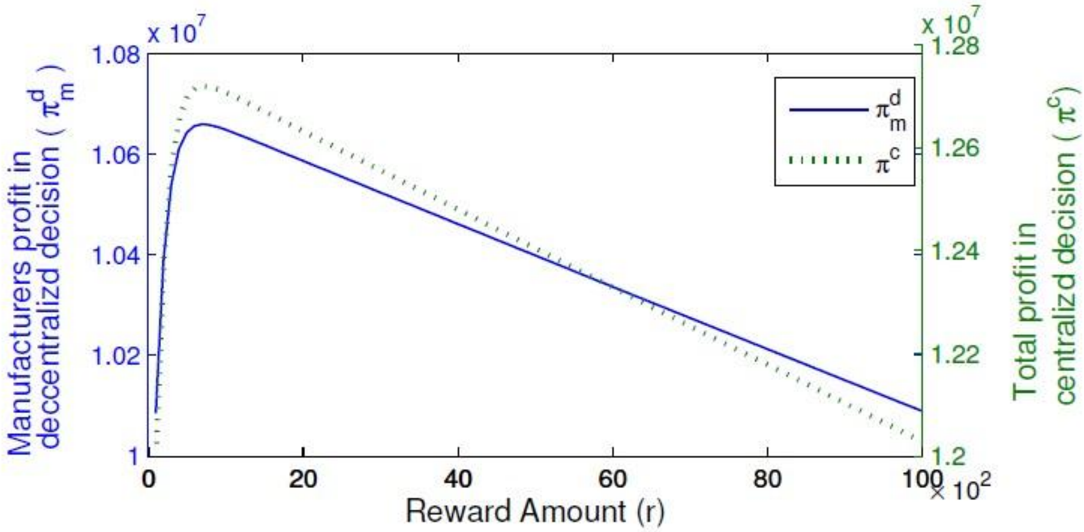

Figure 1: *Effect of* r *on* π_m^d *and* π^c

Table 1: *Optimal solutions under decentralized and centralized decision*

Decision	w^*	p^*	p^*	e^*	r^*	π^*	π^*	Total Profit
Decentralized	8180	17300	17900	360	751	5.39×10^7	1.15×10^6	5.5×10^7
Centralized	—	53000	56800	30900	751	—	—	1.77×10^8

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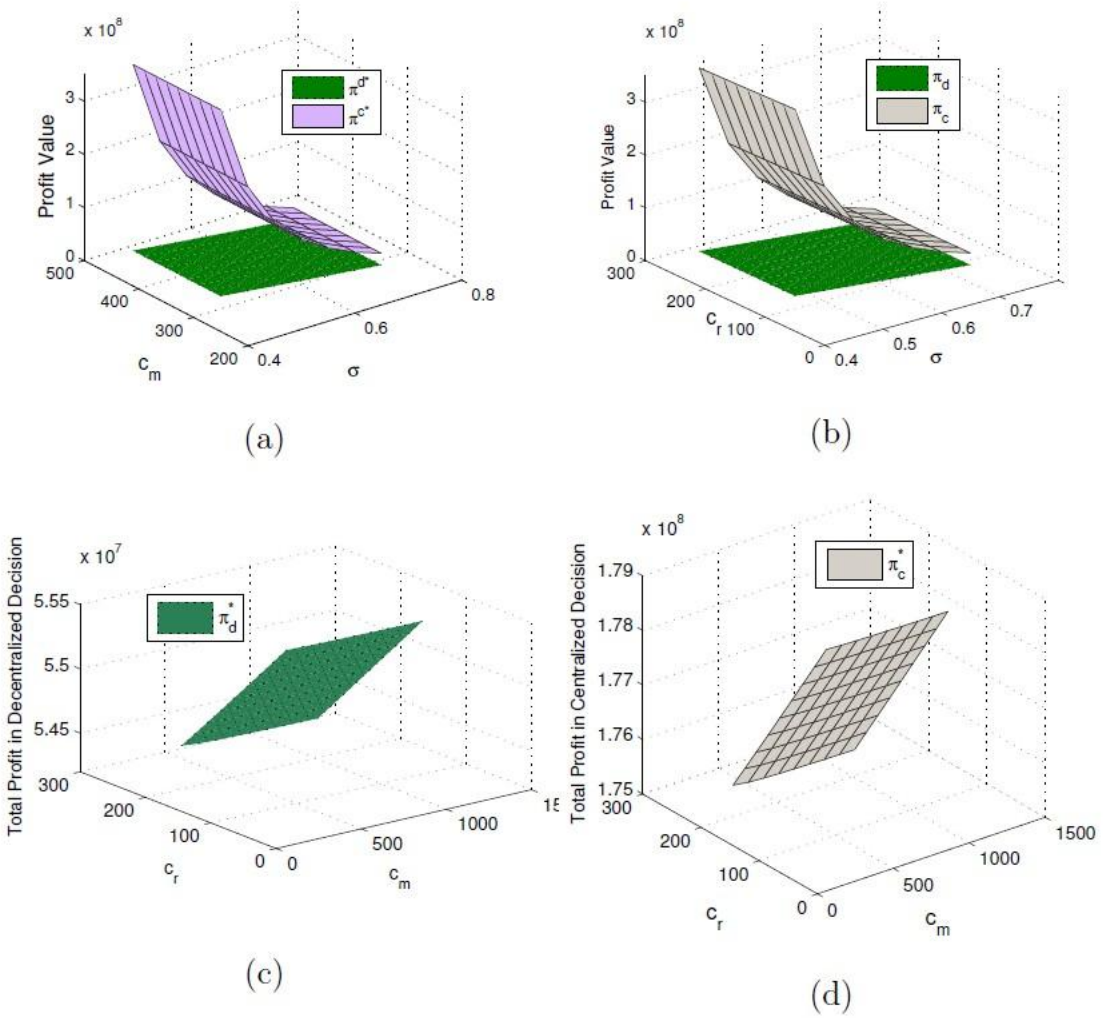

Figure 2: *(a) Effect of* σ *and* c_m *on* $\backslash\pi^{d^*}$ *and* π^{c^*} *(b)Effect of* σ *and* c_m *on* π^{d^*} *and* π^{c^*}*(c) Effect of* c_m *and* c_r *on total profit under decentralized decision (b) Effect of* c_m *and* c_r *on total profit under centralized decision*

Figure 2 shows that: (1) The overall supply chain profit under centralized decision-making is higher than under decentralized decision-making. (2) Under both decision-making approaches, the overall profit decreases as the return rate, manufacturing cost, and remanufacturing cost increase. (3) The overall profit function under centralized decision-making is more sensitive than under decentralized decision-making. (4) The profit function under both decision-making approaches is most sensitive to the return rate, followed by the remanufacturing cost and manufacturing cost.

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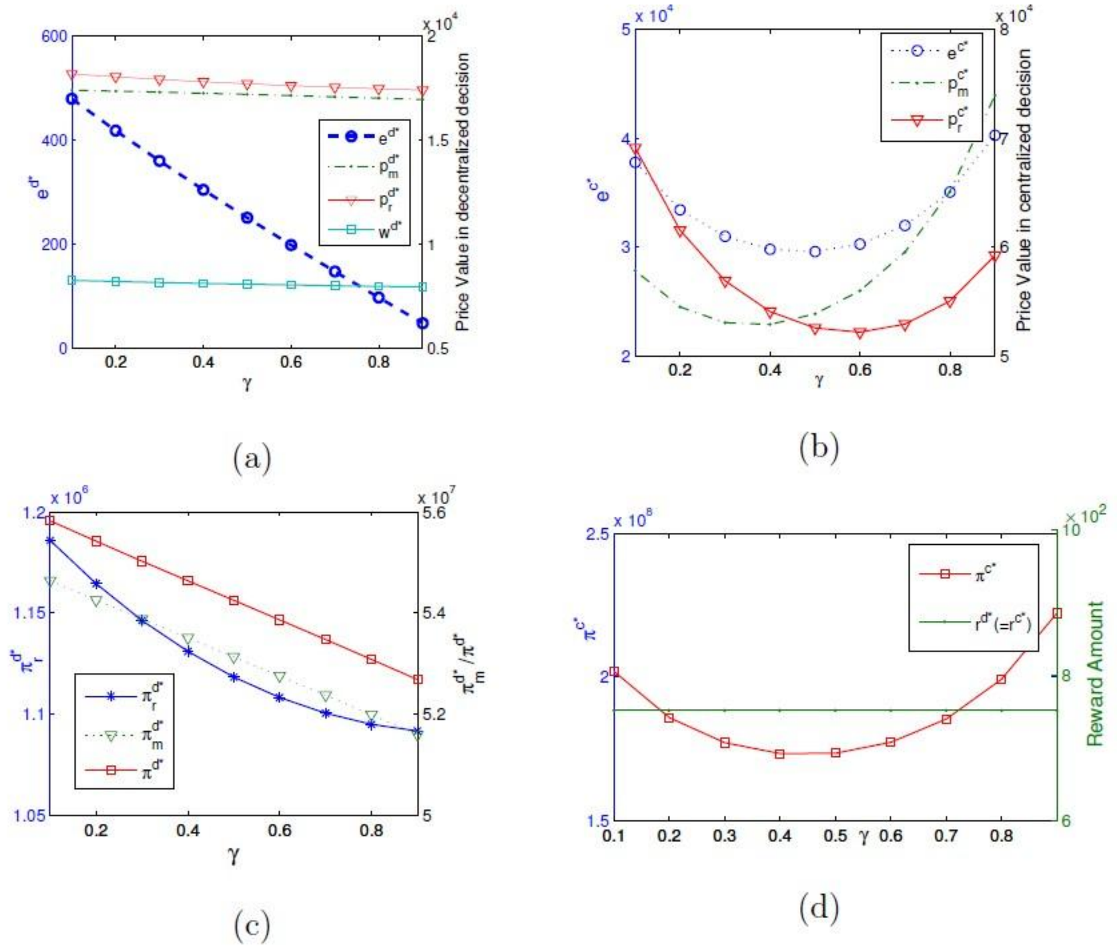

Figure 3: *Effect of γ under decentralized decisions and centralized decisions*

Figure 3 shows the following insights: (1) Under decentralized decision-making, the online and offline selling prices, wholesale price, and sales effort level all decrease as the customer's free-riding behavior coefficient increases. This leads to a decline in both the manufacturer's and retailer's profit functions. The reason is that as γ increases, the retailer lowers their selling price. In response, the manufacturer also reduces their online selling price to remain competitive. Therefore, an increase in \gamma does not benefit either party. Notably, the retailer's offline selling price always remains higher than the manufacturer's online selling price. (2) Under centralized decision-making, the decision variables exhibit opposite behavior when the manufacturer and retailer cooperate. In this case, the online and offline selling prices and sales effort level initially decrease and then increase as γ rises. As a result, the overall supply chain profit first decreases and then increases beyond a certain threshold of γ. This occurs because, under centralized decision-making, the manufacturer and retailer are not in competition and make joint decisions to optimize the entire supply chain. (3) However, under both decision-making structures, the reward amount offered by the manufacturer to collect used products remains unchanged as γ increases.

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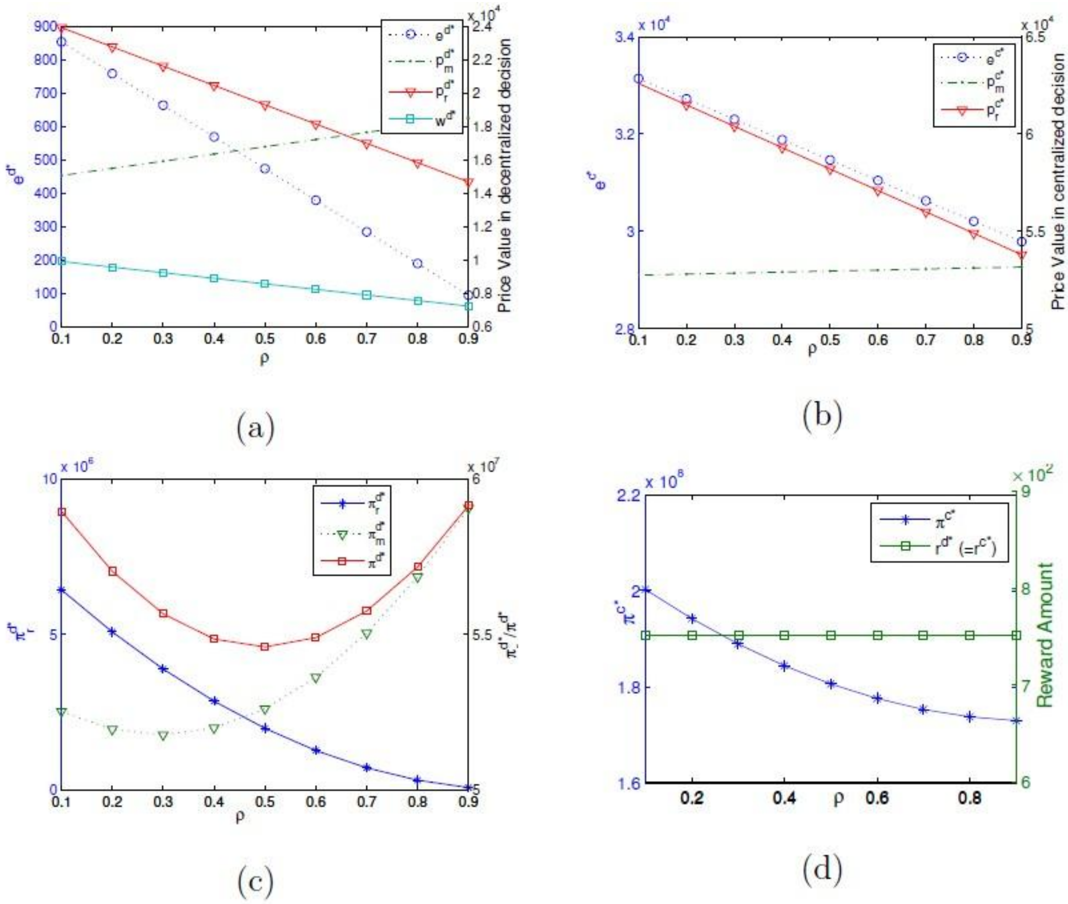

***Figure 4**: Effect of ρ under decentralized decisions and centralized decisions*

It can be seen from Figure 4 that: (1) As customer preference for the online channel increases, under decentralized decision-making, the manufacturer takes advantage by raising the online selling price. Simultaneously, due to a decline in offline channel preference, the retailer lowers their offline selling price to attract customers. Consequently, the manufacturer's profit increases while the retailer's profit decreases with increasing ρ, which is expected. As a result, the overall profit initially decreases due to the decline in the retailer's profit, and then increases as the manufacturer's profit continues to grow. (2) Under centralized decision-making, an increase in ρ leads to a reduction in the overall supply chain profit. This is primarily due to a significant decrease in the retailer's offline selling price, despite a slight increase in the manufacturer's online selling price. (3) Similar to the parameter γ, the parameter ρ does not influence the reward amount offered by the manufacturer for collecting used products under either centralized or decentralized decision-making structures.

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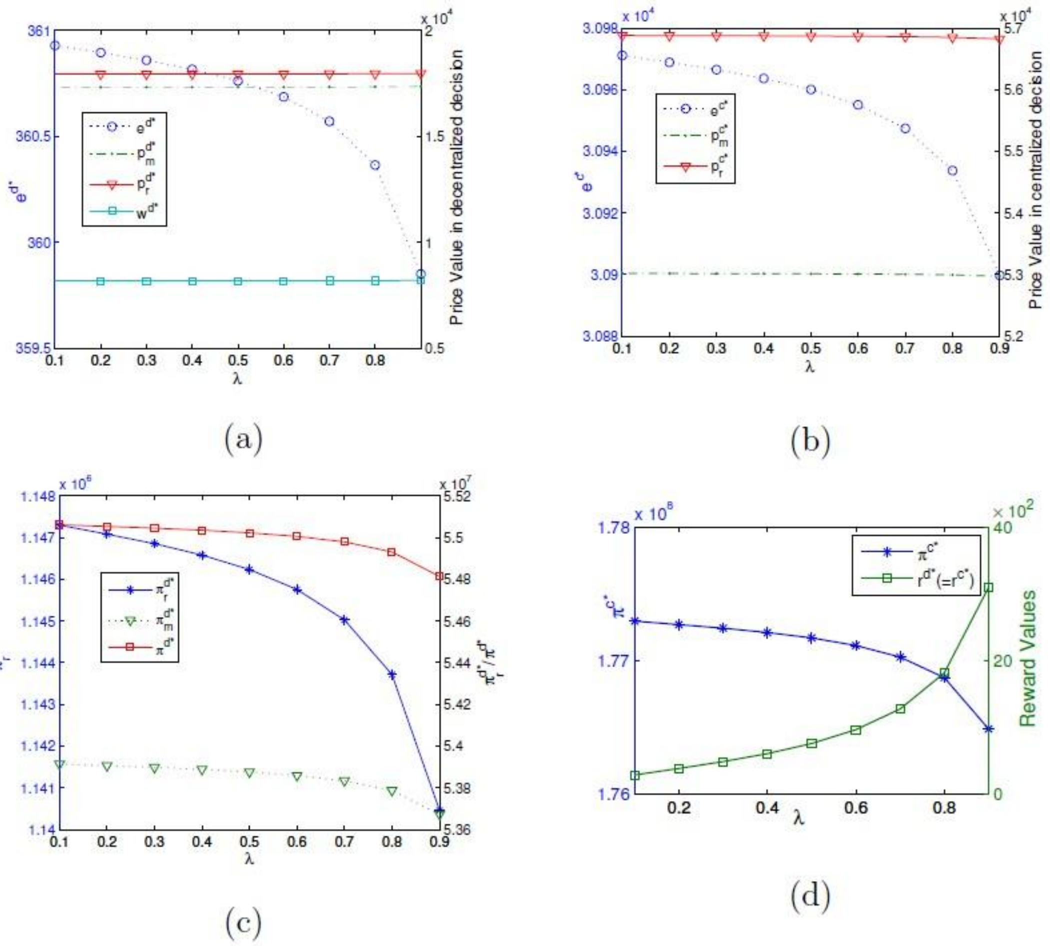

Figure 5: *Effect of λ under decentralized decisions and centralized decisions*

Figure 5 indicates the following: (1) As λ increases, all prices experience a marginal decline under both decision-making scenarios. However, once λ exceeds a certain threshold, the sales effort level drops significantly. This leads to a reduction in the manufacturer's profit, the retailer's profit, and the overall supply chain profit. (2) To counteract the impact of rising λ, the manufacturer increases the reward amount offered for collecting used products in order to boost the return rate.

4. Conclusion

In this study, we develop a Closed-Loop Dual-Channel Supply Chain (CLDSC) model that integrates an offline retail channel and an online direct channel, incorporating both free-riding behavior (FRB) and a full refund return policy. In the proposed model, the manufacturer collects used products from customers by offering a monetary incentive for remanufacturing purposes.

The study presents several key findings: (1) As customer FRB increases under a decentralized decision-making structure, the service effort level of the offline channel declines, leading to a reduction in product demand. Consequently, FRB adversely affects both the manufacturer's profit and the overall supply chain performance, indicating that free riding has a negative impact in a decentralized setting. However, under centralized decision-making, when

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FRB surpasses a certain threshold, more customers shift from the offline to the online channel. Since the retailer and manufacturer make decisions cooperatively in this scenario, the retailer increases their service effort level, resulting in improved profitability for the entire supply chain. (2) The offline price is typically higher than the online price in nearly all cases, except when the customer's preference for the online channel exceeds a specific threshold. (3) An important observation is that the reward offered for returning used products remains largely insensitive to changes in remanufacturing and manufacturing costs. However, the reward amount is influenced by changes in the parameter that affects the collection rate in response to the reward. (4) An increase in the return rate leads to a decline in overall supply chain profitability in both centralized and decentralized decision-making scenarios.

Finally, this study has several limitations. First, it does not consider the cross-channel return scenario, where customers return products purchased from the online channel to the offline channel. Second, it assumes that the return rates for offline and online channels are identical, whereas, in practice, these rates often differ significantly. Third, the model assumes that the manufacturer is solely responsible for collecting used products, although in reality, third-party agencies are sometimes employed for this purpose. These limitations will be addressed in future research to enhance the model's applicability and accuracy.

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