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# BIJECTIVE FUZZY PLANAR AND NON-PLANAR GRAPH IN SMART CITY TRAFFIC NETWORK

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**Abstract**

The Bijective condition ensures the proper association of fuzzy relations, which is essential for precise graph representation. In this paper a new concept of bijective fuzzy planar and non-planar graphs are introduced. Considerable and effective edges of bijective fuzzy planar and non-planar graphs have been identified. Fuzzy planarity values have been calculated. Based on this value Smart city traffic network system is analyzed. The concept of Bijective function for vertices and edges help in prioritizing traffic flow.

**Keywords**: Bijective fuzzy graph, Bijective Planar fuzzy graph, Bijective Non Planar fuzzy graph.

## 1. Introduction

The concept of fuzzy graph was introduced by Rosenfield, A. [8] using fuzzy relation, represents the relationship between the objects by preciously indicating the level of the relationship between the objects of the function set. Abdul-jabbar et al. [1] introduced the concept of fuzzy planar graph. Also, Nirmala and Dhanabal [7] defined special fuzzy planar graphs. Pal et. al [9,10] defined fuzzy planar graph in a different concept where crossing of edges are allowed. They also studied different properties of fuzzy planar graph. McAllister [6] characterized the fuzzy intersection graphs. Many works have been done on fuzzy graphs in [2,3,4,5,11]. In this paper Bijective fuzzy planar graph and Bijective fuzzy non planar graphs are defined. Bijective fuzzy planar graphs retain the structural property of planarity with fuzziness. Bijective fuzzy non-planar Graphs are inherently complex and cannot be represented on a flat plane without some level of edge crossing. In this paper we also discussed about bijective fuzzy graphs with considerable and effective edges. It would be unattainable to prioritize which areas need better traffic flow management, if roads have the same membership values, so bijection plays an important role in certain cases like traffic flow of a city, route planning for laying roads and urban mobility. Also city planners can distribute funds efficiently based on traffic significance.

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## 2 Bijective fuzzy planar graphs

**2.1 Definition[4]** Let $V$ be a non-empty set and $\sigma : V \rightarrow [0, 1]$ be a mapping. Also, let $E = \{((x, y), (x, y_\mu j), j = 1, 2, \ldots, p_{xy} | (x, y) \in V \times V\}$ be a fuzzy multiset of $V \times V$ such that $(x, y)_\mu j \leq \min\{\sigma(x), \sigma(y)\}$ for all $j = 1, 2, \ldots, p_{xy}$, where $p_{xy}$=max$\{j|(x, y)_\mu j /= 0\}$. Then $G = (V, \sigma, E)$ is denoted as fuzzy multigraph where $\sigma(x)$ and $(x, y)_\mu j$ represent the membership value of the vertex $x$ and the membership value of the edge $(x, y)$ in $G$. There may be more than one edge between the vertices $x$ and $y$. Hence, $(x, y)_\mu j$ denotes the membership value of the $j$-th edge between the vertices $x$ and $y$. Let $p_{xy}$represents the number of edges between the vertices $x$ and $y$.

**2.2 Definition:[4]** Let $G = (V, \sigma, E)$ be a fuzzy multigraph . When two edges intersect at a point, a value is assigned to that point in the following way. Let $E$ contains two edges $((a, b), (a, b)_\mu k)$ and $((c, d), (c, d)_\mu l)$ which are intersected at a point $P$, where $k$ and $l$ are fixed integers. Intersecting value at the point $P$ is denoted by $I_p$=I(a,b)+I(c,d)2 .

**2.3 Definition:[4]** Let $G$ be a fuzzy graph and assume that points of intersections between the edges are $P_1, P_2, \ldots, P_z$. Then $G$ is said to be fuzzy graph with fuzzy planarity value $f$, where

$$f = \frac{1}{1 + \{I_{P1} + I_{P2} + \ldots + I_{Pz}\}}$$

$f$ is bounded and $0 < f \leq 1$.

**2.4 Definition:** A graph G=σ,μ is said to be a *Bijective fuzzy graph*, if σ:V→0,1 and μ:V×V→0,1 is bijective such that the membership value of edges and vertices are distinct and μu,v<σu,σv ∀ u,v∈V.

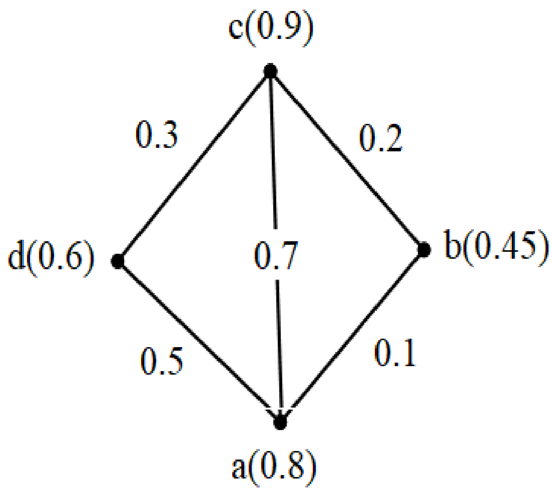


**Fig 1: Bijective fuzzy graph**

**2.5 Definition:** If σ and μ are bijective and μu,v<σu,σv ∀ u,v∈V with planarity value 1 such that G* is embeddable, then the graph G is said to be a *bijective fuzzy planar graph*.

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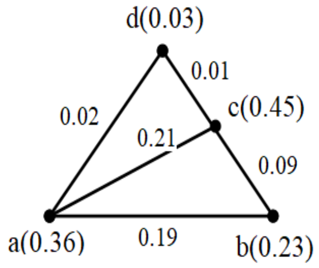


**Fig. 2:** ***Bijective*** **fuzzy planar graph**

I=μσ

Ia,b=0.190.23=0.8260

Ib,c=0.090.023=0.3913

Ia,d=0.020.03=0.6667

Ic,d=0.010.03=0.3333

Ia,c=0.210.36=0.5833

The fuzzy planarity value for the above graph is 1 since it has no intersection points (IP=0) i.e., f=11+0=1

**2.6 Definition:** A graph G is referred to as a *fuzzy non-planar Bijective graph* if σ and μ are bijective and μu,v<σu,σv ∀ u,v∈V with some planarity value *f* such that 0≤f<1, in such a way that G* is non-embedable

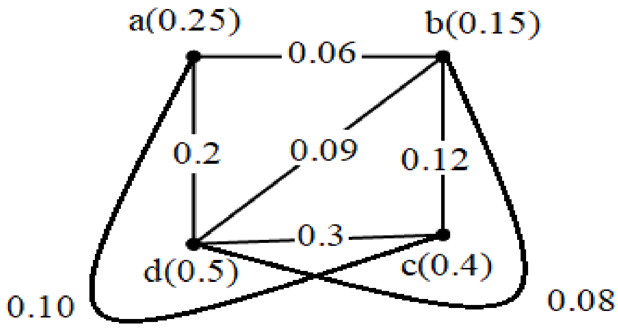


**Figure 3 : Bijective fuzzy non-planar graph**

I=μσ

Ib,dμ1=0.080.15=0.53

Ia,cμ1=0.100.25=0.4

IP1=Ib,dμ1+Ia,cμ12

=0.53+0.42

=0.47

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f=11+0.47=0.68.

**2.7 Definition:** Assume that, E={(x,y,x,yμj, j=1,2,…,pxy|x,y ϵ V×V} where pxy=max{j|x,yμj≠0} and G=(V,σ,E) is a Bijective planar fuzzy graph i.e., a fuzzy graph with fuzzy planarity value 1. A region bounded by the set of fuzzy edges E'⊂E (without any fuzzy edge inside the region) is known as a *fuzzy face f* of G. The membership value of the fuzzy face *F* is given by F=x,y∈E' .

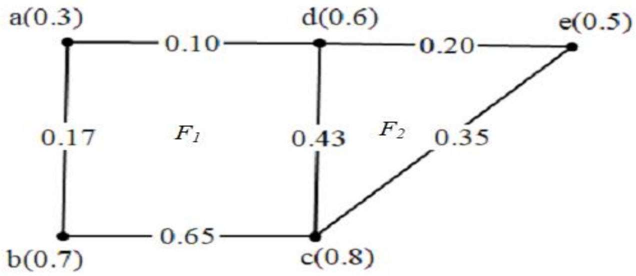


**Figure 4 : Bijective fuzzy Planar graph (i.e., *f*=1)**

I=μσ

Ia,d=0.10.3=0.3333

Ia,b=0.170.3=0.5667

Ib,c=0.650.7=0.9286

Ic,d=0.430.6=0.7167

Ic,e=0.350.5=0.7

Id,e=0.200.5=0.4

F1=min0.9286, 0.7167, 0.5667, 0.3333

F1=0.3333

F2=min{0.7167, 0.4, 0.7}

F2=0.4

**3 Bijective fuzzy graphs with considerable and effective edges:**

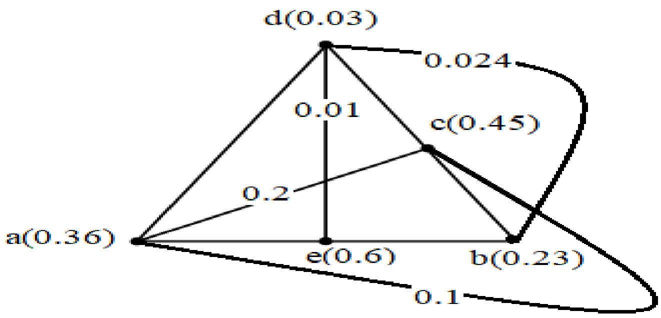


**Figure 5: Non planar Bijective fuzzy graph**

I=μσ

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Consider, c=0.1

Id,e=0.010.03=0.3333 C

Ia,cμ1=0.20.36=0.56 E

Ia,cμ2=0.10.36=0.2778 C

Id,b=0.0240.03=0.8 [E]

Here, the edges Id,e and Id,b are effective edges, since their values are greater than 0.5. And the edges Ia,cμ1 and Ia,cμ2 are considerable edges, since their values are greater than c=0.1 and lesser than 0.5.

**Theorem 3.1:** If G is a bijective fuzzy non-planar graph with planarity value f>0.5 and considerable number *c*, then the number of points of intersection between considerable edges in G is atmost 1c or 1c-1.

**Proof.** Let G=(V,σ,E) be a bijective fuzzy non-planar graph f>*0.5* and

E={(x,y,x,yμj), j=1,2,…,pxy|(x,y)ϵV×V} and

pxy=max{j|x,yμj≠0}.

Let *f* be a fuzzy planarity value with considerable number $0 < c < 0.5$ for any considerable edge a,b,a,bμj, a,bμj≥c×min{σa,σ(b)}.

In this case Ia,b≥c.

The considerable edges between intersection of *k* points P1,P2,…,Pk .

Let IP1=Ia,b+Ic,d2≥c

Here, intersecting between two considerable edges a,b,a,bμj andc,d,c,dμiof point P1 .

So,i=1kIpi ≥kc.

Given that G is bijective fuzzy non-planar graph f>0.5,

we know that

0.5<f≤11+kc

0.5<11+kc

k={1c, if 1c is not an integer. 1c-1, if1c is an integer}

k<1c

Hence proved.

**Theorem 3.2:** If G is a bijective fuzzy non planar graph with f>0.5, the number of points of intersection between effective edges in G is atmost one.

**Proof.** Suppose that G = (, , ) is a bijective fuzzy non-planar graph with fuzzy planarity value greater than 0.5. Assume that G has at least two points 1and 2formed by the intersection of effective edges in G.

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Let ((, ), (, )μ) , (,) ≥ 0.5 be any effective edge, by intersection of effective edges ((, ), (, )μ)and ((, ), (, )μ),we get IP1=Ia,b+Ic,d2≥0.5

Similarly, 2 ≥ 0.5

Then 1+IP1+IP2≥2.

f=11+IP1+IP2≤0.5.

This becomes a contradiction to the fuzzy planarity value is greater than 0.5. The effective edges between intersection points cannot be two.

From this the fuzzy planarity value decreases then the number of points of intersection of effective fuzzy edges increases.

If the number of points of intersection of effective edges is one, then $f > 0.5$.

Any bijective fuzzy non-planar graph without any crossing between edges is a bijective fuzzy planar graph with $f > 0.5$.

Hence maximum number of points of intersection between the effective edges in G is one.

**4. α- arcs and δ- arcs in Bijective Fuzzy planar and Non Planar Graph:**

α-arcs and δ arcs are different types of arcs which are used to study about the edges of fuzzy graph. In this section the relation between these arcs, considerable edges and effective edges are observed.

**Theorem 4.1:** If the intersecting arcs of non-planar bijective fuzzy graph are δ arcs then its fuzzy planarity value is greater than 0.5.

The above theorem is justified with the following example.

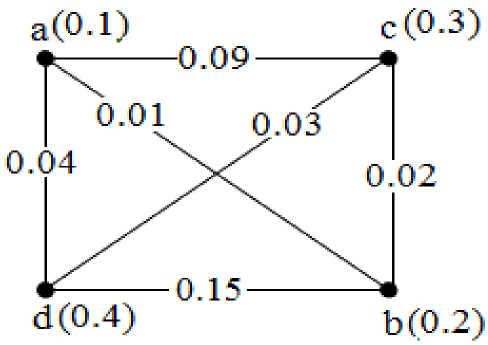


**Figure 6: Non- Planar Bijective fuzzy graph**

In the above graph the point of intersection is between the arcs (a,b) and (c,d).

Here, and . Therefore by the definition of arcs, (a,b) and (c,d) are arcs of this graph.

$I_{(a,b)}$= $I_{(c,d)}$= $I_P$ =

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Hence the planarity value f = .

Converse part of the above theorem need not be true. This can also be verified with the following example.

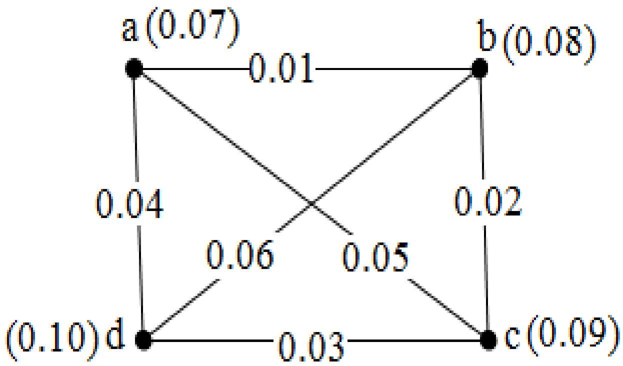


**Figure 7: Non- Planar Bijective fuzzy graph**

For the above graph the point of intersection is between two strong arcs (a,c) and (b,d). $I_{(a.c)} = 0.71$ , $I_{(b,d)} = 0.75$ and

$I_P =$ .

The planarity value f =

But the corresponding arcs of intersection are not arcs. In the above two examples the number of point of intersection is taken as one but if it is more than one also the result is true.

Similarly one can easily verify that if the intersection is between two δ- arcs or between one α-arc and one δ arcs also the planarity value is greater than 0.5

**Observations:**

1. In a Bijective Planar Fuzzy Graph with effective edges if v is a f-cut node and if it is a common node of any two α- strong arcs, then atleast one of the α- strong arc is effective.
2. If G is a Bijective fuzzy tree with effective edges then its spanning subgraph F has atleast one effective edge which is α- strong
3. No β – arcs exist in Bijective fuzzy planar and Bijective fuzzy non-planar graph
4. Every α-strong arc in a bridge is an effective edge
5. Considerable edge of any Bijective non planar fuzzy graph cannot be a bridge.

**4 Application in Smart city traffic network**

Consider the important places (hospitals) of a smart city to be the vertices of a fuzzy non-planar graph and the roads connecting the important places to be the edges, such that edge crossing represents intersection of two roads. If the intersecting value is 0.8, then it has moderate congestion risk and if the intersecting value is 0.4, then it has low impact. Further if the planarity value f 0.5, then it indicate efficient, low congestion traffic design. If the planarity value f < 0.5 signals potential traffic bottlenecks due to critical crossings. Therefore high planarity value routes are prioritized for emergency vehicles.

## 5 Conclusion

Effective and Considerable edges of bijective fuzzy planar and non-planar graphs were identified. Its planarity values have been calculated. Based on this values one can decided whether crossing is allowed or not. Further, these concepts can be applied to many real line situation for taking decision.

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