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SUPER MEAN LABELING: CODING AND EPIDEMIC MODELING WITH FOUR-STAR GRAPHS

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Abstract

This paper presents two approaches for coding messages using super mean labeling on a four-star graph $K_{1,\chi_1} \cup K_{1,\chi_2} \cup K_{1,\chi_3} \cup K_{1,\chi_4}$, where $\chi_1 \leq \chi_2 \leq \chi_3 \leq \chi_4$. We provide two illustrations for each star graph, employing techniques such as Even Forward and Odds Backward (EFOB), Odds Backward

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Evens Forward (OBEF), Addition of Two Numbers, and Multiplication of Two Numbers via C programming, all with alphabetical mappings. After observations on super mean labeling categories, we describe methods to recover the labeling on any graph using four stars, enabling combined use of the labeling and graphs.

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Key Words and Phrases: Super Mean Labeling, Four-Star Graph, EFOB, OBEF, Coding Techniques, C Programming

1 Introduction

Graph labeling is a vibrant area of graph theory where vertices or edges are assigned labels according to specific conditions, with applications in coding theory, network design, and more [1]. Among various labeling techniques, mean labeling has garnered significant attention, where edge labels are derived as the mean of adjacent vertex labels [21, 22, 20]. Super mean labeling extends this concept by requiring vertex labels to be distinct integers from 0 to the number of vertices, with edge labels being the ceiling of the average and also distinct [2, 3, 15]. Research on super mean labeling has explored its existence and properties across various graph classes. For instance, studies have investigated super mean labeling on star graphs, identifying conditions for existence and non-existence [19]. Further results on super mean graphs include analyses of H-graphs, coronas, and subdivisions [14, 16]. Variants such as k-super mean labeling and super root square mean labeling have also been introduced to generalize the framework [17, 23]. Applications of super mean labeling in coding techniques have been particularly promising. Previous works have demonstrated coding methods using two-star and three-star graphs with super mean labeling [6, 7, 8, 9, 10, 11, 12, 13]. Additionally, integrations with difference cordial labeling and other cordial variants have expanded the coding capabilities [4, 5, 25, 26]. Recent investigations have delved into related mean labelings, such as super Lehmer-3 mean labeling for theta graphs and super classical mean labeling [24, 18]. These advancements provide a foundation for more complex labeling schemes. In this paper, two strategies for coding messages employing four stars on a graph with super mean labeling, $K_{1,\chi_1} \cup K_{1,\chi_2} \cup K_{1,\chi_3} \cup K_{1,\chi_4}$, $\chi_1 \leq \chi_2 \leq \chi_3 \leq \chi_4$ are presented. In order to use the labeling and the graphs together, methods for repairing the super mean labeling on any graph with four star graphs are given after

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a few observations are performed in order to assign them a super mean designation. Rules for assigning the really cruel label on four star graphs are provided. Procedure for encoding the message is stated prior to illustrations. To ensure a clear understanding of the labeling process, we first outline a practical rule of thumb for applying super mean labeling to four-star graphs [7].

2 Foundational Definitions

The following definitions are required for the work presented in this paper.

Definition 1. A graph G is an ordered triple $(V(G), E(G), \varphi_G)$, where $V(G)$ is a nonempty set of vertices, $E(G)$ is a set of edges disjoint from $V(G)$, and φ_G is an incidence function that associates each edge with an unordered pair of vertices of G, which are not necessarily distinct. If e is an edge connecting vertices a and b, then a and b are called the ends of e [1].

Definition 2. A one-star graph, denoted K_{1,χ_1}, is a tree with one interior node and χ_1 leaves. It is a complete bipartite graph [1].

Definition 3. A two-star graph is formed by the disjoint union of two one-star graphs, K_{1,χ_1} and K_{1,χ_2}, and is denoted $K_{1,\chi_1} \cup K_{1,\chi_2}$ [1].

Definition 4. A three-star graph is the disjoint union of three one-star graphs, K_{1,χ_1}, K_{1,χ_2}, and K_{1,χ_3}, denoted $K_{1,\chi_1} \cup K_{1,\chi_2} \cup K_{1,\chi_3}$ [1].

Definition 5. A four-star graph is the disjoint union of four one-star graphs, K_{1,χ_1}, K_{1,χ_2}, K_{1,χ_3}, and K_{1,χ_4}, denoted $K_{1,\chi_1} \cup K_{1,\chi_2} \cup K_{1,\chi_3} \cup K_{1,\chi_4}$ [1].

Definition 6. A graph G is a mean graph if there exists a function $f : V(G) \to \{0, 1, 2, 3, \dots, q\}$ that induces a map $f^* : E(G) \to \{0, 1, 2, 3, \dots, q\}$ defined by $f^*(e) = \lceil (f(a) + f(b))/2 \rceil$ for an edge $e = \{a, b\}$. The resulting edge labels are distinct and drawn from $\{1, 2, 3, \dots, q\}$ [3, 2].

Definition 7. Let G be a (k, q) graph and $f : V(G) \to \{0, 1, 2, \dots, k\}$ an injective function. For every edge $e = \{a, b\}$, define $f^*(e) = \lceil (f(a) + f(b))/2 \rceil$. If the edge labels are distinct, then f is called a super mean labeling, and G is a super mean graph [2, 19].

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A program written in the C programming language is a procedural, general-purpose program known for its efficiency and versatility. It is used in numerous applications, including operating systems, databases, and web browsers. With these foundational concepts in place, we can now proceed to outline the step-by-step procedure for encoding messages using super mean labeled graphs.

3 Guidelines for Super Mean Labeling on Four-Star Graphs

1. Consider the four-star graph $K_{1,\chi_1} \cup K_{1,\chi_2} \cup K_{1,\chi_3} \cup K_{1,\chi_4}$, where $\chi_1 \leq \chi_2 \leq \chi_3 \leq \chi_4$. Let k denote the number of vertices and l the number of edges, where:

   $$k = 4+\chi_1+\chi_2+\chi_3+\chi_4, \quad l = \chi_1+\chi_2+\chi_3+\chi_4, \quad k+l = 4+2\chi_1+2\chi_2+2\chi_3+2\chi_4.$$

   Assign distinct labels from $\{1, 2, \dots, k+l\}$ to the central and pendant vertices. The central vertices of the stars are denoted a, b, c, and d, with pendant vertices labeled a_i, b_j, c_k, and d_l. The edges are denoted aa_i, bb_j, cc_k, and dd_l, with labels $\rho(aa_i)$, $\rho(bb_j)$, $\rho(cc_k)$, and $\rho(dd_l)$.

2. The edge label is the ceiling of the mean of the vertex labels, i.e., $\rho(aa_i) = \lceil(\rho(a) + \rho(a_i))/2\rceil$, if $\rho(a)$ and $\rho(a_i)$ are both odd or both even; otherwise, adjust to ensure distinct labels.

3. To avoid duplicate edge labels, ensure that assignments like $\rho(a_i) = 2k$, $\rho(a_{i+1}) = 2k+1$ (yielding the same edge value, e.g., $\lceil(1+5)/2\rceil = \lceil(1+6)/2\rceil = 3$) are avoided. Instead, use $\rho(a_i) = 2k+1$, $\rho(a_{i+1}) = 2k+2$ to produce distinct edge values (e.g., $\lceil(1+5)/2\rceil = 3$, $\lceil(1+6)/2\rceil = 4$).

Step 1: Assign $\rho(a) = 1$, $\rho(b) = \lfloor(k + l + 1)/2\rfloor$, $\rho(c) = k + l$, and $\rho(d)$ appropriately to ensure distinct labels. For illustration, if $\rho(a_1) = 2$, $\rho(b_1) = 4$, adjust subsequent assignments to avoid conflicts.

Step 2: Assign labels to the first star, then the second, third, and fourth stars, ensuring all edge labels are distinct.

Building on this rule, we now present an illustration of super mean labeling applied to a four-star graph.

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3.1 Illustration of Super Mean Labeled Four-Star Graph

The first, second, third, and fourth stars are denoted by the Greek letters γ, δ, ϵ, and ϕ, respectively. For instance, $\gamma(P_i)$ indicates the digit attributed up to the i-th pendant first star's vertex. The uppermost vertex, i-th pendant vertex, and value of the i-th edge are denoted by the letters O, P_i, and E_i respectively.

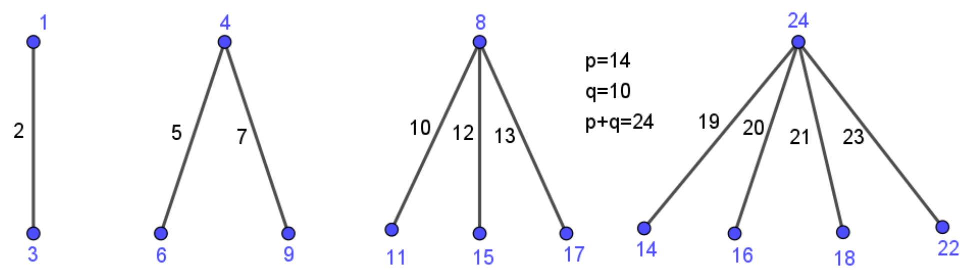

Figure 1: Super mean labeling applied to the four-star graph $K_{1,1} \cup K_{1,2} \cup K_{1,3} \cup K_{1,4}$, showing vertex and edge labels.

The figure illustrates the assignment of distinct labels to vertices and the resulting mean-based edge labels, ensuring no repetitions. Before delving into the coding procedures and illustrations, it is essential to review the fundamental definitions that underpin this work.

4 Message Encoding Procedure

1. You must take an appropriate graph. To locate the graph that should be used, a mathematical or non-mathematical indication is given.
2. Using the provided explanation, labeling is completed on an appropriate graph.
3. The English alphabets are separated in some manner, and the numbers that correspond to each alphabet are recorded.
4. Each letters coding scheme is explained.

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5. Compose the message that requires coding.
6. The coding is written in some fashion using the notations from step 4.
7. By rearranging the letter order, the coding is shown in the appropriate shape.
8. The coding instructions and an understanding of graph labeling on any graph are necessary for decoding the message.

To demonstrate the practical application of this procedure, we present several illustrations using the four-star graph with different labeling and numbering schemes.

4.1 Illustration on Four-Star Graphs Using Distinct Numbers

1. **Message:** SUPER MEAN LABELING IS USED FOR VARIATION OF PURPOSES.
2. **Graph:** The four-star graph $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5}$.
3. **Labeling:** Fig.2 displays the extremely cruel designation for $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5}$.

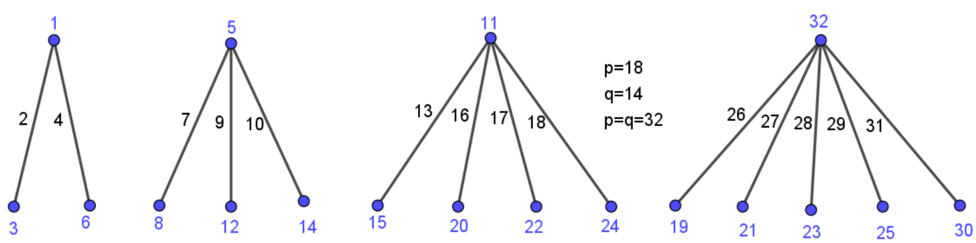

Figure 2: Super mean labeling of the four-star graph $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5}$ for the first illustration, highlighting vertex assignments and distinct edge means.

The figure shows a specific configuration of labels ensuring all edge means are unique.

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4. **Alphabetical Numbering: Even Forward and Odds Backward (EFOB):**

26	1	25	2	24	3	23	4	22	5	21	6	20
A	B	C	D	E	F	G	H	I	J	K	L	M
7	19	8	18	9	17	10	16	11	15	12	14	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Between B, D, and F and so forth, the even-positioned alphabets are assigned the digits 1 through 13. Additionally, the odd-positioned alphabets Y, W, U, and so forth are assigned the digits 14 through 26. This approach is known as EFOB.

5. **EFOB**:

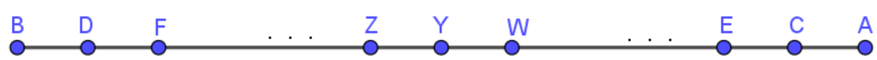

Figure 3: Visualization of the Even Forward and Odds Backward (EFOB) alphabetical numbering scheme, mapping letters to numerical values.

The plot depicts the forward and backward assignment patterns for even and odd positions.

We use a function for encoding to represent the alphabets numbering. The function h for alphabets in even positions is provided by:

$$h(2f) = f, \quad f = 1, 2, 3, \ldots, 13.$$

For alphabets in odd positions, the function h is supplied by:

$$h(2f + 1) = 26 - f, \quad f = 1, 2, 3, \ldots, 12.$$

We do the opposite for decoding. To find the numbers for any letter and look for it in the four-star label, EFOB is used to code each word individually. Additionally, super mean labeling on $K_{1,2} \cup K_{1,3} \cup K_{1,4} \cup K_{1,5}$ is used to encode the message.

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6. **Coding (Word-wise):**
   - SUPER: $\epsilon(E_3)\epsilon(E_2)\delta(P_1)\epsilon(P_4)\delta(E_2)(1,1)$.
   - MEAN: $\epsilon(P_2)\epsilon(P_4)\phi(P_1)\delta(E_1)(1,1)$.
   - LABELING: $\gamma(P_2)\phi(P_1)\gamma(O)\epsilon(P_4)\gamma(P_2)\epsilon(P_3)\delta(E_1)\phi(P_3)(1,1)$.
   - IS: $\epsilon(P_3)\epsilon(E_3)(1,1)$.
   - USED: $\epsilon(E_2)\epsilon(E_3)\epsilon(P_4)\gamma(E_1)(1,1)$.
   - FOR: $\gamma(P_1)\phi(P_1)\delta(E_2)(1,1)$.
   - VARIATION: $\epsilon(O)\phi(P_1)\delta(E_2)\epsilon(P_3)\phi(P_1)\delta(E_3)\epsilon(P_3)\phi(P_1)\delta(E_1)(1,1)$.
   - OF: $\phi(P_1)\gamma(P_1)(1,1)$.
   - PURPOSES: $\delta(P_1)\epsilon(E_2)\delta(E_2)\delta(P_1)\phi(P_1)\epsilon(E_3)\phi(P_1)\epsilon(E_3)$.
7. **Horizontal String:**

$$
\begin{aligned}
&\epsilon(E_3)\epsilon(E_2)\delta(P_1)\epsilon(P_4)\delta(E_2)(1,1)\epsilon(P_2)\epsilon(P_4)\phi(P_1)\delta(E_1)(1,1)\\
&\quad\gamma(P_2)\phi(P_1)\gamma(O)\epsilon(P_4)\gamma(P_2)\epsilon(P_3)\delta(E_1)\phi(P_3)(1,1)\\
&\quad\epsilon(P_3)\epsilon(E_3)(1,1)\epsilon(E_2)\epsilon(E_3)\epsilon(P_4)\gamma(E_1)(1,1)\\
&\quad\gamma(P_1)\phi(P_1)\delta(E_2)(1,1)\epsilon(O)\phi(P_1)\delta(E_2)\epsilon(P_3)\\
&\quad\phi(E_1)\delta(E_3)\epsilon(P_3)\phi(P_1)\delta(E_1)(1,1)\phi(P_1)\gamma(P_1)(1,1)\\
&\quad\delta(P_1)\epsilon(E_2)\delta(E_2)\delta(P_1)\phi(P_1)\epsilon(E_3)\phi(P_1)\epsilon(E_3).
\end{aligned}
$$

8. **Picture Coding (Visual Coding):**

Continuing with variations in numbering schemes, the next illustration incorporates a simple C program for addition to assign alphabetical numbers.

4.2 illustration Using Addition of Two Numbers in C Program

1. **Message:** SUPER MEAN LABELING IS A TECHNIQUE USED IN GRAPH THEORY.
2. **Graph:** The four-star pattern $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6}$.

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$$\varepsilon(E_3)\varepsilon(E_2)\delta(P_1)\varepsilon(P_4)\delta(E_2)(1,1)\varepsilon(P_2)\varepsilon(P_4)\varphi(P_1)\delta(E_1)(1,1)$$
$$\gamma(P_2)\varphi(E_1)\gamma(O)\varepsilon(P_4)\gamma(P_2)\varepsilon(P_3)\delta(E_1)\varphi(P_3)(1,1)\ \varepsilon(P_3)\varepsilon(E_3)$$
$$(1,1)\varepsilon(E_2)\varepsilon(E_3)\varepsilon(P_4)\gamma(E_1)(1,1)\gamma(P_1)\varphi(P_1)\delta(E_2)(1,1)\varepsilon(O)$$
$$\varphi(P_1)\delta(E_2)\varepsilon(P_3)\varphi(P_1)\delta(E_3)\varepsilon(P_3)\varphi(P_1)\delta(E_1)(1,1)\varphi(P_1)\gamma(P_1)$$
$$(1,1)\delta(P_1)\varepsilon(E_2)\ \delta(E_2)\ \delta(P_1)\varphi(P_1)\ \varepsilon(E_3)\varphi(P_1)\ \varepsilon(E_3).$$

3. **Attaching Labels:** Fig.4 displays the extremely derogatory labeling for $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6}$.

   This configuration demonstrates an alternative labeling that maintains the super mean properties.

4. **Addition of Two Numbers in C Program:**

```
#include <stdio.h>
int main() {
    int a = 5, b = 10, c;
    c = a + b;
    printf("Addition of %d, %d is %d.\n", a, b, c);
    return 0;
}
```

   Output:

```
Addition of 5, 10 is 15.
```

5. **Numbering of Alphabets:**

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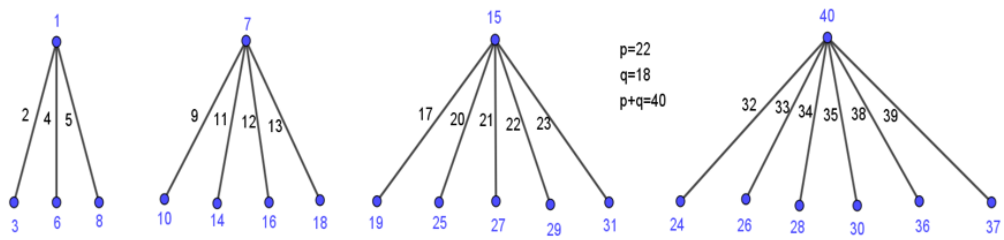

Figure 4: Super mean labeling configuration for the second illustration on the four-star graph $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6}$ with unique edge labels derived from vertex means.

4	5	6	7	1	8	9	10	11	2	12	13	14
A	B	C	D	E	F	G	H	I	J	K	L	M
15	3	16	17	18	19	20	21	22	23	24	25	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Alphabetically, the numbers 1 through 3 are attributed to the 5, 10, and 15 positions, utilizing the super mean labeling on $K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6}$ and the addition of two numbers in a C program.

6. **Coding (Word-wise):**

- SUPER: $\epsilon(P_1)\epsilon(E_3)\delta(E_3)\gamma(O)\delta(E_4)(1,1)$
- MEAN: $\delta(P_2)\gamma(O)\gamma(E_2)\epsilon(O)(1,1)$
- LABELING: $\delta(E_4)\gamma(E_2)\gamma(E_3)\gamma(O)\delta(E_4)\delta(E_2)\epsilon(O)\delta(E_1)(1,1)$
- IS: $\delta(E_2)\epsilon(P_1)(1,1)$
- A: $\gamma(E_2)(1,1)$
- TECHNIQUE: $\epsilon(E_2)\gamma(O)\gamma(P_2)\delta(P_1)\epsilon(O)\delta(E_2)\epsilon(E_1)\epsilon(E_3)\gamma(O)(1,1)$
- USED: $\epsilon(E_3)\epsilon(P_1)\gamma(O)\delta(O)(1,1)$
- IN: $\delta(E_2)\epsilon(O)(1,1)$
- GRAPH: $\delta(E_1)\delta(E_4)\gamma(E_2)\delta(E_3)\delta(P_1)(1,1)$

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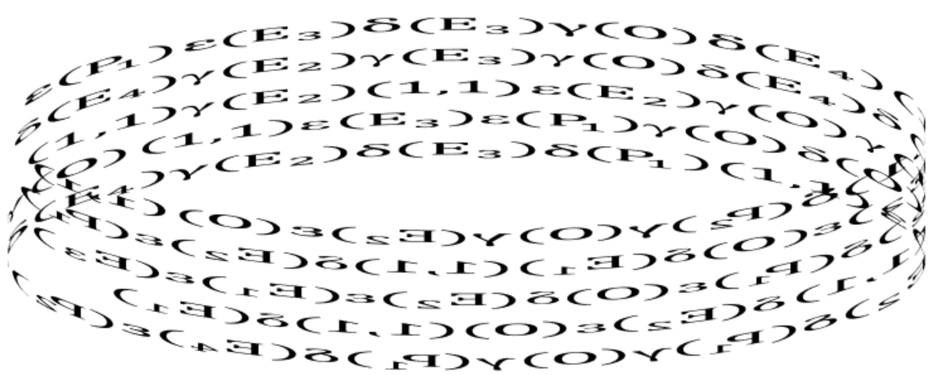
- THEORY: $\epsilon(E_2)\delta(P_1)\gamma(O)\gamma(P_1)\delta(E_4)\epsilon(P_2)$.

7. **Horizontal String:**

$$\begin{aligned}&\epsilon(P_1)\epsilon(E_3)\delta(E_3)\gamma(O)\delta(E_4)(1,1)\delta(P_2)\gamma(O)\gamma(E_2)\epsilon(O)(1,1)\\ &\quad\delta(E_4)\gamma(E_2)\gamma(E_3)\gamma(O)\delta(E_4)\delta(E_2)\epsilon(O)\delta(E_1)(1,1)\\ &\quad\delta(E_2)\epsilon(P_1)(1,1)\gamma(E_2)(1,1)\epsilon(E_2)\gamma(O)\gamma(P_2)\delta(P_1)\\ &\quad\epsilon(O)\delta(E_2)\epsilon(E_1)\epsilon(E_3)\gamma(O)(1,1)\epsilon(E_3)\epsilon(P_1)\gamma(O)\delta(O)(1,1)\\ &\quad\delta(E_2)\epsilon(O)(1,1)\delta(E_1)\delta(E_4)\gamma(E_2)\delta(E_3)\delta(P_1)(1,1)\\ &\quad\epsilon(E_2)\delta(P_1)\gamma(O)\gamma(P_1)\delta(E_4)\epsilon(P_2).\end{aligned}$$

8. **Picture Coding (Visual Coding):**



Shifting to another approach, the following illustration uses the same numerical base but with an alternative alphabetical mapping.

4.3 illustration Using Consistent Numbers

1. **Message:** SUPER MEAN LABELING IS A STUDY OF THE PROPERTIES IN GRAPH.

2. **Graph:** The four-star graph $K_{1,3} \cup K_{1,3} \cup K_{1,3} \cup K_{1,3}$.

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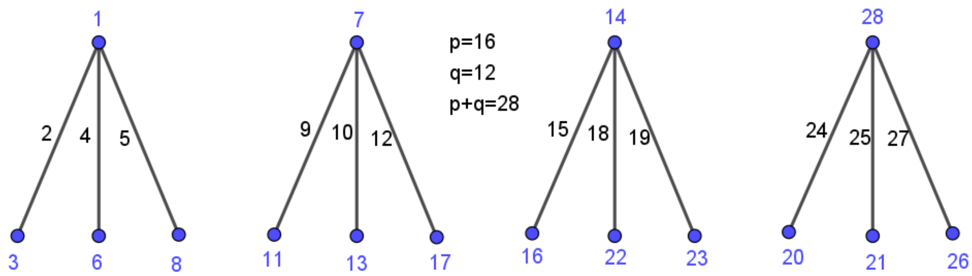

Figure 5: Super mean labeling for the third illustration on the four-star graph $K_{1,3} \cup K_{1,3} \cup K_{1,3} \cup K_{1,3}$, featuring distinct mean-based edge labels.

3. **Labeling:** Fig.5 displays the extremely derogatory labeling for $K_{1,3} \cup K_{1,3} \cup K_{1,3} \cup K_{1,3}$.

   This labeling variant ensures uniqueness in edge values while using consistent numerical foundations.

4. **Alphabetical Numbering: Odds Backward Evens Forward (OBEF):**

13	14	12	15	11	16	10	17	9	18	8	19	7
A	B	C	D	E	F	G	H	I	J	K	L	M
20	6	21	5	22	4	23	3	24	2	25	1	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

   In reverse order, the odd-numbered letters Y, W, U, and so forth, are assigned the numerals 1 through 13, and continuing from B to D to F and so on, the even-positioned alphabets are given the numbers 14 through 26. OBEF is the name of this technique.

5. **OBEF:**

   The plot highlights the backward assignment for odds and forward for evens.

   The alphabets numbering is expressed in terms of an encoding function. The function m for odd-positioned alphabets is:

$$m(l) = \frac{27 - l}{2}, \quad l = 1, 3, 5, \ldots, 25.$$

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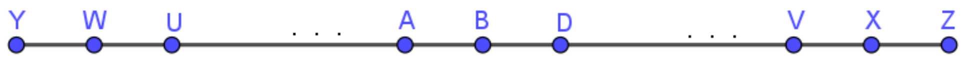

Figure 6: Visualization of the Odds Backward Evens Forward (OBEF) alphabetical numbering scheme, illustrating reverse and forward mappings.

For even-positioned alphabets:

$$m(l) = \frac{l}{2} + 13, \quad l = 2, 4, 6, \ldots, 26.$$

We do the opposite for decoding. To locate any letters number and locate it on the four-star graph, OBEF is used to code each word individually.

6. **Coding (Word-wise):**
   - THE: $\vartheta(P_3)\theta(P_3)\theta(P_1)(1,1)$.
   - SUPER: $\tau(E_2)\tau(P_1)\omega(P_2)\theta(P_1)\vartheta(P_2)(1,1)$.
   - MEAN: $\theta(O)\theta(P_1)\theta(P_2)\omega(P_1)(1,1)$.
   - LABELING: $\vartheta(E_3)\theta(P_2)\vartheta(O)\theta(P_1)\vartheta(E_3)\theta(E_1)\omega(P_1)\theta(E_2)(1,1)$.
   - IS: $\theta(E_1)\tau(E_2)(1,1)$.
   - A: $\theta(P_2)(1,1)$.
   - STUDY: $\tau(E_2)\vartheta(P_3)\tau(P_1)\vartheta(E_1)\tau(O)(1,1)$.
   - OF: $\tau(P_2)\vartheta(P_1)(1,1)$.
   - THE: $\vartheta(P_3)\theta(P_3)\theta(P_1)(1,1)$.
   - PROPERTIES: $\omega(P_2)\vartheta(P_2)\tau(P_2)\omega(P_2)\theta(P_1)\vartheta(P_2)\vartheta(P_3)\theta(E_1)\theta(P_1)\tau(E_2)(1,1)$.
   - IN: $\theta(E_1)\omega(P_1)(1,1)$.
   - GRAPH: $\theta(E_2)\vartheta(P_2)\theta(P_2)\omega(P_2)\theta(P_3)$.

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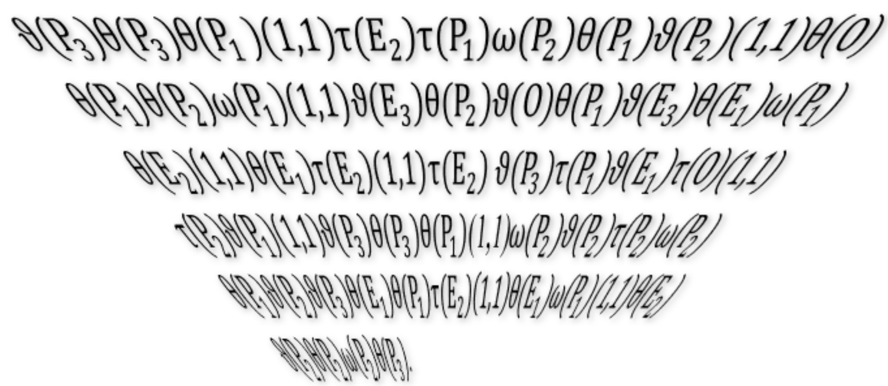
7. **Horizontal String:**

$$\vartheta(P_3)\theta(P_3)\theta(P_1)(1,1)\tau(E_2)\tau(P_1)\omega(P_2)\theta(P_1)\vartheta(P_2)(1,1)$$
$$\theta(O)\theta(P_1)\theta(P_2)\omega(P_1)(1,1)\vartheta(E_3)\theta(P_2)\vartheta(O)$$
$$\theta(P_1)\vartheta(E_3)\theta(E_1)\omega(P_1)\theta(E_2)(1,1)theta(E_1)\tau(E_2)(1,1)$$
$$\theta(P_2)(1,1)\tau(E_2)\vartheta(P_3)\tau(P_1)\vartheta(E_1)\tau(O)\tau(P_2)$$
$$\vartheta(P_1)(1,1)\vartheta(P_3)\theta(P_3)\theta(P_1)(1,1)\omega(P_2)\vartheta(P_2)$$
$$\tau(P_2)\omega(P_2)\theta(P_1)\vartheta(P_2)\vartheta(P_3)\theta(E_1)\theta(P_1)\tau(E_2)(1,1)$$
$$\theta(E_1)\omega(P_1)(1,1)\theta(E_2)\vartheta(P_2)\theta(P_2)\omega(P_2)\theta(P_3).$$

8. **Picture Coding (Visual Coding):**



To further explore computational integrations, the final illustration employs a C program for multiplication in the numbering process.

4.4 illustration Using Multiplication of Two Numbers in C Program

1. **Message:** A ONE STAR GRAPH IS THE COMPLETE BIPARTITE GRAPH.

2. **Graph:** The four-star graph $K_{1,4} \cup K_{1,4} \cup K_{1,4} \cup K_{1,4}$.

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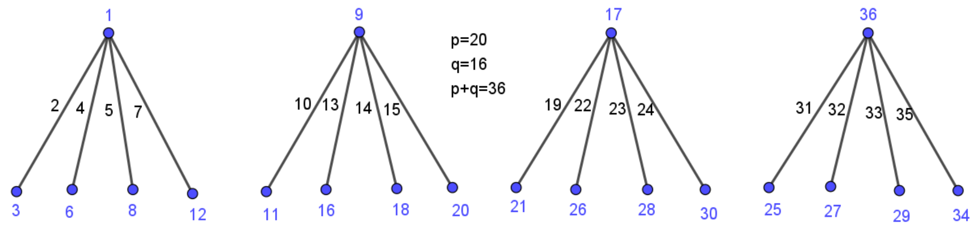

Figure 7: Super mean labeling for the fourth illustration on the four-star graph $K_{1,4} \cup K_{1,4} \cup K_{1,4} \cup K_{1,4}$, demonstrating vertex and edge label uniqueness.

3. **Labeling:** Fig.7 displays the extremely harsh labeling for $K_{1,4} \cup K_{1,4} \cup K_{1,4} \cup K_{1,4}$.

   This setup provides yet another valid super mean labeling for encoding purposes.

4. **Multiplication of Two Numbers in C Program:**

```
#include <stdio.h>
int main() {
    int a, b;
    printf("Enter 2 numbers for multiplication");
    scanf("%d %d", &a, &b);
    printf("Multiplication of %d and %d is %d\n", a, b, a*b);
    return 0;
}
```

   Output:

```
Enter 2 numbers for multiplication: 4, 6
Multiplication of 4 and 6 is 24.
```

5. **Numbering of Alphabets:**

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4	5	6	1	7	2	8	9	10	11	12	13	14
A	B	C	D	E	F	G	H	I	J	K	L	M
15	16	17	18	19	20	21	22	23	24	3	25	26
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Number 1, 2, and 3 are allotted to the 4, 6, and 24 positioned alphabets, using the method in multiplication of two numbers in C program.

6. **Coding (Word-wise):**
   - A: $\tau(E_2)(1,1)$.
   - ONE: $\theta(P_2)\theta(E_4)\tau(E_3)(1,1)$.
   - STAR: $\theta(P_4)\vartheta(P_1)\tau(E_2)\vartheta(E_1)(1,1)$.
   - GRAPH: $\tau(P_3)\vartheta(E_1)\tau(E_2)\vartheta(O)\theta(O)(1,1)$.
   - IS: $\theta(E_1)\theta(P_4)(1,1)$.
   - THE: $\vartheta(P_1)\theta(O)\tau(E_3)(1,1)$.
   - COMPLETE: $\tau(P_2)\theta(P_2)\theta(E_3)\vartheta(O)\theta(E_2)\tau(E_3)\vartheta(P_1)\tau(E_3)(1,1)$.
   - BIPARTITE: $\tau(E_2)\theta(E_1)\vartheta(O)\tau(E_2)\vartheta(E_1)\vartheta(P_1)\theta(E_1)\vartheta(P_1)\tau(E_3)(1,1)$.
   - GRAPH: $\tau(P_3)\vartheta(E_1)\tau(E_2)\vartheta(O)\theta(O)$.

7. **Horizontal String:**

$$
\begin{aligned}
&\tau(E_2)(1,1)\theta(P_2)\theta(E_4)\tau(E_3)\\
&\theta(P_4)\vartheta(P_1)\tau(E_2)\vartheta(E_1)\\
&\tau(P_3)\vartheta(E_1)\tau(E_2)\vartheta(O)\theta(O)\\
&(1,1)\theta(E_1)\theta(P_4)(1,1)\vartheta(P_1)\theta(O)\\
&\tau(E_3)(1,1)\tau(P_2)\theta(P_2)\theta(E_3)\vartheta(O)\theta(E_2)\\
&\tau(E_3)\vartheta(P_1)\tau(E_3)(1,1)\tau(E_2)\theta(E_1)\vartheta(O)\tau(E_2)\\
&\vartheta(E_1)\vartheta(P_1)\theta(E_1)\vartheta(P_1)\tau(E_3)(1,1)\tau(P_3)\\
&\vartheta(E_1)\tau(E_2)\vartheta(O)\theta(O).
\end{aligned}
$$

8. **Picture Coding (Visual Coding):**

Beyond these coding illustrations, the concepts of super mean labeling on four-star graphs extend to practical applications, such as in epidemic modeling.

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$$\tau(E_2)(1,1)\theta(P_2)\theta(E_4)\tau(E_3)(1,1)\theta(P_4)\vartheta(P_1)\tau(E_2)\vartheta(E_1)(1,1)$$
$$\tau(P_3)\vartheta(E_1)\tau(E_2)\vartheta(0)\theta(0)(1,1)\theta(E_1)\theta(P_4)(1,1)\ \vartheta(P_1)\theta(O$$
$$\tau(E_3)(1,1)\tau(P_2)\theta(P_2)\theta(E_3)\vartheta(0)\theta(E_2)\tau(E_3)\vartheta(P_1)\tau(E_3)(1,1)$$
$$\tau(P_3)\theta(E_1)\vartheta(0)\tau(E_2)\vartheta(E_1)\vartheta(P_1)\theta(E_1)\vartheta(P_1)\tau(E_3)(1,1)$$
$$\tau(P_3)\vartheta(E_1)\tau(E_2)\vartheta(0)\theta(0).$$

5 Applications in Epidemic Modeling

The super mean labeling technique applied to four-star graphs offers valuable insights in epidemic modeling. Here, the graph can model contact networks featuring multiple hubs, such as key individuals or locations like offices, schools, or gatherings linked to people. Each star represents a central point with pendant vertices as connected entities. By assigning unique labels to vertices and computing distinct edge means via ceilings of averages, these can symbolize varying transmission risks or probabilities, ensuring diverse pathways for more precise simulations.

Within graph-based Susceptible-Infected-Recovered (SIR) models, labels help customize infection parameters. Vertex labels $f(v)$ could indicate levels of vulnerability or contagiousness, while edge labels $f^*(e)$ represent transmission efficiencies between nodes. This uniqueness supports detailed tracking of spread routes, aiding in evaluating scenarios and planning responses.

For a discrete-time SIR setup on the four-star graph, define $S_t(v)$, $I_t(v)$, and $R_t(v)$ as probabilities for a vertex v being susceptible, infected, or recovered at time t. Updates integrate labels like this:

For a pendant p linked to hub h in a star:

$$I_{t+1}(p) = I_t(p) + S_t(p) \cdot \beta \cdot f^*(e_{h,p}) \cdot I_t(h),$$

with β as the baseline rate, and $f^*(e_{h,p})$ adjusting for contact variations.

Hubs aggregate similarly across pendants, weighted accordingly.

A C program can simulate this by setting up labels and running SIR iterations. Here's a sample code for such a model:

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```c
// Simple SIR simulation on four-star graph with super mean labels.
#include <stdio.h>
#include <stdlib.h>

#define NUM_STARS 4
#define PENDANTS_PER_STAR 1  // For K_{1,1}

typedef struct {
    int hub_label;
    int pendant_label;
    int edge_label;  // Ceiling of (hub + pendant)/2
} Star;

void initialize_stars(Star stars[]) {
    // illustration labels from one illustration
    stars[0].hub_label = 1; stars[0].pendant_label = 3;
    stars[0].edge_label = 2;
    stars[1].hub_label = 4; stars[1].pendant_label = 2;
    stars[1].edge_label = 3;
    stars[2].hub_label = 5; stars[2].pendant_label = 6;
    stars[2].edge_label = 6;
    stars[3].hub_label = 7; stars[3].pendant_label = 8;
    stars[3].edge_label = 8;
}

void simulate_sir(Star stars[], double beta, int steps) {
    double S[NUM_STARS * 2], I[NUM_STARS * 2], R[NUM_STARS * 2];
    // Initialize: All susceptible except one infected hub
    for (int i = 0; i < NUM_STARS * 2; i++) {
        S[i] = 1.0; I[i] = 0.0; R[i] = 0.0;
    }
    I[0] = 1.0; S[0] = 0.0;  // Infect first hub

    for (int t = 0; t < steps; t++) {
        for (int s = 0; s < NUM_STARS; s++) {
            int hub_idx = s * 2;
            int pend_idx = hub_idx + 1;
            // Update pendant from hub
            double delta_I_pend = S[pend_idx] * beta *
            stars[s].edge_label * I[hub_idx] / 10.0;
            I[pend_idx] += delta_I_pend;
            S[pend_idx] -= delta_I_pend;
```

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```
            // Recovery (simple gamma=0.1)
            double delta_R = 0.1 * I[hub_idx];
            R[hub_idx] += delta_R;
            I[hub_idx] -= delta_R;
            delta_R = 0.1 * I[pend_idx];
            R[pend_idx] += delta_R;
            I[pend_idx] -= delta_R;
        }
        printf("Step %d: Total Infected %.2f\n", t, I[0]+I[1]+I[2]+
        I[3]+I[4]+I[5]+I[6]+I[7]);
    }
}

int main() {
    Star stars[NUM_STARS];
    initialize_stars(stars);
    simulate_sir(stars, 0.05, 10);
    return 0;
}
```

This program loads a super mean label setup, models spread adjusted by edges, and reports infected counts per step. Models like this forecast dynamics in networks with hubs, such as city transit during outbreaks, using distinct labels for varied transmissions.

Overall, embedding super mean labeling in epidemic frameworks improves handling of diverse contact risks, supporting better simulations and strategies.

6 Conclusion

In this paper, we explored super mean labeling across various contexts. We utilized four- and five-star graphs to transmit messages through different alphabetical numbering methods like EFOB, OBEF, Addition of Two Numbers, and Multiplication of Two Numbers with C programs. Four distinct messages were encoded using two alphabet numbering systems.

Looking ahead, additional C-program techniques and repeated letters for alphabet numbering will enhance coding on five-star graphs. Hints, mathematical or otherwise, align with messages, and coded outputs in images add complexity to decoding.

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