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A MATHEMATICAL MODEL FOR OBESITY IN YOUNG WOMEN AGED 18 TO 25 YEARS

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Abstract

A mathematical model for obesity in young women aged 18 to 25 years is formulated. The boundedness of the system is verified using Gronwall's inequality. Equilibrium points are found. The local stability of the model is analysed around the equilibrium points using the Routh-Hurwitz Criteria. The global stability of the model is analysed using Lyapunov's theorem on stability. Numerical simulations are carried out using MATLAB.

Keywords: Boundedness, Equilibrium Analysis, Global Stability, Gronwall's inequality, Local stability, Lyapunov's theorem, Obesity, Routh-Hurwitz Criteria

1. Introduction

Overweight and obesity are known as irregular or disproportionate fat growth that creates health risks. Body fat is measured using Body Mass Index (BMI). Body Mass Index of an adult is calculated based on height and weight. If the BMI is higher than or equal to 25, it is considered overweight. If it is higher than or equal to 30, it is considered as obesity [6].

The prevalence of overweight and obesity is due to energy imbalance. Many factors such as lack of physical activity, unhealthy food habits, poor quality sleep, stress, genetics and certain medicines can contribute to obesity [7]. Obesity may be with or without complications. It increases the risk of high blood pressure, high or low level LDL cholesterol, type 2 diabetes, coronary heart disease, stroke, etc.[8]

Many researchers have formulated and analyzed mathematical models for obesity. Nourrridine Siewe et al.[5] developed an ODE model of glucose dynamics to illustrate the contribution of obesity to type 2 diabetes mellitus. Enrique Lozano-Ochoa et al. [2] presented an ODE model for obesity to highlight the impact of social contagion on the spread of obesity. Adam Ben Taieb et al. [1] studied the need for better risk prediction models for obesity related complications and emphasized that the accuracy and application of current equations are limited since they frequently lack external validation and do not distinguish between BMI classes, especially class III obesity. Nasim S. Sabounchi et al. [4] presented a system dynamics model for obesity in women of reproductive age which emphasized the problems such as morbidity and death rates among pregnant obese women and the effects of weight loss and reproductive outcomes. Kasinathan Kasi Gowthami et al. [3] conducted a study on obesity

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among young female college students between the ages of 18 and 25 at Vellore district, Tamil Nadu and emphasized the necessity of prevention and treatment strategies.

In general, obesity is becoming a more significant public health issue affecting all age groups, particularly young women. Given the notable rise in the prevalence of obesity in the Vellore district, it is imperative to comprehend the underlying variables contributing to this trend. In this paper, a mathematical model for obesity in young women aged 18 to 25 years in Vellore district is formulated.

2. Formulation of Mathematical Model

A system of four ordinary differential equations representing the obesity in young women aged 18 to 25 years is as follows:

$$\frac{dN_w}{dt} = \eta - \alpha N_w + \lambda O_W - \mu N_w \qquad (1)$$

$$\frac{dO_w}{dt} = \alpha N_w - \mu O_w - \beta O_w - \gamma O_w - \lambda O_w + \omega O_0 \qquad (2)$$

$$\frac{dO_0}{dt} = \beta O_w - \mu O_0 - \theta O_0 + \varepsilon O_1 - \omega O_0 \qquad (3)$$

$$\frac{dO_1}{dt} = \gamma O_w + \theta O_0 - \mu O_1 - \delta O_1 - \varepsilon O_1 \qquad (4)$$

with the initial conditions $N_w(0) > 0, O_w(0) \geq 0, O_0(0) \geq 0, O_1(0) \geq 0$. Also $\eta, \alpha, \lambda, \mu, \beta, \gamma, \omega, \theta, \varepsilon, \delta$ are all positive.

Here $N_w(t), O_w(t), O_0(t), O_1(t)$ are the normal weight, overweight, obesity without complications, obesity with complications states respectively. $A(t)$-Total population, η-Recruitment rate, α-Transition rate from normal weight to overweight, λ-Clearance rate, μ-Natural death rate, β-Progression rate from overweight to obese without complications, γ- Progression rate from overweight to obese with complications, ω-Partial recovery rate, θ −Rate of developing complications, ε-Rate of controlling complications and δ-Disease-induced death rate.

The flow of the model is given as

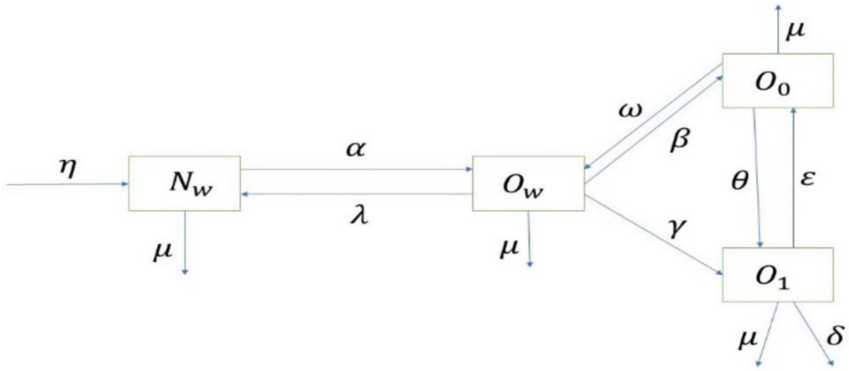

Figure. Mathematical Model for Obesity in Young Women Aged 18 to 25 Years

International Journal of Applied Mathematics
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3. Boundedness of the System

Let $S(t) = N_w(t) + O_W(t) + O_0(t) + O_1(t), \vartheta > 0$

$\frac{dS}{dt} + \vartheta S = \frac{dN_w}{dt} + \frac{dO_w}{dt} + \frac{dO_0}{dt} + \frac{dO_1}{dt} + \vartheta S$

$= (\eta - \alpha N_w + \lambda O_W - \mu N_w) + (\alpha N_w - \mu O_w - \beta O_w - \gamma O_w - \lambda O_w + \omega O_0) +$
$(\beta O_w - \mu O_0 - \theta O_0 + \varepsilon O_1 - \omega O_0) + (\gamma O_w + \theta O_0 - \mu O_1 - \delta O_1 - \varepsilon O_1) +$
$\vartheta(N_w(t) + O_W(t) + O_0(t) + O_1(t))$

$= \eta - \mu(N_w + O_W + O_0 + O_1) + \vartheta(N_w(t) + O_W(t) + O_0(t) + O_1(t))$

$\leq \eta + \vartheta(N_w(t) + O_W(t) + O_0(t) + O_1(t)) = \rho$(say)

By using Gronwall's inequality,

$0 < S(N_w(t) + O_W(t) + O_0(t) + O_1(t)) \leq \frac{\rho}{\vartheta}\left(1 - e^{-\vartheta t}\right) +$
$\left(S(N_w(0) + O_W(0) + O_0(0) + O_1(0))\right)e^{-\vartheta t}$

When $t \to \infty, 0 < S(t) \leq \frac{\rho}{\vartheta}$

Hence the solutions of the system (1)-(4) with the initial conditions in the positive quadrant are bounded.

4. Equilibrium Analysis

The system has two equilibrium points. First one is the disease-free equilibrium point i.e. $E_0(\overline{N_w}, 0, 0, 0)$ which is obtained by setting $O_w = O_0 = O_1 = 0$ in $\frac{dN_w}{dt} = 0$. That is,

$$\eta - \alpha N_w - \mu N_w = 0 \tag{5}$$

Solving (5) we get,

$$\overline{N_w} = \frac{\eta}{\alpha + \mu}$$

and the other is the endemic equilibrium point $E^*(N_W^*, O_W^*, O_0^*, O_1^*)$ which is obtained by setting $\frac{dN_w}{dt} = \frac{dO_w}{dt} = \frac{dO_0}{dt} = \frac{dO_1}{dt} = 0$. That is,

$$\eta - \alpha N_w + \lambda O_W - \mu N_w = 0 \tag{6}$$

$$\alpha N_w - \mu O_w - \beta O_w - \gamma O_w - \lambda O_w + \omega O_0 = 0 \tag{7}$$

$$\beta O_w - \mu O_0 - \theta O_0 + \varepsilon O_1 - \omega O_0 = 0 \tag{8}$$

$$\gamma O_w + \theta O_0 - \mu O_1 - \delta O_1 - \varepsilon O_1 = 0 \tag{9}$$

Solving (6)-(9) we get,

$$\begin{aligned} N_w^* = \frac{\eta}{D} [&\beta(\delta\theta + \delta\mu + \theta\mu + \mu^2 + \mu\varepsilon) + \gamma(\delta\theta + \delta\mu + \delta\omega + \theta\mu + \mu^2 + \mu\varepsilon + \mu\omega) \\ &+ \delta(\theta\lambda + \theta\mu + \lambda\mu + \lambda\omega + \mu^2 + \mu\omega) + \theta(\lambda\mu + \mu^2) \\ &+ \lambda(\mu^2 + \mu\varepsilon + \mu\omega + \varepsilon\omega) + (\mu^3 + \mu^2\varepsilon + \mu^2\omega + \mu\varepsilon\omega)] \end{aligned}$$

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$$O_w^* = \frac{\alpha\eta(\delta\theta + \delta\mu + \delta\omega + \theta\mu + \mu^2 + \mu\varepsilon + \mu\omega + \varepsilon\omega)}{D}$$

$$O_0^* = \frac{\alpha\eta(\beta\delta+\beta\mu+\beta\varepsilon+\gamma\varepsilon)}{D}$$

$$O_1^* = \frac{\alpha\eta(\beta\theta+\gamma\theta+\gamma\mu+\gamma\omega)}{D}$$

where the denominator, $D = \alpha\beta\delta\theta + \alpha\beta\delta\mu + \alpha\beta\theta\mu + \alpha\beta\mu^2 + \alpha\beta\mu\varepsilon + \alpha\gamma\delta\theta + \alpha\gamma\delta\mu + \alpha\gamma\delta\omega + \alpha\gamma\theta\mu + \alpha\gamma\mu^2 + \alpha\gamma\mu\varepsilon + \alpha\gamma\mu\omega + \alpha\delta\theta\mu + \alpha\delta\mu^2 + \alpha\delta\mu\omega + \alpha\theta\mu^2 + \alpha\mu^3 + \alpha\mu^2\varepsilon + \alpha\mu^2\omega + \alpha\mu\varepsilon\omega + \beta\delta\theta\mu + \beta\delta\mu^2 + \beta\theta\mu^2 + \beta\mu^3 + \beta\mu^2\varepsilon + \gamma\delta\theta\mu + \gamma\delta\mu^2 + \gamma\delta\mu\omega + \gamma\theta\mu^2 + \gamma\mu^3 + \gamma\mu^2\varepsilon + \gamma\mu^2\omega + \delta\theta\lambda\mu + \delta\theta\mu^2 + \delta\lambda\mu^2 + \delta\lambda\mu\omega + \delta\mu^3 + \delta\mu^2\omega + \theta\lambda\mu^2 + \theta\mu^3 + \lambda\mu^3 + \lambda\mu^2\varepsilon + \lambda\mu^2\omega + \lambda\mu\varepsilon\omega + \mu^4 + \mu^3\varepsilon + \mu^3\omega + \mu^2\varepsilon\omega$

It can be noted that N_w^*, O_w^*, O_0^* and O_1^* is positive

5. Local Stability

The Jacobian matrix of the system (1)-(4) is given by

$$J = \begin{bmatrix} -(\alpha+\mu) & \lambda & 0 & 0 \\ \alpha & -(\mu+\beta+\gamma+\lambda) & \omega & 0 \\ 0 & \beta & -(\mu+\theta+\omega) & \varepsilon \\ 0 & \gamma & \theta & -(\mu+\delta+\varepsilon) \end{bmatrix}$$

At E^* the matrix J is the same as above.

The characteristic equation of J is

$$\begin{vmatrix} -(\alpha+\mu)-k & \lambda & 0 & 0 \\ \alpha & -(\mu+\beta+\gamma+\lambda)-k & \omega & 0 \\ 0 & \beta & -(\mu+\theta+\omega)-k & \varepsilon \\ 0 & \gamma & \theta & -(\mu+\delta+\varepsilon)-k \end{vmatrix} = 0$$

i.e.

$$\begin{vmatrix} (\alpha+\mu)+k & \lambda & 0 & 0 \\ \alpha & (\mu+\beta+\gamma+\lambda)+k & \omega & 0 \\ 0 & \beta & (\mu+\theta+\omega)+k & \varepsilon \\ 0 & \gamma & \theta & (\mu+\delta+\varepsilon)+k \end{vmatrix} = 0$$

i.e. $k^4 + Ak^3 + Bk^2 + Ck + D = 0$ where $A = \alpha + \beta + \gamma + \delta + \varepsilon + \theta + \lambda + 4\mu + \omega$,

$$B = \alpha(\beta+\gamma+\delta+\varepsilon+\theta+\omega) + \beta(\delta+\varepsilon+\theta) + \gamma(\delta+\varepsilon+\theta+\omega) + \delta(\theta+\lambda+\omega) + \varepsilon(\lambda+\omega) + \lambda(\theta+\omega) + 3\mu(\alpha+\beta+\gamma+\delta+\varepsilon+\theta+\lambda+2\mu+\omega),$$

$$C = \alpha\beta\delta + \alpha\beta\varepsilon + \alpha\beta\theta + 2\alpha\beta\mu + \alpha\gamma\delta + \alpha\gamma\varepsilon + \alpha\gamma\theta + 2\alpha\gamma\mu + \alpha\gamma\omega + \alpha\delta\theta + 2\alpha\delta\mu + \alpha\delta\omega + 2\alpha\varepsilon\mu + \alpha\varepsilon\omega + 2\alpha\theta\mu + 3\mu^2\alpha + 2\alpha\mu\omega + \beta\delta\theta + 2\beta\delta\mu + 2\beta\varepsilon\mu + 2\beta\theta\mu + 3\mu^2\beta + \gamma\delta\theta + 2\gamma\delta\mu + \gamma\delta\omega + 2\gamma\varepsilon\mu + 2\gamma\theta\mu + 3\mu^2\gamma + 2\gamma\mu\omega + \delta\theta\lambda + 2\delta\theta\mu + 2\delta\lambda\mu + \delta\lambda\omega + 3\mu^2\delta + 2\delta\mu\omega + 2\varepsilon\lambda\mu + \varepsilon\lambda\omega + 3\mu^2\varepsilon + 2\varepsilon\mu\omega + 2\theta\lambda\mu + 3\mu^2\theta + 3\mu^2\lambda + 2\lambda\mu\omega + 4\mu^3 + 3\mu^2\omega,$$

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$$D = \alpha\beta\delta\theta + \alpha\beta\delta\mu + \alpha\beta\varepsilon\mu + \alpha\beta\theta\mu + \mu^2\alpha\beta + \alpha\gamma\delta\theta + \alpha\gamma\delta\mu + \alpha\gamma\delta\omega + \alpha\gamma\varepsilon\mu + \alpha\gamma\theta\mu + \mu^2\alpha\gamma + \alpha\gamma\mu\omega + \alpha\delta\theta\mu + \mu^2\alpha\delta + \alpha\delta\mu\omega + \mu^2\alpha\varepsilon + \alpha\varepsilon\mu\omega + \mu^2\alpha\theta + \mu^3\alpha + \mu^2\alpha\omega + \beta\delta\theta\mu + \mu^2\beta\delta + \mu^2\beta\varepsilon + \mu^2\beta\theta + \mu^3\beta + +\gamma\delta\theta\mu + \mu^2\gamma\delta + \gamma\delta\mu\omega$$

Here $A > 0, C > 0, D > 0$ and $ABC - C^2 > A^2D$.

Hence by Routh-Hurwitz criteria the system (1)-(4) is locally asymptotically stable around E^*.

6. Global Stability

$V(N_w, O_W, O_0, O_1) = (N_w - N_w^*) - N_w^* \ln\frac{N_w}{N_w^*} + (O_w - O_w^*) - O_w^* \ln\frac{O_w}{O_w^*} + (O_0 - O_0^*) - O_0^* \ln\frac{O_0}{O_0^*} + (O_1 - O_1^*) - O_1^* \ln\frac{O_1}{O_1^*}$

It can be easily shown that the function V is 0 at the equilibrium point $E^*(N_w^*, O_w^*, O_0^*, O_1^*)$ and is positive for other positive values of N_w, O_W, O_0, O_1.

The time derivative of V along the solutions of (6)-(9) is

$$\frac{dV}{dt} = \left(1 - \frac{N_w^*}{N_w}\right)\frac{dN_w}{dt} + \left(1 - \frac{O_w^*}{O_w}\right)\frac{dO_w}{dt} + \left(1 - \frac{O_0^*}{O_0}\right)\frac{dO_0}{dt} + \left(1 - \frac{O_1^*}{O_1}\right)\frac{dO_1}{dt} =$$

$$= \left(1 - \frac{N_w^*}{N_w}\right)(\eta + \lambda O_w - (\alpha + \mu)N_w) + \left(1 - \frac{O_w^*}{O_w}\right)(\alpha N_w + \omega O_0 - (\mu + \beta + \gamma + \lambda)O_w) + \left(1 - \frac{O_0^*}{O_0}\right)(\beta O_w + \varepsilon O_1 - (\mu + \theta + \omega)O_0) + \left(1 - \frac{O_1^*}{O_1}\right)(\gamma O_w + \theta O_0 - (\mu + \delta + \varepsilon)O_1)$$

$$= -(\alpha + \mu)\frac{(N_w - N_w^*)^2}{N_w} - (\mu + \beta + \gamma + \lambda)\frac{(O_w - O_w^*)^2}{O_w} - (\mu + \theta + \omega)\frac{(O_0 - O_0^*)^2}{O_0} - (\mu + \delta + \varepsilon)\frac{(O_1 - O_1^*)^2}{O_1}$$

$$\frac{dV}{dt} < 0$$

i.e. $\frac{dV}{dt}$ is negative definite.

By Lyapunov's theorem on stability, it can be concluded that the endemic equilibrium $E^*(N_w^*, O_w^*, O_0^*, O_1^*)$ is globally asymptotically stable provided if $N_w < N_w^*, O_w < O_w^*, O_0 < O_0^*$ and $O_1 < O_1^*$.

7. Numerical Simulations

The parameter values were calculated based on the study on obesity among young female college students between the ages of 18 and 25 at Vellore district, Tamil Nadu, conducted by Kasinathan Kasi Gowthami et al. [3]. The recruitment rate $\eta = 0.91$, Transition rate from normal weight to overweight $\alpha = 0.94$, Clearance rate $\lambda = 0.29$, Natural death rate $\mu = 0.83$,

International Journal of Applied Mathematics
Volume 38 No. 6s, 2025
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Progression rate from overweight to obese without complications $\beta = 0.64$, Progression rate from overweight to obese with complications $\gamma = 0.52$, Partial recovery rate $\omega = 0.74$, Rate of developing complications $\theta = 0.14$, Rate of controlling complications $\varepsilon = 0.66$ and Disease-induced death rate $\delta = 0.71$.

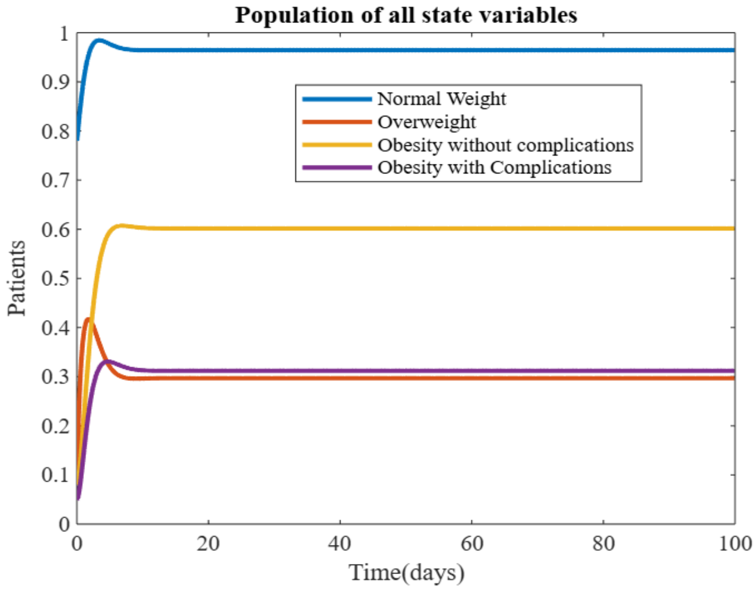

Figure 1. Flow of all populations

In figure 1, the graph depicts all the populations, where the population of obesity without complications is higher than the population of obesity with complications.

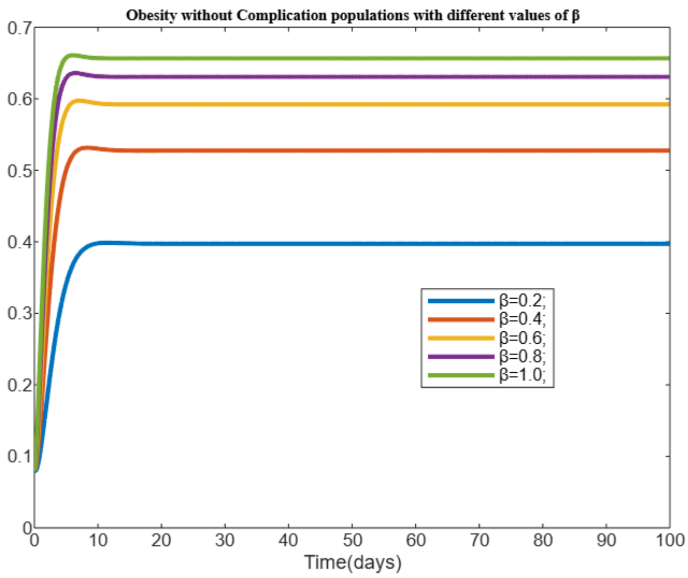

Figure 2. Obesity without complications with different values of β

In figure 2, the graph depicts the population of the obesity without complications increases if the β value increases.

International Journal of Applied Mathematics
Volume 38 No. 6s, 2025
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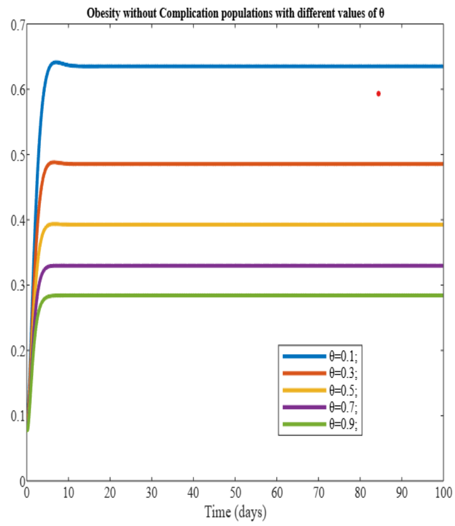

Figure 3. Population of obesity without complications for different values of θ

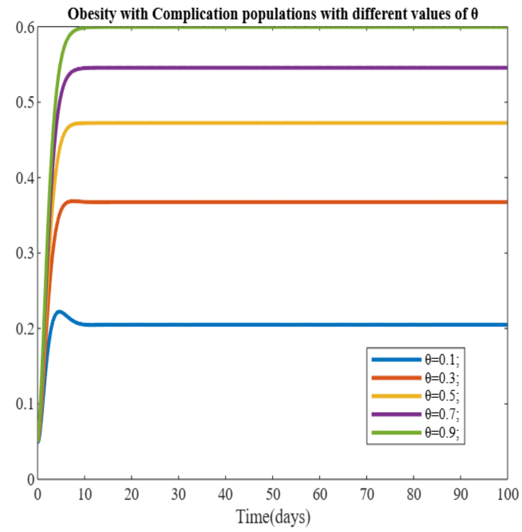

Figure 4. Population of obesity with complications for different values of θ

Comparing the graphs in figures 3 and 4, it is understood that the population of obesity without complications declines when the θ value increases, but the population of obesity with complications increases when the θ value increases.

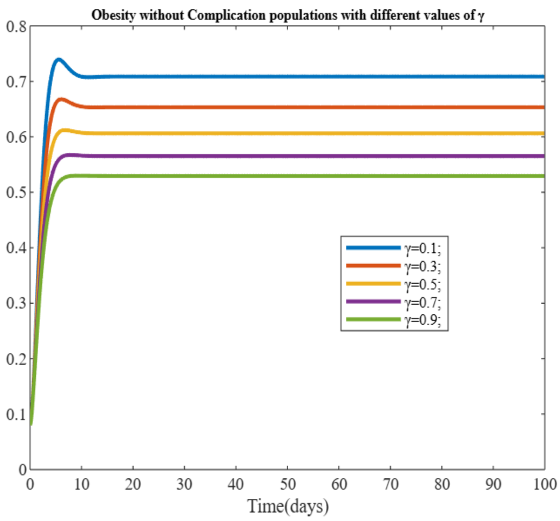

Figure 5. Population of obesity without complications for different values of γ

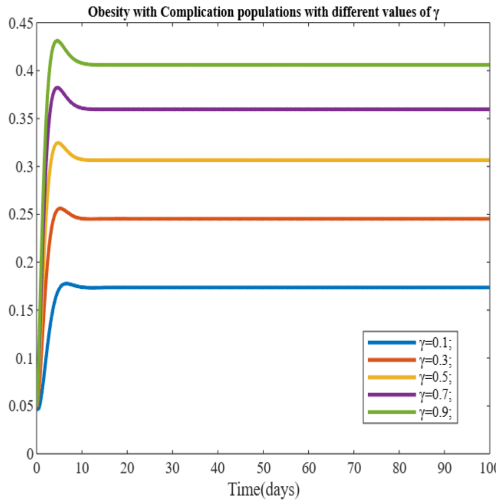

Figure 6. Population of obesity with complications for different values of γ

Comparing the graphs in figures 5 and 6, it is understood that the population of obesity without complications declines when the γ value increases, but the population of obesity with complications increases when the γ value increases.

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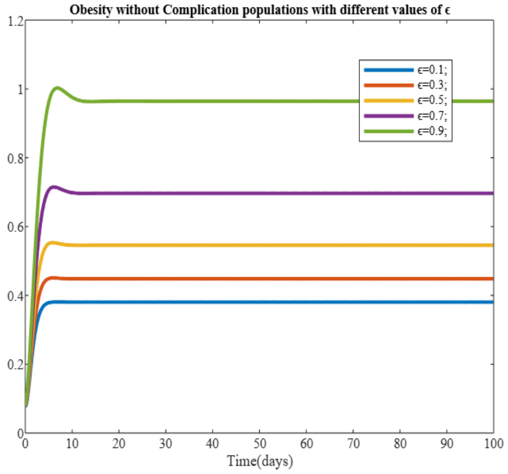

Figure 7. Population of Obesity without complications for different values of ε

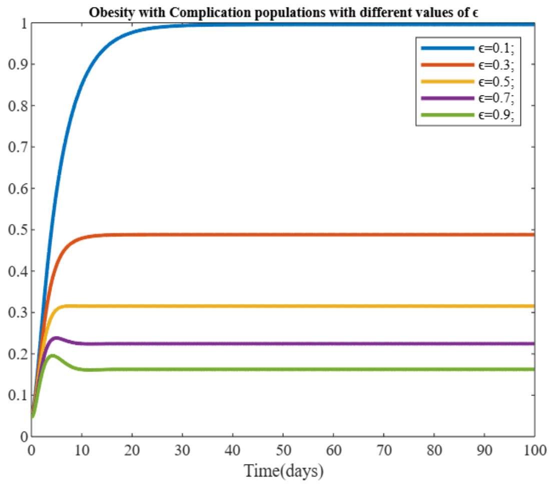

Figure 8. Population of Obesity with complications for different values of ε

Comparing the graphs in figures 7 and 8, it is understood that the population of obesity without complication increases when the ε value increases, but the population of obesity with complication declines when the ε value increases.

Conclusion

This paper provides important insights into the dynamics of weight growth and the effect of parameters on it. The model provides a solid framework for the better understanding of obesity in this population. The boundedness of the system was verified which offers a way for successful intervention tactics by concentrating on constrained solutions. The disease-free equilibrium and endemic equilibrium points were found. The system is found to be locally asymptotically stable around the endemic equilibrium using the Routh-Hurwitz criteria and globally asymptotically stable using Lyapunov's stability theorem. This guarantees that the solutions are applicable in real-world scenarios and long-lasting. Numerical simulations were carried out for the flow of all populations, obesity with complications and obesity without complications with respect the parameters β-Progression rate from overweight to obese without complications, θ-Rate of developing complications, γ-Progression rate from overweight to obese with complications and ε-Rate of controlling complications. The population of obesity without complications is higher than the population of obesity with complications and it increases if the β value increases. The population of obesity without complications declines when the θ value increases, but the population of obesity with complications increases when the θ value increases. The population of obesity without complications declines when the γ value increases, but the population of obesity with complications increases when the γ value increases. The population of obesity without complication increases when the ε value increases, but the population of obesity with complication declines when the ε value increases. This model gives an idea that helps to reduce obesity rate in Vellore district and beyond by encouraging young women to lead healthy lifestyle as communities and medical professionals work together to combat obesity.

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