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ADVANCED SOFT TOPOLOGICAL STRUCTURES FOR SITE SELECTION: INTEGRATING GRILL TOPOLOGY AND B#D-AXIOMS

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Abstract: Site selection in civil engineering involves decisions under uncertainty, vague- ness and incomplete information. We present a hybrid framework that combines soft topology and grill topology to model flexibility and robustness, and we in- troduce *soft b#D-sets* together with related separation axioms (soft b#D_k) for refined discrimination among sites. A hospital site selection case study illustrates the approach and demonstrates how b#D-axioms influence final choices.

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1. Introduction

Modern civil engineering site selection often requires combining qualitative expert judg- ment and incomplete quantitative data. Soft set theory and its topological extensions

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provide flexible mathematical tools for handling parametric uncertainty. Grill topology adds a filter-like robustness notion. Soft b#D-sets and corresponding separation axioms provide further discriminating power for distinguishing candidate sites. This paper integrates these concepts and demonstrates their use in a worked case study.

2. Preliminaries

2.1 Soft Set Theory

Let X be a universe (set of sites) and E a set of parameters. A *soft set* (F, A) over X, where $A \subseteq E$, is a mapping

$F : A \longrightarrow P(X)$,

so each parameter $a \in A$ is associated with $F(a) \subseteq X$ (the sites roughly satisfying a).

2.2 Soft Topological Space

A *soft topology* τ on X is a family of soft sets over X such that: (null soft set) and $\tilde{X}$(absolute soft set) belong to τ,

i. arbitrary unions of members of τ lie in τ,
ii. finite intersections of members of τ lie in τ.

The pair (X, τ) is a *soft topological space*. A member of τ is *soft open*; complements (relative to $\tilde{X}$) of soft open sets are *soft closed*.

2.3 Interior and Closure

For a soft set (F, A) in a soft topology (X, τ) we define:

$$\mathrm{Int}(F, A) = \bigcup \{(G, B) \in \tau \mid (G, B) \subseteq (F, A)\},$$

$$\mathrm{Cl}(F, A) = \bigcap \{(G, B) \mid (F, A) \subseteq (G, B),\ (G, B)^c \in \tau\}.$$

Standard inclusions $\mathrm{Int}(F, A) \subseteq (F, A) \subseteq \mathrm{Cl}(F, A)$ hold.

3. Grill Topology Extension

3.1 Soft Grill

A *grill* G on a classical set is a family of nonempty subsets with the property: if $A \cap G \neq \emptyset$ for every $G \in \mathrm{G}$ then $A \in \mathrm{G}$. We lift this idea to soft sets:

[Soft Grill] Let (F, A) be a soft set over X. A *soft grill* G_s is a family of soft sets such that whenever a soft set (H, B) intersects every member of G_s (componentwise nonempty intersections), then $(H, B) \in \mathrm{G}_s$.

3.2 Grill-Topology Interaction

A soft topological space (X, τ) admits a *soft grill topology* τ_{G} when there exist soft grills G_s whose members interact coherently with τ (members of G_s are comparable with members of τ in neighborhood-like fashion).

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3.3 Theorem (Grill-based Soft Selection Criterion)

Let (X, τ) be a soft topological space with soft grill G_s. A site $x_i \in X$ is *grill-robust* if

$\forall (F, A) \in \tau,\ \exists (G, B) \in \mathrm{G}_s$ such that $x_i \in F(a) \cap G(b)$

for some parameters $a \in A$, $b \in B$.

Proof. Fix arbitrary $(F, A) \in \tau$. By hypothesis, there exists $(G, B) \in \mathrm{G}_s$ with $x_i \in F(a) \cap G(b)$ for some a, b. Thus x_i lies in the union (over grill members) of the intersections with each soft-open set; taking the intersection over all $(F, A) \in \tau$ yields

$$x_i \in \bigcap_{(F,A)\in\tau} \bigcup_{(G,B)\in \mathrm{G}_S} F(a) \cap G(b)\ ,$$

so x_i remains in every such intersection — i.e. it is robust under soft-open sets and grill filters.

$\square$ $\square$

$F(a)$ $G(b)$

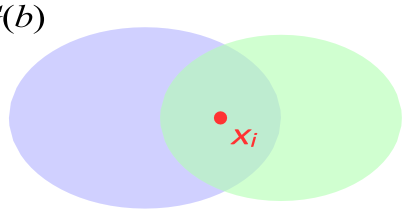

Figure 1: Intersection of a soft open set $F(a)$ and a soft grill set $G(b)$; the point x_i lies in the intersection.

4. Soft b#D-Sets and Related Separation Axioms

Let (X, τ, E) be a soft topological space over universe X with parameter set E. A soft subset S_A of X is called a *soft b#D-set* if there exist soft b#-open sets U_A and V_A such that

$$S_A = U_A \setminus V_A, \qquad U_A \subsetneq \tilde{X}_A,$$

where $\tilde{X}_A$ is the absolute soft set over $A \subseteq E$.

Every proper soft b#-open subset is a soft b#D-set; however a soft b#D-set need not be soft b#-open. In particular:

- The null soft set Φ_A is a soft b#D-set.
- The absolute soft set $\tilde{X}_A$ is not a soft b#D-set.

Hence the collection of all soft b#D-sets is not a minimal topology-like structure.

Let $X = \{a, b, c\}$ and $E = \{e_1, e_2\}$ and suppose a soft topology (see appendices) yields soft b#-open family containing sets such as $\{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ but also proper subsets like $\{(e_1, \{a\}), (e_2, \{a\})\}$. Then the latter can be expressed as $U_A \setminus V_A$ and hence is a soft b#D-set though not soft b#-open.

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Every soft b#D-set is a soft bD-set (a weaker notion), but the converse need not hold.

A soft topological space (X, τ, E) is:

1. *oft b#D*$_0$ if for any distinct $x, y \in X$ there exists a soft b#D-set containing one but not the other.

2. *Soft b#D*$_1$ if for any distinct $x, y \in X$ there exist soft b#D-sets S and T with $x \in S \setminus T$ and $y \in T \setminus S$.

3. *Soft b#D*$_2$ if for any distinct $x, y \in X$ there exist *disjoint* soft b#D-sets F_A and G_A with $x \in F_A$, $y \in G_A$.

For (X, τ, E):

1. If (X, τ, E) is soft b#-T$_k$ then it is soft b#D$_k$ for $k = 0, 1, 2$.

2. If (X, τ, E) is soft b#D$_k$ then it is soft b#D$_{k-1}$ for $k = 1, 2$.

A space is soft b#D$_0$ iff it is soft b#-T$_0$. Also, soft b#D$_1$ is equivalent to soft b#D$_2$. If a space is soft b#D$_1$ then it is soft b#-T$_0$.

Let (X, τ, E) be a soft topological space and $A \subseteq X$ a soft set. Then:

1. If A is soft regular closed, then A is soft b#-open.

2. If A is soft regular open, then A is soft b#-closed.

5. Worked Example: Applying Soft b#D-Axioms to Hospital Site Selection

We revisit the case study with $X = \{x_1, x_2, x_3, x_4, x_5\}$ and parameters

A (Accessibility) $\{x_1, x_2, x_4\}$

S (Soil) $\{x_2, x_3, x_5\}$

E (Environment) $\{x_1, x_3, x_4\}$

C (Cost) $\{x_2, x_4\}$

U (Utilities) $\{x_1, x_2, x_3\}$

Construct the soft sets (F_a, A_a) corresponding to each parameter (abbreviated here by their element-sets).

a. Candidate soft b#-open families

Assume (based on domain expert input) the following soft b#-open sets are considered reasonable neighbourhoods of preference:

$U_1 = \{x_2, x_4\}$ (strong accessibility+cost match)

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$U_2 = \{x_1, x_3\}$ (good environment+utilities)

$U_3 = \{x_2, x_3\}$ (soil + utilities)

Each U_i corresponds to a soft b#-open set U_{A_i} across an appropriate parameter subset A_i.)

Let $V_1 = \{x_4\}$ be a small undesirable pocket (e.g., high environmental sensitivity) represented as a soft b#-open set to subtract.

b. Forming soft b#D-sets

We can form soft b#D-sets by $S = U_1 \setminus V_1 = \{x_2\}$. Concretely:

$S = U_1 - V_1 = \{x_2, x_4\} \setminus \{x_4\} = \{x_2\}$.

Thus $\{x_2\}$ is a soft b#D-set (a single-site b#D-set) — representing a focused, robust candidate.

Similarly, take $T = U_2 \setminus \emptyset = \{x_1, x_3\}$; T is both b#-open and a b#D-set.

c. Checking b#D separation axioms

b#D 0: For any two distinct sites, can we find a soft b#D-set containing one but not the other?

i. Example: x_2 vs x_4. $S = \{x_2\}$ contains x_2 but not x_4.

ii. x_1 vs x_3. $T = \{x_1, x_3\}$ contains both, but we can refine: choose $T_1 = U_2 \setminus \{x_3\} = \{x_1\}$ (assumed constructible) to separate.

Thus, with the available b#D-sets (and mild refinements) the space exhibits b#D 0 behaviour.

b#D 1: For distinct x, y we need two b#D-sets, one containing x but not y and another containing y but not x.

iii. x_2 and x_4: $S = \{x_2\}$ and $S' = \{x_4\}$ (if $\{x_4\}$ can be realized as $U \setminus V$) separate them; earlier V_1 gave $\{x_4\}$ as a candidate if we choose $U' = \{x_4, x_5\}$ and subtract $\{x_5\}$.[1]

If such decompositions exist for every pair, the space meets b#D 1; in our expert-constructed family this is plausible with localized soft b#-open choices.

b#D 2: Requires disjoint soft b#D-sets for any pair.

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iv. x_2 and x_1: choose $S = \{x_2\}$ and $T = \{x_1\}$ (disjoint), both expressible as $U \setminus V$ above.

Hence, with appropriate local refinement of b#-open sets, the model can satisfy b#D 2 for most practical candidate pairs, and the presence of $\{x_2\}$ as a singleton b#D-set signals its strong discrimination power.

Interpretation for decision-making: The existence of singleton soft b#D-sets (e.g. $\{x_2\}$) indicates that certain sites are *isolatable* by robust preference neighborhoods — making them excellent candidates for selection. The stronger the separation axioms hold (b#D 2 strongest), the more discriminating power the model has to yield unambiguous choices.

6. Case Study Results and Discussion

Using soft-open frequency counts earlier, x_2 appears in 4 of 5 parameter sets (high preference) and can be isolated as a soft b#D-set; thus it is both grill-robust and b#D-discriminated. Sites x_1, x_3, x_4 are medium preference and less easily isolatable unless more refined soft b#-open sets are created. x_5 is low preference.

7. Conclusion

We provided a coherent integration of soft topology, grill topology and soft b#D-sets for robust, discriminative site selection in civil engineering. The b#D axioms give a formal way to measure how finely sites can be separated by preference neighborhoods; applied to the hospital case study, they reinforce the selection of x_2 as robust and discriminable. Future work: integrate GIS layers, formalize construction rules for b#-open families from geodata, and test on larger datasets.

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