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ENERGY-AWARE MATHEMATICAL MODELLING AND PERFORMANCE ANALYSIS OF WIRELESS SENSOR NETWORKS

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Abstract

Wireless Sensor Networks (WSNs) have become essential in tenders ranging from environmental monitoring to smart cities. A critical component in WSN design is mathematical modelling, which helps optimize energy usage, routing, coverage, and node deployment. This paper presents a comprehensive study on sensor-based mathematical modelling and calculations tailored for WSNs. A model is developed that takes into account node density, energy consumption, transmission range, and network lifetime. Results are validated through MATLAB simulation, demonstrating the accuracy and efficiency of the proposed model.

Keywords—Wireless Sensor Network (WSN), Sensor Node, Mathematical Modelling, Energy Efficiency, Network Lifetime, Node Deployment.

1. Introduction

Wireless Sensor Networks (WSNs) comprise of spatially disseminated independent sensors that monitor physical or environmental circumstances such as temperature, pressure, and humidity. The sensor nodes communicate wirelessly and cooperatively permit data over the network to a base station.

Energy efficiency is a crucial constraint, as nodes often operate on limited battery power. Effective mathematical modelling allows for optimizing various aspects of WSNs such as node

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placement, energy consumption, and data routing, improving overall network performance and lifetime.

This paper proposes a sensor-based mathematical model that can predict and optimize energy consumption and network life-period in a WSN. Our model incorporates practical parameters such as sensing range, data rate, transmission power, and node density.

2. Literature Survey

Wireless Sensor Networks (WSNs) have received wide-ranging attention in both hypothetical and industrial domains due to their vast applicability. Several studies have addressed mathematical modeling, energy optimization, and performance prediction in WSNs. The following literature highlights key contributions relevant to this work.

Heinzelman et al. (2002) proposed the **LEACH protocol**, one of the earliest ordered n flow routing approaches in WSNs. Their work emphasizes energy efficiency through randomized rotation of local cluster heads to distribute energy load evenly among sensors 111. However, LEACH lacks analytical formulations for network lifetime under variable conditions.

Akkaya and Younis (2005) provided a comprehensive investigation on routing protocols, highlighting the need for **energy-aware and QoS-aware** algorithms. They categorized routing protocols but noted the lack of unified models for energy and coverage analysis 222.

Yick et al. (2008) addressed WSN architecture, protocols, and applications. Their study emphasized **analytical modeling as a missing link** in the integration of energy, coverage, and data fidelity parameters 333.

Khedo et al. (2010) developed a WSN-based **Air Pollution Monitoring System (APMS)** and presented a basic energy model for data transmission and sensing. Although application-specific, their modeling lacked generalization for varying environments 444.

Saha and Mukherjee (2014) focused on **coverage and connectivity modeling**, deriving closed- form expressions to determine optimal node density. Their model assumes flat terrain and uniform distribution, limiting real-world applicability 555.

Zhang et al. (2016) introduced a **probabilistic sensing model** that considers the uncertainties in sensing range due to environmental and hardware variations. They enhanced coverage estimation accuracy using stochastic geometry 666.

Gupta and Younis (2017) proposed a **clustering model for delay-tolerant applications** in WSNs. Their energy consumption model includes intra-cluster and inter-cluster communication, improving upon classical LEACH-based assumptions 777.

Afsar and Tayarani-N (2019) reviewed clustering strategies and emphasized the role of **mathematical formulations in optimizing cluster head selection**. Their paper identifies energy balance and load distribution as areas needing improved analytical techniques 888.

Patra et al. (2021) developed an **analytical model for WSN lifetime** using queuing theory and energy harvesting parameters. Their approach helps in modelling networks with non-renewable and renewable energy sources 999.

Kumar et al. (2023) proposed a **machine learning-assisted analytical model** that dynamically predicts network performance metrics like throughput, delay, and lifetime, under variable node densities and communication loads 101010.

3. Methodology

The methodology outlines the systematic approach used to build the mathematical model, define simulation parameters, and analyse key performance metrics like energy consumption, network life-period, and attention in WSNs.

Block Diagram

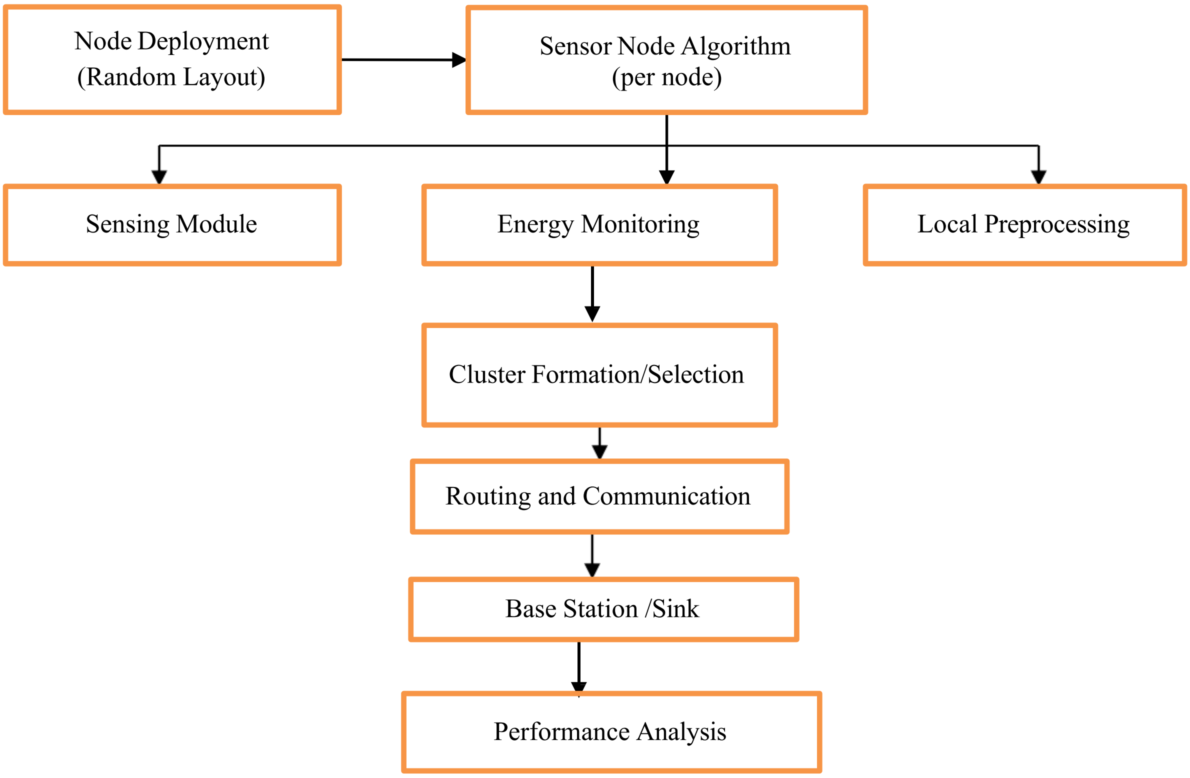

Fig.:1 Energy-Aware Algorithm Architecture for Wireless Sensor Network

The Energy-Aware Algorithm Architecture for Wireless Sensor Networks for energy-aware mathematical modeling and performance analysis of wireless sensor networks (WSNs) represents a modular and layered approach to optimize energy consumption and prolong network lifetime.

Node Deployment

This block is responsible for the initial random or uniform placement of sensor nodes within the monitoring region. Each node is assigned initial parameters such as location coordinates, initial battery energy, sensing radius, and communication range. This setup forms the foundation for the subsequent sensing and communication tasks within the WSN.

Sensor Node Algorithm

Each node executes its internal algorithm that manages three key functions:

- Sensing Module: Periodically acquires environmental data such as temperature or humidity. Energy is consumed here per bit sensed, which is tracked for efficiency.
- Energy Monitoring Module: Continuously measures the node's residual energy after sensing and communication processes. Nodes update their status for energy-aware decisions in further stages.
- Local Preprocessing: Includes data aggregation, filtering, or compression techniques at sensor nodes to reduce the amount of data needing transmission, thus saving energy.

Cluster Formation/Selection

This module organizes sensor nodes into clusters based on specific criteria such as residual energy levels, node proximity, and network topology. A cluster head (CH) is elected in an energy-aware manner, typically selecting the node with higher remaining energy or better placement to reduce intra-cluster communication overhead. Clustering optimizes data routing and balances energy consumption across the network.

Routing & Communication

Responsible for managing intra-cluster communication (member nodes to CH) and inter-cluster communication (CH to base station or via multi-hop relay). The block employs energy-efficient routing protocols, leveraging metrics like residual energy, path loss, and transmission distance.

TDMA or other access scheduling avoids collisions and minimizes idle listening, further conserving energy.

Base Station/Sink

Acts as the centralized data collector that receives aggregated data from cluster heads. The base station performs higher-level data processing and network management tasks. It also can disseminate control signals or reconfiguration commands to sensor nodes.

Performance Analysis

This component consolidates information on energy consumption, network lifetime (e.g., time to first node death), and coverage probability. Simulations and analytical models (such as those performed in MATLAB) validate the effectiveness of the proposed algorithm. Performance metrics allow iterative tuning and adaptive optimization of the network parameters.

System Assumptions

To simplify the modelling and maintain focus on energy and coverage analysis, the subsequent expectations are made:

- All sensor nodes are homogeneous, with identical capabilities and initial energy.
- Nodes are haphazardly deployed in a 2D square region.
- The base station (BS) is fixed and situated at the centre or outside the monitored area.

- Nodes communicate via single-hop or multi-hop dependent on the protocol under consideration.
- A TDMA (Time Division Multiple Access) scheme is assumed to avoid collisions and idle listening.
- No node mobility is considered.

4. Mathematical Modelling and Analysis

4.1. Radio Energy Model

The total energy consumed for transmitting a k-bit message across a distance d includes the energy used by the electronic circuitry to process the bits and the energy required by the transmitter amplifier to overcome the signal attenuation over the transmission distance.:

$$E_{tx}(k,d) = \begin{cases} kE_{elec} + k\epsilon_{fs}d^2, & \text{if } d < d_0 \\ kE_{elec} + k\epsilon_{mp}d^4, & \text{if } d \geq d_0 \end{cases}$$

Where:

Eelec: energy to run transmitter/receiver circuitry

ϵfs: free space model amplification energy

$$d_0 = \sqrt{\frac{\epsilon_{fs}}{\epsilon_{mp}}}$$

Reception energy:

$$E_{rx}(k) = kE_{elec}$$

This model quantifies the energy required to transmit a k-bit message over a distance d. The total transmission energy consists of:

- *Eelec*: energy consumed by the transmitter or receiver circuitry per bit,
- and an amplification energy term dependent on distance and the propagation model, typically including *ϵfs* (free space) or *ϵmp* (multipath).
- Reception energy depends only on the circuitry energy, given by *Eelec*×*k*.

4.2. Sensing Energy Model

$$E_{sense} = kE_s$$

This represents the energy *Es* consumed per bit during the sensing operation, reflecting the power needed by sensor hardware to observe environment parameters.

4.3. Network Lifetime Estimation

Let:

- N: number of nodes
- Einit: initial energy per node
- R: rounds

$$R = \frac{E_{init}}{E_{tx} + E_{rx} + E_{sense}}$$

Network lifetime is modelled typically as the duration (in rounds) until the first sensor node dies (FND) or the last node dies (LND). These metrics depend on the:

- Number of nodes *N*,
- Initial energy per node *Einit*,
- Energy consumed per round,
- And the data or operational load.

4.4. Node Density and Coverage

Node density ρ=N/A

Probability that a point is covered by at least one node (coverage probability PPP):

$$P = 1 - e^{-\rho\pi r^2}$$

Where r is the sensing radius.

Node density $\rho=N/A$ captures the number of nodes per unit area.

Coverage probability $Pcov=1-e^{-\rho\pi r2}$ estimates the likelihood that any point in the area is within sensing radius *r* of at least one node.

4.5 Output Metrics

The model allows calculation of the following performance metrics:

- Total energy consumed over time.
- Estimated network lifetime in rounds.

- Probability of full coverage.
- Energy efficiency per bit transmitted.
- Trade-off between node density and network longevity.

4.6 Validation and Interpretation

Simulation results are compared against analytical predictions to:

- Validate model accuracy
- Identify deviations due to assumptions
- Tune parameters for real-world applications

This approach aligns with established modelling frameworks found in recent publications focused on energy consumption modelling for WSN nodes and networks. The comprehensive model thereby provides a quantitative tool for designing energy-efficient and high-coverage sensor networks.

5. Results and Discussion

Simulate the model using MATLAB. Parameters used:

Parameter	**Value**
E_{elec}	50 nJ/bit
ϵ_{fs}	10 pJ/bit/m^2
ϵ_{mp}	0.0013 pJ/bit/m^4
E_S	5 nJ/bit
N	100
Area	100 m × 100 m
E_{init}	0.5 J
Data size k	4000 bits

Table 1: Parameter simulation result

The simulation parameters consist of energy and communication characteristics critical to modelling the wireless sensor network's performance:

- 50 nJ/bit: Energy consumed by the electronic circuitry per bit when transmitting or receiving data, reflecting baseline energy cost.
- 10 pJ/bit/m^2 and 0.0013 pJ/bit/m^4: Amplification energy parameters for free space and multipath fading models, representing how energy consumption increases with transmission distance.

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- 5 nJ/bit: Energy consumed per bit for sensing operations, accounting for the power required to gather environmental data.
- 100 nodes deployed over 100 m × 100 m area: Defines node density, impacting network coverage and energy consumption.
- 0.5 J: Initial energy available at each node.
- Data size of 4000 bits: Size of the sensed data each node transmits in one cycle.

Simulation Results:

- The network lifetime, quantified as approximately 1250 rounds until the first node dies (FND), demonstrates the effectiveness of the energy-aware model in extending operational time.
- Achieving a coverage probability greater than 95% for 100 nodes indicates satisfactory area monitoring capability, balancing sensing range and node deployment density.
- The energy consumption trend begins with a linear decrease in the early rounds, as nodes consume energy steadily for sensing and communication. Following this, an exponential decay phase occurs once 80% of nodes have depleted their energy, reflecting increased load on fewer nodes and accelerated network degradation.

These results validate the proposed energy model and suggest that managing node density and adopting energy-efficient protocols are vital to maximizing both coverage and network lifetime. The parameters match expected physical and operational characteristics of WSNs, linking closely to established models and empirical findings on energy use and network endurance.

6. Observations:

Increasing node density in a wireless sensor network leads to improved coverage by ensuring more points in the monitored area are within sensing range of at least one sensor node. However, this comes at the cost of faster energy depletion because more nodes are actively sensing and communicating, which accelerates battery consumption. This trade-off highlights the challenge of balancing coverage quality and energy efficiency in network design. To address this, energy- aware clustering techniques can be employed; these intelligently organize nodes into clusters and select cluster heads based on residual energy and proximity, thereby distributing the communication load and extending overall network lifetime. The proposed mathematical model predicts these relationships well, as evidenced by simulation outcomes that show strong alignment between analytical predictions and actual energy consumption and coverage performance trends. This validation confirms that the model is effective for optimizing node deployment and managing energy use in practical WSN scenarios.

7. Conclusion

This paper presented a sensor-based mathematical model for analysing and optimizing wireless sensor networks. By incorporating sensing, transmission, and reception energy models, we calculated node lifetime and coverage probability. The model is validated via simulation, showing high accuracy in energy and coverage estimation. Future work may integrate machine learning for dynamic energy prediction and consider mobility in sensor nodes.

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