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A HYBRID MULTI-RESPONSE OPTIMIZATION FOR THE BEST BIOFUEL BLEND SELECTION USING AHP–TOPSIS AND AHP-GRA METHODS

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Abstract

The selection of an appropriate biofuel blend is multi-criteria decision-making (MCDM) dilemma based on various qualitative and inconsistent criteria, which are crucial for determining the feasibility of new energy sources. This paper presents a hybrid methodology using the analytical hierarchy process (AHP) to compute the relative criteria weights, whereas the technique for order of preference by similarity to ideal solution (TOPSIS) and grey relational analysis (GRA) was used to rank the available alternatives. The results indicated that brake thermal efficiency (BTE) and nitric oxides (NOx) are the two most important criteria for rating the performance of a biofuel blend. The following preferences were attained for the blends by using the hybrid AHP–TOPSIS method: BD20CeO200 > BD100CeO200 > D > BD20 > BD100 and the hybrid AHP-GRA method: BD20CeO200 > D > BD100CeO200 > BD20 > BD100. Hence, after comparing both hybrid MCDM methods for various biofuel blends, the BD20 with Cerium oxide nanoparticles (200 ppm) was selected as the best biofuel blend for operating CI engines.

Keywords: MCDM, AHP, TOPSIS, GRA, Best blend

Major Findings: Following a comparative analysis using two hybrid multi-criteria decision-making (MCDM) methods—AHP-TOPSIS and AHP-GRA—BD20 blended with 200 ppm cerium oxide nanoparticles was identified as the best biofuel blend for use in compression ignition (CI) engines.

1. Introduction

Multi-criteria decision making (MCDM) combines mathematical and computational tools to determine the most appropriate alternative in terms of numerous performance criteria. Many studies have employed MCDM to solve complex and tedious problems in fields such as engineering, science, technology, and economics [1-11]. The selection of appropriate alternative fuel for IC engine is one such multi-response optimization problem. Contradictory objectives such as BTE, BSFC, HC, CO, NOx, etc have always been a research interest for optimization. The proposed AHP provided comparative weights of the decision criteria sets present and indicated their inter-relations, whereas TOPSIS and GRA were used to identify the optimal alternative. The advantages of the hybrid MCDM method include simplicity,

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consistency, lucidity, and good computational efficiency [2].

Determining the relative significance of different criteria for evaluating the best biofuel blend is a tedious task. Therefore, AHP is a very useful tool for capturing the variability in a decision. AHP is a subjective data-oriented MCDM method which combined the qualitative as well as quantitative criteria in the multi-response decision-making process. However, it does not explore the uncertain relationships and interdependency between the factors in a system. A hierarchical structure and pair-wise comparisons are used in AHP to prioritize the criteria according to their eigen values. The eigen value method is employed for the same which specifies that the eigen vector consequent to the largest eigen value of the pair-wise comparison matrix provides the comparative priorities of the factors and maintains ordinal preferences among the alternatives. Then AHP method assigns relative weights to each evaluation criterion based on the decision maker’s pairwise comparisons. Greater relative weights indicate the importance of the associated criterion. Thus AHP can be employed as a constructive method that integrating subjective human judgments with objective assessment through the Eigenvector method. This method ensures consistency in the decision maker's evaluations, minimizing biases in the decision-making process. In recent studies, the hybrid AHP has been proposed as a replacement for the classical AHP [3-5]. The hybrid method relies on the comprehensive role while selecting an appropriate solution from a finite set of options. The selected option is closer to the positive ideal solution and farthest from the negative ideal solution. TOPSIS, which was developed by Hwang and Yoon, is a conventional method for resolving an MCDM problem. In the TOPSIS method, both the performance ratings and the relative criteria weights are expressed as crisp values [5]. As in other MCDA tools, the characteristics and options should be fixed before the commencement of TOPSIS.

Grey relational analysis (GRA) was proposed by Deng in 1982, to systematically solve the complex unascertained problem containing uncertainty, vagueness and discrete data. GRA requires comparably less data resulted in less computational time as well as able to analyze the data variability through multi-responses, thus overcome the drawbacks of the other statistical method. Nowadays, such a method worldwide used in different research areas in the last decade. By considering the multi-response and uncertainty in the selection of best biofuel blend, an innovative hybrid AHP-GRA method used to attain the systematic and scientific results for decision-makers. Since different criteria may hold varying levels of significance, distinct relative weights should be applied while calculating the grey relational grades, rather than assigning equal weights to all criteria. Thus in this work, the AHP method is used to assign the relative criteria weights and the GRA method for ranking purposes, to convert the complex multi-response optimization into a single response grey relational grade.

Tzeng et al. used AHP to assigns the relative criteria weights while selecting the optimal alternative fuel. They also used the TOPSIS and VIKOR methods for comparison. Hybrid electric buses are the most suitable mode of public transport in the urban areas of Taiwan [6]. Sakthivel et al. used a hybrid MDCA, in which an analytic network process (ANP) was integrated with the TOPSIS and VIKOR methods to assess the optimal biodiesel blend. The

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BD20 fish oil biodiesel blend was found to be optimal for IC engines [7]. Ren and Liang used the fuzzy logarithmic least squares approach and fuzzy TOPSIS to determine the relative weights of subcriteria for evaluating the sustainability of marine fuels. In their study, hydrogen fuel was found to be the appropriate marine fuel for shipping [8].

Wu et al. proposed an integration of ANP and TOPSIS to evaluate and identify the most suitable marketing strategy. [9]. Choudhary et al. propose a framework combining fuzzy AHP and TOPSIS to evaluate and select the best locations for Thermal Power Plants in India, considering social, technical, economic, environmental, and political (STEEP) factors. Also, a sensitivity analysis was conducted to interpret and validate the results [10]. Prakash et al. focus on identifying and ranking the solutions of reverse logistics adoption in the electronics industry to overcome its barriers using the Fuzzy AHP-TOPSIS methodology. Finally, sensitivity analysis is performed to illustrate the robustness of the method [11]. Patil et al. put forward a stepwise scientific framework based on fuzzy AHP -TOPSIS to identify and rank the solutions of knowledge management adoption in supply chain using an empirical case study to overcome its barriers [12]. Pitchipoo et al. introduced the hybridization of AHP and GRA for systematically best supplier selection. They also evaluated the effectiveness of the integrated AHP-GRA model [13]. Mu Rui et al. developed a new hierarchy-based GRA, by utilizing AHP, HHOQ, and GRA methods for expediting the related decision-making process [14]. Ghorabaee et al. studied several publications related to multi-criteria decision-making techniques, including book chapters and papers from peer-reviewed journals and reputable conferences from 2001 to 2016; this review illustrated numerous major combinations of hybrid multi-criteria decision-making techniques, and the results revealed that AHP-TOPSIS was the most popular technique than AHP-GRA [15]. G Sunil Chaitanya et al. employed the AHP-TOPSIS method on various biodiesel blends and found that the B20 blend is the best option for low engine loads while B80 shows up as the best option at high engine loads [16]. Amit R. Patil et al validated composite fuel additives for CI engines using Taguchi, TOPSIS, GRA, and AHP optimization methods; all optimization methods yielded consistent results, all confirming the superiority of sample D8EH6E4 [17]. A. R. Patil et al address the stringent emission norms and the uncertainty of diesel vehicles; a novel multi-additive fuel blend (dimethyl carbonate, 2-ethylhexyl nitrate, ethyl acetate) was developed. Sixteen combinations were tested using Taguchi DOE and optimized via TOPSIS, identifying blend D8EH6E4. This blend achieved a 19% average NOx reduction and notable smoke reduction at high loads without compromising performance or fuel economy [18].

Very few studies have been conducted on selecting an appropriate biofuel blend based on the overall engine characteristics by using a hybrid MCDM method. Therefore, in this study, hybrid MCDM methodological tools such as AHP-TOPSIS and AHP-GRA were utilized to achieve better engine performance, combustion, and environmental benefits by decreasing harmful emissions. The objective of this study was to select the best biofuel blend for diesel engines using multi-response optimization of engine operating parameters. To achieve this aim, a hierarchy based hybrid models were proposed by considering eight sub-criteria and five alternatives.

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2. Proposed research methodology

The schematic of the proposed hybrid MCDM method is displayed in Fig.1. In the first stage, after the problem is defined and the goal is determined, the identification of the available alternatives and their evaluation criteria is a major task. Then, based on the identification, the decision hierarchy was framed, which percolates their interconnectivity as shown in Fig.2. The decision hierarchies generally have four basic levels, such as the objective of the problem, main criteria, sub-criteria, and the alternatives, which are situated at the level by level respectively. In the second stage, a diesel engine was operated at a constant speed (1500 rpm) with various biofuel blends at different loading conditions to obtain the experimental data. Then, the pair-wise comparison matrix was prepared by using Saaty's preference scale to calculate the relative weights of the sub-criteria [19]. An unbiased weight is assigned for all decisive factors of the hierarchy, permitting varied and incommensurable criteria to be evaluated with each other realistically and consistently, which differentiates AHP from other similar methods, such as personal experience and linear-weighting models. In this work, the author has used the literature survey and experts' opinions to identify the evaluation criteria and their relative weights for selection of best blend. Finally, the TOPSIS, as well as GRA ranking methods, were used to rank the available alternatives based on the experimentally measured data and relative weights of the evaluation criteria [20,21].

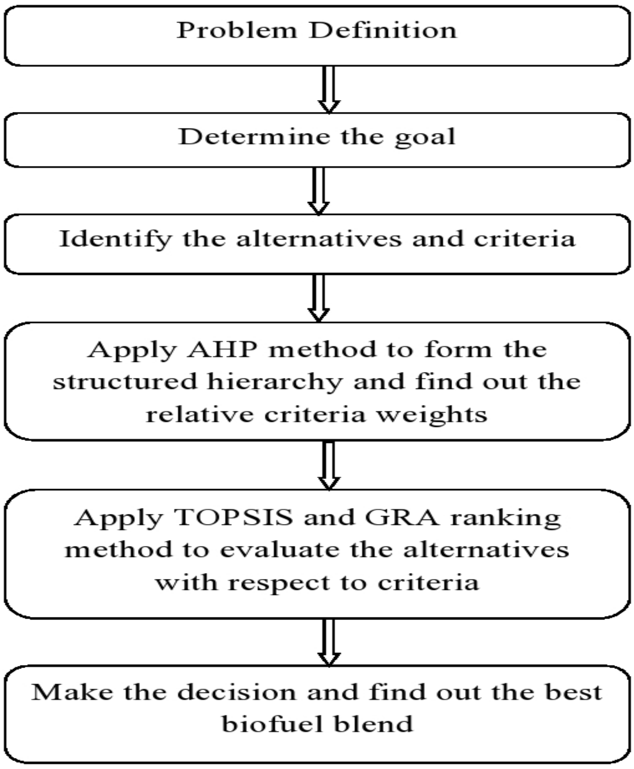

Fig. 1 Steps of the proposed hybrid MCDM methods

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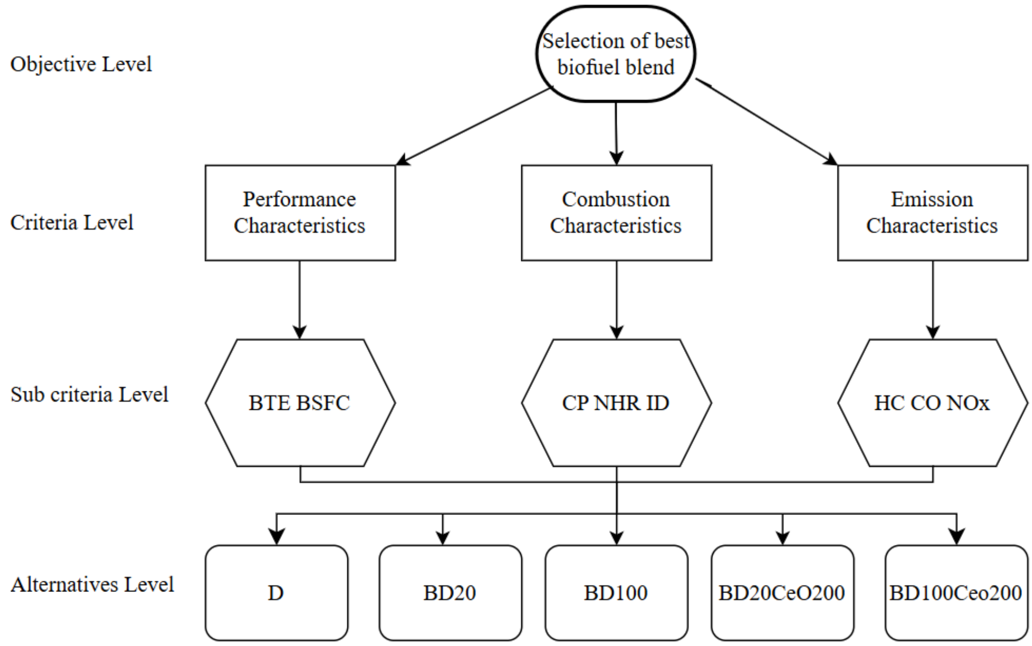

Fig. 2 Decision hierarchy for the proposed AHP model

2.1 The stepwise procedure of the AHP model

Step - 1

Structuring the decision problem as a hierarchy of goals, criteria, sub-criteria, and alternatives.

Step - 2

Prepare a pair-wise comparison matrix and establish priorities among the elements in the hierarchy using Saaty's nine-point preference scale.

Let X be an *n x n* pair-wise comparison matrix:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix}$$

Here, all diagonal elements are equal to 1.

Step - 3

Normalize the matrix as follows:

$$W_j = \left[\frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \right]^T, \quad j = 1, 2, \ldots, n$$

Step - 4

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Calculate the weighted average rating for each decision alternative by dividing the weighted sum vector by the criterion weight. Choose the one with the highest score.

Step - 5

Calculate the consistency index (CI):

$$\boxed{CI = \frac{(\lambda_{\max} - n)}{n - 1}}$$

Where:

- λmax = maximum eigenvalue of the matrix
- The lower value of CI indicates a smaller deviation from consistency.

Step - 6

Determine the consistency ratio (CR):

$$\boxed{CR = \frac{CI}{RI}}$$

Where:

- RI = Random Index, which depends on the matrix size

For consistency, the CR value should be less than or equal to 0.1.

2.2 The stepwise procedure of the TOPSIS model

Step 1: Construct the Decision Matrix

- List all alternatives and criteria.
- Construct a matrix showing the performance of each alternative with respect to each criterion.

Step 2: Normalize the Decision Matrix

- Use the normalization formula:

$$\boxed{r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}}$$

Where Xij is the value of the i^{th} alternative with respect to the j^{th} criterion.

Step 3: Construct the Weighted Normalized Decision Matrix

- Multiply each element of the normalized matrix by the corresponding criterion weight:

$$\boxed{v_{ij} = w_j \cdot r_{ij}}$$

Where Wj is the weight of the j^{th} criterion.

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Step 4: Determine the Positive Ideal and Negative Ideal Solutions

- Positive Ideal Solution (PIS):

$$\boxed{A^{+} = \{\max(v_{ij}) | j \in J_{benefit}; \min(v_{ij}) | j \in J_{cost}\}}$$

- Negative Ideal Solution (NIS):

$$\boxed{A^{-} = \{\min(v_{ij}) | j \in J_{benefit}; \max(v_{ij}) | j \in J_{cost}\}}$$

Step 5: Calculate the Separation Measures

- Separation from Positive Ideal Solution:

$$\boxed{S_i^{+} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^{+})^2}}$$

- Separation from Negative Ideal Solution:

$$\boxed{S_i^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^{-})^2}}$$

Step 6: Calculate the Relative Closeness to the Ideal Solution

$$\boxed{C_i^{*} = \frac{S_i^{-}}{S_i^{+} + S_i^{-}}}$$

Where Ci* is the relative closeness of the i^{th} alternative to the ideal solution.

Step 7: Rank the Alternatives

- Rank the alternatives in descending order of Ci*.
- The alternative with the highest value Ci* is considered the best.

2.3 The stepwise procedure of the GRA model

Step 1: Data Normalization (Grey Relational Generation)

Normalize the decision matrix to ensure comparability among different criteria. Depending on the nature of the criteria, different normalization methods are applied [22]:

- Higher-the-better:

$$\boxed{x_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})}}$$

- Lower-the-better:

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$$x_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})}$$

- Nominal-the-best:

$$x_{ij} = 1 - \frac{|x_{ij} - x_j^*|}{\max(|x_{ij} - x_j^*|)}$$

Where xj* is the target value for criterion j.

Step 2: Reference Sequence Determination

Identify the ideal (reference) sequence, which consists of the best performance values for each criterion after normalization. This sequence serves as a benchmark for comparison [23].

Step 3: Grey Relational Coefficient Calculation

Compute the Grey Relational Coefficient (GRC) to express the relationship between the reference sequence and each comparability sequence:

$$\xi_{ij} = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{ij} + \zeta \cdot \Delta_{\max}}$$

Where:

- $\Delta ij = |x_{0j} - x_{ij}|$
- $\Delta min = \min \Delta ij$
- $\Delta max = \max \Delta ij$
- ζ (zeta) is the distinguishing coefficient, typically set to 0.5.

Step 4: Grey Relational Grade Calculation

Calculate the Grey Relational Grade (GRG) for each alternative by averaging the GRCs across all criteria:

$$\gamma_i = \frac{1}{n} \sum_{j=1}^{n} \xi_{ij}$$

Where:

- γi is the GRG for alternative i
- n is the number of criteria

Step 5: Ranking of Alternatives

Rank the alternatives based on their GRG values. A higher GRG indicates a closer relationship to the ideal sequence, signifying better performance.

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3. Results and discussions

Find out the best biofuel blend plays a critical role in biofuel usage in diesel engines. Researchers have found it difficult to recommend the most appropriate blend among the various proportions of biofuel blends because these blends have similar performance and emission characteristics, thus producing an inconsistency in attaining the stringent emission norms and overall engine performance without any modifications. To eradicate this problem, the hybrid AHP–TOPSIS and AHP-GRA models were used to evaluate and rank the available biofuel blends. In this study, three criteria, eight sub-criteria, and five options were considered based on the available literature and expert opinions to create a performance-based model for determining the best biofuel blend.

By using AHP, a pair-wise comparison matrix was prepared and the relative criteria weights were obtained (Tables 1–3). To determine the reliability of the matrix, significant consistency measures such as the maximum eigen value (λmax = 8.1470), random index (RI = 1.41 was taken for 8 number of parameters as per Saaty's scale), consistency index (= 0.021), and consistency ratio (CR) were calculated (CR = 0.01415). Based on a study by Saaty, if CR is less than 0.1, the matrix is consistent [16]. In this study, the CR value was 0.01415, which indicated that the normalized weighted estimated matrix for numerous criteria was consistent. The result of AHP indicated that the relative significance of each sub-criteria was different, which ultimately helps to minimize the subjectivity and randomness. As presented in Table 4, two sub-criteria, namely, the brake thermal efficiency and nitrogen oxides attained the highest priority weights of 0.2569 and 0.2291, respectively. These values indicated their relative impact while ranking the best biofuel blend.

Table 1 Pair wise comparison matrix for criteria

	BTE	BSFC	HC	CO	NO_X	Peak Pressure	ID	NHR max
BTE	1	3	5	5	1	3	3	2
BSFC	0.33	1	1	1	0.33	1	1	0.5
HC	0.2	1	1	1	0.33	0.5	0.5	0.33
CO	0.2	1	1	1	0.33	0.5	0.5	0.33
NO_X	1	3	3	3	1	3	3	2
Peak Pressure	0.33	1	2	2	0.33	1	2	0.5
ID	0.33	1	2	2	0.33	0.5	1	0.5
NHR max	0.5	2	3	3	0.5	2	2	1

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Table 2 Matrix for relative criteria weights

	BTE	BSFC	HC	CO	NO_X	Peak Pressure	ID	NHR max	Sum
BTE	0.2570	0.2307	0.2777	0.2777	0.2409	0.2608	0.2307	0.2793	0.2570
BSFC	0.0848	0.0769	0.0555	0.0555	0.0795	0.0869	0.0769	0.0698	0.0848
HC	0.0514	0.0769	0.0555	0.0555	0.0795	0.0434	0.0384	0.0460	0.0514
CO	0.0514	0.0769	0.0555	0.0555	0.0795	0.0438	0.0384	0.0460	0.0514
NO_X	0.2570	0.2307	0.1666	0.1666	0.2409	0.2608	0.2307	0.2793	0.2570
Peak Pressure	0.0848	0.0769	0.1111	0.1111	0.0795	0.0869	0.1538	0.0698	0.0848
ID	0.0848	0.0769	0.1111	0.1111	0.0795	0.0434	0.0769	0.0698	0.0848
NHR Max	0.1285	0.15385	0.16667	0.16667	0.12048	0.17391	0.15385	0.13966	0.1285

Table 3 Results obtained with the AHP

Sr. No.	Criteria	Weights	λ max, CI, RCI	CR
1	BTE	0.2569		
2	BSFC	0.0733		
3	HC	0.0559		
4	CO	0.0559	λ max = 8.1470 CI = 0.021 RCI = 1.41	CR = 0.01415
5	NO_X	0.2291		
6	Peak Pressure	0.0968		
7	ID	0.0817		
8	NHR	0.1505		

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Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	28.25	0.261	50	0.14	659	73.1	7.3	69.44
BD20	100	27.19	0.28	47	0.135	679	71.92	5.65	66.1
BD100	100	26.88	0.29	43	0.13	748	71.14	5.5	64.45
BD20 CeO200	100	28.38	0.225	39	0.12	639	73.68	5	67
BD100 CeO200	100	27.89	0.23	36	0.12	715	72.89	4.9	65.51

max

The experimental readings at 100% load were used to demonstrate the computational procedure of the proposed hybrid AHP–TOPSIS and AHP-GRA analysis, as shown in Table 4. After computing the relative weights of numerous sub-criteria, the TOPSIS method was used for evaluating the available options, based on the one closest to the positive ideal solution (PIS) while maintaining the greatest possible distance from the negative ideal solution (NIS). The normalization of a decision matrix and a weighted normalized decision matrix is presented in Tables 5 and 6. The closeness coefficient factor was considered for determining the ranking order of the available alternatives. The alternative BD20CeO200 achieved the highest rank among the available alternatives and had a closeness coefficient of 0.8920. Moreover, the alternative BD100 achieved the lowest rank among the available alternatives and had a closeness coefficient of 0.3721. The preference obtained for the blends by using the hybrid AHP–TOPSIS method was as follows: BD20CeO200 > BD100CeO200 > BD20 > D > BD100. The closeness coefficient of all the available alternatives for the various blends is presented in Table 7.

Table 4. Experimental readings observed from the engine for various alternative blends

Table 5. Normalized decision matrix

Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	0.455	0.451	0.516	0.484	0.427	0.450	0.569	0.466
BD20	100	0.438	0.484	0.484	0.467	0.440	0.443	0.440	0.444
BD100	100	0.433	0.501	0.444	0.449	0.485	0.438	0.428	0.433
BD20CeO 200	100	0.457	0.389	0.402	0.415	0.414	0.454	0.389	0.450
BD100CeO	100	0.449	0.397	0.371	0.415	0.464	0.449	0.382	0.440

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Table 6. Weighted normalized decision matrix

Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	0.117	0.029	0.032	0.030	0.089	0.052	0.029	0.083
BD20	100	0.113	0.031	0.030	0.029	0.092	0.052	0.022	0.079
BD100	100	0.111	0.033	0.028	0.028	0.101	0.051	0.022	0.077
BD20CeO 200	100	0.118	0.025	0.025	0.026	0.087	0.053	0.020	0.080
BD100CeO 200	100	0.116	0.026	0.023	0.026	0.097	0.052	0.019	0.078

Table 7. Positive and Negative Ideal Solutions, Ideal Closeness and Ranking

Blends	% Load	SI+	SI-	CI	RANK
Diesel	100	0.01855	0.01574	0.45901	4
BD20	100	0.01387	0.01503	0.52014	3
BD100	100	0.02078	0.01231	0.3721	5
BD20CeO200	100	0.00308	0.02546	0.89204	1
BD100CeO200	100	0.01218	0.02039	0.6261	2

The GRA methodology involves numerous stages, such as normalization of the experimental data set to eradicate the effect of different units and variability as shown in Table 8, then computation of the deviation sequence as shown in Table 9, and grey relational coefficient for all criteria as shown in Table 10, and finally utilizing the relative criteria weights for determining the grey relational grade for all alternatives and ranking accordingly, as shown in Table 11. The following preferences were attained for the blends by using the hybrid AHP-GRA method: BD20CeO200 > D > BD100CeO200 > BD20 > BD100. The higher value of the grey relational grade implies superior performance. This result is employed to optimize the multiple responses by consolidating them into a single grade.

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Table 8. Normalized decision matrix

Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	0.913	0.446	0.000	0.000	0.817	0.772	0.000	1.000
BD20	100	0.207	0.154	0.214	0.250	0.633	0.307	0.688	0.331
BD100	100	0.000	0.000	0.500	0.500	0.000	0.000	0.750	0.000
BD20 CeO 200	100	1.000	1.000	0.786	1.000	1.000	1.000	0.958	0.511
BD100 CeO 200	100	0.673	0.923	1.000	1.000	0.303	0.689	1.000	0.212

Table 9. Deviation sequences of each alternative

Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	0.09	0.55	1.00	1.00	0.18	0.23	1.00	0.00
BD20	100	0.79	0.85	0.79	0.75	0.37	0.69	0.31	0.67
BD100	100	1.00	1.00	0.50	0.50	1.00	1.00	0.25	1.00
BD20 CeO200	100	0.00	0.00	0.21	0.00	0.00	0.00	0.04	0.49
BD100 CeO200	100	0.33	0.08	0.00	0.00	0.70	0.31	0.00	0.79

Table 10. Grey relation coefficient of each alternative

Blends	% Load	BTE	BSFC	HC	CO	NOx	Peak Pressure	ID	NHR max
Diesel	100	0.852	0.474	0.333	0.333	0.732	0.686	0.333	1.000
BD20	100	0.387	0.371	0.389	0.400	0.577	0.419	0.615	0.428
BD100	100	0.333	0.333	0.500	0.500	0.333	0.333	0.667	0.333
BD20 CeO200	100	1.000	1.000	0.700	1.000	1.000	1.000	0.923	0.506
BD100 CeO200	100	0.605	0.867	1.000	1.000	0.418	0.617	1.000	0.388

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Table 11. Grade and rank of the grey relation of each alternative

Blends	% Load	GRG	RANK
Diesel	100	0.71763	2
BD20	100	0.44876	4
BD100	100	0.371	5
BD20CeO200	100	0.88941	1
BD100CeO200	100	0.6151	3

The consistency between the AHP-TOPSIS and AHP-GRA methods highlights their reliability, with minor deviations in rankings due to variations in their underlying methodologies. Although both the hybrid MCDM methods give the first ranking to BD20ZnO200, the AHP-TOPSIS has the following limitations over the AHP-GRA. It requires more computational timing due to its lengthy calculations to obtain the positive ideal and negative ideal solutions of the criteria and separation measures of the alternatives. Thus, the application of the hybrid AHP-GRA method is providing more valuable assistance for multi-response optimization problems like best blend selection.

4. Conclusion

The systematic evaluation using exploratory methodological tools are the basis for the subsequent selection of the best biofuel blend for diesel engines. In this study, a multifaceted perspective of various engine characteristics was used while selecting the best biofuel blend. This paper presents a hybrid methodology using AHP to compute the relative criteria weights, whereas TOPSIS and GRA were used to rank the available alternatives. The results indicated that the BTE and NOx are the two most crucial criteria for rating the performance of biofuel blends. The preferences were attained for the blends by using the hybrid AHP–TOPSIS method: BD20CeO200 > BD100CeO200 > BD20 > D > BD100 and the hybrid AHP-GRA method: BD20CeO200 > D > BD100CeO200 > BD20 > BD100. Hence, after comparing the various biofuel blends, the BD20 with cerium oxide nanoparticles (200 ppm) was selected as the best biofuel blend for operating CI engines. The proposed hybrid decision method enables researchers to gain new insight to recognize the most appropriate blend for enhancing the energy competency of engines by quantitatively and qualitatively studying the problem. It also offers sophisticated and systematic statistical tools that lean towards the decision makers' support to solve complex engineering problems. For future research work, the results of the present study can be evaluated with those of other multi-criteria techniques like ELECTRE, PROMETHEE, or VIKOR.

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