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THE NEUTROSOPHIC MINIMUM SPANNING TREE PROBLEM IN CONNECTED UNDIRECTED WEIGHTED COMPLETE STRONG NEUTROSOPHIC GRAPHS

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Abstract

N = (S, R) is a Single Valued Neutrosophic Graph (SVNG) andCSN = (S, R) is aComplete StrongNeutrosophic Graph (CSNG).A complete strongneutrosophic graph provides neutrosophic evaluation on both vertices and edges, making it ideal for modeling highly uncertain, complex and inconsistent environments. In this research study, determining theneutrosophic minimum spanning tree (NMST) is introduced in anconnected undirected weighted complete strong neutrosophic graph (CUWCSNG) using Kruskal's Algorithm. Illustrativeexample is given to explain the proposed algorithm.

Keywords: Single-valued neutrosophic graph, Complete strong neutrosophic graph, Minimum spanning tree.

1. Introduction

The concept of Neutrosophic set was introduced by F. Smarandache [1] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The goal of the MST problem [2] is to identify the minimum-weight spanning tree in a connected, weighted graph.Traditionally, MST problems assume that arc lengths (or weights) are precise and fixed. However, in practical applications, arc lengths often correspond to uncertain parameters such as distance, time, traffic conditions, or capacity. Over the past several decades, the MST problem has attracted significant attention from researchers.

This paper is organized as follows.

- In Section 2, we discuss the fundamental concepts related to single valued neutrosophic graph, complete single valued neutrosophic graph,strong single valued neutrosophicgraph, complete strong single valued neutrosophic graph and score function.
- In Section 3, we propose an algorithm to determine theshortest path and its length in a connected, undirected,weighted, complete strong neutrosophicgraph.

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- In section 4 an illustrative example is presented to demonstrate the process of findingthe shortest path and shortest distance between the sourceand destination nodes.
- Section 5summarizes the key findings and concludes the paper.

2. PRELIMINARIES

Defnition 2.1.[3] A Single Valued Neutrosophic Graph (SVNG) is of the form N = (S, R) where S: $P \rightarrow [0,1]$ is a neutrosophic set in P and R: $P \times P \rightarrow [0,1]$ is a neutrosophic relation on P such that

1. The functions $T_S: P \rightarrow [0,1]$, $I_S : P \rightarrow [0,1]$ and $F_S : P \rightarrow [0,1]$ denote the degree of membership, degree of indeterminacy and non-membership of the element $p_w \varepsilon P$; respectively, and $0 \le T_S(p_w) + I_S(p_w) + F_S(p_w) \le 3$ for every $p_w \varepsilon P$, (w = 1, 2,………, n)

2. $E \subseteq P \times P$ where $T_R : P \times P \rightarrow [0, 1]$, $I_R : P \times P \rightarrow [0, 1]$ and $F_R : P \times P \rightarrow [0, 1]$ are such that

$T_R(p_w, p_z) \le \min \{T_S(p_w), T_S(p_z)\}$,

$I_R(p_w, p_z) \le \min \{I_S(p_w), I_S(p_z)\}$,

$F_R(p_w, p_z) \le \max \{F_S(p_w), F_S(p_z)\}$ and

$0 \le T_R(p_w, p_z) + I_R(p_w, p_z) + F_R(p_w, p_z) \le 3$ for every $(p_w, p_z) \varepsilon E$ (w, z = 1, 2, ………, n).

Definition 2.2. [3]A SVNG N = (S,R) is termed as complete provided that the following conditions are met:

$T_R(p_w, p_z) = \min \{T_S(p_w), T_S(p_z)\}$,

$I_R(p_w, p_z) = \min \{I_S(p_w), I_S(p_z)\}$,

$F_R(p_w, p_z) = \max \{F_S(p_w), F_S(p_z)\}$ forall p_w, p_z in P.

Definition 2.3. :A SVNG N = (S, R) is known as strong neutrosophic graph provided that the following conditions are met:

$T_R(p_w, p_z) = \min \{T_S(p_w), T_S(p_z)\}$,

$I_R(p_w, p_z) = \min \{I_S(p_w), I_S(p_z)\}$,

$F_R(p_w, p_z) = \max \{F_S(p_w), F_S(p_z)\}$ for all (p_w, p_z) in E.

Definition 2.4. A SVNG N = (S, R) is known as complete strong neutrosophic graph provided that the following conditions are met:

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$T_R(p_w, p_z) = \min\{T_S(p_w), T_S(p_z)\}$,

$I_R(p_w, p_z) = \min\{I_S(p_w), I_S(p_z)\}$,

$F_R(p_w, p_z) = \max\{F_S(p_w), F_S(p_z)\}$ for all p_w, p_z in P and for all (p_w, p_z) in E.

Definition 2.5. [4] : Let N = (S, R) be a SVNG. Then a score function is defined as follows

SF= $\dfrac{2+T-I-F}{3}$

HereT, I and F represent the degree of truth membership value, indeterminacy membership value and falsity membership values of S.

3. PROPOSED ALGORITHM

Theorem 3.1 Every connected complete strong neutrosophic graph contains a spanning tree.

Proof:A spanning tree of a graph is a subgraph that is a tree and includes all the vertices of the original graph.

A CSNG is a graph where every pair of distinct vertices is connected by a strong neutrosophic edge. Since the graph is connected, there exists a path between every pair of vertices.

Given a connected CSNG, we can construct a spanning tree by selecting a subset of edges that connect all vertices without forming any cycles.

When selecting edges for the spanning tree, we can prioritize edges with high truth membership values (T) and low indeterminacy membership values (I) and falsity membership values (F). This ensures that the spanning tree is composed of strong neutrosophic edges.

Since a connected CSNG has a path between every pair of vertices, and we can select a subset of strong neutrosophic edges to form a tree that spans all vertices, we can conclude that every connected CSNG contains a spanning tree.

Proposed Kruskal's Algorithm:

Input

- A connected, undirected, weighted complete strong neutrosophic graphCSN = (S, R)
- Neutrosophic edge weights represented using neutrosophic numbers
- A function to compute score values of neutrosophic numbers

Output

- NMST, a neutrosophicminimum spanning tree

Steps to achieve the output

Step 1: Draw the connected, undirected, weighted complete strong neutrosophic graph with n vertices.

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Step 2: Compute the neutrosophic numbers corresponding to the neutrosophic edge weights.

Step 3: Calculate the score function of each edge of the graph and tabulate the same.

Step 4: Start with a small weighted edge, proceed sequentially by selecting one edge at a time such thatno cyclic is formed.

Step 5: Stop the process of step 4 when $n - 1$ edges are selected.

These $n - 1$ edges make a neutrosophicminimum spanning tree of the connected undirected weighted complete strong neutrosophic graph.

4. Illustrativeexample

The following graph is the connected undirected weighted complete strong neutrosophic graphCUWCSNG with vertices as Navagraha Temples in Tamil Nadu.

Temples are considered as the vertices of the graph as follows:

P1 - SuryanarKovil (Surya)

P2 - Kailasanathar Temple, Thingalur (Chandra)

P3 - VaitheeswaranKovil (Mars)

P4 - Swetharanyeswarar Temple, Thiruvenkadu (Budha)

P5 - Apatsahayesvarar Temple, Alangudi (Guru)

P6 - Agniswarar Temple, Kanjanur (Shukra)

P7 - TirunallarSaneeswaran Temple (Shani)

P8 - Naganathaswamy Temple, Thirunageswaram (Rahu)

P9 - KethuSthalam, Keezhperumpallam (Ketu)

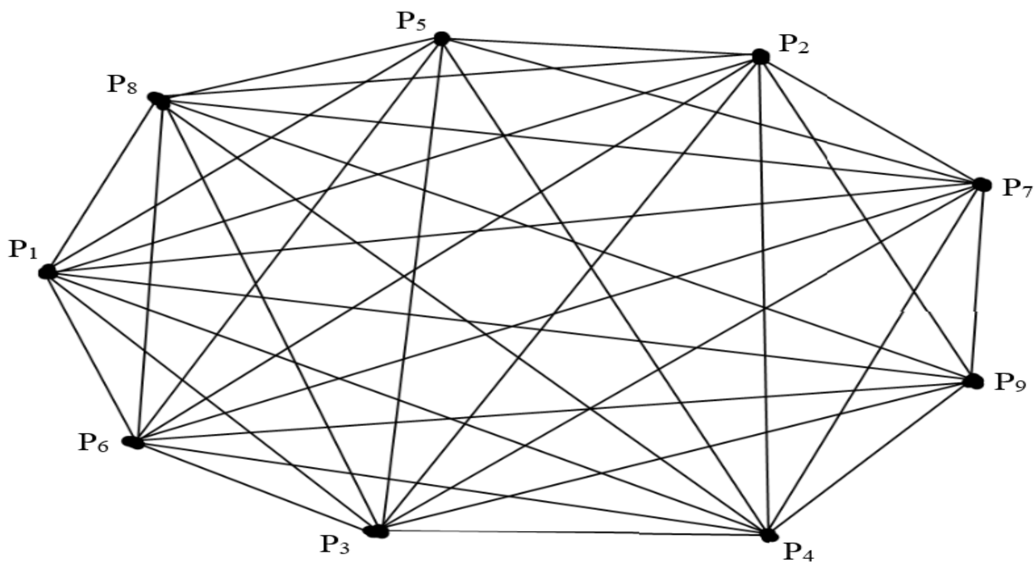

Fig. 1. Complete Strong Neutrosophic Graph of Navagraha Temples

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Table 1. Navagraha Temples Distance (in km from Google maps)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
P_1	0	45	36	46	31	4	45	11	46
P_2	45	0	81	92	43	48	86	40	92
P_3	36	81	0	14	66	34	39	45	20
P_4	46	92	14	0	76	44	38	56	8
P_5	31	43	66	76	0	34	64	20	76
P_6	4	48	34	44	34	0	45	14	44
P_7	45	86	39	38	64	45	0	46	32
P_8	11	40	45	56	20	14	46	0	55
P_9	46	92	20	8	76	44	32	55	0

Table 2. Navagraha Temples Distance asNeutrosophic Number

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
P_1	(1, 0, 0)	(0.511, 0.05, 0.489)	(0.609, 0.05, 0.391)	(0.5, 0.05, 0.5)	(0.663, 0.05, 0.337)	(0.957, 0.05, 0.043)	(0.511, 0.05, 0.489)	(0.88, 0.05, 0.12)	(0.5, 0.05, 0.5)
P_2	(0.511, 0.05, 0.489)	(1, 0, 0)	(0.12, 0.05, 0.88)	(0, 0.05, 1)	(0.533, 0.05, 0.467)	(0.478, 0.05, 0.522)	(0.065, 0.05, 0.935)	(0.565, 0.05, 0.435)	(0, 0.05, 1)
P_3	(0.609, 0.05, 0.391)	(0.12, 0.05, 0.88)	(1, 0, 0)	(0.848, 0.05, 0.152)	(0.283, 0.05, 0.717)	(0.63, 0.05, 0.37)	(0.576, 0.05, 0.424)	(0.511, 0.05, 0.489)	(0.783, 0.05, 0.217)
P_4	(0.5, 0.05, 0.5)	(0, 0.05, 1)	(0.848, 0.05, 0.152)	(1, 0, 0)	(0.174, 0.05, 0.826)	(0.522, 0.05, 0.478)	(0.587, 0.05, 0.413)	(0.391, 0.05, 0.609)	(0.913, 0.05, 0.087)

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P_5	(0.663, 0.05, 0.337)	(0.533, 0.05, 0.467)	(0.283, 0.05, 0.717)	(0.174, 0.05, 0.826)	(1, 0, 0)	(0.63, 0.05, 0.37)	(0.304, 0.05, 0.696)	(0.783, 0.05, 0.217)	(0.174, 0.05, 0.826)
P_6	(0.957, 0.05, 0.043)	(0.478, 0.05, 0.522)	(0.63, 0.05, 0.37)	(0.522, 0.05, 0.478)	(0.63, 0.05, 0.37)	(1, 0, 0)	(0.511, 0.05, 0.489)	(0.848, 0.05, 0.152)	(0.522, 0.05, 0.478)
P_7	(0.511, 0.05, 0.489)	(0.065, 0.05, 0.935)	(0.576, 0.05, 0.424)	(0.587, 0.05, 0.413)	(0.304, 0.05, 0.696)	(0.511, 0.05, 0.489)	(1, 0, 0)	(0.5, 0.05, 0.5)	(0.652, 0.05, 0.348)
P_8	(0.88, 0.05, 0.12)	(0.565, 0.05, 0.435)	(0.511, 0.05, 0.489)	(0.391, 0.05, 0.609)	(0.783, 0.05, 0.217)	(0.848, 0.05, 0.152)	(0.5, 0.05, 0.5)	(1, 0, 0)	(0.402, 0.05, 0.598)
P_9	(0.5, 0.05, 0.5)	(0, 0.05, 1)	(0.783, 0.05, 0.217)	(0.913, 0.05, 0.087)	(0.174, 0.05, 0.826)	(0.522, 0.05, 0.478)	(0.652, 0.05, 0.348)	(0.402, 0.05, 0.598)	(1, 0, 0)

Table 3. Navagraha Temples Distance Score Function(SF= $\frac{2+T-I-F}{3}$)

	P_1	**P_2**	**P_3**	**P_4**	**P_5**	**P_6**	**P_7**	**P_8**	**P_9**
P_1	1	0.657	0.723	0.65	0.759	0.955	0.657	0.903	0.65
P_2	0.657	1	0.397	0.317	0.672	0.635	0.36	0.693	0.317
P_3	0.723	0.397	1	0.882	0.505	0.737	0.701	0.657	0.839
P_4	0.65	0.317	0.882	1	0.433	0.665	0.708	0.577	0.925
P_5	0.759	0.672	0.505	0.433	1	0.737	0.519	0.839	0.433
P_6	0.955	0.635	0.737	0.665	0.737	1	0.657	0.882	0.665
P_7	0.657	0.36	0.701	0.708	0.519	0.657	1	0.65	0.751
P_8	0.903	0.693	0.657	0.577	0.839	0.882	0.65	1	0.585
P_9	0.65	0.317	0.839	0.925	0.433	0.665	0.751	0.585	1

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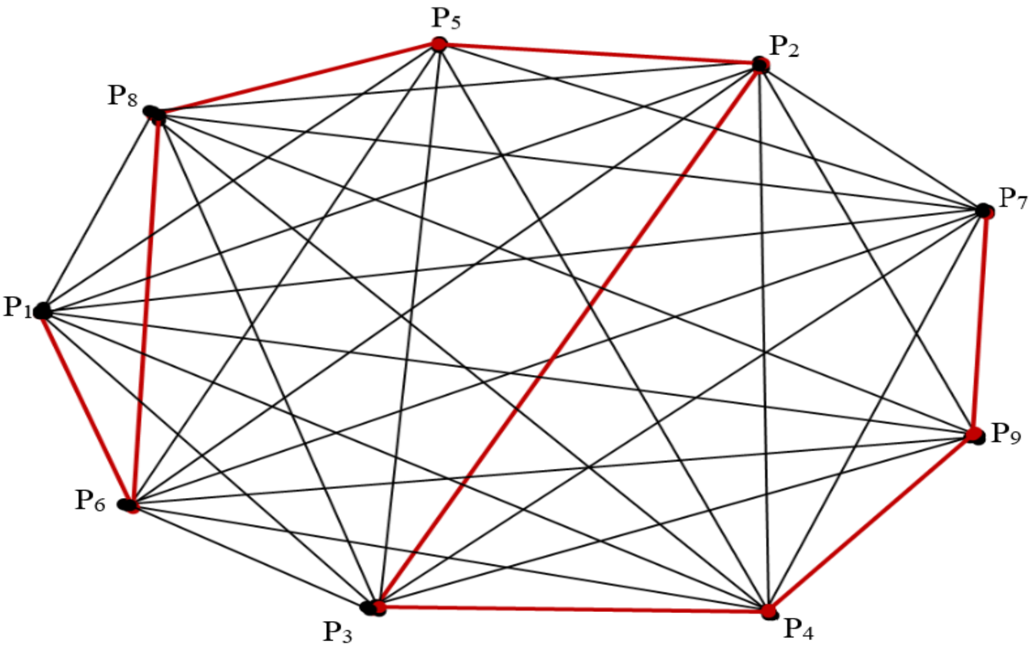

Fig. 2. Complete Strong Neutrosophic Graph of Navagraha Temples with NMST

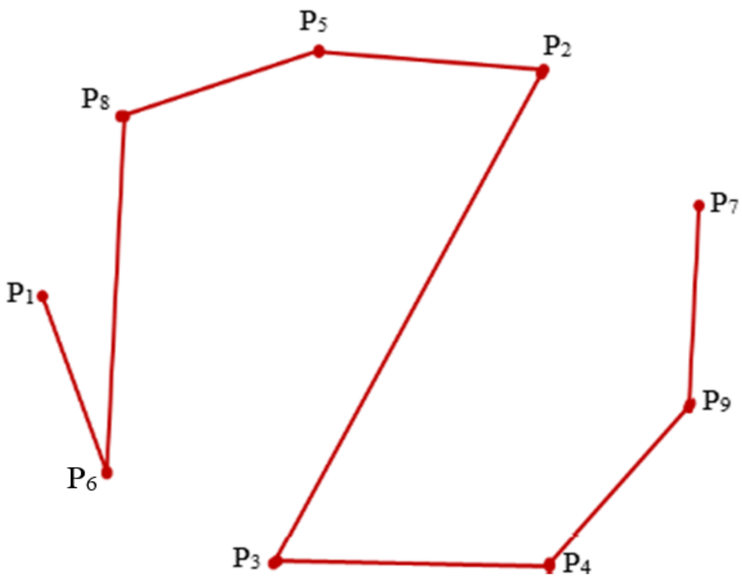

Fig. 3. NeutrosophicMinimum Spanning Tree of Navagraha Temples

Table 4. NeutrosophicMinimum Spanning TreeofNavagraha Temples

Edge	Distance (km)	(T, I, F)	Score Function
SuriyanarP_1 – KanjanurP_6	4	(0.957, 0.05, 0.043)	0.955
KanjanurP_6 –ThirunageswaramP_8	14	(0.848, 0.05, 0.152)	0.982

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ThirunageswaramP_8 –AlangudiP_5	20	(0.783, 0.05, 0.217)	0.839
AlangudiP_5 – ThingalurP_2	43	(0.533, 0.05, 0.467)	0.672
ThingalurP_2 – VaitheeswaranKoilP_3	81	(0.12, 0.05, 0.88)	0.397
VaitheeswaranKoilP_3 –ThiruvengaduP_4	14	(0.848, 0.05, 0.152)	0.882
ThiruvengaduP_4 – KeezhperumpallamP_9	8	(0.913, 0.05, 0.087)	0.925
KeezhperumpallamP_9 –ThirunallarP_7	32	(0.652, 0.05, 0.348)	0.751
Total	**216**	**-**	**6.3**

Hence the NeutrosophicMinimum Spanning Tree is $P_1 - P_6 - P_8 - P_5 - P_2 - P_3 - P_4 - P_9 - P_7$ with score function 6.3 and distance 216km.

5. Conclution

This study proposes an algorithm for identifying the shortest path and its length between a source and destination node with edge weights expressed as neutrosophic numbers. The methodology has been elaborated, and its application is validated with an illustrative example.

References

[1] F. Smarandache (editor), Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics, University of New Mexico, Gallup, NM 87301, USA (2002).

[2] Graham, R.L., Hell, P.: 'On the history of the minimum spanning tree problem',Ann. Hist. Comput., 1985, 7, (1), pp. 43–57

[3] Muhammad Akram and GulfamShahzadi, Operations on Single Valued NeutrosophicGraphs , Journal of uncertain systems vol.11, No.1 , pp 1-26, 2017.

[4] Zhang H.Y, Wang J.Q, Chen X.H.(2014)"Interval neutrosophic sets and their application in Multi criteria decision making problems." Sci. World J. 2014, 645953.

[5] Broumi S, Bakali A, Talea M, and Smarandache F, and Kishore Kumar P.K.(2017) "Shortest Path Problem on Single Valued Neutrosophic Graphs." 2017 International Symposium on Networks, Computers and Communications (ISNCC), (in press)

[6] Gary Chartrand and Ping Zhang, 'A First Course in Graph Theory' Dover Publications, Inc. Mineola, New York, 2012.

[7] R. Divya, Minimum spanning tree on world famous seven statues with prim's and kruskalalgorithm using neutrosophic fuzzy graphs, Aryabhatta Journal of Mathematics & Informatics Vol. 15, No.2, July-Dec., 2023.