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MATHEMATICAL MODELING OF FINGERO-IMBIBITION UNDER GRAVITATIONAL, HETEROGENEITY, AND MAGNETIZATION EFFECTS

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Abstract

This investigation mathematically models the fingero-imbibition process in a heterogeneous porous medium under magnetic influences. Many natural and manmade systems rely on fingero-imbibition, or the preferential flow of a wetting liquid through porous surfaces when magnetic fields are present. The study focuses on vertical heterogeneity, considering how magnetization and gravity-driven flow impede fluid movement. The governing equations are solved using the Variational Iteration Method (VIM) under appropriate initial and boundary conditions. MATLAB-generated graphical representations and numerical data are used to demonstrate how the system performs under various parameter situations.

Keywords: Mathematical model, Variational iteration method, Fingero-imbibition, Non-linear partial differential equation, magnetization parameter, porous medium.

1. Introduction

Understanding the phenomenon of fingero-imbibition is essential for a variety of applications, particularly in secondary oil recovery. In the oil industry, optimizing recovery processes

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requires a detailed understanding of fluid transport through porous rock formations. By analyzing the dynamics of fingero-imbibition, researchers can develop strategies to enhance oil recovery efficiency while minimizing environmental impacts. This study examines the fingero-imbibition phenomenon within a vertically oriented heterogeneous porous medium. Fingero-imbibition occurs when a wetting fluid infiltrates a porous medium, such as soil or rock, and preferential flow paths emerge due to variations in the medium's properties. These variations may include differences in pore size, permeability, or capillary forces. As the wetting fluid advances, it tends to follow these preferential pathways, forming finger-like patterns within the porous structure. When a porous medium saturated with one phase (e.g., oil) comes into contact with another phase (e.g., water) that exhibits a higher wetting preference, the wetting phase penetrates the medium naturally, displacing the resident phase without external forces a phenomenon known as imbibition. Conversely, when the displacing fluid has a lower viscosity than the displaced phase, the displacement front becomes unstable, leading to the formation of finger-like protrusions, a process referred to as viscous fingering. Scheidegger and Johnson (1961) investigated the statistical behavior of instabilities arising during displacement processes in porous media.

The simultaneous occurrence of imbibition and viscous fingering was termed fingero-imbibition by Galaktionov and Posashkov (1989). Vyas et al. (2011) solved the double phase instability phenomenon under the influence of a magnetic field using power series Several researchers have contributed to the analytical and numerical investigation of fingero-imbibition. Parikh et al. (2013) investigated the phenomenon in vertically oriented homogeneous porous matrices using the Functional Separable Choksi and Singh (2017) explored the impact of a magnetic field on two-phase fingero-imbibition in homogeneous porous media. Prajapati and Desai (2017) presented an approximate analytical solution for the fingero-imbibition phenomenon using the optimal homotopy analysis method. . Swaroop and Mehta (2002) used the finite element approach to numerically simulate the fingero-imbibition process in porous medium with magnetic field effect. Patel and Meher (2020) studied the modeling of imbibition phenomena in fluid flow across heterogeneous inclined porous media made of various porous materials. He (2007) developed the variational iteration method: some current findings and novel interpretations. Juhi and Ramakanta (2020) addressed the computational investigation of the time-fractional porous medium equation emerging from fluid flow through a water-wet porous media. Shah and Verma (2000) examined multiphase flow in slightly dipping porous media using magnetic fluid. Hatiboglu and Babadagli (2007) investigated oil recovery by counter-current spontaneous imbibition: The effects of matrix form factor, gravity, IFT, and oil viscosity. Lyiola and Folarin (2014) conducted an approximate analytical research on fingero-imbibition phenomena of the time-fractional type in double phase flow through porous media. Meher et al. (2012) applied the adomian decomposition method to study two-phase fingero-imbibition within porous media. Pathak and Singh (2018) analyzed fingero-imbibition in inclined porous media using the optimal homotopy analysis method. Cheng (2017) established reservoir simulation: mathematical tools for oil recovery. Desai et al. (2015) used the homotopy perturbation method to approximate the analytical solution of a nonlinear equation in one-dimensional imbibitions in homogeneous porous

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media.Liyanage and Juanes (2022) examined the effects of gravity fingering control on deep drainage and evaporation in a three-dimensional porous media.

The present work investigates fingero-imbibition in a heterogeneous porous medium, where two immiscible fluids (oil and water) interact. Water is introduced vertically downward under the combined influence of gravitational and capillary forces. The present mathematical model's goal is to use the generalized separable method to find the analytical solution of the fingero-imbibition portent under the effects of gravity and magnetic fields for the capacity of injected conductive fluid occupied by the cross-sectional area of schematic fingers of average length.

2. Mathematical model

In a heterogeneous porous media and magnetization, characteristics like porosity and permeability change over space. Taking into account the effects of magnetization, these properties are examined in this work as functions of the vertical coordinate z. It is assumed that the flow behavior complies with Darcy's law in order to formulate the fingero-imbibition phenomenon mathematically. Analytical simplicity leads to the idealization of the fingers as rectangular structures, notwithstanding their variable shapes and sizes. Without taking into account individual differences in finger geometry, only the average cross-sectional area occupied by the fingers is taken into account. Thus, the average cross-sectional area occupied by the wetting fluid at a specific time t and depth z is the saturation (Sw) of the injected water during fingero-imbibition.

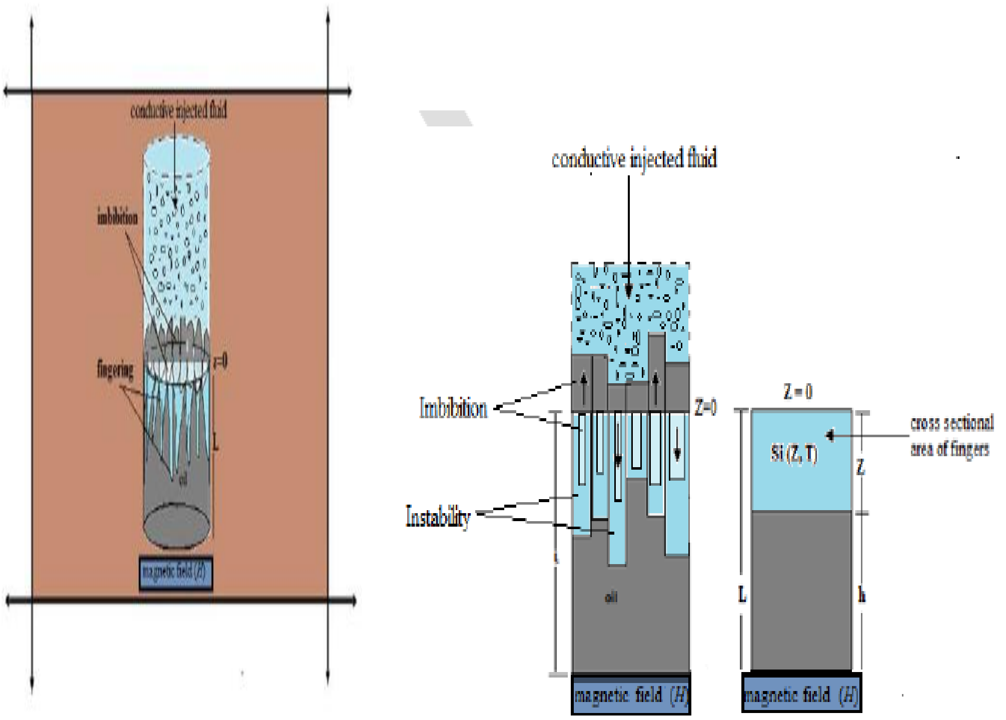

Figure 1. Fingero Imbibition in vertical downward porous matrix with magnetic field effects

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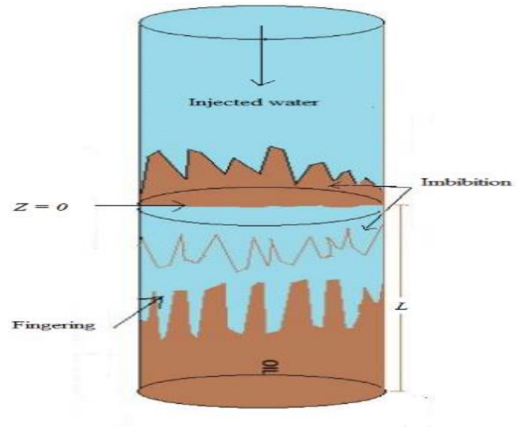
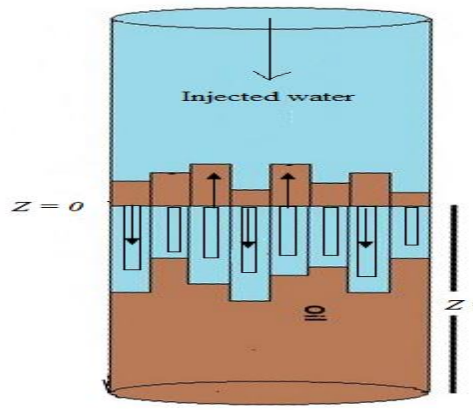


Figure – 2 : Phenomenon of Fingero-imbibition



igure – 3: Schematic diagram of Phenomenon of Fingero-imbibition

3. Formulation of the Problem

Assuming the Darcy rule applies to the flow system under analysis, the velocities of injected water (V_w) and oil (V_o) in the secondary oil retrieval process can be represented as follows [1, 6]:

$$V_w = -\frac{K_w}{\mu_w} K\left(\frac{\partial P_w}{\partial z} + \rho_w g\right) \tag{1}$$

$$V_o = -\frac{K_o}{\mu_o} K\left(\frac{\partial P_o}{\partial z} + \rho_o g\right) \tag{2}$$

The injected water's continuity equation can be written as

$$\varphi\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial V_w}{\partial z} = 0$$
(3)

The capillary pressure represents the difference in pressure between the wetting phase (water) and the non-wetting phase (oil).

$$P_c\left(S_w\right) = P_o - P_w \tag{4}$$

A continuous linear function of the kind is capillary pressure [10].

$$P_c = -\beta S_w \tag{5}$$

According to Scheidegger and Johnson [1], the standard relationship between relative permeability and phase saturation is considered as follows:

$$K_w = S_w,\ K_o = 1 - \alpha S_w \quad \left(\alpha = 1.11\right) \tag{6}$$

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For a uniformly heterogeneous porous material, we treat the porosity and permeability as functions that depend only on the variable z [3].

$$K(z) = K_0(1+bz),\ \varphi(z) = \frac{1}{a_1 - a_2 z}$$

where K_0, b, a_1 and a_2 are positive constants.

Here $\varphi(z)$ is incapable to surpass unity, let $a_1 - a_2 z \geq 1$.

On simplification, $K \propto \varphi$ [15]

Thus, $K = K_c \varphi$ (7)

At the common interface, condition for Counter-current imbibition phenomenon is presented by, $V_w + V_o = 0$

From (1) and (2),

$$\left(\frac{K_w}{\mu_w}\right) K \left(\frac{\partial P_w}{\partial z} + \rho_w g\right) + \left(\frac{K_o}{\mu_o}\right) K \left(\frac{\partial P_o}{\partial z} + \rho_o g\right) - Mg = 0 \tag{8}$$

where M is Hartmann number and From (8) and (4), we obtain

$$\left(\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}\right)\frac{\partial P_w}{\partial z} + \left(\frac{K_o}{\mu_o}\right)\frac{\partial P_c}{\partial z} = -\left(\frac{K_o}{\mu_o}\rho_o + \frac{K_w}{\mu_w}\rho_w\right) g - Mg$$

Therefore,

$$\frac{\partial P_w}{\partial z} = -\left[\frac{\left(\frac{K_w}{\mu_w}\rho_w + \frac{K_o}{\mu_o}\rho_o\right) g + \left(\frac{K_o}{\mu_o}\right)\frac{\partial P_c}{\partial z} - Mg}{\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}}\right] \tag{9}$$

When we apply equation (9) to equation (1), we obtain

$$V_w = -\left(\frac{K_w}{\mu_w}\right) K \left[\frac{\left(\frac{K_o}{\mu_o}\right)(\rho_w - \rho_o) g - \left(\frac{K_o}{\mu_o}\right)\frac{\partial P_c}{\partial z} - Mg}{\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}}\right] \tag{10}$$

Using the equation (10) into equation (3), we get

$$\varphi\left(\frac{\partial S_w}{\partial t}\right) + \frac{\partial}{\partial z}\left[K\left(\frac{\frac{K_w K_o}{\mu_w \mu_o}}{\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}}\right)\frac{\partial P_c}{\partial S_w}\frac{\partial S_w}{\partial z}\right] - \frac{\partial}{\partial z}\left[K(\rho_w - \rho_o) g \left(\frac{\frac{K_w K_o}{\mu_w \mu_o}}{\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}}\right)\right] - Mg = 0 \tag{11}$$

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Since the current analysis includes water and viscous oil, we have [1],

$$\frac{\frac{K_w K_o}{\mu_w \mu_o}}{\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}} \approx \frac{K_o}{\mu_o}=\frac{1-\alpha S_w}{\mu_o} \tag{12}$$

In equation (11), we get the values from (5), (6), (7), and (12),

$$\varphi\left(\frac{\partial S_w}{\partial t}\right)=\frac{K_c \beta}{\mu_o}\frac{\partial}{\partial z}\left[\varphi\left(1-\alpha S_w\right)\frac{\partial S_w}{\partial z}\right]+\frac{K_c\left(\rho_w-\rho_o\right)g}{\mu_o}\frac{\partial}{\partial z}\left[\varphi\left(1-\alpha S_w\right)\right] \tag{13}$$

For simplification, taking $S=1-\alpha S_w$ in equation (13), we find

$$\frac{\partial S}{\partial t}=\frac{K_c \beta}{\mu_o}\left[\frac{\partial}{\partial z}\left(S\frac{\partial S}{\partial z}\right)+S\frac{\partial S}{\partial z}\frac{a_2}{a_1}\right]-\frac{\alpha K_c\left(\rho_w-\rho_o\right)g}{\mu_o}\left(\frac{\partial S}{\partial z}+S\frac{a_2}{a_1}\right)+Mg \tag{14}$$

$\left(\because \frac{1}{\varphi}\frac{\partial \varphi}{\partial z}=\frac{\partial\left(\log\varphi\right)}{\partial z}=\frac{\partial}{\partial z}\left(-\log a_1+\frac{a_2}{a_1}z\right)=\frac{a_2}{a_1}\right)$ (Omitting higher order terms of z)

Now Non dimensional quantities,

$$Z=\frac{z}{L} \text{ and } T=\frac{K_c \beta t}{\mu_w L^2}$$

The equation (14) reduces to,

$$\frac{\partial S}{\partial T}=\frac{\partial}{\partial Z}\left(S\frac{\partial S}{\partial Z}\right)-A\frac{\partial S}{\partial Z}+BS\frac{\partial S}{\partial Z}-ABS \tag{15}$$

where $S\left(Z,\, T\right)=1-\alpha S_w\left(Z,\, T\right)$, $A=\frac{\alpha L\left(\rho_w-\rho_o\right)g}{\beta}$, and $B=L\frac{a_2}{a_1}$

4. Numerical Method

The correction functional of equation (15) is given as follows, using the variational iteration method [[8]:

$$S_{n+1}\left(Z,T\right)=S_n+\int_0^T \lambda\left(T\right)\left[\frac{\partial S_n}{\partial \tau}-\frac{\partial}{\partial z}\left(S_n\frac{\partial S_n}{\partial z}\right)+A\frac{\partial S_n}{\partial z}-BS_n\frac{\partial S_n}{\partial z}+ABS_n\right]d\tau$$

Where λ is a Lagrange's multiplier, and calculated as follow :

As, $S_n\left(Z,\tau\right)$ is showed restricted variation, $\delta S_n\left(Z,\tau\right)=0$.

Calculating deviation with respect to S_n, stating that $\delta S_n\left(0\right)=0$, gives

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$$\delta S_{n+1}(Z,T)=\delta S_n+\delta\int_0^T \lambda(T)\left[\frac{\partial S_n}{\partial \tau}-\frac{\partial}{\partial z}\left(S_n\frac{\partial S_n}{\partial z}\right)+A\frac{\partial S_n}{\partial z}-BS_n\frac{\partial S_n}{\partial z}+ABS_n+M\right]d\tau$$

Using integration by parts, we get

$$\delta S_{n+1}(Z,T)=\delta S_n+\left[\lambda(z)\delta S_n\right]_{\tau=T}-\int_0^T \lambda'(T)\delta S_n d\tau+\int_0^T \lambda AB\delta S_n d\tau$$

At the stationary conditions are: :

$$\left[1+\lambda(\tau)\right]_{\tau=T}=0,$$

$$\left[-\lambda'(\tau)+AB\lambda(\tau)\right]_{\tau=T}=0$$

The Lagrange multiplier is defined as

$$\lambda=-e^{AB(\tau-T)}$$

The provided iteration formula is derived as follows:

$$S_{n+1}(Z,T)=S_n-\int_0^T e^{AB(\tau-T)}\left[\frac{\partial S_n}{\partial \tau}-\frac{\partial}{\partial z}\left(S_n\frac{\partial S_n}{\partial z}\right)+A\frac{\partial S_n}{\partial z}-BS_n\frac{\partial S_n}{\partial z}+ABS_n+M\right]d\tau \qquad (16)$$

We choose the initial estimate

$$S_0=e^{-Z} \qquad (17)$$

Using the above assessment (17) in an iterative formula, we attain the following approximations. $S_1=e^{-Z}+\frac{1}{AB}\left(2e^{-2Z}-Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\left(1-e^{-ABT}\right)$

$$\begin{aligned}
S_2 &= e^{-Z}+\frac{1}{AB}\left(2e^{-2Z}-Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\left(1-e^{-ABT}\right)\\
&-\frac{1}{(AB)^2}\Big[\left(18e^{-3Z}-9Be^{-3Z}+4Ae^{-2Z}-4ABe^{-2Z}\right)+A\left(4e^{-2Z}-2Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\\
&-B\left(6e^{-3Z}-3Be^{-3Z}+2Ae^{-2Z}-2ABe^{-2Z}\right)\Big]\left(1-e^{-ABT}\right)^2\\
&+\frac{1}{(AB)^3}\Big[\left(2e^{-2Z}-Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\left(8e^{-2Z}-4Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\\
&+\left(4e^{-2Z}-2Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)^2\\
&-B\left(2e^{-2Z}-Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\left(4e^{-2Z}-2Be^{-2Z}+Ae^{-Z}-ABe^{-Z}\right)\Big]\left(1-e^{-ABT}\right)^3
\end{aligned} \qquad (18)$$

Similarly, the iterative formula can be used to obtain more iterations (16). Equation (18) represents an analytical approximation of the answer to equation (15) .

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Now, $S = 1 - \alpha S_w \quad \therefore \; S_w = \dfrac{1-S}{\alpha}$

Thus, the desired approximate analytical solution to the current problem is produced.

$$S_{w_1} = \frac{1}{\alpha}\left\{1 - \left[e^{-Z} + \frac{1}{AB}\left(2e^{-2Z} - Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)\left(1 - e^{-ABT}\right)\right]\right\}$$

$$\begin{aligned} S_{w_2} = \frac{1}{\alpha}\Bigg\{1 - \Bigg[& e^{-Z} + \frac{1}{AB}\left(2e^{-2Z} - Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)\left(1 - e^{-ABT}\right) \\ & - \frac{1}{(AB)^2}\Big[\left(18e^{-3Z} - 9Be^{-3Z} + 4Ae^{-2Z} - 4ABe^{-2Z}\right) + A\left(4e^{-2Z} - 2Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right) \\ & - B\left(6e^{-3Z} - 3Be^{-3Z} + 2Ae^{-2Z} - 2ABe^{-2Z}\right)\Big]\left(1 - e^{-ABT}\right)^2 \\ & + \frac{1}{(AB)^3}\Big[\left(2e^{-2Z} - Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)\left(8e^{-2Z} - 4Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right) \\ & + \left(4e^{-2Z} - 2Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)^2 \\ & - B\left(2e^{-2Z} - Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)\left(4e^{-2Z} - 2Be^{-2Z} + Ae^{-Z} - ABe^{-Z}\right)\Big]\left(1 - e^{-ABT}\right)^3\Bigg]\Bigg\} \end{aligned} \tag{19}$$

5. Result and Discussion

For numerical values, we used standard values for many parameters obtained from standard literature. The solution's numerical numbers and graphical representations were developed with MATLAB. MATLAB (19) is used to do numerical analysis and generate graphical illustrations of the result. The constants listed below were chosen from traditional literature.

Due to gravity$(g) = 9.8$, Density of injected water$(\rho_w) = 0.1$, Density of native oil$(\rho_o) = 0.8$, $\beta = 10$, $L = 1$,

Here $A = \dfrac{\alpha L(\rho_w - \rho_o) g}{\beta} \approx 1$.

Table 1 presents the numerical values of water saturation (S_w) at various depths (Z) for a fixed time (T). Figure 4 shows the variation of water (S_w) saturation with depth for the same time (T), while Figure 5 illustrates its variation with time at a fixed depth (Z). Figure 6 provides a 3D representation of the solution. From figures 4, 5, and 6, it is observed that water saturation decreases over time and increases with depth (Z).

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Table – 1: Computed results for water saturation during injection (S_w)

Z	$T=0$	$T=0.02$	$T=0.04$	$T=0.06$	$T=0.08$	$T=0.1$
0.1	0.857E – 1	0.711E – 1	0.568E – 1	0.428E – 1	0.290E – 1	0.155E – 1
0.2	1.633E – 1	1.513E – 1	1.396E – 1	1.281E – 1	1.169E – 1	1.058E – 1
0.3	2.335E – 1	2.237E – 1	2.141E – 1	2.047E – 1	1.955E – 1	1.864E – 1
0.4	2.970E – 1	2.890E – 1	2.811E – 1	2.734E – 1	2.659E – 1	2.585E – 1
0.5	3.545E – 1	3.479E – 1	3.415E – 1	3.352E – 1	3.290E – 1	3.229E – 1
0.6	4.065E – 1	4.011E – 1	3.958E – 1	3.907E – 1	3.856E – 1	3.807E – 1
0.7	4.535E – 1	4.491E – 1	4.448E – 1	4.406E – 1	4.364E – 1	4.324E – 1
0.8	4.961E – 1	4.925E – 1	4.890E – 1	4.855E – 1	4.821E – 1	4.788E – 1
0.9	5.346E – 1	5.317E – 1	5.288E – 1	5.254E – 1	5.232E – 1	5.205E – 1
1.0	5.695E – 1	5.671E – 1	5.647E – 1	5.624E – 1	5.601E – 1	5.579E – 1

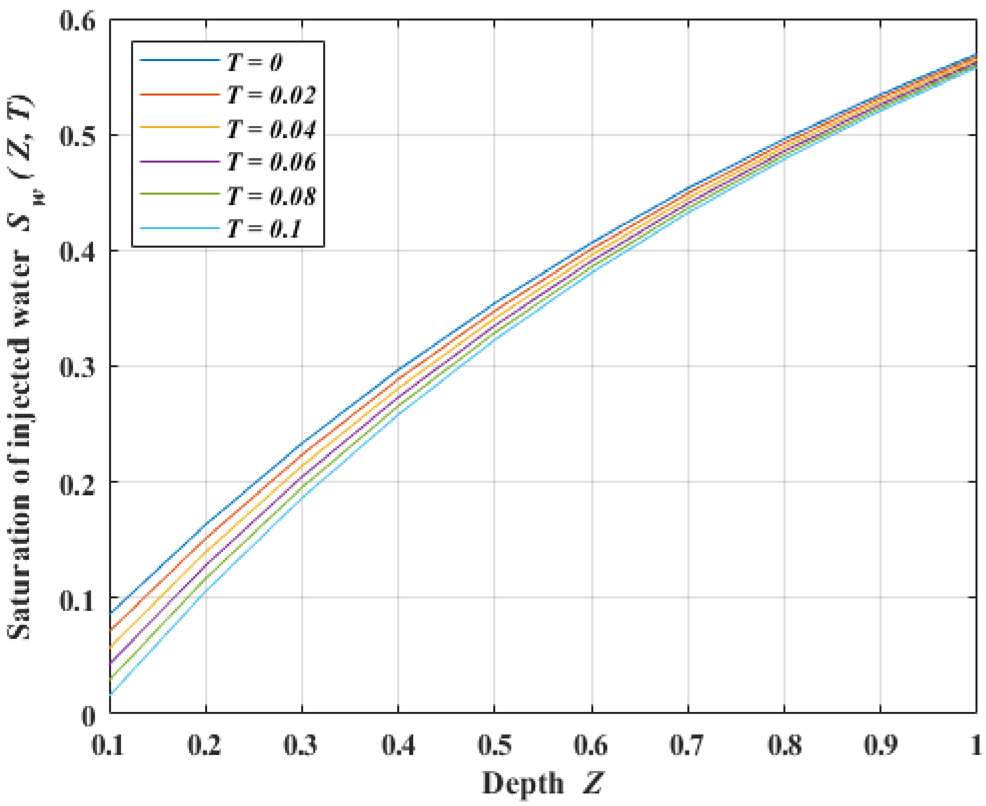

Figure- 4: Saturation of injected water Vs. Depth Z

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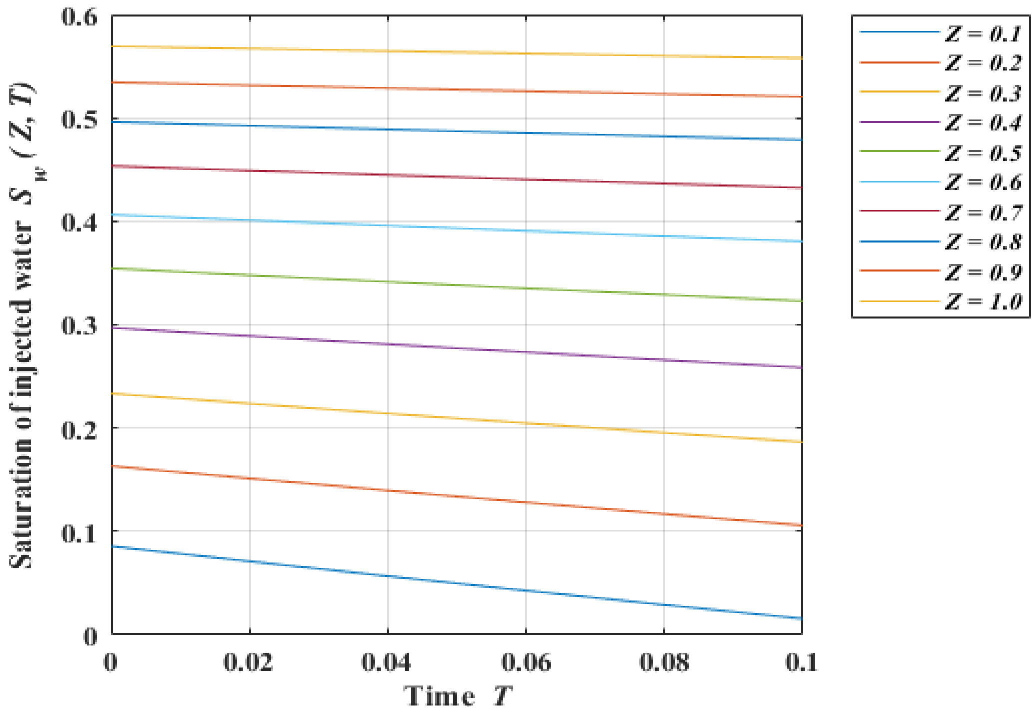

Figure - 5: Saturation of injected water Vs. Time T

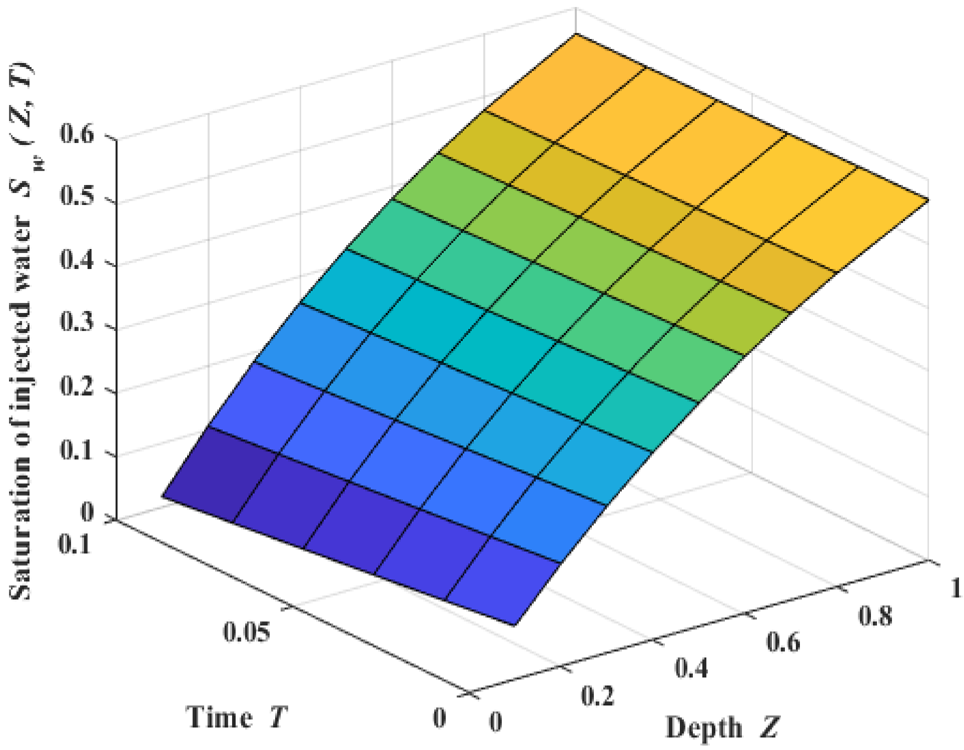

Figure - 6: 3D plot of the Saturation of injected water

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6. Conclusion

In this study, a mathematical model is formulated to investigate the fingero-imbibition phenomenon in a vertically oriented heterogeneous porous medium. The Variational Iteration Method (VIM) is applied to derive an approximate analytical solution to the governing equation, ensuring adherence to both initial and boundary conditions. Numerical and graphical analyses demonstrate that water saturation decreases with time and increases with depth, in agreement with the expected physical behaviour of the system. The proposed analytical framework enhances the understanding of fingero-imbibition in heterogeneous porous media and provides a reliable tool for predicting and improving fluid transport processes.

These findings have significant implications for soil science, petroleum engineering, and environmental remediation, where accurate estimation of fluid flow dynamics is essential for practical applications. The governing equation, a one-dimensional nonlinear partial differential equation, is solved using the Variational Iteration Method under appropriate initial and boundary conditions. The primary objective of this study is to characterize the saturation profile of the wetting phase (water) during fingero-imbibition, thereby providing valuable insights for enhancing oil recovery processes.

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