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RESOLVING FUZZY TRANSPORTATION PROBLEM USING RANKING OF PENTAGONAL FUZZY NUMBER

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Abstract

An approach constructed on the fuzzy one point method be situated is presented in this article to handle fuzzy transportation issues in which all of the restrictions remain assumed to be Pentagonal uncertain facts. In-depth numerical examples are provided to demonstrate the suggested methodology.

Key Words: Fuzzy number, Fuzzy Transportation problem, Pentagonal fuzzy numbers, Ranking technique, Fuzzy one point method, Pentagonal fuzzy transportation problem.

Introduction

Lotfi A. Zadeh first proposed the idea of fuzzyness in 1965[1]. The approximation of analytical functions, the membership functions of algebraic operations are divided into a left side and a representation of the right side by a straight forward in an analytical manner was also analyzed. In 1985, S.H. Chen and Klement

[2] were both researching operations on fuzzy numbers with real-valued functions. Michael Hanss [3] first proposed the triangular fuzzy number in 2004. Fuzzy numbers are utilized in numerous fields, including approximation theory, control theory, and signal processing.

The transportation problem (TrPb), which seeks to gratify the spreading of possessions from

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contractors to customers although minimalizing general conveyance costs, is a basic challenge trendy the field of logistics besides source shackle administration. Much work has been done over the years to provide effective algorithms and techniques for resolving a variety of transportation issues with exact and predictable supply and demand unit values, like transportation prices.

Nevertheless, in practical situations, it is challenging to forecast supply and demand levels "a priori" because they are closely related to the time of year, consumer patterns, and each company's financial situation. Comparable circumstances arise with regard to transportation costs, which typically rely on variables like weather, dynamic

market conditions, fluctuating fuel prices, and different routes. Researchers have used uncertain lucidity, a scientific background that enables the depiction of ambiguity then vagueness in policymaking procedures, to deal with these difficulties. More specifically, uncertain conventional concept offers a strong basis for simulating and evaluating issues involving imprecise data, making it a viable approach to transportation problem optimization.

A variant of the conventional conveyance problematic, the fuzzy conveyance problem (FuTrPb) involves fuzzy numbers for supply, demand, and transportation cost [4]. Fuzzy settings have been used in some studies, demonstrating the growing importance of the fuzzy decision-making technique. The concept of policymaking in uncertain contexts was first proposed by Bellman [5]. New findings surfaced after their investigation, with the most recent research being particularly notable.

The notion of an ideal explanation aimed at the TrPb through uncertain coefficients expressed as uncertain figures was first described by Chanas et al. in 1996. They also created an procedure toward getting an optimum resolution [6]. As soon as the conveyance environment comprises mutually uncertain and brittle integers, Ahmed introduced a novel approach for determining the TrPb's first basic feasible solution [7]. In 2009, Chakraborty and Chakraborty presented another fascinating respective [8]. Their approach was centered on minimizing transportation expenses and time when supply, demand, and carriage costs apiece element of measures are all uncertain. Basirzadeh presented a new method for turning fuzzy quantities into distinct values that might be used to solve a variety of transportation-related issues [4]. Nagoor Gani and colleagues proposed their own variety of the simplex algorithm for resolving the fuzzy transportation problem [9].

Additionally, Shanmugasundaram worked on fuzzy exact algorithms. For the purpose of producing a uncertain initial basic solution and a fuzzy optimal solution, correspondingly, he effectively created the fuzzy versions of Vogel's besides MODI procedures [10]. A different approach was developed by Balasubramanian and Subramanian to handle a particular kind of FuTrPb, where the price is represented as a trilateral uncertain number. To assess the uncertain impartial standards of the objective function then identify the best option, they used ranking techniques for numbers in their research [11]. To attain a uncertain finest result aimed at the FuTrPb, Pandian devised a innovative approach called the uncertain zero fact technique, in which supply, demand, and conveyance charges are characterized through trapezoidal uncertain integers [12]. cost

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Gani developed a two-stage charge-minimizing FuTrPb in which the expenses stay crisp but the source then request stand trapezoidal uncertain values [13]. Malini and Kennedy presented an approach that uses octagonal fuzzy numbers to solve the FuTrPb [14]. Yager's robust ranking system served as the foundation for Ekanayake and Ekanayake's study [15]. They put out a different algorithm for determining the fuzzy triangular and trapezoidal transportation problem's first fundamental solution [15]. Numerous surveys with problems that are resolved in different ways using generalized trapezoidal fuzzy numbers can be found in the literature. By adding extra parameters to create a more flexible representation of uncertainty, generalized fuzzy numbers specifically expand on the idea of classical fuzzy numbers. Until then, no novel method for utilizing generalized fuzzy numbers to solve the FuTrPb has been used until Kaur and Kuman's proposal [16]. Ali Ebrahimnejad aimed towards reduce the computational complication of the current approach by expanding on the ideas of Kaur and Kuman. In comparison to the approach suggested by Kaur and Kuman [17], the method used in his work was more straightforward, computationally efficient, and innovative. Chen added to the previously discussed study by putting forth the knowledge of comprehensive fuzzy statistics in situations when the membership function's normal form has no prescribed bounds [2].

One important component in the field of computational intelligence algorithms is the FuTrPb approximation. Numerous optimization strategies, such as mathematical programming techniques and metaheuristic algorithms, have been modified or created to successfully address fuzzy transportation issues. The fuzzy one-point technique was created by Elizabeth and Sujatha [12] to identify the fuzzy optimal solution for

FuTrPb. The optimum result to a pentagonal FuTrPb was found in this article by employing a ranking strategy and the one-point method.

Preliminaries

Fuzzy Sets [18]**:**

A fuzzy set allows for the incremental evaluation of an element's membership; it is an allowance of the conventional set with essentials that take a gradation of association. Given a universal set A, a uncertain set '$\widetilde{M}$' in A remains distinct as the collection of ordered pairs of an in'$\widetilde{M}$', wherever $\mu^{\widetilde{M}}$: A → [0, 1].

Fuzzy Numbers [18]**:**

Fuzzy statistics demarcated scheduled the customary R of real figures are particularly significant among the several kinds of fuzzy sets. These sets' membership functions, which take the form $\mu^{\widetilde{M}}$: R → [0, 1], are thought of as fuzzy numbers.

Pentagonal Fuzzy Numbers: If the association purpose has the procedure everywhere the central fact w has the evaluation of association 1 then z1 and z2 remain the corresponding rankings of opinions v and l, then the fuzzy number $\widetilde{M}$ = (u, v, w, l, n) is known as a pentagonal fuzzy number. Be aware that each PeFuNu has two weights, z1 and z2.

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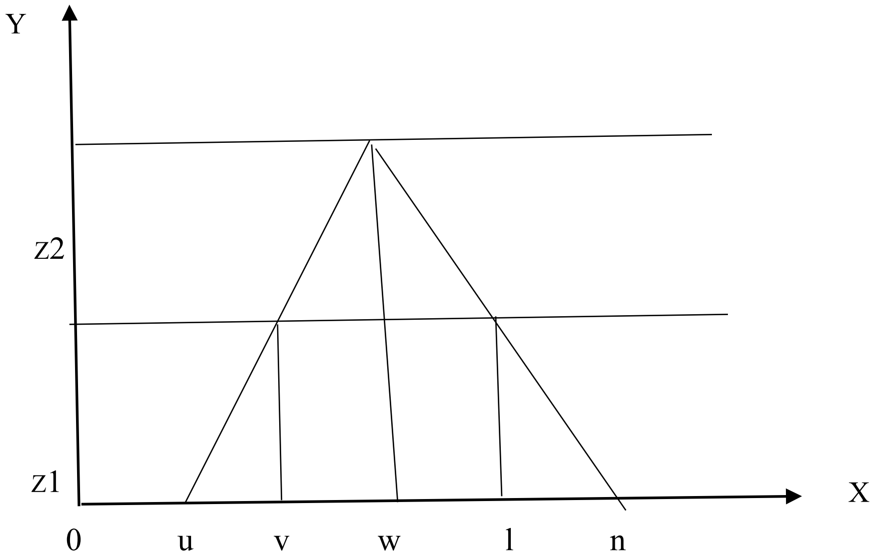

$$\mu_{\widetilde{M}}(M) = \begin{cases} \frac{1}{2}\left(\frac{m-u}{v-m}\right), u \le m \le v \\ \frac{1}{2} + \frac{1}{2}\left(\frac{m-v}{w-m}\right), v \le m \le w \\ 1 - \frac{1}{2}\left(\frac{m-w}{l-m}\right), w \le m \le l \\ \frac{1}{2}\left(\frac{n-m}{n-w}\right), l \le m \le n \\ 0, m < u \text{ and } m > n \end{cases}$$

Let $\widetilde{M}$ = (u1, v1, w1, l1, n1) and $\widetilde{N}$ = (u2, v2, w2, l2, n2) be two PeFuNu. Formerly the grading of PeFuNu is

$$R(\widetilde{M}) = \frac{u+v+w+l+n}{5}$$

Scientific Construction of the Balanced FuTrPb

$$Z = Min\, Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij} a_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{p} a_{ij} = r_i, i = 1 \ to \ q$$

$$\sum_{i=1}^{q} a_{ij} = s_j, j = 1 \ to \ p$$

$$\sum_{i=1}^{q} r_i = \sum_{j=1}^{p} s_j;\ a_{ij} \ge 0 \ for \ all \ i, j$$

Scientific Construction of the UnBalanced FuTrPb

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$$Z = Min\, Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij}a_{ij} - \sum_{i=1}^{q} a_{ij}$$, j=q is a dummny column

$$Z = Min\, Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij}a_{ij} - \sum_{i=1}^{p} a_{ij}$$, j=p is a dummny row

Table 1- Scientific Construction of the FuTrPb

	Dem 1	….	Dem q	Supply
Sup 1	a_{11}	….	a_{1q}	r_1
⋮	⋮	⋮	⋮	⋮
Sup p	a_{p1}	….	a_{pq}	r_p
Demand	s_1	….	s_q	

Procedure for FuTrPb

1. Pentagonal fuzzy numbers are used to represent the fuzzy transportation costs in step one. 2.The ranking algorithm is used in step two to defuzzify the pentagonal fuzzy numbers. 3.Verify that the problem is balanced. If the problem is not balanced, create a dummy column or dummy row with a single cost item to make it balanced. 4.In the transportation table, every row entry should be divided by the row minimum. For example, if ui is the minimum of the ith row of the table [fij], then the ith row entries should be divided by ai. The final table will be [fij /ai].

5. Following the use of Step 4, each column entry in the table [fij/ai] should be divided by the column minimum. In other words, if bj is the minimum of the table's jth column [fij /ai], then bj should be divided by bj. The final table will be [(fij/ai)/bj]. For every i, j, it is observed that (fij /ai)/bj >= 1. Every row and column in the resultant table [(fij /ai)/bj] should have a minimum of one fuzzy entry. 6.Choose the column or row with the only fuzzy one, and give that cell the lowest possible source and demand. Proceed to Step 8 once all fuzzy demand points have been fully received and all fuzzy supply points have been used. If not, proceed to Step 7.

7.Cover all of them (all the ones) by drawing a minimum amount of lines both vertically and horizontally. The remaining elements remain unaltered after you choose the least revealed element, divide all of the unveiled elements by it, and multiply the result at the line junction. Verify that there is at least one fuzzy input in each row and column. In such case, proceed to step 6; if not, proceed to step 4, then step 5. 8.This mapping gives the FuTrPb, given the objective function, the best possible answer

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$$Min\ Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij}a_{ij}$$ is PeFuTrPb is balanced, without establishing a dummy coloumn or a dummy row

$$Z = Min\ Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij}a_{ij} - \sum_{i=1}^{q} a_{ij},$$

If the PeFuTrPb is balanced by establishing a dummy coloumn j= q.

$$Z = Min\ Z = \sum_{i=1}^{q}\sum_{j=1}^{p} f_{ij}a_{ij} - \sum_{i=1}^{p} a_{ij},$$

If the PeFuTrPb is balanced by establishing a dummy row i= p.

All the objective functions are subject to the conditions

$$\sum_{i=1}^{q} r_i \sum_{=j=1}^{p} s_j;\ a_{ij} \geq 0\ for\ all\ i,j$$

Then finally we get the FuTr cost of the problem.

Arithmetical Illustrations

(a) Let's look at the PeFuTrPb, which has three bases (Sup 1, *S*up 2, *S*up 3) besides three termini (*D*em1, *D*em2, *D*em3). Using pentagonal fuzzy numbers as the elements, Table 2 displays the expenses of moving a unit of products since the ith foundation to the jth terminus. Calculate the bare minimum for entirely uncertain transports.

Table 2 – PeFuTr Table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	(3,5,11,9,12)	(8,6,7,11,13)	(7,6,10,15,17)	(30,40,70,80,90)
Sup 2	(3,5,8,10,9)	(5,7,8,11,14)	(4,6,9,11,15)	(35,45,55,95,100)
Sup 3	(4,6,9,12,14)	(6,12,11,14,17)	(5,7,8,10,15)	(40,60,70,80,100)
Demand	(35,45,55,75,90)	(50,60,65,80,95)	(45,50,65,85,105)	

The transportation table is now clear after applying a ranking approach to estimated the provided PeFu source, charge, and request.

Table 3 – Crisp Tr Table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	8	9	11	62
Sup 2	7	9	9	66

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Sup 3	9	12	9	70
Demand	60	68	70	198

Then spread over step-4 besides step-5, obtain the succeeding stand,

Table 4 – Consequential table through at least one uncertain one access in individually row in addition column

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	1	1	1.38	62
Sup 2	1	1.29	1.29	66
Sup 3	1	1.33	1	70
Demand	60	68	70	198

Give the commotion or column's cell through individual one uncertain source and require the bare minimum.

Table 5 – Fuzzy one Allotment table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1		62		62
Sup 2	60			66
Sup 3			70	70
Demand	60	68	70	198

Both fuzzy demand points and fuzzy supply points are not completely exploited in this situation.
Repeat step 7 until every fuzzy mount point has been used and every fuzzy demand point has been received.

Table 6 – Final Optimal Table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1		62		62
Sup 2	60	6		66
Sup 3			70	70
Demand	60	68	70	198

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The minimum transportation cost is $Min\, Z = \sum_{i=1}^{q}\sum_{i=1}^{p} f_{ij} a_{ij}$

$Z = (62 \times 9) + (60 \times 7) + (6 \times 9) + (70 \times 9)$ = 1284

(i) Take a look at a fuzzy transportation problem where pentagonal fuzzy numbers are used to represent supply at the source, demand at the destination, transit costs, etc.

Table 7 – Pentagonal Fuzzy Transportation table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	(7,12,15,16,20)	(3,4,5,6,7)	(4,8,10,12,16)	(12,21,33,44,55,33)
Sup 2	(5,6,7,8,9)	(3,7,8,9,13)	(3,6,7,11,18)	(12,49,54,67,78)
Sup 3	(5,8,11,14,17)	(4,7,9,10,10)	(3,3,3,3,3)	(13,28,66,86,97)
Demand	(21,26,61,77,85)	(30,50,70,90,110)	(25,40,55,85,120)	

The FuTrPb is not balanced. Proceed to transform the instable FuTrPb into a stable one.

Table 8 – Balanced FuTrPb

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	(7,12,15,16,20)	(3,4,5,6,7)	(4,8,10,12,16)	(12,21,33,44,55,33)
Sup 2	(5,6,7,8,9)	(3,7,8,9,13)	(3,6,7,11,18)	(12,49,54,67,78)
Sup 3	(5,8,11,14,17)	(4,7,9,10,10)	(3,3,3,3,3)	(13,28,66,86,97)
Sup 4	(1,1,1,1,1)	(1,1,1,1,1)	(1,1,1,1,1)	(2,6,8,13,16)
Demand	(21,26,61,77,85)	(30,50,70,90,110)	(25,40,55,85,120)	

The transportation table is now clear after applying a ranking approach to estimated the provided pentagonal uncertain supply, charge, and request.

Table 9 – Crisp Tr Table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	14	5	10	33
Sup 2	7	8	9	53
Sup 3	11	8	3	58
Sup 4	1	1	1	45

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Demand	54	70	65	189

Then spread over step-4 besides step-5, obtain the succeeding stand,

Table 4 – Consequential table through at least one uncertain one access in individually row in addition column

	Dem 1	Dem 2	Dem 3	Supply
Sup 1	2.8	1	2	33
Sup 2	1	1	1.29	53
Sup 3	3.67	2.67	1	58
Sup 4	1	1	1	45
Demand	54	70	65	189

Give the commotion or column's cell through individual one uncertain source and require the bare minimum.

Table 11 – Fuzzy one Allotment table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1		33		33
Sup 2	53			53
Sup 3			58	58
Sup 4		37	7	45
Demand	54	70	65	189

Both fuzzy demand points and fuzzy supply points are not completely exploited in this situation.
Repeat step 7 until every fuzzy mount point has been used and every fuzzy demand point has been received.

Table 12 – Fuzzy Optimal table

	Dem 1	Dem 2	Dem 3	Supply
Sup 1		33		33
Sup 2	53			53
Sup 3			58	58
Sup 4	1	37	7	45

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Demand	54	70	65	189

The minimum transportation cost is $Min\,Z = \sum_{i=1}^{q}\sum_{i=1}^{p} f_{ij} a_{ij}$

$Z = (33 \times 5) + (53 \times 7) + (58 \times 3) + (1 \times 1) + (37 \times 1) + (7 \times 1)$

$= 755$

Conclusion

In this work, a ranking technique for the representative value of the pentagonal fuzzy numbers is used to build a systematic procedure for addressing balanced and unbalanced pentagonal fuzzy transportation issues. The best option, which is the lowest transportation cost, is found using the fuzzy one-point approach.

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