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INVENTORY MODEL WITH SALVAGE VALUE FOR WEIBULL DETERIORATION RATE UNDER CREDIT POLICY AND PRESERVATION OF NON-INSTANTANEOUS DETERIORATION

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Abstract:

This study introduces an inventory model that incorporates the effects of preservation technology, which plays a crucial role in significantly lowering the rate of deterioration. The deterioration rate is assumed to follow a two-parameter Weibull distribution and varies over time. The model assumes a quadratic demand pattern, where demand is a non-linear function of time and captures more realistic market behaviours, such as initial growth followed by decline. Salvage value for items that have deteriorated is also allowed. Salvage cost is considered not only for unsold inventory, but also for partially deteriorated products and helps in reducing overall loss. In addition, trade credit policy is also considered for the retailer, which is a common financial practice in supply chain of business. To find the optimal inventory policy, the total cost is minimized through analytical methods. Numerical examples and graphical illustrations are included to demonstrate the practicality and robustness of the model.

Keywords: salvage value, weibull distribution deterioration, preservation, credit system, economic order quantity, quadratic demand.

1. Introduction:

Inventory management is essential for the efficient operation of any business. Poor stock maintenance can increase inventory holding costs, ultimately reducing overall profits. On the other hand, if a retailer orders less inventory than needed, it can lead to stock shortages, which may result in : i)a decline in profits, and ii) a loss of customer goodwill. Hence, it is crucial for a retailer to find the optimal answers to two key inventory management questions i) When is the right time to place an order? ii) What is the appropriate quantity to order? Items can degrade over time due to multiple factors. For instance, pharmaceuticals are effective only for a limited period, while perishable goods like foods and vegetables demand careful storage and handling. As a result, implementing preservation methods is crucial to maintain the quality and usability of the product.

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When items undergo quality degradation, they may still possess a portion of their original value, called as salvage value. In other words, salvage value is the resale value of the product at the end of an inventory cycle. This is the amount that can be recovered through discounted sales and thereby helps retailers to obtain maximum profit from revenue from sales and also from left over inventory. This approach helps limit financial loss while also meeting the needs of budget conscious buyers, like the local eateries, who are willing to accept a lower grade in exchange for a reduced price. Thus salvage value is essential. Demand for certain products are seasonal, like dolls purchased during golu festival for Navrathri in Tamilnadu. The demand of these products gradually decreases, specifying quadratic demand. The Weibull distribution has a broad application in different forms of functions. This versatility makes it effective for representing various stages of an item's life cycle. Credit not only enhances the business of a retailer , but it also ensures its free flow. Few researchers have studied the concept of salvage value of inventory of deterioration items. O.C.Ogundare[10] derived inventory model for deteriorating items with demand following negative exponential distribution, three parameter weibull distribution as deterioration rate, partial backlogging and salvage value. Uttam Kumar[18] calculated the total inventory cost and inventory cycle time for quadratic time dependent demand and pareto distribution as deterioration. Shortages were completely backlogged and salvage value was also considered in the paper. K.Thirugnanasambandam[6] framed mathematical model for decaying inventory with cubic holding cost, quadratic demand and salvage value. Deterioration rate was taken fixed and advertisement cost incorporated. In the above papers , neither preservation nor credit policy were explored. Poonam Mishra[12] integrated inventory system of manufacturer and retailer under preservation technology, credit period sensitive quadratic demand with shortages partially backlogged. Salvage value was considered but demand of items were advertisement and price dependent. Priyamvada[13] maximized the total profit of the seller and maintained environmental performance by green preservation. Shortages were not allowed. Sourav kumar Patra[16] developed model both in crisp and fuzzy environment to obtain optimum preservation technology for power pattern demand and completely backlogged shortages. Mishra V.K[7] found an optimal solution for deteriorating items inventory with preservation technology, constant demand rate, two-parameter weibull distribution deterioration rate, completely backlogged shortages and salvage value. Credit policy was not considered. Abu Hashan [1] studied the joined effect of trade credit, preservation and advertisement policy, without shortages. R.P.Tripathi[15] analyzed EOQ model for deteriorating items under quadratic demand, salvage value and no shortages. Amisha Patel[2] developed an algorithm to optimize retailer's net income with salvage price, trade credit, without shortages for deteriorating inventory and price-sensitive. Preservation cost was not included in the paper. Mishra V.K[8] developed a deterministic inventory model for deteriorating items with constant demand, salvage value, two-parameter weibull distribution as deterioration rate and holding cost as a linear function. Shortages were also completely backlogged, but credit policy was not incorporated. Vinod Kumar Mishra[20] studied the effect of deterioration products for ramp type demand. Deterioration rate was two parameter weibull distribution with shortages partially backlogged. Salvage value was also taken into account, without preservation and credit period. Venkateswarlu.R [19] studied inventory model

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for deteriorating items with quadratic demand rate, constant deterioration with salvage value. R.Mohan [14] attempted to study deteriorating inventory for quadratic demand, variable holding cost and salvage value without shortages. N.K.Sahoo[9] incorporated salvage value, deterioration rate and holding cost as trapezoidal fuzzy numbers in his mathematical model to obtain optimum inventory cost. Pavan kumar[11] proposed inventory model for both uniform and triangular distributions, with salvage cost, time linked holding cost and partially backlogged shortages . [3],[4],[5],[17] discussed for three parameter weibull distribution of deteriorated items with salvage value, without preservation.

2. Research Gap and Contribution

Existing literature on inventory management of deteriorating items primarily focusses on optimizing order quantities, replenishment policies and preservation investments. However, the interplay between preservation costs, trade credit and salvage value has received limited attention. This paper aims to bridge these gaps by investigating the salvage value of deteriorating items under preservation cost and trade credit for quadratic demand and two parameter weibull distribution deterioration. By addressing these research gaps, our contribution in this study will provide valuable insights for the retailer dealing with inventory management, to reduce loss and improve his profit.

3.Notations and Assumptions of the model

3.1 Assumptions

1. Single item is considered in the inventory cycle.
2. Infinite replenishment rate.
3. Demand rate is quadratic , $G(t) = \ a + bt + ct^2$ where initial rate of demand $a \geq 0$, demand rate increases at the rate of $b \neq 0$ and change in demand rate increases at the rate of $c \neq 0$.
4. Time dependent deterioration rate follows two parameter Weibull distribution given by $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter, $\beta > 0$ is the shape parameter, $t > 0$ is the deterioration time.
5. Shortages are not allowed and lead time is zero.
6. Salvage value δ $(0 < \delta < 1)$ of deteriorated units depends on deterioration cost.
7. Preservation cost is included.
8. Retailer receives credit period A, from supplier.
9. No repair or replacement of deteriorated products occurs during the cycle.

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3.2 Notations

τ	inventory cycle length (decision variable)
μ	preservation cost
L	holding cost per unit per unit time
U	ordering cost per cycle
p	purchasing cost per unit per unit time
s	selling cost per unit per unit time ($s > p$)
i_e	earned interest
i_p	paid interest ($i_p > i_e$)

4.Mathematical Formulation:

Inventory model is formulated for non-instantaneous deteriorating items with quadratic demand and two-parameter weibull distribution as deterioration rate.

Differential equation of Inventory level V at time t when $0 < t < \tau$ is

$$\frac{dV(t)}{dt} + \theta(t)V(t) = -G(t) \tag{4.1}$$

$$\frac{dV(t)}{dt} + \alpha\beta t^{\beta-1}\, V(t) = -(a + bt + ct^2) \tag{4.2}$$

General solution is

$$V(t)e^{\alpha t^{\beta}} = \int -(a + bt + ct^2)e^{\alpha t^{\beta}}dt + k \tag{4.3}$$

$$V(t)e^{\alpha t^{\beta}} = -\left[at + \frac{bt^2}{2} + \frac{ct^3}{3} + \alpha\left[\frac{at^{\beta+1}}{\beta+1} + \frac{bt^{\beta+2}}{\beta+2} + \frac{ct^{\beta+3}}{\beta+3}\right]\right] + k \tag{4.4}$$

when $t = 0,\ V(t) = V_0$

substituting above value in (4.4)

$$V(t)e^{\alpha t^{\beta}} = V_0 - \left[at + \frac{bt^2}{2} + \frac{ct^3}{3} + \alpha\left[\frac{at^{\beta+1}}{\beta+1} + \frac{bt^{\beta+2}}{\beta+2} + \frac{ct^{\beta+3}}{\beta+3}\right]\right] \tag{4.5}$$

when $t = \tau,\ V(t) = 0$

Total order quantity is

$$V_0 = a\tau + \frac{b\,\tau^2}{2} + \frac{c\,\tau^3}{3} + \alpha\left[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\right] \tag{4.6}$$

Using (4.6) in (4.5)

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$$V(t) = a(\tau - t) + \frac{b\,(\tau^2-t^2)}{2} + \frac{c\,(\tau^3-t^3)}{3} + \alpha\left[\frac{a(\tau^{\beta+1}-\beta\tau t^{\beta}+\beta t^{\beta+1}-\tau t^{\beta})}{\beta+1} + \frac{b(2\tau^{\beta+2}-\beta\,\tau^2\,t^{\beta}+\beta t^{\beta+2}-2\,\tau^2\,t^{\beta})}{2(\beta+2)} + \frac{c(3\tau^{\beta+3}-\beta\,\tau^3\,t^{\beta}+\beta t^{\beta+3}-3\,\tau^3\,t^{\beta})}{3(\beta+3)}\right] \tag{4.7}$$

Total cost of the inventory cycle are the summation of the following costs:

1.Preservation cost is CP= μ

(4.8)

2.Holding cost is CH = $L\int_0^{\tau} V(t)dt$

$$\text{CH}= L\left[\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4} + \alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right) + \frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3} - \frac{1}{\beta+1}\right) + \frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4} - \frac{1}{\beta+1}\right)\right]\right]$$

(4.9)

3.Deterioration cost is

$$\text{DC}= p\left[V_0 - \int_0^{\tau} G(t)dt\right]$$

$$\text{DC}= \alpha p\left[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\right] \tag{4.10}$$

4.Salvage value of units deteriorated is

VS = δ(deterioration cost)

$$= \alpha p\delta\left[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\right]$$

(4.11)

5. Sales revenue is R = $s\int_0^{\tau} G(t)dt$

$$\text{R} = s\left(a\tau + \frac{b\tau^2}{2} + \frac{c\tau^3}{3}\right)$$

(4.12)

6.Ordering cost is CO = U (4.13)

7. Interest paid and Interest earned

Two cases are considered to calculate the interest paid and interest earned by the retailer in the inventory cycle.

Case(i) : when he settles his due within the inventory period, $A \leq \tau$

During the interval $[0, A]$, the retailer sells his products at selling price s, and deposits his income in an interest bearing account. Thus , sales revenue is generated with an interest i_e.

Interest earned in $[0, A]$ is

$$\text{IE1}= s\,i_e\int_0^{A} G(t)tdt$$

$$\text{IE1}= si_e\left(\frac{aA^2}{2} + \frac{bA^3}{3} + \frac{cA^4}{4}\right) \tag{4.14}$$

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During the interval $[A, \tau]$, the retailer has to pay interest for his left-out stock at cost price p at an interest rate of i_p.

Interest paid in $[A, \tau]$ is

$$IP1 = p\, i_p \int_A^\tau V(t)dt$$

$$IP1 = p\, i_p \left[\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4} + \alpha \left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right) + \frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3} - \frac{1}{\beta+1}\right) + \frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4} - \frac{1}{\beta+1}\right)\right] - \left[a\left(\tau A - \frac{A^2}{2}\right) + \frac{b}{2}\left(\tau^2 A - \frac{A^3}{3}\right) + \frac{c}{3}\left(\tau^3 A - \frac{A^4}{4}\right) + \alpha \left[\frac{a}{\beta+1}\left(\tau^{\beta+1}A - \tau A^{\beta+1} + \frac{\beta A^{\beta+2}}{\beta+2} - \right) + \frac{b}{2(\beta+2)}\left(2\tau^{\beta+2}A - \frac{(\beta+2)\,\tau^2 A^{\beta+1}}{\beta+1} + \frac{\beta A^{\beta+3}}{\beta+3}\right) + \frac{c}{3(\beta+3)}\left(3\tau^{\beta+3}A - \frac{(\beta+3)\,\tau^3 A^{\beta+1}}{\beta+1} + \frac{\beta A^{\beta+4}}{\beta+4}\right)\right]\right]\right]$$

(4.15)

Retailer's Income is

Total profit $= TP_1 = \frac{1}{\tau}\,[VS + R + IE1 - CO - CH - CP - CD - IP1]$ (4.16)

$$TP_1 = \frac{1}{\tau}\left[\alpha p \delta \left[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\right] + s\left(a\tau + \frac{b\tau^2}{2} + \frac{c\tau^3}{3}\right) + s i_e\left(\frac{aA^2}{2} + \frac{bA^3}{3} + \frac{cA^4}{4}\right) - U - L\left[\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4} + \alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right) + \frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3} - \frac{1}{\beta+1}\right) + \frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4} - \frac{1}{\beta+1}\right)\right]\right] - \mu - \alpha p\left[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\right] - p\, i_p\left[\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4} + \alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right) + \frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3} - \frac{1}{\beta+1}\right) + \frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4} - \frac{1}{\beta+1}\right)\right] - \left[a\left(\tau A - \frac{A^2}{2}\right) + \frac{b}{2}\left(\tau^2 A - \frac{A^3}{3}\right) + \frac{c}{3}\left(\tau^3 A - \frac{A^4}{4}\right) + \alpha\left[\frac{a}{\beta+1}\left(\tau^{\beta+1}A - \tau A^{\beta+1} + \frac{\beta A^{\beta+2}}{\beta+2}\right) + \frac{b}{2(\beta+2)}\left(2\hat{o}^{\beta+2}A - \frac{(\beta+2)\,\tau^2 A^{\beta+1}}{\beta+1} + \frac{\beta A^{\beta+3}}{\beta+3}\right) + \frac{c}{3(\beta+3)}\left(3\tau^{\beta+3}A - \frac{(\beta+3)\,\tau^3 A^{\beta+1}}{\beta+1} + \frac{\beta A^{\beta+4}}{\beta+4}\right)\right]\right]\right]\right] \quad (4.17)$$

Case(ii) : when he settles his inventory at the end of the inventory period , $A = \tau$

During the interval $[0, \tau]$, the retailer sells his products at selling price s, and deposits his income at the rate of interest i_e. Since, he repays his due at the end of the cycle, he does not pay any interest.

IP2 = 0 (4.18)

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IE2= $s\, i_e \int_0^{\tau} G(t)tdt$

IE2= $si_e\left(\frac{a\tau^2}{2}+\frac{b\tau^3}{3}+\frac{c\tau^4}{4}\right)$

(4.19)

Retailer's Income is

Total profit = $TP_2 = \frac{1}{\tau}\ [VS+R+IE2-CO-CH-CP-CD-IP2]$ (4.20)

$$TP_2 = \frac{1}{\tau}\Bigg[\alpha p\delta\left[\frac{a\,\tau^{\beta+1}}{\beta+1}+\frac{b\,\tau^{\beta+2}}{\beta+2}+\frac{c\,\tau^{\beta+3}}{\beta+3}\right]+s\ \left(a\tau+\frac{b\tau^2}{2}+\frac{c\tau^3}{3}\right)+si_e\left(\frac{a\tau^2}{2}+\frac{b\tau^3}{3}+\frac{c\tau^4}{4}\right)- U-L\left[\frac{a\tau^2}{2}+\frac{b\tau^3}{3}+\frac{c\tau^4}{4}+\alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right)+\frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3}-\frac{1}{\beta+1}\right)+\frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4}-\frac{1}{\beta+1}\right)\right]\right]-\mu- \alpha p\left[\frac{a\,\tau^{\beta+1}}{\beta+1}+\frac{b\,\tau^{\beta+2}}{\beta+2}+\frac{c\,\tau^{\beta+3}}{\beta+3}\right]\Bigg]$$

(4.21)

5. Optimal Solution of the Mathematical Model

Theorem 5.1 :

If $A \le \tau$, there exists a unique maximum solution TP_1^* for the total cost function TP_1 .

Proof : Necessary condition for TP_1 to be maximum is $\frac{\partial TP_1}{\partial \tau}=0$ and $\frac{\partial^2 TP_1}{\partial \tau^2}<0$

Differentiating TP_1 w.r.t τ

$$\frac{\partial TP_1}{\partial \tau}==-\frac{1}{\tau^2}\Bigg[\alpha p\delta\left[\frac{a\,\tau^{\beta+1}}{\beta+1}+\frac{b\,\tau^{\beta+2}}{\beta+2}+\frac{c\,\tau^{\beta+3}}{\beta+3}\right]+s\ \left(a\tau+\frac{b\tau^2}{2}+\frac{c\tau^3}{3}\right)+si_e\left(\frac{aA^2}{2}+\frac{bA^3}{3}+\frac{cA^4}{4}\right)-U-L\left[\frac{a\tau^2}{2}+\frac{b\tau^3}{3}+\frac{c\tau^4}{4}+\alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right)+\frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3}-\frac{1}{\beta+1}\right)+\frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4}-\frac{1}{\beta+1}\right)\right]\right]-\mu-\alpha p\left[\frac{a\,\tau^{\beta+1}}{\beta+1}+\frac{b\,\tau^{\beta+2}}{\beta+2}+\frac{c\,\tau^{\beta+3}}{\beta+3}\right]-p\,i_p\Bigg[\frac{a\tau^2}{2}+\frac{b\tau^3}{3}+\frac{c\tau^4}{4}+\alpha\left[\frac{a\,\tau^{\beta+2}}{\beta+1}\left(\frac{\beta}{\beta+2}\right)+\frac{b\,\tau^{\beta+3}}{2}\left(\frac{3}{\beta+3}-\frac{1}{\beta+1}\right)+\frac{c\tau^{\beta+4}}{3}\left(\frac{4}{\beta+4}-\frac{1}{\beta+1}\right)\right]-\Bigg[a\left(\tau A-\frac{A^2}{2}\right)+\frac{b}{2}\left(\tau^2A-\frac{A^3}{3}\right)+\frac{c}{3}\left(\tau^3A-\frac{A^4}{4}\right)+\alpha\left[\frac{a}{\beta+1}\left(\tau^{\beta+1}A-\tau A^{\beta+1}+\frac{\beta A^{\beta+2}}{\beta+2}\right)+\frac{b}{2(\beta+2)}\left(2\tau^{\beta+2}A-\frac{(\beta+2)\,\tau^2A^{\beta+1}}{\beta+1}+\frac{\beta A^{\beta+3}}{\beta+3}\right)+\frac{c}{3(\beta+3)}\left(3\tau^{\beta+3}A-\frac{(\beta+3)\,\tau^3A^{\beta+1}}{\beta+1}+\frac{\beta A^{\beta+4}}{\beta+4}\right)\right]\Bigg]\Bigg]\Bigg]+\frac{1}{\tau}\Bigg[\alpha p\delta\left[a\tau^{\beta}+b\tau^{\beta+1}+c\tau^{\beta+2}\right]+s\,(a+b\tau+c\tau^2)-L\Bigg[(\,a\tau+b\tau^2+c\tau^3)+\alpha\left[\frac{a\beta\tau^{\beta+1}}{\beta+1}+\frac{b(\beta+3)\tau^{\beta+2}}{2}\left(\frac{3}{\beta+3}-\frac{1}{\beta+1}\right)+\frac{c(\beta+4)\tau^{\beta+3}}{3}\left(\frac{4}{\beta+4}-\right.\right.$$

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$\frac{1}{\beta+1}\Big)\Big]\Big] - \alpha p\big[a\tau^{\beta} + b\tau^{\beta+1} + c\tau^{\beta+2}\big] - p\, i_p \Big[\big(a\tau + b\tau^2 + c\tau^3\big) + \alpha\Big[\frac{a\,\beta\tau^{\beta+1}}{\beta+1} +$
$\frac{b(\beta+3)\tau^{\beta+2}}{2}\Big(\frac{3}{\beta+3} - \frac{1}{\beta+1}\Big) + \frac{c(\beta+4)\tau^{\beta+3}}{3}\Big(\frac{4}{\beta+4} - \frac{1}{\beta+1}\Big)\Big] - \Big[aA + b\tau A + c\tau^2 A + \alpha\Big[\frac{a}{\beta+1}\big((\beta +$
$1)\tau^{\beta}A - A^{\beta+1}\big) + b\Big(\tau^{\beta+1}A - \frac{\tau A^{\beta+1}}{\beta+1}\Big) + c\Big(\tau^{\beta+2}A - \frac{\tau^2 A^{\beta+1}}{\beta+1}\Big)\Big]\Big]\Big]\Big] = 0$

$\frac{\partial^2 TP_1}{\partial\tau^2} < 0$ when $A \leq \tau$.

Hence, TP_1 is maximum at TP_1^* .

Theorem 5.2:

If $\tau = A$, there exists a unique maximum solution T_2^* for the total cost function TP_2 .

Proof:

Necessary condition for TP_2 to be maximum is $\frac{\partial TP_2}{\partial\tau} = 0$ and $\frac{\partial^2 TP_2}{\partial\tau^2} < 0$

Differentiating TP_2 w.r.t τ

$\frac{\partial TP_2}{\partial\tau} = -\frac{1}{\tau^2}\Big[\alpha p\delta\Big[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\Big] + s\,\Big(a\tau + \frac{b\tau^2}{2} + \frac{c\tau^3}{3}\Big) + si_e\Big(\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4}\Big) -$
$U - L\Big[\frac{a\tau^2}{2} + \frac{b\tau^3}{3} + \frac{c\tau^4}{4} + \alpha\Big[\frac{a\,\tau^{\beta+2}}{\beta+1}\Big(\frac{\beta}{\beta+2}\Big) + \frac{b\,\tau^{\beta+3}}{2}\Big(\frac{3}{\beta+3} - \frac{1}{\beta+1}\Big) + \frac{c\tau^{\beta+4}}{3}\Big(\frac{4}{\beta+4} - \frac{1}{\beta+1}\Big)\Big]\Big] - \mu -$
$\alpha p\Big[\frac{a\,\tau^{\beta+1}}{\beta+1} + \frac{b\,\tau^{\beta+2}}{\beta+2} + \frac{c\,\tau^{\beta+3}}{\beta+3}\Big]\Big] + \frac{1}{\tau}\Big[\alpha p\delta\big[a\tau^{\beta} + b\tau^{\beta+1} + c\tau^{\beta+2}\big] + s\,(a + b\tau + c\tau^2) +$
$si_e\big(a\tau + b\tau^2 + c\tau^3\big) - L\Big[\big(a\tau + b\tau^2 + c\tau^3\big) + \alpha\Big[\frac{a\beta\,\tau^{\beta+1}}{\beta+2} + \frac{b(\beta+3)\tau^{\beta+2}}{2}\Big(\frac{3}{\beta+3} - \frac{1}{\beta+1}\Big) +$
$\frac{c(\beta+4)\tau^{\beta+3}}{3}\Big(\frac{4}{\beta+4} - \frac{1}{\beta+1}\Big)\Big]\Big] - \alpha p\big[a\tau^{\beta} + b\tau^{\beta+1} + c\tau^{\beta+2}\big]\Big] = 0$

$\frac{\partial^2 TP_2}{\partial\tau^2} < 0$ at $\tau = A$.

Hence, there exists a unique and maximum value TP_2^* for TP_2.

6.Numeric Illustrations, Sensitivity Analysis and Graphical Illustrations:

Let us consider the following numerical examples

Example 1 : Consider the following values in appropriate unit value for the inventory to illustrate case(i), $A \leq \tau$.

$a = 100$ units, $b = 5$ units, $c = 0.05$ units, α =0.02, $\beta = 1.5$, $p = \$10$, $s = \$18$, $\delta = \$3$,
$i_e = 5\%$ (0.05), $i_p = 10\%(0.1)$, $L = \$1.50$, $U = \$120$, $\mu = \$2$, $A =$ 0.4years

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Optimal total profit $TP_1 = \$1539.18$, $\tau = 0.5$years.

Example 2: Consider the following values in appropriate unit value for the inventory to illustrate case(ii), $A = \tau$.

$a = 100$ units, $b = 5$ units, $c = 0.05$ units, $\alpha = 0.02$, $\beta = 1.5$, $p = \$10$, $s = \$18$, $\delta = \$3$, $i_e = 5\%\ (0.05)$, $i_p = 10\%(0.1)$, $L = \$1.50$, $U = \$120$, $\mu = \$2$, $A = 0.5$years

Optimal Total profit $TP_2 = \$1541.30$, $\tau = 0.5$years.

6.1 Sensitivity Analysis:

For Case(i) , sensitivity analysis is given in the following table by keeping $\tau = 0.5$ to be fixed and change in profit is calculated with $\pm 20\%$ variation in parameters A, a, δ and μ .

Table 1: Sensitivity Analysis when $A \leq \tau = 0.5$

Change in %	**A**	**a**	**δ**	**μ**	**Total profit TP_1(with change in)**			
					A	**a**	**δ**	**μ**
-20%	0.32	80	2.4	1.60	1535.05	1187.31	1537.45	**1539.58**
-15%	0.34	85	2.55	1.70	1535.91	1275.28	1537.88	1539.48
-10%	0.36	90	2.70	1.80	1536.88	1363.25	1538.31	1539.38
-5%	0.38	95	2.85	1.90	1537.97	1451.21	1538.75	1539.28
0	0.40	100	3.00	2.00	**1539.18**	**1539.18**	**1539.18**	**1539.18**
5%	0.42	105	3.15	2.10	1540.50	1627.14	1539.61	1539.08
10%	0.44	110	3.30	2.20	1541.94	1715.11	1540.04	1538.98
15%	0.46	115	3.45	2.30	1543.49	1803.08	1540.47	1538.88
20%	0.48	120	3.60	2.40	**1545.16**	**1891.04**	**1540.90**	1538.78

For Case(ii) , sensitivity analysis is given in the following table by keeping $\tau = A$ and change in profit is calculated with $\pm 20\%$ variation in parameters A, a, δ and μ simultaneously.

Table 2: Sensitivity Analysis when $A = \tau$

Change in %	**A**	**a**	**δ**	**μ**	**Total profit TP_2(with change in)**			
					A	**a**	**δ**	**μ**
-20%	0.400	80	2.4	1.60	1536.52	1187.80	1538.77	**1540.07**
-15%	0.425	85	2.55	1.70	1537.61	1275.70	1539.11	1539.96
-10%	0.450	90	2.70	1.80	1538.77	1363.61	1539.45	1539.84
-5%	0.475	95	2.85	1.90	1540.00	1451.59	1539.82	1539.71
0	0.500	100	3.00	2.00	**1541.30**	**1541.30**	**1541.30**	**1541.30**
5%	0.525	105	3.15	2.10	1542.66	1626.95	1540.54	1539.45
10%	0.550	110	3.30	2.20	1544.09	1714.00	1540.91	1539.31
15%	0.575	115	3.45	2.30	1545.58	1803.23	1541.29	1539.16
20%	0.600	120	3.60	2.40	**1547.13**	**1891.67**	**1541.67**	1539.00

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6.2 Graphical Illustrations : The following figures gives the effect of the parameters A, a, δ and μ on both total profits, TP_1 and TP_2.

Figure 1

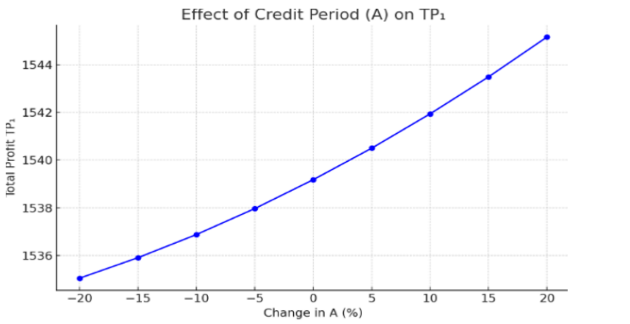


Figure 2

Figure 3

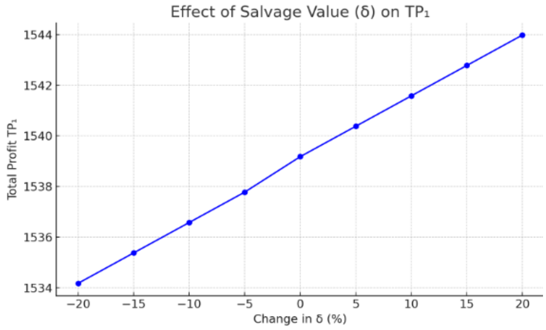


Figure 4

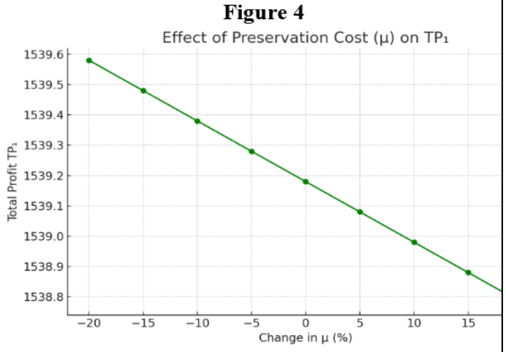


Figure 5

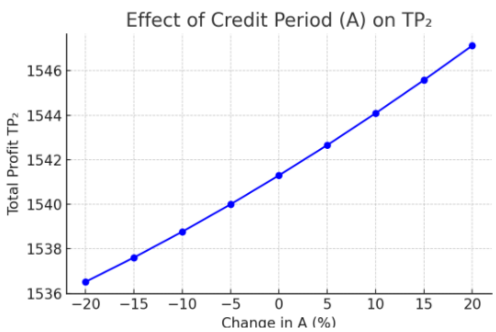
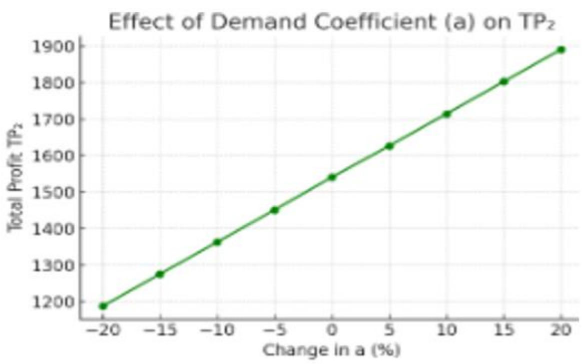


Figure 6



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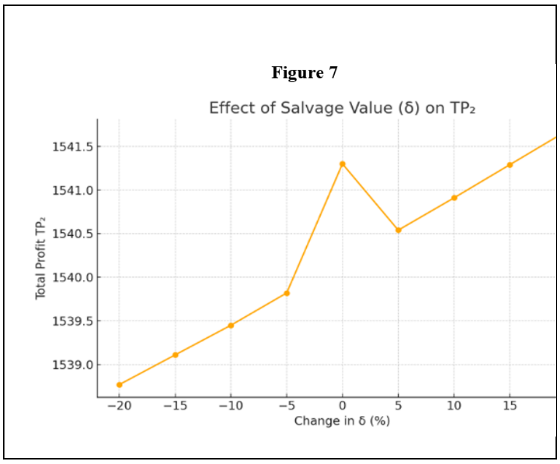

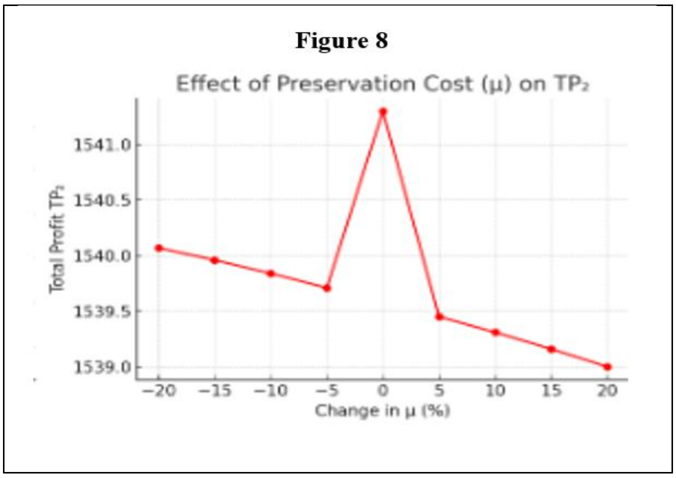

7.Observations:

From Figures 1 to 8, the following observations are made

1.Credit period : Increasing the credit period (A) in figures 1 and 5, shows moderate improvement in the total profits, TP_1 and TP_2, as it delays payment to the supplier. Receiving or offering longer credit period, enhances cash flow and profit, but its effect is less compared to salvage value.

2.Demand coefficient(a) : Significant increase in total profits TP_1 and TP_2 are seen as demand increases in figures 2 and 4. Higher demand demonstrates higher sales and hence sharp increase in profits.

3.Salvage value (δ) : As salvage value increases, total profits TP_1 and TP_2 also increases steadily. Higher salvage value enhances profit, because profit is also recovered from unsold or leftover items at the end of the inventory. The analysis from figures 3 and 7 shows that increasing δ results in a higher profit, though the impact is less pronounced than demand of the product.

4.Preservation cost (μ) : As preservation cost increases in figures 4 and 8, both total profits TP1 and TP_2 decreases. Increased investment in preservation reduces total profit, due to diminishing returns from excessive preservation expenditures. Increasing this parameter also enhances time value of the product for a retailer. From figures 4 and 8, we see that increase in μ decreases profit, confirming that investment in preservation must be optimized, under low deterioration scenarios.

8. Conclusion:

Retailers strategy for maximizing profit under the influence of salvage value, preservation cost and credit policy was formulated and illustrated through example for quadratic demand and two parameter weibull distribution as deterioration rate. Profit was maximum under the influence of salvage cost, when demand rate is highest and was also observed that the preservation cost has to use in moderation.

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