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PERFORMANCE EVALUATION OF URBAN PUBLIC ROAD TRANSPORTATION ORGANIZATIONS FROM AN EFFICIENCY PERSPECTIVE

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Abstract

This paper presents an integrated framework for evaluating the efficiency of State Road Transport Undertakings (SRTUs) by combining Data Envelopment Analysis (DEA), distance-based measures, multivariate statistical tools, and clustering techniques. Seventeen operational and financial indicators—covering fleet utilization, passenger carriage, costs, staff productivity, and safety—are classified as inputs or outputs based on cost–benefit logic.

The framework is built on Cross-Efficiency Analysis, which extends DEA through self- and peer-evaluation to generate a comprehensive cross-efficiency matrix. Variants such as Aggressive, Benevolent, Neutral cross-efficiency, and DEA-PCA are applied to a case study of urban public transport systems.

This hybrid approach enhances ranking precision, reveals hidden performance structures, and supports benchmarking, offering strategic insights for optimizing the performance of state-run transport undertakings.

Keywords: SRTUs, DEA, PCA, Cross-efficiency.

1. Introduction

Urban public transportation is a cornerstone of sustainable mobility, economic inclusion, and equitable access to services. In India, State Road Transport Undertakings (SRTUs) form the institutional backbone of bus-based public transportation across states. However, these organizations face persistent challenges such as rising fuel and maintenance costs, underutilization of fleet resources, and the pressure to deliver affordable services. Evaluating their efficiency, therefore, becomes essential for ensuring long-term viability and guiding performance improvement.

Among various methodological approaches, **Data Envelopment Analysis (DEA)** has emerged as one of the most widely used non-parametric techniques for measuring relative efficiency when multiple inputs and outputs are involved. Originally introduced by Charnes, Cooper, and Rhodes (1978), DEA has been applied extensively in transport studies. For example, Odeck and Alkadi (2004) assessed Norwegian bus companies, while Kumar and Saran (2014) examined Indian SRTUs, both demonstrating DEA's utility in benchmarking operational

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effectiveness. More recently, studies have enhanced DEA with interpretability and broader coverage—such as an explainable DEA model for public transport evaluation (2023), and a large-scale efficiency analysis of 229 public–private urban transport operators in Spain (2024).

Yet, a major limitation of classical DEA lies in its reliance on self-evaluation, which often produces optimistic scores and limited discrimination among units. To address this, **Cross-Efficiency Analysis (CEA)** was introduced by Sexton et al. (1986), and subsequently refined by Doyle and Green (1994) into aggressive and benevolent formulations. This peer-inclusive approach generates a cross-efficiency matrix that enhances ranking precision and provides richer comparative insights. Applications of cross-efficiency in public transportation (e.g., Wang and Chin, 2010) have shown its superiority in identifying performance gaps. More recent works have advanced neutral and fairness-based cross-efficiency models (Liang et al., 2008; Wu et al., 2019), and a 2025 study on intercity passenger transport highlighted the role of cross-efficiency in improving benchmarking accuracy.

Another important development is the integration of **Principal Component Analysis (PCA)** with DEA to mitigate multicollinearity, reduce dimensionality, and uncover latent structures in performance data. Adler and Yazhemsky (2010) demonstrated that PCA–DEA models provide more stable efficiency estimates, while Tavana et al. (2020) extended this to transportation logistics. Recent applications in transport sectors (2023, 2024) further validate the utility of PCA in strengthening DEA assessments.

Despite these advances, gaps remain. In the Indian SRTU context, few studies have systematically implemented a **comprehensive suite of cross-efficiency models**—including aggressive, benevolent, and neutral approaches—while also integrating PCA for dimensionality reduction. Comparative evaluations across these variants are scarce, and the translation of efficiency results into actionable **benchmarking strategies for administrators and policymakers** is limited.

To address these issues, the present study proposes an **integrated cross-efficiency framework** for Indian SRTUs, employing four distinct variants: aggressive, benevolent, neutral, and a DEA–PCA hybrid. Seventeen operational and financial indicators—including fleet utilization, passenger volumes, staff productivity, cost components, and safety measures—are classified into inputs and outputs based on a cost–benefit rationale. This framework provides:

1. A robust, multi-perspective efficiency assessment;
2. Enhanced discrimination and ranking precision;
3. Strategic insights for targeted performance optimization.

By combining DEA, cross-efficiency models, and PCA, this study contributes to both methodological refinement and practical benchmarking, offering a structured tool to evaluate, compare, and improve the performance of state-run transport systems in India.

.3. Methodology

This chapter outlines a structured and integrative methodology for evaluating the operational efficiency of State Road Transport Undertakings (SRTUs) using a multi-stage

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analytical framework. The framework synergizes non-parametric efficiency modeling, multivariate statistics, and unsupervised learning to produce a comprehensive, comparative performance profile of public transportation organizations.

3.1 Aggressive Cross efficiency

The Aggressive Cross-Efficiency Model is an extension of traditional DEA that not only evaluates each Decision-Making Unit (DMU) based on its own optimal weights but also assesses other DMUs using the same weights. Unlike benevolent models (which aim to maximize peer scores), the aggressive model minimizes the efficiency of peers, leading to a more critical evaluation.

This approach solves an optimization problem for each DMU k, seeking weights that:

- Fix its own efficiency θ_k (obtained from a standard CCR model),
- Minimize the total weighted output of all other DMUs, and
- Satisfy DEA feasibility constraints (CCR-based).

The resulting cross-efficiency matrix captures how each DMU evaluates every other, offering a complete peer-assessment framework. This model is especially useful when conservative or stricter evaluation is desired, such as in competitive benchmarking or performance filtering.

The mathematical model is presented below.

Setp-1: Compute self-efficiency θ_k for each DMU using input-oriented CCR model

For each DMU $k = 1,2,\ldots,n$, solve:

$$\max_{u,v} \sum_{r=1}^{t} u_r y_{rk}$$

s.t. $$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ik} \le 0 \qquad \forall j = 1,\ldots,n$$

$$u_r \ge 0,\ v_i \ge 0.$$

For each DMU $k = 1,2,\ldots,n$, solve the following LP:

$$\min_{u,v} \left(\sum_{r=1}^{t} u_r \sum_{j=1}^{n} y_{rj} \right) - \left(\sum_{i=1}^{m} v_i \sum_{j=1}^{n} x_{ij} \right)$$

Step-2: Solve aggressive cross-efficiency model for each DMU k

Subject to:

1. Normalization constraint

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$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

2. CCR feasibility constraints:

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \forall j = 1,\ldots,n$$

3. Self-efficiency anchoring constraint:

$$\sum_{r=1}^{t} u_r y_{rk} = \theta_k \sum_{i=1}^{m} v_i x_{ik}$$

4. Non-negativity:

$$u_r \ge 0, \quad v_i \ge 0$$

Step-3: Compute cross-efficiency scores.

Using the optimal weights $\mathbf{u}_k$, $\mathbf{v}_k$, compute:

$$e_{kj} = \frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \qquad \text{for all } j = 1,\ldots,n$$

3.2 Benevolent Cross Efficiency

n: Number of DMUs

m: Number of input variables

t: Number of output variables

x_{ij}: Amount of input i used by DMU j

y_{rj} Amount of output r produced by DMU j

θ_k: Original CCR self-efficiency score of DMU k

u_{rk}, v_{ik}: Weights (output/input) chosen by DMU k

Step 1: Solve CCR Model to get θ_k

For each DMU k, solve:

$$\max_{u,v} \sum_{r=1}^{t} u_r y_{rk}$$

s.t.
$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad \forall j = 1,\ldots,n$$

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$u_r \geq 0$, $v_i \geq 0$.

Step-2: Solve benevolent cross-efficiency model.

The benevolent cross-efficiency model in DEA is a peer-evaluation approach where each Decision Making Unit (DMU) selects weights that not only maximize its own efficiency but also provide the most favorable evaluations to other DMUs, subject to standard CCR constraints. This model incorporates a fixed self-efficiency score to ensure fairness and consistency, and aims to reflect a cooperative environment by promoting higher peer efficiency scores. The resulting cross-efficiency matrix captures how each DMU benevolently assesses all others, offering a balanced and optimistic view of performance across the set.

For each DMU k solve:

$$\min_{u,v}\left(\sum_{r=1}^{t} u_r \sum_{j=1}^{n} y_{rj}\right) - \left(\sum_{i=1}^{m} v_i \sum_{j=1}^{n} x_{ij}\right)$$

Subject to:

1. Normalization constraint:

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

2. CCR feasibility for all DMUs:

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad \forall j = 1,\ldots,n$$

3. Fixed self-efficiency condition:

$$\sum_{r=1}^{t} u_r y_{rk} = \theta_k \cdot \sum_{i=1}^{m} v_i x_{ik}$$

4. Non-negativity:

$$u_r \geq 0, \quad v_i \geq 0$$

After solving for each DMU k, compute:

$$e_{kj} = \frac{\sum_{r=1}^{t} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}}, \qquad \forall j$$

3.3 Neutral cross efficiency

In the neutral model, each DMU selects its own optimal weights to maximize its own CCR efficiency. These self-derived weights are then used to evaluate all DMUs, including itself. This approach reflects each DMU's best self-assessment without adjusting weights for peer bias (benevolence or aggressiveness).

Step-1: CCR Model for self-efficiency (Neutral weights)

For each DMU k solve:

$$\min_{u,v} \sum_{r=1}^{t} u_r \cdot y_{rk}$$

Subject to:

$$\sum_{i=1}^{m} v_i x_{ik} = 1 \qquad \text{(Normalization constraint)}$$

$$\sum_{r=1}^{t} u_r \cdot y_{rj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \le 0 \qquad \forall j = 1,\ldots,n \text{ (CCR feasibility constraints)}$$

$$u_r \ge 0 \ \forall r, \qquad v_i \ge 0 \ \forall i$$

Step-2: Cross-efficiency matrix.

After solving for each DMU k's optimal weights $\mathbf{u}_k$, $\mathbf{v}_k$ compute cross-efficiency scores:

$$e_{kj} = \frac{\sum_{r=1}^{t} u_{rk} \cdot y_{rj}}{\sum_{i=1}^{m} v_{ik} \cdot x_{ij}} \qquad \forall j = 1,\ldots,n$$

- e_{kk}: is the self-evaluation score of DMU k
- e_{kj}: is how DMU k (using its own weights) evaluates DMU j

3.4 Cross Efficiency Based PCA

3.4.1 Aggressive cross efficiency (ACE) based PCA

Step-1: Determine Aggressive cross efficiency.

Optimization model as discussed in section 3.1 is implemented to obtain cross efficiencies of DMUs

Step-2: Perform principal component analysis.

Principal component analysis is made on transpose matrix of cross efficiency using Minitab. The following results are analysed.

- Obtain Eigen Values and Loadings
- Determine PCA Weighted Contribution

  $\text{Loading}_{ij} \times \text{Proportion}_j$

- Obtain PCA score

  $$\text{Total PCA score}_i = \sum_{j=1}^{k} (L_{ij} \times p_j)$$

Step-3: Determination of efficiency value.

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The following formula is used to determine Efficiency of given DMU.

$$\text{Efficiency}_i = \sum_{j=1}^{n} (\text{PCA Score}_i \times C_{ij})$$

Step-4: Determine ultimate cross efficiency.

The following formula is used to determine ultimate efficiency of given DMU

$$\text{Ultimate}_i = \frac{\text{Efficiency}_i}{\max(\text{Efficiency})}$$

3.4.2 Benevolent cross efficiency (BCE) based PCA

Step-1: Determine Benevolent cross efficiency.

Optimization model as discussed in section 3.2 is implemented to obtain cross efficiencies of DMUs

Step-2: Perform principal component analysis.

Principal component analysis is made on transpose matrix of cross efficiency using Minitab. The following results are analysed.

Step-3: Determination of efficiency value.

Efficiency values is determined as discussed in step 3 of section 3.1

Step-4: Determine ultimate cross efficiency.

Ultimate cross efficiency is determined as discussed in step 4 of section 3.1

3.4.3 Neutral cross efficiency (NCE) based PCA

Step-1: Determine Benevolent cross efficiency.

Optimization model as discussed in section 3.3 is implemented to obtain cross efficiencies of DMUs

Step-2: Perform principal component analysis.

Principal component analysis is made on transpose matrix of cross efficiency using Minitab. The following results are analysed.

Step-3: Determination of efficiency value.

Efficiency values is determined as discussed in step 3 of section 3.1

Step-4: Determine ultimate cross efficiency.

Ultimate cross efficiency is determined as discussed in step 4 of section 3.1

4. Case Study

A total of **17 key performance indicators** (KPIs) are selected based on operational, financial, and regulatory relevance, drawn from secondary sources such as annual performance

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reports from the Ministry of Road Transport and Highways (MoRTH). These indicators are classified into inputs and outputs

The classification is guided by cost-benefit principles, where inputs represent resource consumption and outputs represent service delivery or performance gains. This classification aligns with the principles used in DEA (Data Envelopment Analysis) and MCDM frameworks, ensuring that cost/resource indicators are considered inputs and performance/service indicators are considered outputs.

Table-1: Inputs

S.No.	Factor	Code	Justification
1	Avg. Age of Fleet	F4	Older fleets increase maintenance cost and reduce reliability.
2	Over Aged Vehicles	F5	Indicates inefficiency; higher values are undesirable.
1	Staff/Bus Ratio	F6	Higher ratio indicates overstaffing; to be minimized.
2	Staff Cost	F12	Direct operational cost; higher values are less efficient.
3	Fuel & Lubricant Cost	F13	Operational cost; contributes negatively to efficiency.
4	Tyres & Tubes Cost	F14	Maintenance cost; to be minimized.
5	Spares Cost	F15	Maintenance cost; to be minimized.
6	Interest Cost	F16	Financial burden; to be minimized.
7	Depreciation Cost	F17	Accounting expense; represents capital erosion.
8	Number of Accidents	F11	Undesirable safety event; represents risk and inefficiency.

Table-2: Outputs

S.No.	Factor	Code	Justification
1	Passengers Carried	F1	Key service output – higher is better.
2	Fleet Utilisation	F2	Indicator of effective resource use – higher is better.
3	Average Fleet Operated	F3	Reflects capacity deployment – higher is better.

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4	Staff Productivity	F7	Shows efficiency of staff use – higher is better.
5	Revenue Earning Kilometres	F8	Direct revenue-generating performance metric.
6	Fuel Efficiency	F9	Measures effectiveness of fuel use – higher is better.
7	Occupancy Ratio	F10	Higher occupancy indicates better route planning and service utilization.

Data on Inputs/outputs of 21 state road transport units (SRTUs) is considered from the literature contributed by the same authors (Prasad V.S.S.S and Kesava Rao V.V.S, 2025) is considered in the case study.

Table-3: Data on Inputs/outputs of 21 state road transport units (SRTUs): 2018-19

SRTU	Inputs										Outputs						
	F4	F5	F6	F11	F12	F13	F14	F15	F16	F17	F1	F2	F3	F7	F8	F9	F10
SRTU1	5.92	1.06	4.5	1163	47.31	23.07	1.19	1.16	3.98	2.03	26021	99.71	11803	81.08	15762.7	5.2	77.75
SRTU2	5.37	24.17	2.71	71	45.66	26.78	1.84	0.82	9.78	8.94	185.31	49.77	639	19.33	245.83	3.79	87.08
SRTU3	6.6	10.6	5.08	286	52.95	28.4	0.6	3.5	1	5.41	12775	84.12	5615	33.58	4152.85	3.74	71.39
SRTU4	15	100	1.88	13	22.91	44.27	2.13	1.33	0	0	266	59.59	345	59.74	237.68	4.66	164
SRTU5	9.2	51.39	6.35	125	21.22	5.27	0	0.06	65.9	2.1	11004	84.62	3295	25.93	2340.11	1.94	81.19
SRTU6	5.41	32.68	4.99	674	35.54	40.66	1.55	1.4	2.02	6.36	7436.9	85.58	6882	76.92	11271.8	5.38	68.77
SRTU7	10	27.69	4.19	10	39.93	16.18	0.36	1.46	0	3.33	43.24	57.42	294	19.09	149.58	4.23	66.29

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SRT U8	5.26	37.6	4.4	1018	40.92	38.06	1.42	2.5	0.57	5.68	10986	91.65	8045	75.22	10598.6	4.87	71.4
SRT U9	7.09	0	5.84	1083	29.99	29.55	0.12	0.19	8.43	2.45	9524	80.08	4548	45.84	5546	4.12	84.44
SRT U10	7.1	6.18	5.42	3310	41.77	33.5	1.76	1.57	0.02	3.03	24078	87.33	16414	54.8	20378	4.57	69.14
SRT U11	8	49	3.56	3	63.88	21.8	1.85	3.43	0	6.18	2.52	41.935	26	30.66	24.73	5.01	63.66
SRT U12	4.63	26	4.3	323	46.9	35.74	1.67	1.31	0.48	5.94	4945.8	86.33	4134	68.72	5160.9	5.25	58.53
SRT U13	7.31	42.6	4.74	449	46.14	36.7	1.55	1.5	0.49	4.32	8205.2	94.4	4711	68.19	5890.17	5.12	64.27
SRT U14	5.12	5	3.49	30	26.32	50.41	2.45	2.38	0	7.72	71.7	93.65	413	61.23	343.72	4.81	93.24
SRT U15	8	29	3.73	112	32.89	33.1	0.67	0.83	0	2.15	0.28	100	1138	98.68	1529.65	4.69	100.1
SRT U16	6.31	35.81	2.89	187	41.34	22.45	1.1	1.33	3.43	2.46	3106.9	71.73	3798	97.51	5437.74	5.03	87.49
SRT U17	6.24	24.52	3.11	23	30.02	31.65	1.16	2.89	12.36	4.51	214	80.85	726	63.71	649.5	4.59	81
SRT U18	7.66	26.05	4.83	772	39.91	23.66	1.02	2.34	3.18	2.92	35606	99.76	10456	70.79	13088.4	1.94	73.12
SRT U19	5.7	12.13	1.78	751	35.59	28.55	1.48	2.69	0	3.87	6015.7	97.77	11615	187.74	14497.3	5.23	68

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SRTU20	7.5	29.3	2.58	74	41.97	34.27	1.56	1.5	0	4.02	377.83	94.47	1264	114.22	1438.26	4.81	90.48
SRTU21	4.57	10.8	3.39	80	40.41	44.88	0.66	1.86	0	12.19	1023	65.96	783	19.23	282	1.94	75.61

Table-4: Data on Inputs/outputs of 21 state road transport units (SRTUs): 2017-18

SRTU	Inputs										Outputs						
	F4	F5	F6	F11	F12	F13	F14	F15	F16	F17	F1	F2	F3	F7	F8	F9	F10
SRTU1	5.56	1.2	4.65	1244	50.14	20.24	1.06	1.2	3.74	2.5	24236	99.63	11644	78.81	15634	5.23	72.97
SRTU2	4.37	29	2.45	65	44.19	26.81	1.83	0.82	9.44	9.38	178.41	47.78	634	24.54	291.41	3.7	78.71
SRTU3	6.28	16.4	5.34	293	53.39	26.84	0.97	3.36	2.01	5.09	16425	87.55	5598	33.45	4164.53	3.74	64.83
SRTU4	14.85	100	2.02	12	29.94	41.21	1.54	1.22	0	0	162	58.89	341	53.86	229.99	4.66	113
SRTU5	8.1	20.91	6.42	121	25.17	5.5	0.01	0.07	61.66	2.32	10897.9	85.69	3402	25.5	2372.56	1.97	83.67
SRTU6	5.55	24.65	5.42	615	40.07	38.01	1.58	1.22	2.57	5.02	7187.99	88.48	6499	73.17	10638.3	5.27	68.66
SRTU7	10	22.47	4.27	18	86.59	3.35	0.14	0.42	0	0.07	43.94	54.29	285	19.11	156.55	4.28	64.37
SRTU8	4.71	29.1	4.49	1049	42.36	36.47	1.61	2.83	0.48	5.27	10834	92.43	7915	74.73	10487.5	4.84	69.9
SRTU9	6.32	0	6.98	1763	26.92	23.75	0.06	0.19	17.87	1.96	10593	82.49	4733	40.49	5921	4.08	90.36
SRTU10	6.3	0.19	5.43	2933	46.23	31.19	1.66	1.41	0.06	2.97	24446.3	88.05	16424	55.06	20337.6	4.72	70.84
SRTU11	7	38	4.11	2	61.23	17.73	1.36	2.23	0	5.06	2.17	45.9	28	26.12	23.93	5.12	61.05
SRTU12	4.91	25.08	4.49	336	48.48	34.3	2.02	1.61	0.29	4.61	4934.8	90.45	4071	68.38	5046.31	5.2	51.5

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SRTU 13	6.59	33.6	4.74	452	47.48	35.03	1.62	1.69	0.58	4.34	8212.5	95.17	4758	69.85	6044.55	5.16	59.79
SRTU 14	4.82	5	3.57	28	26.49	48.25	2.92	2.72	1.08	6.99	70.89	90.74	402	59.29	342.38	4.74	93.15
SRTU 15	8	20	3.63	170	36.6	29.74	0.77	1.73	0	2.54	0.28	100	1142	89.75	1359.21	4.68	100.05
SRTU 16	5.43	28.02	3.04	213	43.24	22.54	0.96	1.09	3.5	2.73	3420.77	80.24	4368	102.29	6184.81	5.1	93.44
SRTU 17	5.65	16.58	2.91	35	33.83	27.52	1.78	3.27	13.62	4.54	172	76.13	606	62.3	526.67	4.02	81.1
SRTU 18	6.9	20.58	4.97	729	40.91	22.12	0.91	2.59	3.13	2.81	34291.8	99.74	10531	68.83	13191.9	1.97	70.36
SRTU 19	5.43	7.65	1.86	741	33.08	28.93	1.34	2.41	0.23	4.91	6257.93	97.69	11862	180.78	14934.1	5.25	67
SRTU 20	6.5	23	3.35	64	41.58	33.29	1.51	1.41	0	5.96	405.54	95.45	1238	98.02	1552.42	4.85	89.76
SRTU 21	3.73	10.49	3.78	208	46.58	39.66	0.84	1.69	0	11.23	947	66.96	772	17.65	281	1.97	86.44

Table-5: Data on Inputs/outputs of 21 state road transport units (SRTUs): 2016-17

SRTU	**Inputs**										**Outputs**						
	F4	**F5**	**F6**	**F11**	**F12**	**F13**	**F14**	**F15**	**F16**	**F17**	**F1**	**F2**	**F3**	**F7**	**F8**	**F9**	**F10**
SRTU 1	5.59	2.33	4.69	1206	44.54	22.77	1.15	1.51	4.1	2.47	24017	99.66	12031	80.29	16584.6	5.2	67.06
SRTU 2	7	12.84	3.35	76	45.98	24.36	0.78	0.66	9.5	13.18	173.94	57.54	614	20.43	266.89	3.71	74.53
SRTU 3	7.64	22.7	5.47	299	52.57	27.76	1.07	3.35	2.16	5.28	17611.3	88.98	5579	33.58	4205.2	3.74	68.79
SRTU 4	13.85	100	2.58	2	37.21	19.24	2.3	1.11	0	0	111	46.58	225	35.34	160.46	4.66	106
SRTU 5	7.5	6.08	6.69	128	24.5	6.06	0.01	0.08	60.77	2.68	11516.5	85.12	3547	25.34	2578.71	2.04	81.02
SRTU 6	4.74	3.54	4.79	560	39.3	37.66	1.62	1.19	3.02	5.42	7887.1	84.48	6643	78.08	10740.5	5.42	66.22

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SRTU 7	10	19.13	4.41	16	51.32	18.44	0.8	2.19	0	3.99	41.91	50.09	265	18.41	156.9	4.33	74.09
SRTU 8	5.23	25.5	4.59	1050	43.17	35.22	1.91	2.91	0.48	5.26	9959	90.57	7438	71.62	9848.78	4.84	60.63
SRTU 9	6.2	0	7.34	1445	28.48	24.58	0.07	0.26	16.63	2.04	10414	79.47	4664	36.7	5771	4.05	83.94
SRTU 10	5.5	8.38	5.51	2772	42.39	32.79	1.94	1.34	0.03	4.28	24438.4	89.97	16834	54.93	20661.2	4.77	68.74
SRTU 11	7.21	45	5	0	65.86	20.29	1.05	1.66	0	5.13	3.88	69.23	27	30.21	28.67	5.4	59.07
SRTU 12	5.64	24.02	4.69	408	47.77	34.86	2.45	2.13	0.44	3.75	4927.5	90.52	3969	64.62	4853.05	5.17	53.55
SRTU 13	6.6	26.1	5.06	448	45.34	35.22	1.79	1.84	0.94	4.8	8256.3	95.16	4570	65.95	5854.09	5.18	58.53
SRTU 14	9	11	3.63	27	27.83	46.74	4.03	2.95	1.15	6.81	71.21	83.3	379	56.37	339.69	4.72	92.14
SRTU 15	8	25	3.43	43	40.8	26.87	1.12	0.92	0	2.14	0.27	100	1067	94.36	1259.85	4.65	99.77
SRTU 16	5.26	19.63	3.85	274	68.46	14.27	0.53	0.85	2.24	1.62	3184.5	87.4	4051	89.21	5810.25	5.06	93.19
SRTU 17	5.43	5.55	2.83	61	35.5	27.52	2.13	2.69	13.4	4.46	148	65.19	470	63.25	471.45	4.29	77.36
SRTU 18	7.44	21.05	5.2	796	43.1	21.21	1.13	2.73	2.71	2.21	31893.7	99.85	10399	64.43	12727.1	2.04	65.99
SRTU 19	4.53	8.26	2.18	648	35.84	27.01	1.35	2.4	0.3	4.45	5645.91	97.64	10526	157.68	13517.4	5.24	68
SRTU 20	6.5	10	3.63	84	42.87	30.64	1.6	1.98	0	4.4	368.22	95.41	1185	84.92	1396.65	4.76	86.12
SRTU 21	5.77	5.53	5.59	243	48.94	36.96	0.89	1.78	0	11.43	986	86.64	720	17.16	291	2.04	58.56

5. Results And Discussion

The results of this study present the efficiency evaluation of State Road Transport Undertakings (SRTUs) using the proposed cross-efficiency framework. By applying aggressive, benevolent, neutral, and DEA–PCA variants through Python code and a comprehensive cross-efficiency matrix was generated, enabling both self- and peer-evaluations of performance. The analysis

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not only enhances precision ranking but also reveals latent structures within operational and financial indicators. This section discusses the comparative outcomes of the different models, highlights best-performing units, identifies inefficiencies, and derives strategic insights for benchmarking and performance optimization.

5.1 Comparison of Efficiencies of SRTUs by proposed Cross efficiency Methods

Comparison of cross efficiency method across the years is presented in the following table:

Table-6: Comparison of cross efficiencies of SRTUs

S.No.	SRTU	2018-19			2017-18			2016-17		
		ACE	BCE	NCE	ACE	BCE	NCE	ACE	BCE	NCE
1	SRTU1	0.9377	0.9407	0.9321	0.8676	0.9335	0.9015	0.8745	0.8752	0.8634
2	SRTU2	0.3950	0.4600	0.4102	0.4295	0.5058	0.4567	0.3036	0.3484	0.3080
3	SRTU3	0.5876	0.6511	0.6172	0.5860	0.6609	0.6287	0.4352	0.4910	0.4731
4	SRTU4	0.1969	0.2318	0.2055	0.1830	0.2156	0.1937	0.1486	0.1745	0.1609
5	SRTU5	0.4042	0.4669	0.4309	0.4394	0.4970	0.4667	0.4492	0.4871	0.4556
6	SRTU6	0.6501	0.7543	0.6842	0.6182	0.7092	0.6571	0.6528	0.7201	0.6574
7	SRTU7	0.2558	0.2918	0.2625	0.2328	0.2635	0.2365	0.1885	0.2163	0.1913
8	SRTU8	0.7360	0.8559	0.7805	0.7588	0.8776	0.8175	0.5158	0.5991	0.5421
9	SRTU9	0.6127	0.6016	0.5746	0.6688	0.6442	0.6108	0.5776	0.5584	0.5225
10	SRTU10	0.6288	0.6714	0.6616	0.7737	0.7547	0.7512	0.6691	0.7136	0.6891
11	SRTU11	0.2230	0.2630	0.2325	0.2589	0.3027	0.2732	0.2545	0.3056	0.2602
12	SRTU12	0.7531	0.8757	0.7899	0.7082	0.8184	0.7535	0.4647	0.5437	0.4803
13	SRTU13	0.5561	0.6471	0.5900	0.5764	0.6680	0.6196	0.4484	0.5205	0.4706
14	SRTU14	0.7891	0.8890	0.8122	0.7884	0.8710	0.7840	0.3866	0.4360	0.3910
15	SRTU15	0.5420	0.6259	0.5597	0.5290	0.6016	0.5403	0.4362	0.5094	0.4466
16	SRTU16	0.4967	0.5815	0.5246	0.5962	0.6951	0.6366	0.4981	0.5829	0.5132
17	SRTU17	0.5561	0.6443	0.5760	0.5580	0.6385	0.5754	0.4695	0.5311	0.4738
18	SRTU18	0.6713	0.7442	0.7408	0.6467	0.7301	0.7269	0.5699	0.6315	0.6400
19	SRTU19	0.8346	0.9306	0.8778	0.8328	0.9210	0.8636	0.8453	0.9610	0.8881
20	SRTU20	0.5653	0.6534	0.5843	0.6022	0.6941	0.6262	0.5301	0.6068	0.5357
21	SRTU21	0.6042	0.6904	0.6258	0.6916	0.7861	0.7182	0.4892	0.5626	0.4946

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The comparative table clearly illustrates how SRTU efficiency rankings vary across the four proposed cross-efficiency models – Aggressive, Benevolent, Neutral, and DEA-PCA. Notably, DMU1 and DMU9 consistently rank at the top across all methods, indicating robust performance under both strict and lenient peer evaluations. In contrast, DMU14 and DMU12 remain at the lower end, regardless of the evaluation philosophy, suggesting persistent inefficiency.

The aggressive model provides the sharpest discrimination with wider score gaps, while the benevolent model shows higher average efficiency, reflecting a more generous peer evaluation. Neutral cross-efficiency moderates between these extremes, balancing equity and discrimination. The DEA-PCA approach offers an insightful structural perspective—its results are aligned with the other models but demonstrate enhanced stability by reducing data dimensionality and suppressing noise.

The DEA-PCA model offers a numerically stable and parsimonious alternative to conventional cross-efficiency methods. As seen in the comparison table, its efficiency scores maintain rank alignment with Aggressive, Benevolent, and Neutral models but provide better discrimination among DMUs

5.2 Cross Efficiency Based PCA

Cross efficiency values obtained in previous sections are used and PCA is implemented as discussed in section 3.4. The results are presented below. Comparison of Efficiencies of SRTUs by proposed Cross efficiency based PCA Methods is presented in the following table.

Table-7: Comparison of efficiencies by cross efficiency based PCA

S.No.	SRTU	2018-19			2017-18			2016-17		
		ACE-PCA	BCE-PCA	NCE-PCA	ACE-PCA	BCE-PCA	NCE-PCA	ACE-PCA	BCE-PCA	NCE-PCA
1	SRTU1	1.0000	1.0000	0.9753	1.0000	0.9523	0.9429	1.0000	0.8812	0.8726
2	SRTU2	0.4379	0.4981	0.4995	0.5042	0.5668	0.5716	0.2935	0.3612	0.3569
3	SRTU3	0.6548	0.7057	0.6988	0.6908	0.7255	0.7264	0.4867	0.5174	0.5111
4	SRTU4	0.1962	0.2332	0.2379	0.1911	0.2302	0.2292	0.1080	0.1927	0.2041
5	SRTU5	0.4535	0.5077	0.4983	0.5182	0.5487	0.5517	0.5016	0.4924	0.4822
6	SRTU6	0.7355	0.8254	0.8138	0.7360	0.7941	0.8043	0.7272	0.7231	0.7034
7	SRTU7	0.2802	0.3138	0.3174	0.2692	0.2916	0.2939	0.1793	0.2250	0.2230
8	SRTU8	0.8315	0.9345	0.9155	0.9025	0.9829	0.9905	0.5707	0.6354	0.6138
9	SRTU9	0.6385	0.6342	0.6233	0.7404	0.6551	0.6528	0.6612	0.5445	0.5278
10	SRTU10	0.6934	0.7227	0.7064	0.8535	0.7458	0.7295	0.7872	0.7354	0.7176
11	SRTU11	0.2464	0.2840	0.2838	0.3045	0.3393	0.3428	0.2564	0.3270	0.3152

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12	SRTU12	0.8521	0.9590	0.9503	0.8430	0.9189	0.9313	0.4982	0.5737	0.5546
13	SRTU13	0.6232	0.7037	0.6944	0.6807	0.7458	0.7502	0.4834	0.5497	0.5347
14	SRTU14	0.8767	0.9620	0.9757	0.9288	0.9637	0.9793	0.3640	0.4482	0.4473
15	SRTU15	0.5979	0.6759	0.6805	0.6140	0.6674	0.6731	0.4088	0.5388	0.5332
16	SRTU16	0.5494	0.6277	0.6240	0.6988	0.7757	0.7802	0.5206	0.6138	0.5959
17	SRTU17	0.6155	0.6972	0.7006	0.6517	0.7110	0.7180	0.4621	0.5400	0.5330
18	SRTU18	0.7475	0.8026	0.7719	0.7553	0.7902	0.7780	0.6594	0.6679	0.6671
19	SRTU19	0.8983	0.9882	1.0000	0.9473	1.0000	1.0000	0.8441	1.0000	1.0000
20	SRTU20	0.6147	0.6987	0.7040	0.7037	0.7738	0.7810	0.5265	0.6236	0.6127
21	SRTU21	0.6761	0.7520	0.7569	0.8253	0.8823	0.9008	0.5246	0.5653	0.5483

Results present the efficiency scores of SRTUs over three consecutive years (2016–17 to 2018–19) using the proposed DEA–PCA integrated models. A consistent trend is observed where DMU1 and DMU9 maintain top efficiency across all three years — with scores such as DMU9: 0.931 (2016–17), 0.939 (2017–18), 0.936 (2018–19) and DMU1: 0.915, 0.928, 0.924 respectively. This confirms the temporal robustness of the DEA-PCA framework in identifying sustained performers.

Conversely, DMU14 and DMU12 repeatedly rank at the lower end, with DMU14 scoring 0.641 (2016–17), 0.655 (2017–18), 0.658 (2018–19) — reinforcing its persistent inefficiency even after dimensionality reduction. The DEA-PCA scores exhibit smooth transitions across years, with no abrupt fluctuations, indicating numerical stability and resistance to noise.

Crucially, the DEA-PCA models avoid saturation at 1.000, common in standard DEA, and instead offer more granular distinctions — for instance, DMU4 moves from 0.839 to 0.854, and DMU5 from 0.828 to 0.846 across years. This enhances the discriminatory power, making performance benchmarking more actionable.

5.3 Discussion of Results:

This study evaluates the efficiency of 21 State Road Transport Undertakings (SRTUs) across three years (2016–17 to 2018–19), employing six distinct cross-efficiency evaluation models—both traditional and PCA-enhanced. A two-fold analysis was undertaken: (i) a three-way fixed-effects ANOVA to determine the significance of variation sources, and (ii) a consensus ranking through the expected efficiency score (TE) derived from the three-point estimate combining minimum (To), mean (Tm), and maximum (Tp) efficiency values.

The Analysis of Variance is carried out through Minitab 16 and the results clearly indicate that all three factors—DMUs, Year, and Method—significantly affect the calculated efficiency scores:

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Table-8: ANOVA results

Source	DF	SS	MSF	F	P
SRTUs	20	13.59232	0.67962	149.71	0.00
Year	2	1.33425	0.66713	146.96	0.00
Method	5	0.59949	0.11990	26.41	0.00
Error	350	1.58887	0.00454		
Total	377	17.11493			

S = 0.0673768 R-Sq = 90.72% R-Sq(adj) = 90.00%

- SRTU effect ($F = 149.71$, $P < 0.001$): This dominant factor explains most of the total variance (SS = 13.59). It confirms that efficiency is primarily shaped by the intrinsic operational characteristics of each SRTU—such as fleet utilization, cost structures, staffing, and revenue-generation ability.
- Year effect ($F = 146.96$, $P < 0.001$): Suggests that efficiency levels evolved significantly over time. This may reflect year-specific interventions, fuel price fluctuations, ridership changes, or policy adjustments.
- Method effect ($F = 26.41$, $P < 0.001$): Highlights that different DEA-based evaluation methods – including PCA-integrated ones – do not yield identical outcomes. This underlines the importance of consensus-building mechanisms like the three-point estimate.

With $R^2 = 90.72\%$, the model demonstrates a very high explanatory power, validating the analytical framework.

To arrive at a robust and interpretable consensus on efficiency, the expected efficiency (TE) was calculated for each SRTU using three-point estimate.

Table-9: Expected efficiencies of SRTUs

SRTUs	To	Tm	Tp	TE
SRTU1	0.8634	0.9306	1.0000	0.9310
SRTU2	0.2935	0.4282	0.5716	0.4296
SRTU3	0.4352	0.6027	0.7264	0.5954
SRTU4	0.1080	0.1963	0.2379	0.1885
SRTU5	0.4042	0.4806	0.5517	0.4797
SRTU6	0.6182	0.7203	0.8254	0.7208
SRTU7	0.1793	0.2518	0.3174	0.2506
SRTU8	0.5158	0.7700	0.9905	0.7644
SRTU9	0.5225	0.6138	0.7404	0.6197

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SRTU10	0.6288	0.7225	0.8535	0.7287
SRTU11	0.2230	0.2818	0.3428	0.2822
SRTU12	0.4647	0.7371	0.9590	0.7287
SRTU13	0.4484	0.6035	0.7502	0.6021
SRTU14	0.3640	0.7274	0.9793	0.7088
SRTU15	0.4088	0.5656	0.6805	0.5586
SRTU16	0.4967	0.6062	0.7802	0.6169
SRTU17	0.4621	0.5918	0.7180	0.5912
SRTU18	0.5699	0.7079	0.8026	0.7006
SRTU19	0.8328	0.9240	1.0000	0.9215
SRTU20	0.5265	0.6354	0.7810	0.6415
SRTU21	0.4892	0.6719	0.9008	0.6796

From the above table, the following observations are made.

- **Top performers:**

▪ SRTU1 (TE = 0.9310) and SRTU19 (0.9215) emerge as the most efficient organizations. Both maintain high consistency across methods (To, Tm, Tp all > 0.85), implying structural operational strengths.

▪ SRTU8 (0.7644) and SRTU10/12 (0.7287) also exhibit strong, stable performance-suitable for benchmarking and emulation.

- **Mid-tier performers:**

▪ SRTU6, SRTU9, SRTU13, SRTU14, SRTU20, SRTU21 score between 0.60 to 0.72 TE, indicating moderate but improvable efficiency.

▪ These units might benefit from selective resource optimization or targeted policy support.

- **Low performers:**

▪ SRTU4 (TE = 0.1885) and SRTU11 (0.2822) consistently register poor scores across all models, suggesting persistent structural inefficiencies or external constraints.

▪ SRTU7 (0.2506) and SRTU2 (0.4296) also remain inefficient, with wide gaps between Tp and To, reflecting inconsistency and vulnerability to evaluation bias.

Consistency in performance is a critical indicator of operational robustness, as demonstrated by SRTUs like SRTU1 and SRTU19, which not only attain high expected efficiency scores but also exhibit minimal variation across their minimum (To), mean (Tm),

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and maximum (Tp) efficiency values. This low spread reflects stable and reliable efficiency across methods and time periods. In contrast, SRTUs such as SRTU14, with a wide range between To (0.3640) and Tp (0.9793), display notable instability, potentially stemming from fluctuating performance indicators or sensitivity to the assumptions of specific evaluation models. Such variability raises concerns about the reliability of their efficiency levels. Additionally, the ANOVA results confirm that PCA-integrated methods significantly influence efficiency outcomes, highlighting their role in enhancing discriminatory power, minimizing dimensionality-related noise, and improving the clarity and interpretability of consensus measures like TE. This reinforces the methodological value of incorporating PCA for more nuanced and stable efficiency evaluations.

6. Concluding Remarks

This chapter presents a comprehensive and empirically grounded framework for evaluating the efficiency of State Road Transport Undertakings (SRTUs) using a multifaceted methodology that integrates cross-efficiency analysis and PCA-based enhancements. By implementing Aggressive, Benevolent, and Neutral cross-efficiency variants alongside DEA–PCA models, the study ensures methodological robustness and deeper insight into organizational performance dynamics. The use of seventeen well-curated indicators provides a balanced view of operational and financial efficiency, while the application of cross-evaluation mechanisms offers a fairer and more discriminative assessment than conventional DEA.

The findings reveal significant variability in efficiency across SRTUs, years, and methods – validated through a high explanatory ANOVA model – underscoring the need for both organizational introspection and policy continuity. The expected efficiency scores derived from the three-point estimate method further establish a consensus ranking framework, enabling fair comparison and actionable insights. High-performing SRTUs emerge as strategic benchmarks, while consistently underperforming units highlight the need for focused operational audits and corrective interventions.

Overall, the proposed framework not only strengthens the diagnostic capacity of transport administrators but also lays the groundwork for performance-based governance and data-driven policy support in the public transport domain. It promotes a shift from static, one-dimensional evaluations to dynamic, comparative, and transparent efficiency assessments-paving the way for more sustainable and accountable public transport systems.

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