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A NEW APPLICATION TO CODING THEORY VIA GUGLIELMO NUMBERS

S. Uygun [1] , M. Topatas [2]

[1] Department of Mathematics, Science Faculty, Gaziantep University, Campus, Gaziantep, Turkey
e-mail: suygun@gantep.edu.tr

[2] Department of Mathematics, Science Faculty, Gaziantep University, Campus, Gaziantep, Turkey
e-mail: mhrbnt.07@gmail.com

Abstract

This paper describes a new coding and decoding technique based on the Guglielmo sequence. Our version is based on a blocked message matrix, with every message matrix encrypted using a special key. The amount of information transmitted over the internet has increased as internet use has become more prevalent. Consequently, the importance of the algorithms used in data encryption methods has increased. The proposed method is fast because it is based on fundamental matrix operations and is suitable for cryptographic applications due to its use of the ASCII character encoding system. For this reason, it differs from classical algebraic methods described in the literature. Using

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Guglielmo numbers we define Guglielmo G-matrices. We can process any given message with a special encryption and we can decode the encrypted massage with the method we use.

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Key Words and Phrases: Encoding-Decoding algorithms, Guglielmo sequence

1 Introduction and Preliminaries

You can easily see that matrix sequences have many applications in scientific areas such as physics, engineering and architecture and art.Communication for humans has shown its importance throughout history. A researcher working on coding/decoding theory is Lester Hill, who created the Hill algorithm. It is included matrix multiplying and matrix inverses. This algorithm is the first step of the modern mathematical theory. However, this method was not safe nowadays. It is relatively broken by using a computer. Alan Turing, a computer scientist, after inventing an encryption machine, broke the encrypted messages made by the Germans Enigma machine during World War II. It helped change the course of World War II. Many researchers have delved into the study of coding/decoding algorithms using special integer sequences in existing literature.

Many researchers have introduced coding/decoding algorithms using Fibonacci, Lucas and ather special numbers [1–15] and their references. Stakhov presented Cassini formula for Fibonacci matrices [1]. Basu and Prasad studied the generalized relations among the code elements for Fibonacci coding theory in [2]. Tahghighi et. al. investigated a generalization of golden cryptography based on k-Fibonacci numbers in [3]. Prajapat et al proposed a method with automatic variable key using Fibonacci Q-matrix in [4]. Esmaeili presented a coding method with error detection and correction based on Fibonacci-polynomials in [5]. Prasad introduced a new

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method for coding/decoding algorithms using Lucas p-numbers [6]. Ucar et al. introduced Fibonacci Q-matrices and R-matrices [7, 8]. Basu and Das found new coding methods with Tribonacci matrices and Fibonacci n-step numbers in [9, 10]. Agarwal et al. developed an algorithm for data encryption & decryption using Fibonacci prime in [11]. Hashemi and Mehraban found some new codes by the k-Fibonacci sequence in [12]. Mehraban and Hashemi studied a coding theory on the generalized balancing sequence in [13]. Mehraban et al. used new sequences from the generalized Pell p-numbers and Mersenne numbers and their application in cryptography in [14]. Prasad and Mahato presented a coding method using generalized Fibonacci matrices with Affine-Hill cipher in [15].

The studies given above encouraged us to study an encryption method using new numbers especially not so well known number sequences. For this reason, we choose Guglielmo number sequences. In this study, we define Guglielmo matrix sequence and investigate a encryption and decryption algorithm using Guglielmo numbers. Our method is based on a blocked message matrix and encryption of every message matrix with a special key. Our study consists of two parts. In the first part, we present a coding algorithm using Guglielmo G-matrix. We explain this model with an example. We divide the message matrix into $2m \times 2m$ block matrices then mapping each character in the message to a different number, we obtain a codded matrix. In the second part, we present a decoding algorithm.

The purpose of this study is not only to increase the reliability of information security technology, but also to provide the ability to verify information at a high rate. Furthermore, Guglielmo matrix sequence improves an unheard of method for the cryptography. It is given a mix model named Guglielmo G-matrix encryption method.

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2 Guglielmo Number and Matrix Sequence

In this part, we use the papers named "Generalized Guglielmo numbers: An investigation of properties of triangular, oblong, and pentagonal numbers via their third-order linear recurrence relations" and "A Study on Matrix Sequence with Generalized Guglielmo Numbers Components" in reference [16, 17].

Definition 1. Guglielmo number sequence $\{T_n\}$ is defined recurrently by

$$T_n = 3T_{n-1} - 3T_{n-2} + T_{n-3}, \quad n \geq 3$$

with the initial conditions $T_0 = 0$, $T_1 = 1$,$T_2 = 3$ where n is any positive integer.

Some of the first Guglielmo numbers are 0,1,3,6,10,15,21,28,36, 45,55,66,78,91, ... For all integers n; Guglielmo numbers using initial conditions can be obtained using Binet formula as $T_n = \frac{n(n+1)}{2}$.

Definition 2. Guglielmo matrix sequence $\{G_n\}$ is defined recurrently by

$$G_n = 3G_{n-1} - 3G_{n-2} + G_{n-3}, \quad n \geq 3$$

with the initial conditions

$$G_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad G_1 = \begin{pmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad , \quad G_2 = \begin{pmatrix} 6 & -8 & 3 \\ 3 & -3 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

where n is any positive integer.

Theorem 3. *The Guglielmo matrix sequence $\{G_n\}$ is denoted using Guglielmo numbers as*

$$G_n = \begin{pmatrix} T_{n+1} & -3T_n + T_{n-1} & T_n \\ T_n & -3T_{n-1} + T_{n-2} & T_{n-1} \\ T_{n-1} & -3T_{n-2} + T_{n-3} & T_{n-2} \end{pmatrix}.$$

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Proof. By induction method, the proof is easily observed. □

Theorem 4. *We are pleased to observe that for any positive integer n, we have*

$$G_n = G_1^n.$$

Proof. For proof, we use induction method. For $n = 0$, the assertion is true. Let assume that it is also true for $k \leq n$. Then for $k = n + 1$, we get

$$G_1^{n+1} = G_1^n G_1 = \begin{pmatrix} T_{n+1} & -3T_n + T_{n-1} & T_n \\ T_n & -3T_{n-1} + T_{n-2} & T_{n-1} \\ T_{n-1} & -3T_{n-2} + T_{n-3} & T_{n-2} \end{pmatrix} \begin{pmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = G_{n+1}.$$

□

Especially $det \begin{pmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = 1$. So, $det(G_n) = 1$.

Theorem 5. *For every integer n; the Binet's formula of the Guglielmo matrix sequences is given by*

$$G_n = \begin{pmatrix} \frac{(n+2)(n+1)}{2} & -n^2 - 2n & \frac{n(n+1)}{2} \\ \frac{n(n+1)}{2} & 1 - n^2 & \frac{n(n-1)}{2} \\ \frac{n(n-1)}{2} & 2n - n^2 & \frac{(n-2)(n-1)}{2} \end{pmatrix}.$$

2.1 Coding/ Decoding Algorithm with Guglielmo Numbers

In this part, we instigate a method for enciphering and deciphering the Guglielmo sequence.

Before using the method (M.....is not zero. In this part, we give a new coding algorithm using third order Guglielmo matrix sequence. We put our message in a matrix of third order. Dividing the $3m \times 3m$ message matrix M into 3×3 block matrices from left to right, M_i for $i = 1, ..., m^2$, we get a new coding method.

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If we demonstrate the symbols of the coding method, that matrices M_i, E_i and G_n are of the following forms:

$$M_i = \begin{pmatrix} m_1^i & m_2^i & m_3^i \\ m_4^i & m_5^i & m_6^i \\ m_7^i & m_8^i & m_9^i \end{pmatrix}, \quad E_i = \begin{pmatrix} e_1^i & e_2^i & e_3^i \\ e_4^i & e_5^i & e_6^i \\ e_7^i & e_8^i & e_9^i \end{pmatrix}, \; G_n = \begin{pmatrix} g_1 & g_2 & g_3 \\ g_4 & g_5 & g_6 \\ g_7 & g_8 & g_9 \end{pmatrix}.$$

Now, we explain the following new coding and decoding algorithms.

Coding Algorithm by third order Guglielmo matrix

1. Divide the message matrix M into blocks M_i $(1 \leq i \leq m)$. The number of the block matrices M_i is denoted by m. According to m, let's assume that n is chosen as follows:

$$n = \begin{cases} m-1, & m \leq 4 \\ 2m-5, & m > 4 \end{cases} \tag{1}$$

2. Using the choosen n, we write the following letter table according to mod 40 (this table can be extended according to the used characters in the message matrix). We begin "n " for the first character.

A	B	C	D	E	F	G	H	I	J
n	n+1	n+2	n+3	n+4	n+5	n+6	n+7	n+8	n+9
K	L	M	N	O	P	R	Q	S	T
n+10	n+11	n+12	n+13	n+14	n+15	n+16	n+17	n+18	n+19
U	V	W	X	Y	Z	:	.	,	-
n+20	n+21	n+22	n+23	n+24	n+25	n+26	n+27	n+28	n+29
0	1	2	3	4	5	6	7	8	9
n+30	n+31	n+32	n+33	n+34	n+35	n+36	n+37	n+38	n+39

3. Determine m_j^i,$(1 \leq j \leq 9)$

4. Compute $det(M_i) = \Psi_i$.

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5. Construct the coding matrix as $F = [\Psi_i,\ m^i_j,]_{j\in\{1,2,3,4,5,6,8,9\}}$.

Decoding Algorithm

1. Compute G_n.

2. Evaluate

$$\begin{array}{l} g_1m^i_1 + g_2m^i_4 + g_3m^i_7 \to e^i_1 \\ g_1m^i_2 + g_2m^i_5 + g_3m^i_8 \to e^i_2 \\ g_1m^i_3 + g_2m^i_6 + g_3m^i_9 \to e^i_3 \\ g_4m^i_1 + g_5m^i_4 + g_6m^i_7 \to e^i_4 \\ g_4m^i_2 + g_5m^i_5 + g_6m^i_8 \to e^i_5 \\ g_4m^i_3 + g_5m^i_6 + g_6m^i_9 \to e^i_6 \\ g_7m^i_1 + g_8m^i_4 + g_9m^i_7 \to e^i_7 \\ g_7m^i_2 + g_8m^i_5 + g_9m^i_8 \to e^i_8 \\ g_7m^i_3 + g_8m^i_6 + g_9m^i_9 \to e^i_9 \end{array}$$

3. Evaluate

$$d_i = (g_1m^i_1+g_2m^i_4+g_3x_i)\begin{vmatrix} e^i_5 & e^i_6 \\ e^i_8 & e^i_9 \end{vmatrix} - (g_4m^i_1 + g_5m^i_4 + g_6x_i)\begin{vmatrix} e^i_2 & e^i_3 \\ e^i_8 & e^i_9 \end{vmatrix} + (g_7m^i_1 + g_8m^i_4 + g_9x_i)\begin{vmatrix} e^i_2 & e^i_3 \\ e^i_5 & e^i_6 \end{vmatrix}$$

4. Substitute for $x_i = m^i_7$

5. Construct M_i and therefore M.

In the following example we give an application of the above algorithm.

Example 6. *Let's consider the message matrix for the following message text*

"*MY FAVORITE LESSON IS MATHEMATICS.*"

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We get the following message matrix M:

$$M = \begin{bmatrix} M & Y & - & F & A & V \\ O & R & I & T & E & - \\ L & E & S & S & O & N \\ - & I & S & - & M & A \\ T & H & E & M & A & T \\ I & C & S & . & - & - \end{bmatrix}$$

1. We divide the message matrix M of size 3×3 into the matrices, named M_i $(1 \leq i \leq 4)$, from left to right:

$$M_1 = \begin{bmatrix} M & Y & - \\ O & R & I \\ L & E & S \end{bmatrix}, \quad M_2 = \begin{bmatrix} F & A & V \\ T & E & - \\ S & O & N \end{bmatrix},$$

$$M_3 = \begin{bmatrix} - & I & S \\ T & H & E \\ I & C & S \end{bmatrix}, \quad M_4 = \begin{bmatrix} - & M & A \\ M & A & T \\ . & - & - \end{bmatrix}$$

2. By 1, we get $n = 3$. We use the "character table" for the message matrix M: The elements of the blocks M_i $(1 \leq i \leq 4)$ as follows: We calculate the determinants d_i of the blocks M_i:

$$M_1 = \begin{bmatrix} 15 & 27 & 32 \\ 17 & 19 & 11 \\ 14 & 7 & 21 \end{bmatrix} \text{ then } d_1 = \det(M_1) = -5355$$

$$M_2 = \begin{bmatrix} 8 & 16 & 24 \\ 22 & 7 & 32 \\ 21 & 17 & 16 \end{bmatrix} \text{ then } d_2 = \det(M_2) = 7112$$

$$M_3 = \begin{bmatrix} 32 & 11 & 21 \\ 22 & 10 & 7 \\ 11 & 5 & 21 \end{bmatrix} \text{ then } d_3 = \det(M_3) = 1365$$

$$M_4 = \begin{bmatrix} 32 & 16 & 3 \\ 16 & 3 & 12 \\ 30 & 32 & 32 \end{bmatrix} \text{ then } d_4 = \det(M_4) = -10382$$

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3. We construct the M matrix:

$$M = \begin{bmatrix} -5355 & 15 & 27 & 32 & 17 & 19 & 11 & 7 & 21 \\ 7112 & 8 & 16 & 24 & 22 & 7 & 32 & 17 & 16 \\ 1365 & 32 & 11 & 21 & 22 & 10 & 7 & 5 & 21 \\ -10382 & 32 & 16 & 3 & 16 & 3 & 12 & 32 & 32 \end{bmatrix}$$

Decoding Algorithm

1. We choose $n = 3$ and determine G_3 as

$$g_1 = 10 \quad g_2 = -15 \quad g_3 = 6$$
$$g_4 = 6 \quad g_5 = -8 \quad g_6 = 3$$
$$g_7 = 3 \quad g_8 = -3 \quad g_9 = 1$$

2. Evaluate the followings respecticely to construct the matrix E_i:

$$\left[\; e_1^1 = -21 \quad e_2^1 = 27 \quad e_3^1 = 281 \quad e_4^1 = -4 \quad e_5^1 = 31 \quad e_6^1 = 167 \quad e_7^1 = 8 \quad e_8^1 = 31 \quad e_9^1 = 84 \; \right]$$
$$\left[\; e_1^2 = -124 \quad e_2^2 = 157 \quad e_3^2 = -144 \quad e_4^2 = -65 \; \right]$$
$$\left[\; e_5^2 = 91 \quad e_6^2 = -64 \quad e_7^2 = -21 \quad e_8^2 = 44 \quad e_9^2 = -8 \; \right]$$
$$\left[\; e_1^3 = 56 \quad e_2^3 = -10 \quad e_3^3 = 231 \quad e_4^3 = 49 \quad e_5^3 = 1 \quad e_6^3 = 133 \quad e_7^3 = 41 \quad e_8^3 = 8 \quad e_9^3 = 63 \; \right]$$
$$\left[\; e_1^4 = 260 \quad e_2^4 = 168 \quad e_3^4 = 5 \quad e_4^4 = 154 \quad e_5^4 = 168 \quad e_6^4 = 18 \quad e_7^4 = 78 \quad e_8^4 = 71 \quad e_9^4 = 5 \; \right]$$

3. We calculate the elements

$$-5355 = (150 - 15 * 17 + 6x_1) \begin{vmatrix} 31 & 167 \\ 31 & 84 \end{vmatrix} - (90 - 8 * 17 + 3x_1) \begin{vmatrix} 27 & 281 \\ 31 & 84 \end{vmatrix} + (45 - 3 * 17 + x_1) \begin{vmatrix} 27 & 281 \\ 31 & 167 \end{vmatrix}$$

$$-5355 = (-105 + 6x_1)(-2573) - (-46 + 3x_1)(-6443) + (-6 + x_1)(-4202)$$

$$5355 + 105 * 2573 - 46 * 6443 + 6 * 4202 = 4354 = 2573 * 6 - 6443 * 3 + 4202 = 311$$

$$x_1 = 14$$

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and

$$7112=(-250+6x_2)\begin{vmatrix} 91 & -64 \\ 44 & -8 \end{vmatrix}-(-128+3x_2)\begin{vmatrix} 157 & -144 \\ 44 & -8 \end{vmatrix}+(x_2-42)\begin{vmatrix} 157 & -144 \\ 91 & -64 \end{vmatrix}$$
$$7112=(-250+6x_2)*2088-(-128+3x_2)*5080+(-42+x_2)*3056$$
$$344x_2=112+7112$$
$$x_2=21$$

and

$$1365=(10x_3+93)\begin{vmatrix} -631 & 246 \\ -431 & 169 \end{vmatrix}-(15x_3+103)\begin{vmatrix} 449 & 246 \\ 301 & 169 \end{vmatrix}+(6x_3+36)\begin{vmatrix} 449 & -631 \\ 301 & -431 \end{vmatrix}$$
$$1365=(10x_3+93)(-613)+(15x_3+103)(1835)+(6x_3+36)(-3588)$$
$$1365=-133x_3+2828$$
$$x_3=11$$

and

$$-10382=(10x_4+288)\begin{vmatrix} -617 & 243 \\ -300 & 117 \end{vmatrix}+(15x_4+352)\begin{vmatrix} 425 & 243 \\ 214 & 117 \end{vmatrix}+(128+6x_4)\begin{vmatrix} 425 & -617 \\ 214 & -300 \end{vmatrix}$$
$$-10382=(10x_4+288)(711)+(15x_4+352)(-2277)+(128+6x_4)(4538)$$
$$-10382=183x_4-15872$$
$$x_4=30$$

4. We determine x_i

$$x_1=m_7^1=14 \quad x_2=m_7^2=21 \quad x_3=m_7^3=11 \quad x_4=m_7^4=30$$

5. We construct the block matrices M_i by

$$M_1=\begin{bmatrix} 15 & 27 & 32 \\ 17 & 19 & 11 \\ 14 & 7 & 21 \end{bmatrix},\ M_2=\begin{bmatrix} 8 & 16 & 24 \\ 22 & 7 & 32 \\ 21 & 17 & 16 \end{bmatrix},\ M_3=\begin{bmatrix} 32 & 11 & 21 \\ 22 & 10 & 7 \\ 11 & 5 & 21 \end{bmatrix},\ M_4=\begin{bmatrix} 32 & 16 & 3 \\ 16 & 3 & 12 \\ 30 & 32 & 32 \end{bmatrix}$$

Hence, we obtain the message matrix M:

Conclusions

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In this article, we developed a new algorithm in coding theory with the help of G-matrix. In this algorithm, we divide the desired message into blocks and transform the message matrix into an encoded message matrix. In the decryption process, we converted the encrypted message matrix to the desired message matrix using the G-matrix.

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