International Journal of Applied Mathematics
Volume 38 No. 4s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

EPIDEMIC CONTACT TRACING AND CONTAINMENT IN FUZZY SOCIAL GRAPHS USING STRENGTH-BASED FUZZY LOCATION TOTAL DOMINATION SETS

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Abstract
In this study, we investigate the concept of fuzzy locating total dominating sets (FLTDS) in simple connected fuzzy graphs. A subset $\mathfrak{D}_t \subseteq V(f_G)$ is defined as a FLTDS if every
if each vertex outside of $\mathfrak{D}_t$ is next to at least one $\mathfrak{D}_t$ vertex via a strong arc (fuzzy domination), and each such vertex has a unique strength-based representation vector with respect to $\mathfrak{D}_t$ (fuzzy locating property). The strength-based representation captures the degrees of connectivity between a vertex and the elements of the dominating set, reflecting the inherent fuzziness of relationships in the graph. We define the fuzzy locating total domination number $\gamma_L^t(f_G)$ as the minimum cardinality of a FLTDS in a fuzzy graph f_G. The paper presents foundational results and characterizations of FLTDS in selected classes of fuzzy graphs, providing insights into their structural properties and potential applications in network monitoring, fault detection, and information spread in uncertain environments.
Keywords: fuzzy Locating set, fuzzy dominating set, fuzzy locating dominating set, fuzzy locating total dominating set, fuzzy locating total dominating number, strength of connectedness, strong path.

1.1 Introduction

Fuzzy graph theory has emerged as a powerful mathematical tool for modeling systems where relationships between entities are uncertain, imprecise, or variable. In such contexts, classical graph theory falls short of capturing the real-world complexity of interactions. One important area of study in this domain is the integration of domination and location properties in fuzzy graphs, particularly relevant in monitoring and detection systems such as wireless sensor networks, fault localization, and intrusion detection.

In this paper, we introduce and explore the concept of the Fuzzy Locating Total Dominating Set (FLTDS) in fuzzy graphs. Let $\mathfrak{D}_t \subseteq V(f_G)$ be a fuzzy subset of vertices such that $|\mathfrak{D}_t| \geq 1$. For each $v \in V \setminus \mathfrak{D}_t$ define its strength based representation vector with respect $\mathfrak{D}_t$ as $\mathcal{L}oc(v|\mathfrak{D}_t) = \{\mu^\infty(v, u_1), \ldots \mu^\infty(v, u_k)\}$,
Where $\mathfrak{D}_t = \{u_1, \ldots \ldots u_m\} \subseteq V(f_G)$. Then the set $\mathfrak{D}_t$ is called fuzzy locating total dominating set (FLTDS) of the fuzzy graph f_G if the set $\mathfrak{D}_t$ adheres to each of the following requirements.

1. Fuzzy Domination: For each vertex $v \in V$ there exists at least one vertex $u \in \mathfrak{D}_t$ such that $\mu(u, v) > 0$, and the edge (u, v) is a strong arc.

2. Location Distinguishability: For any two distinct vertices $v_1, v_2 \in V \setminus \mathfrak{D}_t$, $\mathcal{L}oc(v_1|\mathfrak{D}_t) \neq \mathcal{L}oc(v_2|\mathfrak{D}_t)$ (FLTDS). A set $\mathfrak{D}_t$ is referred to as a FLTDS of f_G, The lowest cardinality is its FLTD number is $\gamma_L^t(f_G)$.

1.2 Literature Review

The foundation of fuzzy mathematics was laid by Lotfi Asker Zadeh in 1965 [1], and the formal framework of fuzzy graphs was introduced by Rosenfeld in 1975 [5, 14]. That same year, Slater proposed the concept of a resolving number in graphs [7, 8], marking the beginning of location-based identification in graph structures. Building upon these ideas, Bailey and Cameron (2009) [3] extended the concept of resolving sets to locating-dominating sets, which have since found application in areas like sensor placement**,** fault detection**,** and power grid monitoring. In the realm of fuzzy graphs. The concept of domination was introduced **by** A. Somasundaram and S. Somasundaram [6]. The notion of strong arcs in fuzzy graphs was explored by K. R. Bhutani and A. Rosenfeld [18]. Later, O. T. Manjusha and Sunitha integrated strong arcs into studies of fuzzy domination and total domination [20]. A. Nagoorgani and V. T. Chandrasekaran also made significant contributions to the development of fuzzy domination [17]. In the recent decade, Shanmugapriya R. introduced the fuzzy resolving set in 2019 [13], expanding the field of fuzzy location-based analysis. Building upon this, Amuthini D. and Shanmugapriya R**.** introduced the concept of maximum strength of connectedness in fuzzy graphs [12], providing a foundation for strength-based representations in fuzzy locating structures.

1.3 Motivation

Many real-world networks such as social, communication, and sensor systems exhibit uncertainty in the strength of connections between nodes. Classical graph models fail to capture this fuzziness, making fuzzy graphs a more suitable framework. In such systems, effective monitoring requires not only covering all nodes (domination) but also uniquely identifying each one (location). This motivates the study of Fuzzy Locating Total Dominating Sets (FLTDS), which integrate both goals under uncertain conditions. FLTDS are particularly useful in applications like fault detection, epidemic tracing, and intrusion monitoring, where optimizing resource use under imprecise information is crucial. This work formalizes FLTDS and explores their behavior in special classes of fuzzy graphs.

1.4 Novelty and Contribution of the Study

This study introduces the novel concept of the Fuzzy Locating Total Dominating Set (FLTDS), which unifies domination and location in fuzzy graphs something not previously addressed in existing literature. Unlike prior works that treat these concepts separately or only in crisp graphs, this research defines, characterizes, and computes the fuzzy locating total domination number in uncertain environments. Extension of classical graph parameters into the fuzzy domain, enabling new research directions in AI, cybersecurity, and smart systems. This work bridges a theoretical gap and enhances practical modeling in uncertain, dynamic systems.

1.5 Paper Organization

The paper is structured as follows: Section 2 covers essential definitions and preliminaries. Section 3 introduces the concepts of Fuzzy Locating Total Dominating Sets, their domination number, and strength of connectedness, with examples and theorems. Section 4 explores a real-

International Journal of Applied Mathematics
Volume 38 No. 4s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

world application of the proposed framework. Section 5 concludes with key findings and future research directions. A table of notations and abbreviations is also provided.

Table 1. A few significant notations and acronyms

Notations/Symbols	Meaning
F_G	Fuzzy graph
F_H	Fuzzy subgraph
$FLTDS$	Fuzzy locating total dominating set
$\mu^{\infty}(U,V)$	Strength of connectedness
$CF_G\ (k\sigma)$	Complete fuzzy graph

Fundamental Definitions

Definition 2.1

An ordered triple $G\ (V, \sigma, \mu)$ can be used to describe a fuzzy graph. With V being a non-empty collection of vertices and a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, it can be shown that for any $u, v \in V, \mu\ (u, v) < \sigma\ (u) \Lambda\, \sigma\ (v)$, where Λ indicates the minimum. We refer to the fuzzy vertex set of G as μ and the fuzzy edge set of G as σ.

Definition 2.2

A fuzzy graph The related fuzzy vertex and fuzzy edge functions are denoted by r and ρ, respectively, for the fuzzy subgraph $H = (r, \rho)$ of $G = (\sigma, \mu)$. Both $r\ (u) \leq \sigma\ (u)$ for each u in V and $\rho\ (u, v) \leq \mu\ (u, v)$ for each u, v in V are satisfied by the fuzzy subgraph.

Definition 2.3

A complete fuzzy graph defined by the condition $\mu\ (u, v) = min\ \{\sigma(u), \sigma(v)\}$, for all $u, v \in V$.

Definition 2.4

In a fuzzy graph $G\ (V, \sigma, \mu)$, a path P of length n is a sequence of distinct vertices $y_0, y_1, y_2, \ldots y_n$ such that the membership value between each pair of consecutive vertices is nonzero, $\mu\ (y_{i-1}, y_i) > 0,\ i = 1, 2, \ldots, n.$ When n is one or greater, it denotes the length of the path. Additionally, a path can be seen as a single vertex, denoted as *y,* where the path's length is 0. The arcs of the path are the successive pairs (y_{i-1}, y_i). A path P is identified as a cycle if the first and last nodes are the same $(y_0 = y_n)$ and n is three or more. If a cycle P contains more than one weakest arc, it's termed a fuzzy cycle (f-cycle).

Definition 2.5

The weight of a path's weakest arc is used to calculate the path's strength. It is common practice to specify a path's strength as $\sigma(y_0)$, when it has no length.

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Definition 2.6

In a fuzzy graph $G(V, \sigma, \mu)$, the strength of connectedness between two nodes, u and v, is represented as $CONNG(u, v)$ or written as $\omega(u, v)$. This is the strongest path strength that connects them. If a path P's strength equals $CONNG\ (u, v)$, it is considered the strongest path between u and v.

Definition 2.7

Let G be a f_G on V and $S \subseteq V$. The fuzzy cardinality of S is defined as $\sum_{u \in S} \sigma(u)$. The order p and size q of the fuzzy graph $G(\sigma, \mu)$ are given by
$p = \sum_{u \in V} \sigma(u) \quad q = \sum_{(u,v) \in E} \mu(u, v)$.

3. Fuzzy Locating total dominating set (FLTDS)

3.1 Definition (Fuzzy Dominate)

Let G be a fuzzy graph with two vertices, u and v. If the arc (u, v) is strong, we say that u dominates v. A strong vertex dominating set $\mathfrak{D} \subseteq V(f_G)$ if, for every $v \in V \setminus \mathfrak{D}$, there exists $u \in \mathfrak{D}$, such that u dominates v. If no appropriate subset of a strong vertex dominating set $\mathfrak{D}$, is also a strong vertex dominating set, then $\mathfrak{D}$, is considered minimal. The smallest cardinality among all minimal strong vertex dominating sets is the strong dominating number, represented by $\gamma(f_G)$.

3.2 Definition

A **fuzzy total dominating set** in a fuzzy graph $f_G(V, \sigma, \mu)$ is a subset of vertices such that every vertex in the graph is dominated by at least one vertex in the set, and each vertex in the set is also dominated by at least one other vertex within the set.

3.3 Definition (Fuzzy Locating set/code)

Let $\mathfrak{D} = \{v_1, v_2, , \ldots.. v_k\}$ be an ordered set of k distinct vertices in a connected fuzzy graph. $f_G(V, \sigma, \mu)$. For any vertex $u \in V(f_G)$, define the k-tuple:

$$\mathcal{L}oc(v|\mathfrak{D}) = \{\mu^\infty(u, v_1), \mu^\infty(u, v_2), \ldots.. \mu^\infty(u, v_k)\}$$

Where $\mu^\infty(u, v_i)$ denotes the strength of connectedness between vertex u and v_i, $i = 1,2 \ldots k$). The set $\mathfrak{D}$ is termed a fuzzy locating set if the vector representation $\mathcal{L}oc(v|\mathfrak{D})$ are distinct for every vertex $v \in V(f_G)$, that is, no two vertices share the same k tuple. The FLN denoted by $f_{ln}(G)$, is the lowest cardinality among all such FLS in the fuzzy graph.

3.4 Definition. Fuzzy Locating Total Dominating Set

Consider a simple connected fuzzy graph $f_G(V, \sigma, \mu)$. Let $\mathfrak{D}_t \subseteq V(f_G)$ be a fuzzy subset of vertices such that $|\mathfrak{D}_t| \geq 1$, for each vertex $v \in V \setminus \mathfrak{D}_t$ define its strength based representation vector with respect $\mathfrak{D}_t$ as

$\mathcal{L}oc(v|\mathfrak{D}_t) = \{\mu^\infty(u, v_1), \mu^\infty(u, v_2), \ldots.. \mu^\infty(u, v_k)\}$

where $\mathfrak{D}_t = \{v_1, v_2, , \ldots.. v_m\} \subseteq V(f_G)$. Then the set $\mathfrak{D}_t$ is called fuzzy locating total dominating set (FLTDS) of the fuzzy graph f_G if it satisfies the following two conditions.

For every vertex $v \in V$ there exists at least one vertex $u \in \mathfrak{D}_t$ such that $\mu(u, v) > 0$, and the edge (u, v) is a strong arc, and For any two distinct vertices $v_1, v_2 \in V \setminus \mathfrak{D}_t$, $\mathcal{L}oc(v_1|\mathfrak{D}_t) \neq \mathcal{L}oc(v_2|\mathfrak{D}_t)$ (FLTDS). A set $\mathfrak{D}_t$ is referred to as a FLTDS of f_G, The lowest cardinality is its FLTD number is $\gamma_L^t(f_G)$.

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Example: 1

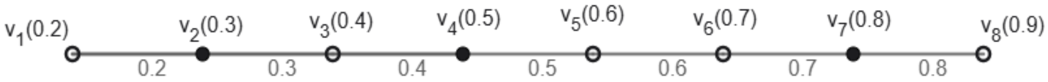
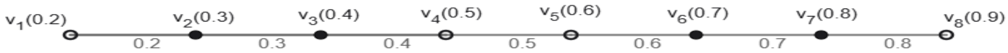


Figure (a) $\mathfrak{D}$ =(v_2, v_4, v_7) forms a FLTDS of P_8
$\gamma^L(f_G)$ =0.3+0.5+0.8=1.6



$\boldsymbol{Table}$: **1**

Vertex $\boldsymbol{v_i}$	**Depiction** $\boldsymbol{v_i\vert\mathfrak{D}_t}$	$\boldsymbol{\mathcal{L}oc(v_i\vert\mathfrak{D}_t)}$
v_1	$v_1\vert\mathfrak{D}_t$	(0.2, 0.2, 0.2, 0.2)
v_2	$v_2\vert\mathfrak{D}_t$	(0, 0.3, 0.3, 0.3)
v_3	$v_3\vert\mathfrak{D}_t$	(0.3, 0, 0.4, 0.4)
v_4	$v_4\vert\mathfrak{D}_t$	(0.3, 0.4, 0.5, 0.5)
v_5	$v_5\vert\mathfrak{D}_t$	(0.3, 0.4, 0.6, 0.6)
v_6	$v_6\vert\mathfrak{D}_t$	(0.3, 0.4, 0, 0.7)
v_7	$v_7\vert\mathfrak{D}_t$	(0.3, 0.4, 0.7, 0)
v_8	$v_8\vert\mathfrak{D}_t$	(0.3, 0.4, 0.7, 0.8)

Figure (b) $\mathfrak{D}_t$ = (v_2, v_3, v_6, v_7) be a FLTDS of P_8
$\gamma_L^t(f_G)$=0.3+0.4+0.7+0.8= 2.2

3.5.1 Known results in fuzzy locating number:

- $1 \le F_{ln}(G) \le n - 1$.
- If H is an induced fuzzy subgraph of G, then
$$F_{ln}(H) \le F_{ln}(G)$$
- Let G be a f_G. Then the $F_{ln}(G) = 1$ if and only if $f_G = K_1$.
- For any CF_G (k_n) with $n \ge 2$, the least FLN satisfies $F_{ln}(G) = n - 1$.
- Let G be a f_G Then $F_{ln}(G) = 1$ if and only if G is a path P_2 or P_3
- Let G be a f_G of order n. Then the FLN $F_{ln}(G) = n - 1$ if and only if G is a CF_G, i.e.,$G = K_n$.
- For any F_g Tree (T) with $n \ge 3$, the least FLN satisfies $F_{ln}(G) = 2$

3.5.2 Theorem (Existence of FLTDS)

Every non-empty fuzzy graph f_G has a FLTDS.

Proof:

Since G is connected fuzzy graph, every vertex is connected by a path with non-zero edge membership. Choose $\mathfrak{D} = v \setminus \{v_0\}$ for some arbitrary$v_0 \in V$. Now for fuzzy Domination, v_0 is adjacent to some $u \in \mathfrak{D}$ and (v_0, u) are strong arc. Also the vertices in the subset of $\mathfrak{D}$ are

Received: August 09, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

not isolated. For fuzzy locating, since$|\mathfrak{D}| = n - 1$, all remaining vertices are distinguishable based on their fuzzy adjacency vectors with respect to $\mathfrak{D}$ unless all edges are equal. In that case choose another vertex or add an extra vertex to $\mathfrak{D}$ until locating condition is met. If all the vertices are distinguishable and the subset $\mathfrak{D}$ are not isolated then fuzzy connected graph G has a FLTDS..

3.5.3 Corollary

For an connected fuzzy graph G, $F_{ln}(G) \leq \gamma_{\mathcal{L}}^{f}(G) \leq \gamma_{\mathcal{L}}^{t}(G)$

3.5.4 Theorem

If G is a connected fuzzy graph such that $\gamma_{\mathcal{L}}^{f}(G) = 2$, then it contains at most 5 vertices.

Proof:

Assume$\gamma_{\mathcal{L}}^{f}(G) = 2$. i.e., the least FLDS of G has size 2. Since there are at least three vertices in G, clearly the number of distinct representation codes must satisfy$|\sigma(G)| \geq 3$. But $\gamma_{\mathcal{L}}^{f}(G) = 1$ is enough if the fuzzy graph is a path. To prove if $\gamma_{\mathcal{L}}^{f}(G) = 2$ it contains$|\sigma(G)| = 5$. Let $\sigma_i \in \sigma(G)$ for $i = 1, 2,3,4,5$, and let $\mathfrak{D} = \{\sigma_1, \sigma_2\}$ be the FLTDS which is dominates its neighbor, and the vertices in subset Đ are not isolated In this case, with only two reference vertices, it is possible to generate a unique representation codes. If $\gamma_{\mathcal{L}}^{t}(G) = 2$, for a connected fuzzy graph G, then $|\sigma(G)| \leq 5$.

3.5.5 Theorem

Let $G\ (\sigma, \mu\ )$ be a connected fuzzy graph of order $n \geq 2$ then

$\gamma_{\mathcal{L}}^{t}(G) = n - 1 \Leftrightarrow f_G = K_n$

Proof:

Assume $f_G = K_4$, the complete fuzzy graph on four vertices, In such a graph, every pair of vertices is connected with a nonzero membership value, meaning any single vertex say v_1, can dominate the entire fuzzy graph. Thus, the fuzzy total domination number $\gamma_{\mathcal{L}}^{t}(G) = 1$. However, the dominating set $\mathfrak{D} = \{v_1\}$ does not serve as a fuzzy total locating dominating set. While v_1 dominating all other vertices, it fails to uniquely locate them. Specifically, the strength vectors relative to Đ are follows

$\wp_{\mathfrak{D}}(v_1) = \{0\}$, $\wp_{\mathfrak{D}}(v_2) = \{0.4\}$, $\wp_{\mathfrak{D}}(v_3) = \{0.4\}$, $\wp_{\mathfrak{D}}(v_4) = \{0.4\}$. Here, vertices $\{v_1, v_2, v_3\}$ share identical location vectors, To ensure that each vertex is uniquely identified by its strength of connectedness to the fuzzy dominating set must include all vertices except one. Let $\mathfrak{D} = \{v_1, v_2, v_3\}$. Each four vertices now has a distinct vector of connection strengths to the set $\mathfrak{D}$, satisfying the fuzzy total locating domination requirement. Therefore, in the complete fuzzy graph K_4 we have

$\gamma_{\mathcal{L}}^{f}(G) = 3 = n - 1$. Since K_4 is complete fuzzy graph, any FLDS must include all the vertices except one, so $\gamma_{\mathcal{L}}^{t}(G) \geq 3$. Therefore $\gamma_{\mathcal{L}}^{\mathrm{f}}(\mathrm{G}) = 3$, conclude for all complete fuzzy graphs $\mathrm{K_n}$, $\gamma_{\mathcal{L}}^{\mathrm{t}}(\mathrm{G}) = \mathrm{n} - 1$.

This logic extends naturally to the general case, for any complete fuzzy graph $\mathrm{K_n}$ all vertices are mutually connected. To uniquely identify each vertex via its strength vector, we must include n −1 vertices in the FLTDS. Hence:

$\gamma_{\mathcal{L}}^{\mathrm{t}}(\mathrm{G}) = \mathrm{n} - 1$ if and only if $\mathrm{G} = \mathrm{K_n}$

For example (Complete Fuzzy Graph k_4)

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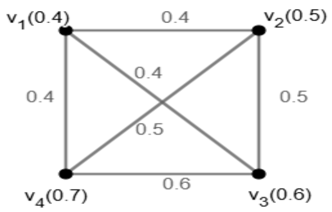

$figure\ c.\ (\boldsymbol{k_4})$

$\boldsymbol{Table}$ **2.**

Vertexv_i	Depiction $v_i\|\mathfrak{D}_t$	$\mathcal{L}oc(v_i\|\mathfrak{D}_t)$
v_1	$v_1\|\mathfrak{D}_t$	(0, 0.4, 0.4)
v_2	$v_2\|\mathfrak{D}_t$	(0.4, 0, 0.5)
v_3	$v_3\|\mathfrak{D}_t$	(0.4, 0.5, 0)
v_4	$v_4\|\mathfrak{D}_t$	(0.4, 0.5, 0.6)

Figure (b) $\mathfrak{D}_t$ = (v_1, v_2, v_3) forms a fuzzy locating total Dominant set of K_n
$\gamma_L^t(f_G)$=0.4+0.5+0.6= 1.5
Hence, for all complete fuzzy graph K_n, the FLTDS is $n-1$.

3.6 Locating total Domination number on special graphs

Definition3.6.1 (N-Sunlet graph)

An **n-sunlet graph**, denoted by S_n, is a graph consisting of 2_n vertices. It is constructed by attaching one pendant (degree-one) vertex to each vertex of a cycle c_n That is, for every vertex in the cycle, a unique pendant edge is added, resulting in a symmetric, flower-like structure with n cycle vertices and n pendant vertices.

Theorem 3.6.2

The FLTDS of a n-sunlet graph, $\gamma_L^t(S_n) = n$.

Proof:

Case (i) Let $n = 3$

There are six vertices in the 3-sunlet graph S_3. Let v_1, v_2, v_3 be the cycle vertices and their respective pendant (leaf) vertices be u_1, u_2, u_3 with u_i exclusively next to v_i. Let the candidate FLTDS be the set $\mathfrak{D}_t = \{v_1, v_2, v_3\}$. To ensure complete dominance, every cycle vertex $v_i \in \mathfrak{D}_t$ is next to its matching pendant u_i. All of the graph's vertices that meet the locating criteria can be used to identify each vertex. Assume now that
$\mathfrak{D}_t' = \mathfrak{D}_t \setminus \{v_2\}$. i.e., $\{v_1, v_3\}$ is a smaller set. In this case, the pendent vertex is not next to any vertex in $\mathfrak{D}_t'$ and is hence not dominated.
$\mathfrak{D}_t'$ is therefore not a total dominating set. Given that $\mathfrak{D}_t$ is a minimum FLTDS and

$$\gamma_L^t(S_3) = \sum_{i=1}^{3} \mathfrak{D}_t\,(v_i)$$

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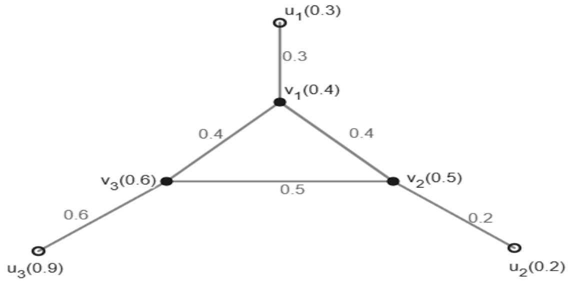


(c) 3-sunlet graph

Table 3

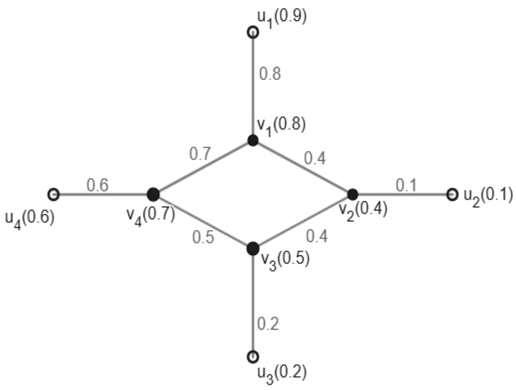
Vertexv_i	Depiction $v_i\|\mathfrak{D}_t$	$\mathcal{L}oc(v_i\|\mathfrak{D}_t)$
v_1	$v_1\|\mathfrak{D}_t$	(0, 0.4, 0.4)
v_2	$v_2\|\mathfrak{D}_t$	(0.4, 0, 0.5)
v_3	$v_3\|\mathfrak{D}_t$	(0.4, 0.5, 0)
u_1	$u_1\|\mathfrak{D}_t$	(0.3, 0.3, 0.3)
u_2	$u_2\|\mathfrak{D}_t$	(0.2, 0.2, 0.2)
u_3	$u_3\|\mathfrak{D}_t$	(0.4, 0.5, 0.6)

$\mathfrak{D}_t(S_3) = \{v_1, v_2, v_3\}$, $\gamma_L^t(S_3) = \sum_{i=1}^{3} \mathfrak{D}_t\,(v_i)$= 0.4+0.5+0.6=1.5

Case (ii) $n = 4$

Similarly, the 4-sunlet graph S_4 has 8 vertices: cycle vertices v_1, v_2, v_3, v_4 and pendant vertices u_1, u_2, u_3, u_4. Let $\mathfrak{D}_t = \{v_1, ., v_4\}$. Each cycle the vertex $v_i \in \mathfrak{D}_t$ adjacent to a unique pendant u_i satisfying fuzzy total domination. . Each vertex in the graph is distinguishable by all its vertices satisfying the fuzzy locating condition. Now, consider the least set $\mathfrak{D}_t'$=$\mathfrak{D}_t\backslash$ $\{v_3\}$=$\{v_1, v_2, v_4\}$. Then, vertex v_3 is not adjacent to any vertex in $\mathfrak{D}_t'$ is not a FLTDS. Hence, $\mathfrak{D}_t$ is minimal and $\gamma_L^t(S_3) = \sum_{i=1}^{4} \mathfrak{D}_t\,(v_i)$

Example:



4-sunlet graph

Table 4

Vertexv_i	Depiction $v_i\|\mathfrak{D}_t$	$\mathcal{L}oc(v_i\|\mathfrak{D}_t)$

v_1	$v_1\|\mathfrak{D}_t$	(0, 0.4, 0.5, 0.7)
v_2	$v_2\|\mathfrak{D}_t$	(0.4, 0, 0.4, 0.4)
v_3	$v_3\|\mathfrak{D}_t$	(0.5, 0.4, 0, 0.4)
v_4	$v_4\|\mathfrak{D}_t$	(0.7, 0.4, 0.5, 0)
u_1	$u_1\|\mathfrak{D}_t$	(0.8, 0.4, 0.5, 0.6)
u_2	$u_2\|\mathfrak{D}_t$	(0.4, 0.1, 0.1, 0.1)
u_3	$u_3\|\mathfrak{D}_t$	(0.2, 0.2, 0.2,0.2)
u_4	$u_4\|\mathfrak{D}_t$	(0.6, 0.4 0.5,0)

$\mathfrak{D}_t(S_4) = \{v_1, v_2, v_3, v_4\}$, $\gamma_L^t(S_4) = \sum_{i=1}^{4} \mathfrak{D}_t(v_i)$= 0.8+0.4+0.5+0.7=2.4

Case (iii) General case $n \geq 3$

Let the n-sunlet graph S_n consist of Cycle vertices: $v_1, v_2, v_3 \ldots. v_n$. Pendant vertices: $u_1, u_2, u_3 \ldots. u_n$ where v_i is adjacent only to u_i

So, the vertex set is $(S_n) = \{v_1, v_2, v_3 \ldots. v_n, u_1, u_2, u_3 \ldots. u_n\}$

Let the set $\mathfrak{D}_t = \{v_1, v_2, v_3 \ldots. v_n\}$. We now show that $\mathfrak{D}_t$ is a minimal FLTDS. Each cycle vertex $v_i \in \mathfrak{D}_t$ is adjacent to a unique pendant u_i satisfying total domination. Each vertex in the graph is distinguishable by all its vertices satisfying the locating condition⇒ FLTDS is satisfied.

$\gamma_L^t(S_n) = \sum_{i=1}^{n} \mathfrak{D}_t(v_i)$.

4. Application

Title: Epidemic Contact Tracing and Containment Using FLTDS in Fuzzy Social Graphs

Context: In managing infectious diseases (like COVID-19 or Monkeypox), contact tracing identifies individuals who have interacted with infected patients to contain spread. In real-life, the intensity of contact (duration, proximity, context) varies — making the connections **fuzzy**, not crisp.

- Nodes = Individuals
- Edges = Social interactions (contact)
- Edge weights = Fuzzified contact levels, like:
- Close contact: 0.9
- Shared environment: 0.6
- Occasional interaction: 0.3

Objective

Use Fuzzy Locating Total Dominating Set to identify a minimal subset of individuals (D) Have no isolated member (they are all socially active or traceable). Dominate all others i.e., all people are in contact with at least one member of D. Allow unique identification of everyone outside D, based on the fuzziness of their interactions with D.

Fuzzy Social Graph Example (Contact Tracing Network)

Let $V = \{v1, v2, v3, v4, v5, v6\}$ be people in a district. Edge strength $\mu(u, v)$ represents the intensity of contact.

International Journal of Applied Mathematics
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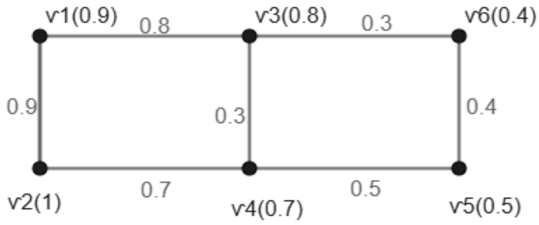
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Person pair	**Contact Strength**
v1 − v2	0.9
v1 − v3	0.8
v2 − v4	0.7
v3 − v4	0.6
v4 − v5	0.5
v5 − v6	0.4
v6 − v3	0.3

Fuzzy Social Graph Example (Contact Tracing Network)



FLTDS Set Selection
Let's pick $D = \{v1, v4, v5\}$, we have to check
Total Dominating: All people in D have at least one connection to another in D.
Dominating: Every person is connected to someone in D.
Locating: Each person $u \notin D$ has a unique fuzzy neighborhood signature in D.

Table 5: FLTDS provide a have unique representation of all vertices

Person	**Depiction** $v_i|\mathfrak{D}_t$	$\mathcal{L}oc(v_i|\mathfrak{D}_t)$
v1	$v1|\mathfrak{D}_t$	(0, 0.7, 0.5)
v2	$v2|\mathfrak{D}_t$	(0.9, 0.7, 0.5)
v3	$v3|\mathfrak{D}_t$	(0.7, 0,7 0.5)
v4	$v4|\mathfrak{D}_t$	(0.7, 0, 0.5)
v5	$v5|\mathfrak{D}_t$	(0.5, 0.5, 0)
v6	$v6|\mathfrak{D}_t$	(0.4, 0.4, 0.4

Now for our Locating: Each person $u \notin D$ has a unique fuzzy neighborhood signature in D.
Table 6

Person	**Depiction** $v_i|\mathfrak{D}_t$	$\mathcal{L}oc(v_i|\mathfrak{D}_t)$
v1	$v1|\mathfrak{D}_t$	(0, 0.7, 0.5)
v4	$v4|\mathfrak{D}_t$	(0.7, 0, 0.5)

Received: August 09, 2025

International Journal of Applied Mathematics
Volume 38 No. 4s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

v5	$v5\|\mathfrak{D}_t$	(0.5, 0.5, 0)

Real-World Benefit:
In Health Departments: Instead of tracking *every* person, health workers only monitor the key people in FLTDS. Those people are socially central (not isolated), and their contact strength profile can uniquely help identify and trace every other person they interacted with. Each individual outside D can be uniquely identified via their fuzzy contact strength with members of D.

Advantages:
Reduced testing and tracing costs
Efficient resource allocation during epidemics
Scalable to large populations with limited public health staff

Sample Public Health Data:

Person	**Contacts**	**Type of Contact**	**Fuzzy Strength**
v_1	v_2, v_3	Household, Co-worker	0.9, 0.8
v_2	v_1, v_4	Family, Grocery Visit	0.9, 0.7
v_3	v_1, v_4, v_6	Co-worker, Clinic, Gym	0.8, 0.6, 0.3
v_4	v_2, v_3, v_5	Grocery, Clinic, Bus	0.7, 0.6, 0.5
v_5	v_4, v_6	Bus, Market	0.5, 0.4
v_6	v_5, v_3	Market, Gym	0.4, 0.3

Summary of Application.
In epidemic management, identifying and monitoring individuals who may spread infection is crucial. A fuzzy social graph models the population, where, Nodes represent individuals. Edges represent social interactions (contacts), with fuzzy weights showing contact intensity (e.g., close, casual, brief). Using a Fuzzy Locating Total Dominating Set (FLTDS). A small subset of individuals D is selected such that: Every person (including those in D) is in contact with someone in D (total domination).
Every person outside D has a unique fuzzy contact pattern with D (locating property).Benefits Reduces resources by focusing only on key individuals in D Ensures no person is left unmonitored. Allows unique identification of potential carriers based on their contact strengths with D. Ideal for targeted testing, quarantine, or vaccine distribution.

5 Conclusion

This paper investigated the application of **Fuzzy Locating Dominating Sets (FLDS)** in optimizing sensor placement within uncertain network environments. By extending the

Volume 38 No. 4s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

classical locating dominating set concept to fuzzy graphs, the approach effectively models real-world uncertainties such as variable signal strengths and intermittent connectivity. The presented case study of a smart office floor demonstrated how $FLDS$ can minimize the number of sensors while ensuring full coverage and unique localization of monitored locations under fuzzy conditions. This framework offers a promising direction for designing robust and cost-efficient sensor networks in domains ranging from smart buildings to fault diagnosis and robotic monitoring. Future work will focus on developing efficient algorithms for finding minimal FLDS in large-scale fuzzy graphs and exploring dynamic sensor adaptation in evolving environments.

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